ALIF HILMI AKBAR - G64160074

1.) $f(x) = e^x - 2 - x$

i.) Bisection

$a = -2{,}1$

$b = -1{,}6$

$c = \frac{b+a}{2}$

a	b	c	f(c)	f(a)
-2,1	-1,6	-2	0,1353	0,4907
-2	-1,6	-1,8	-0,0347	0,1353
-2	-1,8	-1,9	0,0496	-0,0347
-1,9	-1,8	-1,85	0,0072	0,0496

ii.) Posisi palsu

$a = -2{,}1$

$b = -1{,}6$

$c = b - \frac{f(b)(b-a)}{f(b) - f(a)}$

a	b	c	f(c)
-2,1	-1,6	-1,83	$-9{,}52 \cdot 10^{-3}$
-2,1	-1,83	-1,84	$-1{,}04 \cdot 10^{-4}$
-2,1	-1,84	-1,841	$-1{,}707 \cdot 10^{-5}$
-2,1	-1,841	-1,8414	$-7{,}204 \cdot 10^{-7}$

iii.) Newton

$p_0 = -2$

$p_n = p_0 - \frac{f(p_0)}{f'(p_0)}$

p_0	$f(p_0)$	p_n	$p_n - p_0$
-2	$1{,}353 \cdot 10^{-1}$	-1,843	$1{,}565 \cdot 10^{-1}$
-1,843	$1{,}748 \cdot 10^{-3}$	-1,841	$2{,}076 \cdot 10^{-3}$
-1,841	$3{,}414 \cdot 10^{-7}$	-1,841	$4{,}057 \cdot 10^{-7}$

iv.) Secant

$p_0 = -2$

$p_1 = -2{,}5$

$p_{n+1} = p_n - \frac{f(p_n)(p_n - p_{n-1})}{f(p_n) - f(p_{n-1})}$

n	p_n	$f(p_n)$	selisih
0	-2	0,1353	$6{,}515 \cdot 10^{-1}$
1	-2,5	0,582	$6{,}787 \cdot 10^{-3}$
2	-1,849	0,006	$3{,}409 \cdot 10^{-4}$
3	-1,842	0,0003	$2{,}284 \cdot 10^{-7}$

2.) $\begin{aligned} x_1 + x_2 &= 5 \\ 2x_1 - x_2 + 5x_3 &= -9 \\ 3x_2 - 4x_3 + 2x_4 &= 19 \\ 2x_3 + 6x_4 &= 2 \end{aligned}$ $\quad A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & -1 & 5 & 0 \\ 0 & 3 & -4 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix}$ $\quad B = \begin{pmatrix} 5 \\ -9 \\ 19 \\ 2 \end{pmatrix}$

a.) Gauss

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 2 & -1 & 5 & 0 & -9 \\ 0 & 3 & -4 & 2 & 19 \\ 0 & 0 & 2 & 6 & 2 \end{array}\right) \xrightarrow{E_{21}(-2)} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & -3 & 5 & 0 & -19 \\ 0 & 3 & -4 & 2 & 19 \\ 0 & 0 & 2 & 6 & 2 \end{array}\right) \xrightarrow{E_{32}(1)} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & -3 & 5 & 0 & -19 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 & 2 \end{array}\right)$$

$$\downarrow E_{43}(-2)$$

$$\therefore \begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= -2 \\ x_4 &= 1 \end{aligned} \quad \left\{\begin{aligned} x_1 + x_2 &= 5 \\ x_1 &= 2 \end{aligned}\right. \quad \left\{\begin{aligned} -3x_2 + 5x_3 &= -19 \\ -3x_2 - 10 &= -19 \\ x_2 &= 3 \end{aligned}\right. \quad \left\{\begin{aligned} x_3 + 2x_4 &= 0 \\ x_3 + 2 &= 0 \\ x_3 &= -2 \end{aligned}\right. \quad \left\{\begin{aligned} 2x_4 &= 2 \\ x_4 &= 1 \end{aligned}\right. \Leftarrow \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & -3 & 5 & 0 & -19 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right)$$

b.) Gauss-Jordan

lanjutan dari Gauss

$$\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & -3 & 5 & 0 & -19 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right) \xrightarrow{E_{34}(-2)} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & -3 & 5 & 0 & -19 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right) \xrightarrow{E_{23}(-5)} \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 5 \\ 0 & -3 & 0 & 0 & -9 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right)$$

$$\downarrow E_{12}(1/3)$$

$$\therefore \begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= -2 \\ x_4 &= 1 \end{aligned} \Leftarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \xleftarrow[E_4(\frac{1}{2})]{E_2(-\frac{1}{3})} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & -3 & 0 & 0 & -9 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 2 & 2 \end{array}\right)$$

c.) Faktorisasi LU

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 2 & -1 & 5 & 0 \\ 0 & 3 & -4 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix} \xrightarrow{E_{21}(-2)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 5 & 0 \\ 0 & 3 & -4 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix} \xrightarrow{E_{32}(1)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 5 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 6 \end{pmatrix}$$

$$\downarrow E_{43}(-2)$$

- $LY = B$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} 5 \\ -9 \\ 19 \\ 2 \end{pmatrix} \Rightarrow \begin{aligned} &\bullet\ y_1 = 5 \\ &\bullet\ 2y_1 + y_2 = -9 \\ &\quad y_2 = -19 \end{aligned} \quad \left.\begin{aligned} &\bullet\ -y_2 + y_3 = 19 \\ &\quad y_3 = 0 \\ &\bullet\ 2y_3 + y_4 = 2 \\ &\quad y_4 = 2 \end{aligned}\right\}$$

$$A = L.U \qquad \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix}}_{L}\underbrace{\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 5 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}}_{U}$$

- $UX = Y$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & -3 & 5 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ -19 \\ 0 \\ 2 \end{pmatrix} \Rightarrow \begin{aligned} &\bullet\ 2x_4 = 2 \\ &\quad x_4 = 1 \\ &\bullet\ x_3 + 2x_4 = 0 \\ &\quad x_3 = -2 \end{aligned} \quad \begin{aligned} &\bullet\ -3x_2 + 5x_3 = -19 \\ &\quad -3x_2 = -9 \\ &\quad x_2 = 3 \\ &\bullet\ x_1 + x_2 = 5 \\ &\quad x_1 = 2 \end{aligned} \quad \therefore \begin{aligned} x_1 &= 2 \\ x_2 &= 3 \\ x_3 &= -2 \\ x_4 &= 1 \end{aligned}$$

d.) Matriks Invers

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & -1 & 5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & -4 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{E_{21}(-2)} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & 3 & -4 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{E_{32}(1)} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 & 1 & 0 \\ 0 & 0 & 2 & 6 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\downarrow E_{43}(-2)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 & 4 & -2 & -2 & 1 \end{array}\right) \xrightarrow{E_{34}(-1)} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & 5 & 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 2 & 4 & -2 & -2 & 1 \end{array}\right) \xrightarrow{E_{23}(-5)} \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 28 & -14 & -15 & 5 \\ 0 & 0 & 1 & 0 & -6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 2 & 4 & -2 & -2 & 1 \end{array}\right)$$

$$\downarrow E_2(-1/3)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -28/3 & 14/3 & 15/3 & -5/3 \\ 0 & 0 & 1 & 0 & -6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 2 & 4 & -2 & -2 & 1 \end{array}\right) \xrightarrow[E_4(1/2)]{E_{12}(-1)} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 31/3 & -14/3 & -5 & 5/3 \\ 0 & 1 & 0 & 0 & -28/3 & 14/3 & 5 & -5/3 \\ 0 & 0 & 1 & 0 & -6 & 3 & 3 & -1 \\ 0 & 0 & 0 & 1 & 2 & -1 & -1 & 1/2 \end{array}\right) \Rightarrow (I|A^{-1})$$

$$x = A^{-1} \cdot B$$

$$= \begin{pmatrix} 31/3 & -14/3 & -5 & 5/3 \\ -28/3 & 14/3 & 5 & -5/3 \\ -6 & 3 & 3 & -1 \\ 2 & -1 & -1 & 1/2 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -9 \\ 19 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -2 \\ 1 \end{pmatrix} \Rightarrow \therefore \begin{array}{l} x_1 = 2 \\ x_2 = 3 \\ x_3 = -2 \\ x_1 = 1 \end{array}$$

e.) Iterasi Jacobi dan Gauss-Seidel

- Cek syarat konvergen

$|1| > |1| + 0 + 0$ ×

$|-1| > |2| + |5| + 0$ ×

$|-4| > 0 + |3| + |2|$ ×

$|6| > 0 + 0 + |2|$ ✓

Syarat konvergen tidak terpenuhi. Jika SPL dicoba diselesaikan dengan metode iterasi, hasilnya akan divergen.

(baik iterasi Jacobi maupun Gauss-Seidel)

- Hasil iterasi Jacobi

n	x_{1n}	x_{2n}	x_{3n}	x_{4n}
0	0	0	0	0
1	5	9	-4,75	0,33
2	-4	-4,75	2,16	1,92
3	9,75	11,83	-7,35	-0,389
4	-6,83	-8,27	3,93	2,79

⋮

- Hasil iterasi Gauss-Seidel

n	x_{1n}	x_{2n}	x_{3n}	x_{4n}
0	0	0	0	0
1	5	19	9,5	-2,83
2	-14	28,5	15,21	-4,74
3	-23,5	38,04	21,41	-6,8
4	-33,04	49,98	29,34	-9,45

⋮

2.b.) $A = \begin{pmatrix} 1 & 3 & -3 \\ -3 & 7 & -3 \\ -6 & 6 & -2 \end{pmatrix}$ Tentukan nilai eigen dan vektor eigen.

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 3 & -3 \\ -3 & 7-\lambda & -3 \\ -6 & 6 & -2-\lambda \end{vmatrix} = 0$$

$[(1-\lambda)(7-\lambda)(-2-\lambda) + 108] - [18(7-\lambda) - 18(1-\lambda) - 9(-2-\lambda)] = 0$

$-\lambda^3 + 6\lambda^2 - 32 = 0$

$(-\lambda+4)(\lambda^2 - 2\lambda - 8) = 0$

$(-\lambda+4)(\lambda-4)(\lambda+2) = 0$

$\lambda_1 = 4, \ \lambda_2 = 4, \ \lambda_3 = -2$

Nilai-nilai eigennya adalah $\lambda_1 = 4$, $\lambda_2 = 4$, dan $\lambda_3 = -2$, sama dengan hasil di R.

- Untuk $\lambda_1 = \lambda_2 = 4$,

$$\left(\begin{array}{ccc|c} -3 & 3 & -3 & 0 \\ -3 & 3 & -3 & 0 \\ -6 & 6 & -6 & 0 \end{array}\right) \xrightarrow[E_{31(-2)}]{E_{21(-1)}} \left(\begin{array}{ccc|c} -3 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \begin{array}{c} -3x_1 + 3x_2 - 3x_3 = 0 \\ x_1 = -x_2 + x_3 \end{array}$$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_2 + x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

- Untuk $\lambda_3 = -2$

$$\left(\begin{array}{ccc|c} 3 & 3 & -3 & 0 \\ -3 & 9 & -3 & 0 \\ -6 & 6 & 0 & 0 \end{array}\right) \xrightarrow[E_{21(1)}]{E_{31(2)}} \left(\begin{array}{ccc|c} 3 & 3 & -3 & 0 \\ 0 & 12 & -6 & 0 \\ 0 & 12 & -6 & 0 \end{array}\right) \xrightarrow{E_{32(-1)}} \left(\begin{array}{ccc|c} 3 & 3 & -3 & 0 \\ 0 & 12 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

- $3x_1 + 3x_2 - 3x_3 = 0 \longrightarrow 3x_1 + 3x_2 - 6x_2 = 0$
- $12x_2 - 6x_3 = 0$

$2x_2 = x_3$

$3x_1 - 3x_2 = 0$

$x_1 = x_2$

$$\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_1 \\ 2x_1 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$\therefore$ Vektor-vektor eigennya adalah : $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right\}$

3.) $Y = \frac{A}{x}$

$Y = A \cdot \frac{1}{x}$

$\Sigma\left(\frac{1}{x}\right)^2 \cdot A + \Sigma\left(\frac{1}{x}\right) \cdot B = \Sigma\left(\frac{1}{x} \cdot Y\right)$

$\Sigma\left(\frac{1}{x}\right) \cdot A + n \cdot B = \Sigma Y$

$B = 0 \quad \Downarrow$

x	Y	$\frac{1}{x}$	$\left(\frac{1}{x}\right)^2$	$\frac{1}{x} \cdot Y$
0,5	7,1	2	4	14,2
0,8	4,4	1,25	1,56	5,5
1,1	3,2	0,91	0,83	2,91
1,8	1,9	0,56	0,31	1,06
4,0	0,9	0,25	0,06	0,23
Σ 8,2	17,5	4,97	6,76	23,9

$6{,}76A = 23{,}9$

$4{,}97A = 17{,}5$ −

$1{,}79A = 6{,}4$

$A \approx 3{,}58$

$Y = \frac{B}{x^3}$

$Y = B \cdot \frac{1}{x^3}$

$B \Rightarrow A$

$B = 0$

$\Sigma\left(\frac{1}{x^3}\right)^2 \cdot A + \Sigma\left(\frac{1}{x^3}\right) \cdot B = \Sigma\left(\frac{1}{x^3} \cdot Y\right)$

$\Sigma\left(\frac{1}{x^3}\right) \cdot A + n \cdot B = \Sigma Y$

$\Downarrow$

x	Y	$\frac{1}{x^3}$	$\left(\frac{1}{x^3}\right)^2$	$\frac{1}{x^3} \cdot Y$
0,5	7,1	8	64	56,8
0,8	4,4	1,95	3,8	8,58
1,1	3,2	0,75	0,56	2,4
1,8	1,9	0,17	0,03	0,32
4,0	0,9	0,016	$2{,}56 \cdot 10^{-4}$	0,014
Σ	17,5	10,89	68,39	68,11

$68{,}39A = 68{,}11$

$10{,}89A = 17{,}5$ −

$57{,}5A = 50{,}61$

$A \approx 0{,}88$