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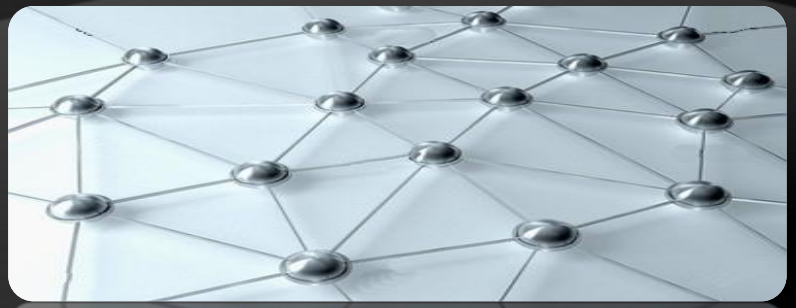
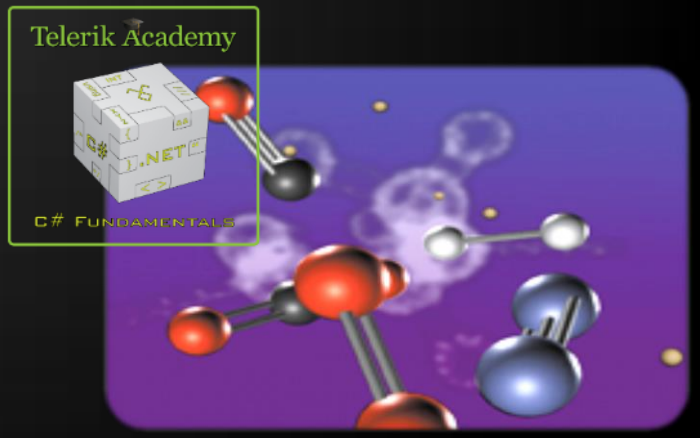


Trees and Traversals

Trees, Tre-Like Structures, Binary Search Trees, Balanced Trees, Tree Traversals, DFS and BFS



Data Structures and Algorithms

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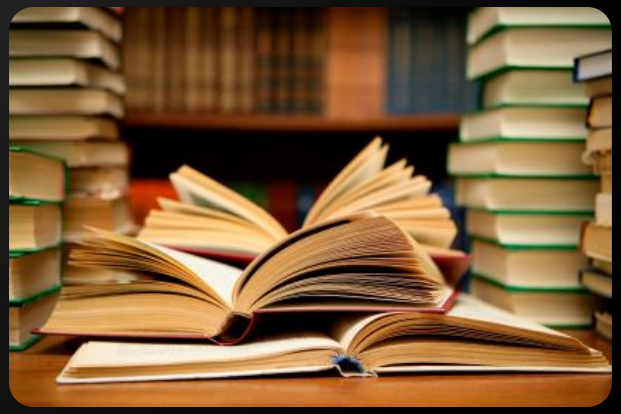
2. Trees and Related Terminology
3. Implementing Trees
4. Traversing Trees
   - DFS and BFS Traversals
5. Balanced Search Trees
   - Balanced Trees in .NET



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Tree-like Data Structures

Trees, Balanced Trees, Graphs, Networks

Tree-like Data Structures

- Tree-like data structures are:
  - Branched recursive data structures
    - Consisting of nodes
    - Each node connected to other nodes
- Examples of tree-like structures
  - Trees: binary, balanced, ordered, etc.
  - Graphs: directed / undirected, weighted, etc.
  - Networks

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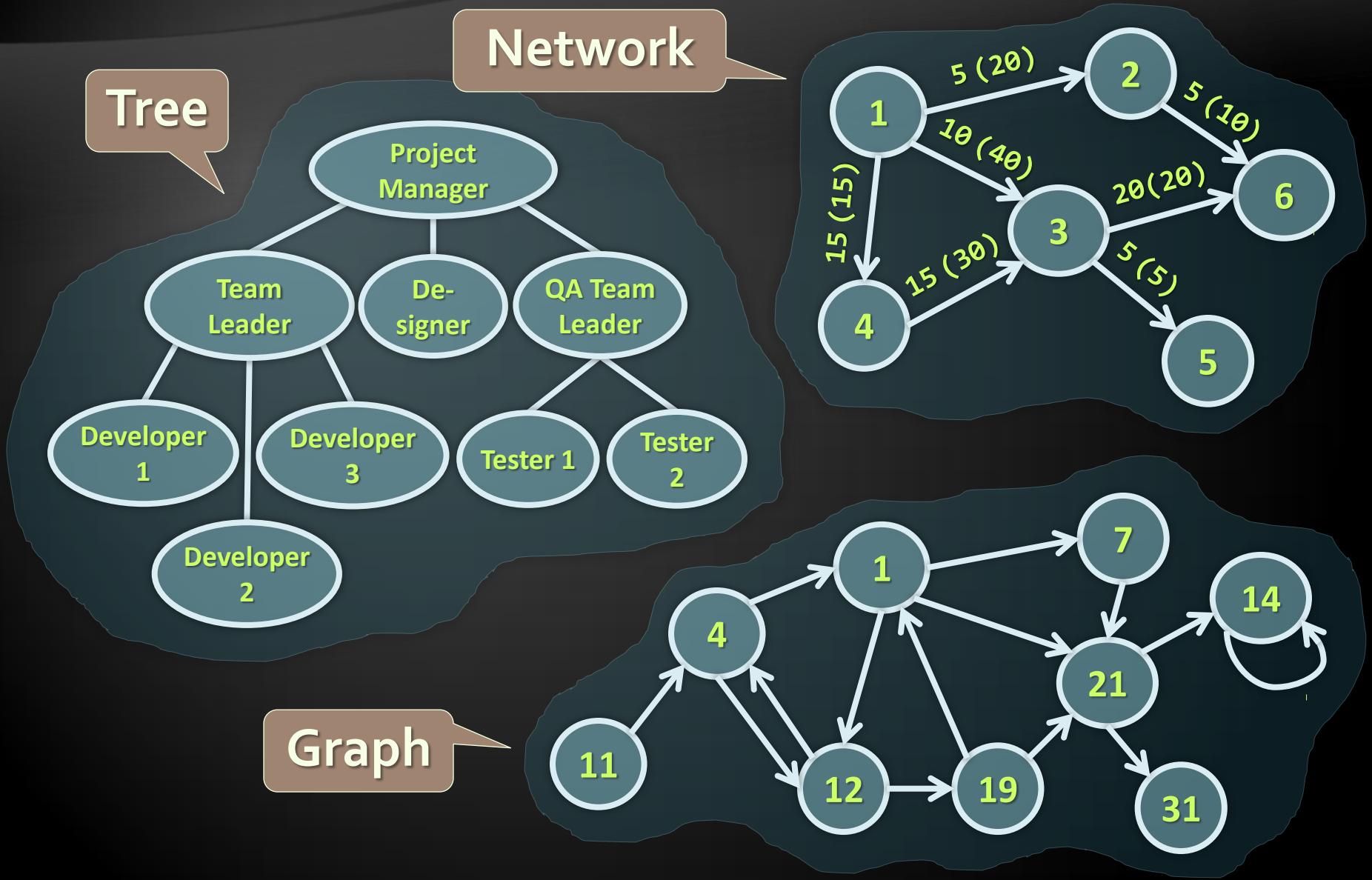
Tree-like Data Structures



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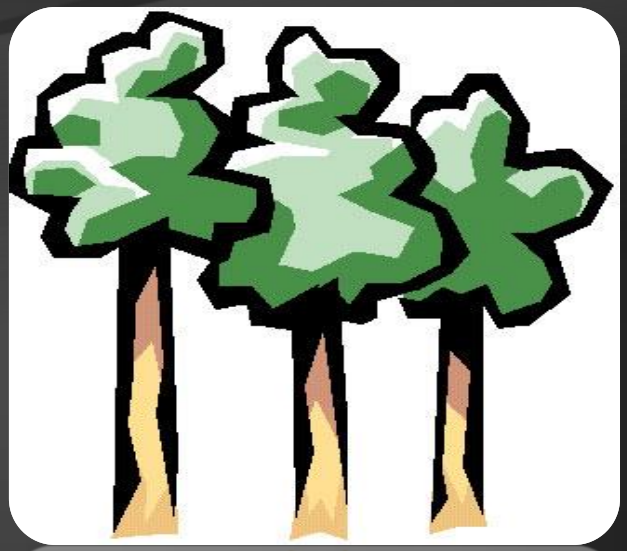
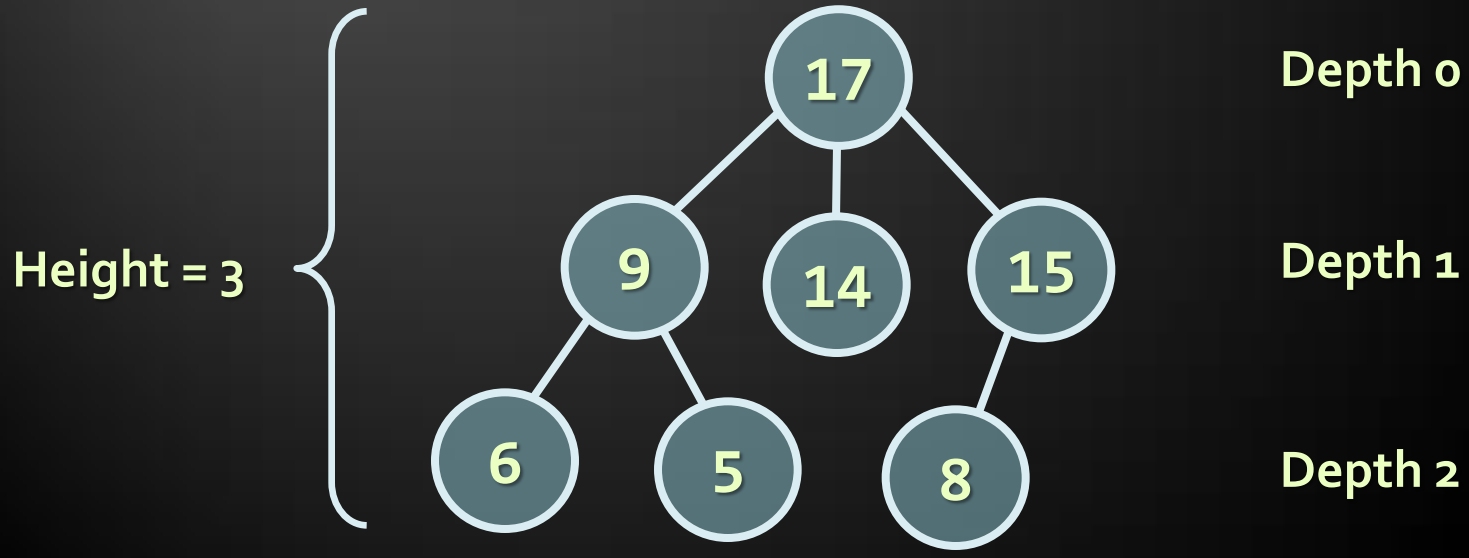


Trees and Related Terminology

Node, Edge, Root, Children, Parent, Leaf,
Binary Search Tree, Balanced Tree

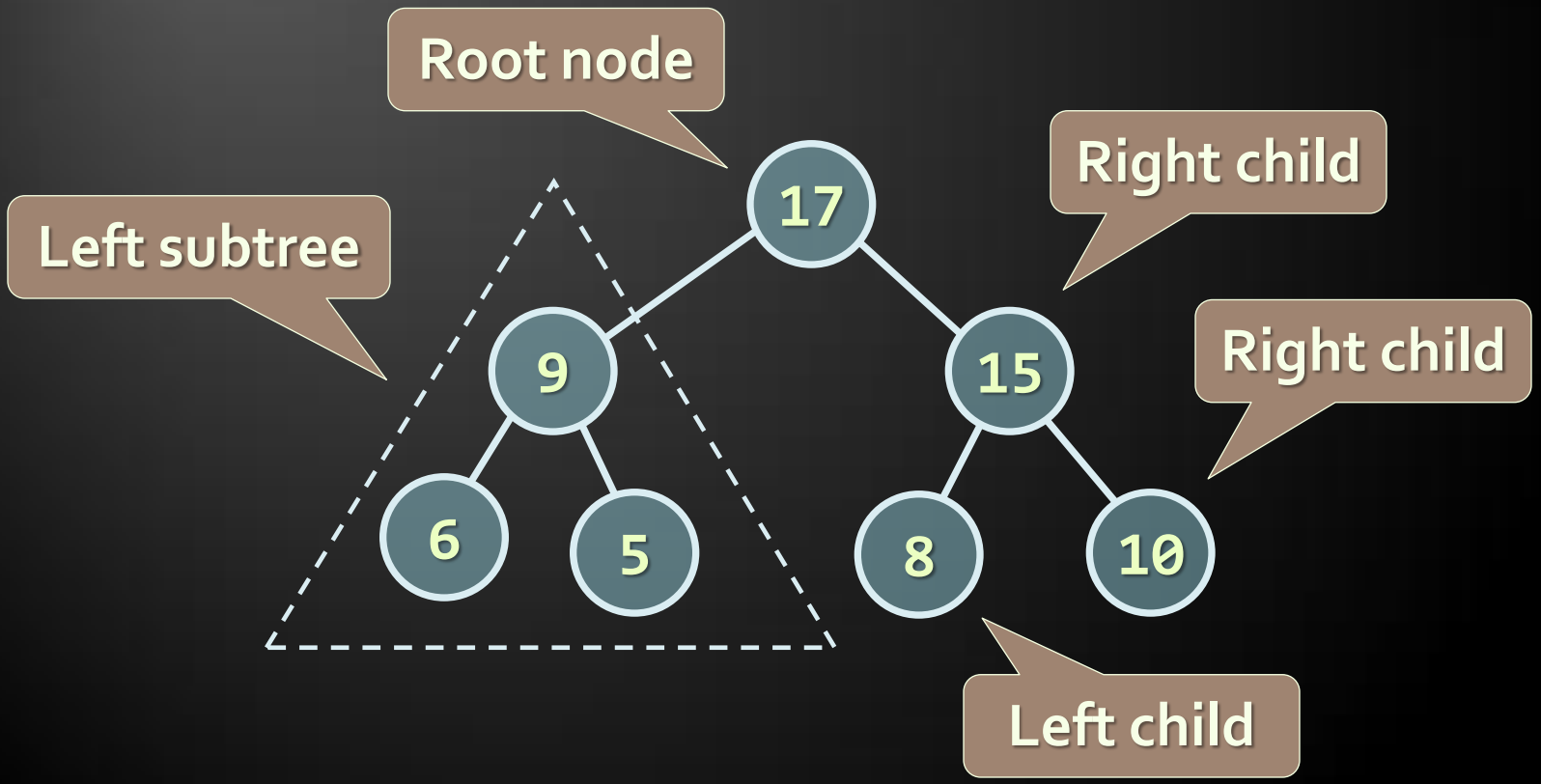
Trees

- Tree data structure – terminology
  - Node, edge, root, child, children, siblings, parent, ancestor, descendant, predecessor, successor, internal node, leaf, depth, height, subtree



Binary Trees

- Binary trees: the most widespread form
  - Each node has at most 2 children

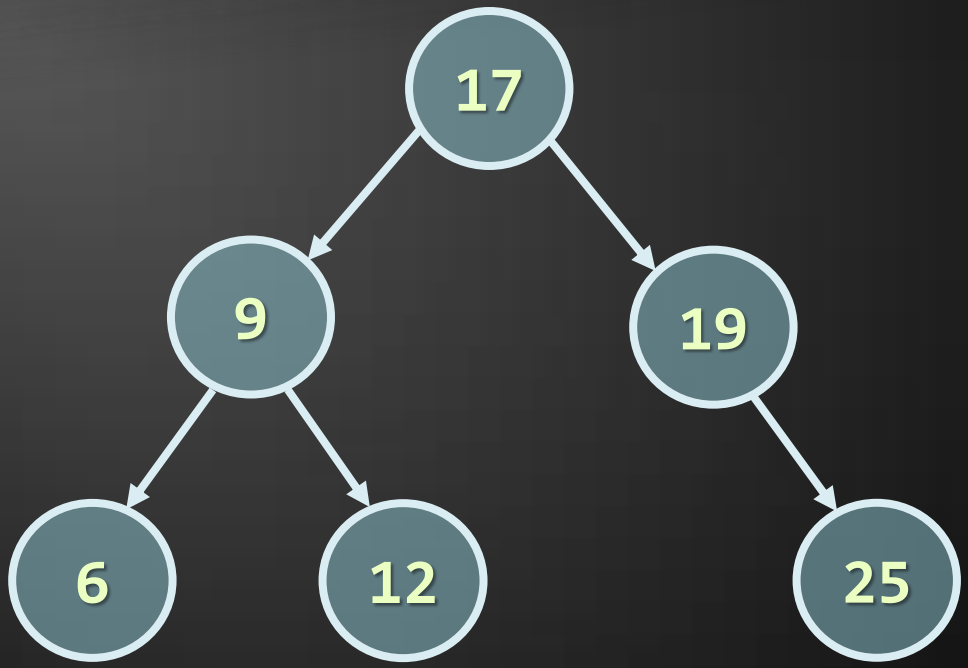


Binary Search Trees

- Binary search trees are ordered
  - For each node *x* in the tree
    - All the elements of the left subtree of *x* are ≤ *x*
    - All the elements of the right subtree of *x* are > *x*
- Binary search trees can be balanced
  - Balanced trees have height of ~ log(*x*)
  - Balanced trees have for each node nearly equal number of nodes in its subtrees

Binary Search Trees (2)

- Example of balanced binary search tree



- If the tree is balanced, add / search / delete operations take approximately log(*n*) steps

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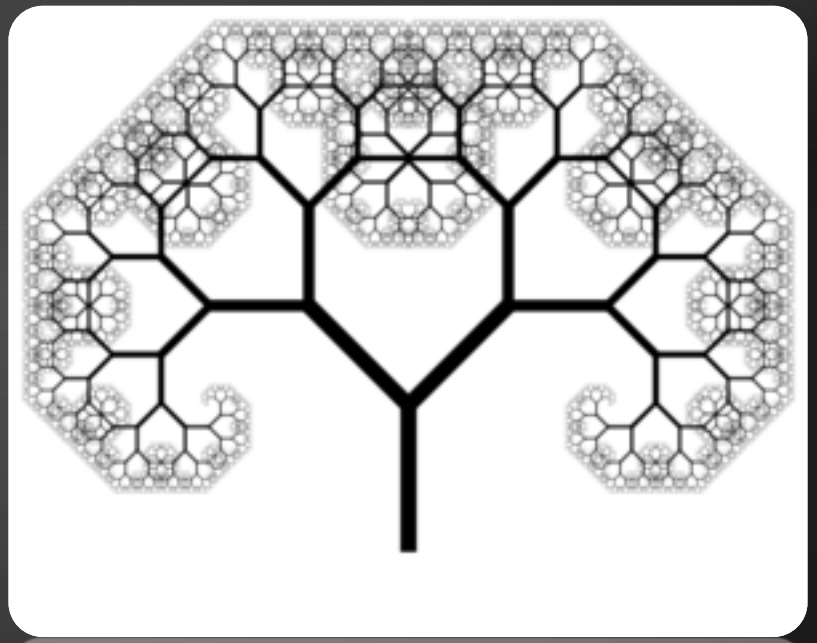


Implementing Trees

Recursive Tree Data Structure

Recursive Tree Definition

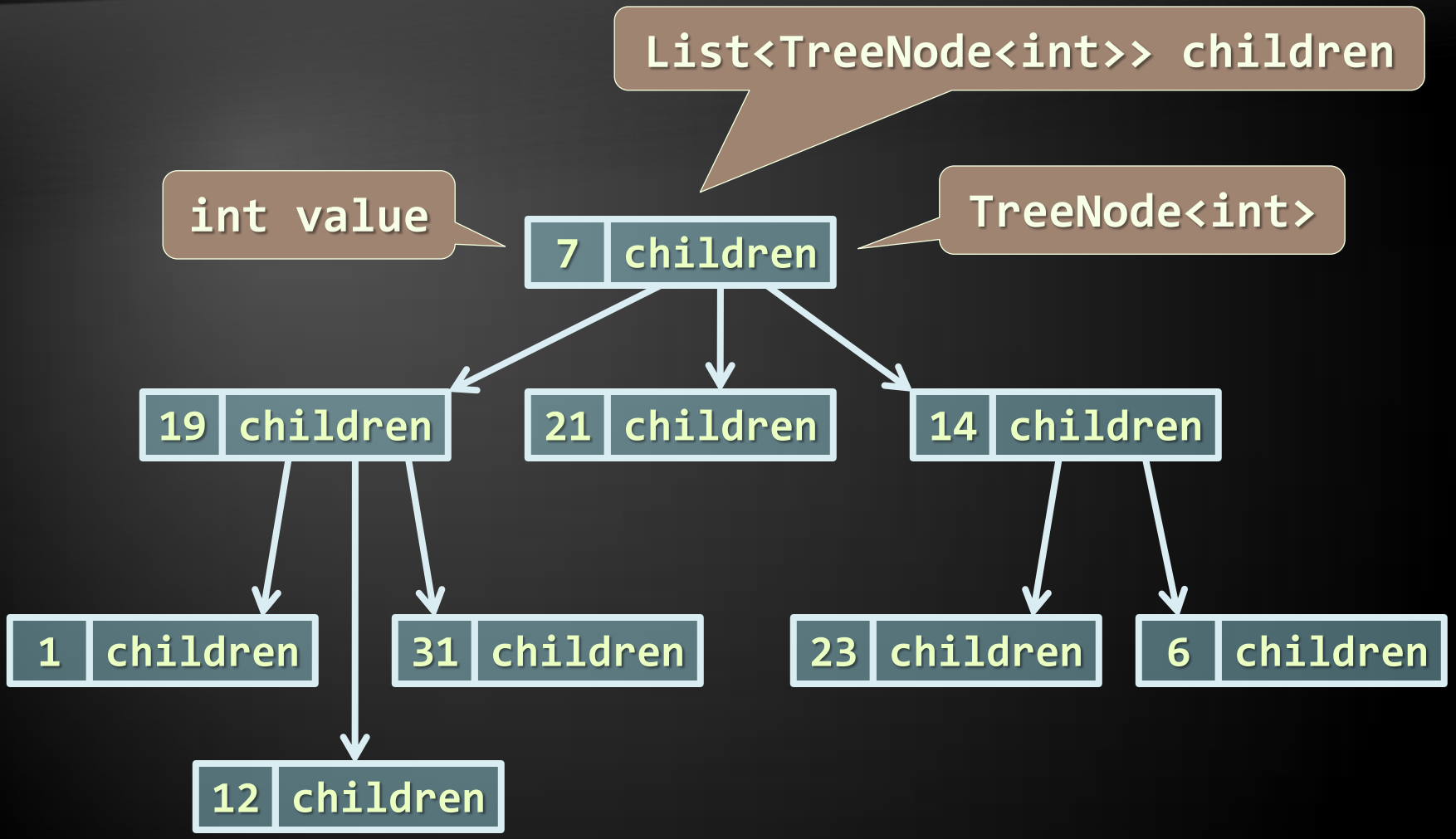
- The recursive definition for tree data structure:
  - A single node is tree
  - Tree nodes can have zero or multiple children that are also trees
- Tree node definition in C#

```
public class TreeNode<T>
{
  private T value;
  private List<TreeNode<T>> children;
  …
}
```

The value contained in the node

List of child nodes, which are of the same type

TreeNode<int> Structure



Implementing TreeNode<T>

```
public TreeNode(T value)
{
  this.value = value;
  this.children = new List<TreeNode<T>>();
}
public T Value
{
  get { return this.value; }
  set { this.value = value; }
}
public void AddChild(TreeNode<T> child)
{
  child.hasParent = true;
  this.children.Add(child);
}
public TreeNode<T> GetChild(int index)
{
  return this.children[index];
}
```

Implementing Tree<T>

- The class Tree<T> keeps tree's root node

```
public class Tree<T>
{
  private TreeNode<T> root;

  public Tree(T value, params Tree<T>[] children) :
    this(value)
  {
    foreach (Tree<T> child in children)
    {
      this.root.AddChild(child.root);
    }
  }

  public TreeNode<T> Root
  {
    get {  return this.root; }
  }
}
```

Flexible constructor for building trees

Building a Tree

- Constructing a tree by nested constructors:

```
Tree<int> tree =
    new Tree<int>(7,
        new Tree<int>(19,
            new Tree<int>(1),
            new Tree<int>(12),
            new Tree<int>(31)),
        new Tree<int>(21),
        new Tree<int>(14,
            new Tree<int>(23),
            new Tree<int>(6))
);
```

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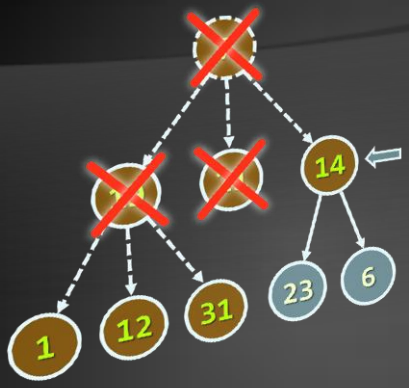
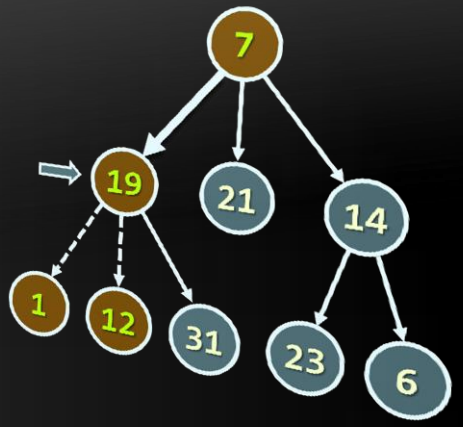


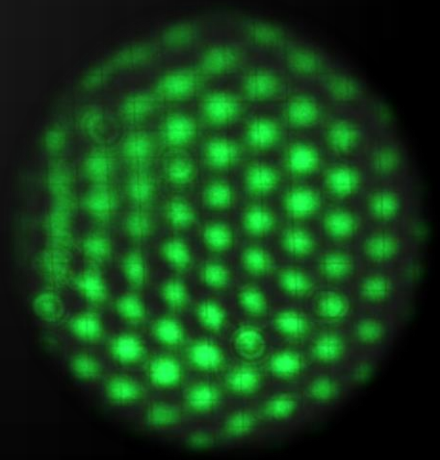


Tree Traversals

DFS and BFS Traversals

Tree Traversal Algorithms

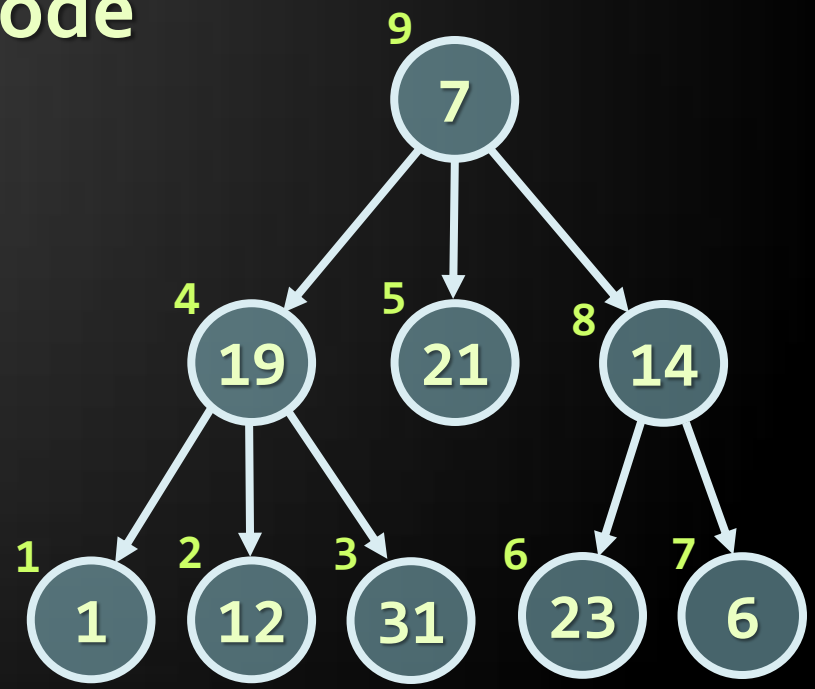
- Traversing a tree means to visit each of its nodes exactly one in particular order
  - Many traversal algorithms are known
  - Depth-First Search (DFS)
    - Visit node's successors first
    - Usually implemented by recursion
  - Breadth-First Search (BFS)
    - Nearest nodes visited first
    - Implemented by a queue



Depth-First Search (DFS)

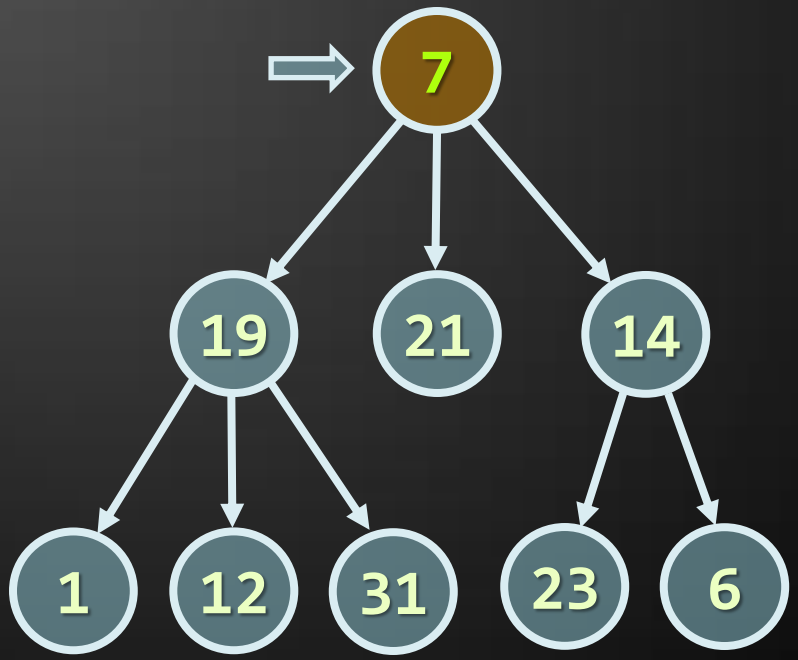
- Depth-First Search first visits all descendants of given node recursively, finally visits the node itself
- DFS algorithm pseudo code

```
DFS(node)
{
  for each child c of node
    DFS(c);
  print the current node;
}
```



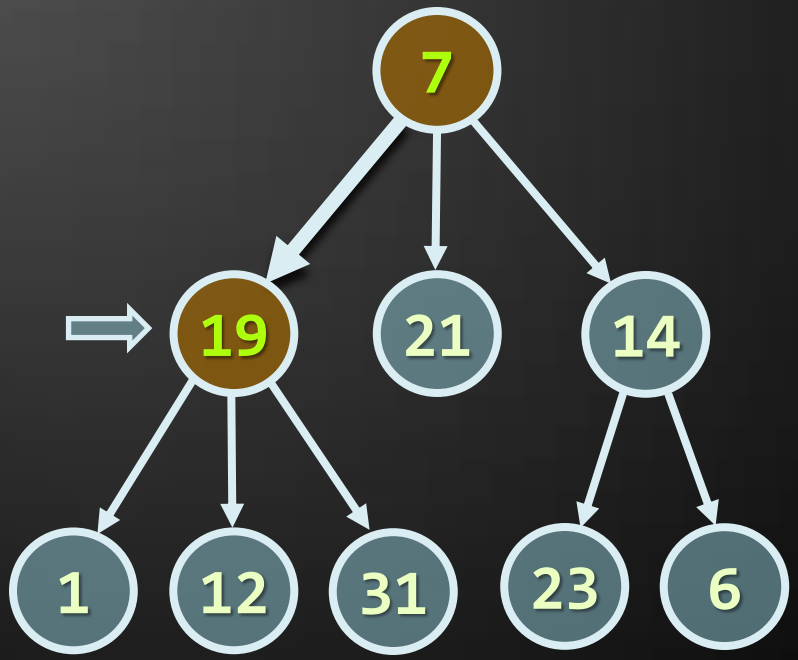
DFS in Action (Step 1)

- Stack: 7
- Output: (empty)



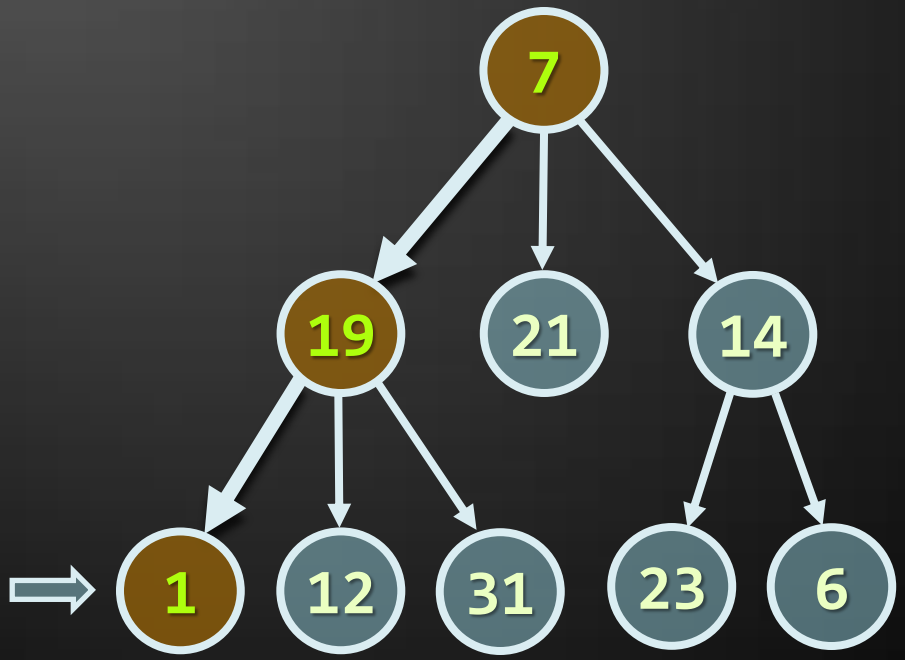
DFS in Action (Step 2)

- Stack: 7, 19
- Output: (empty)



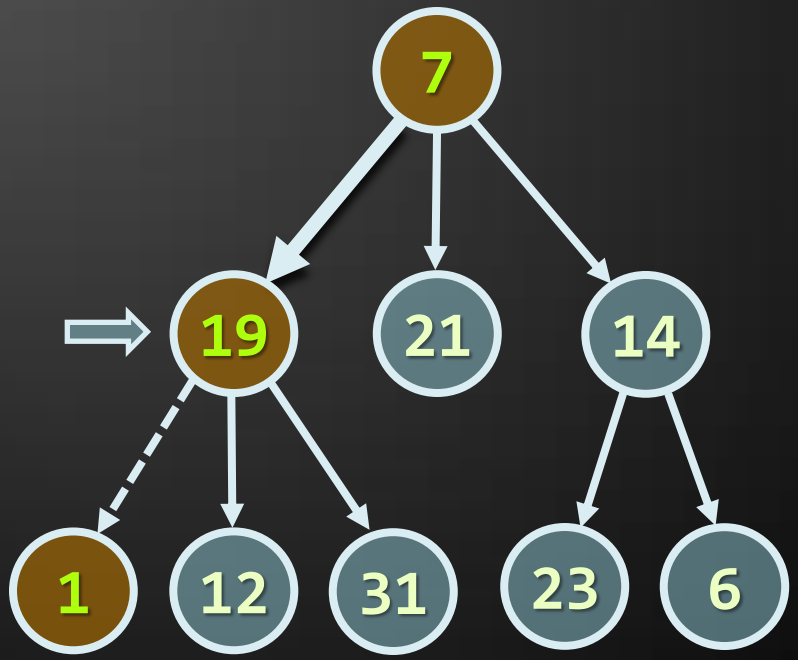
DFS in Action (Step 3)

- Stack: 7, 19, 1
- Output: (empty)



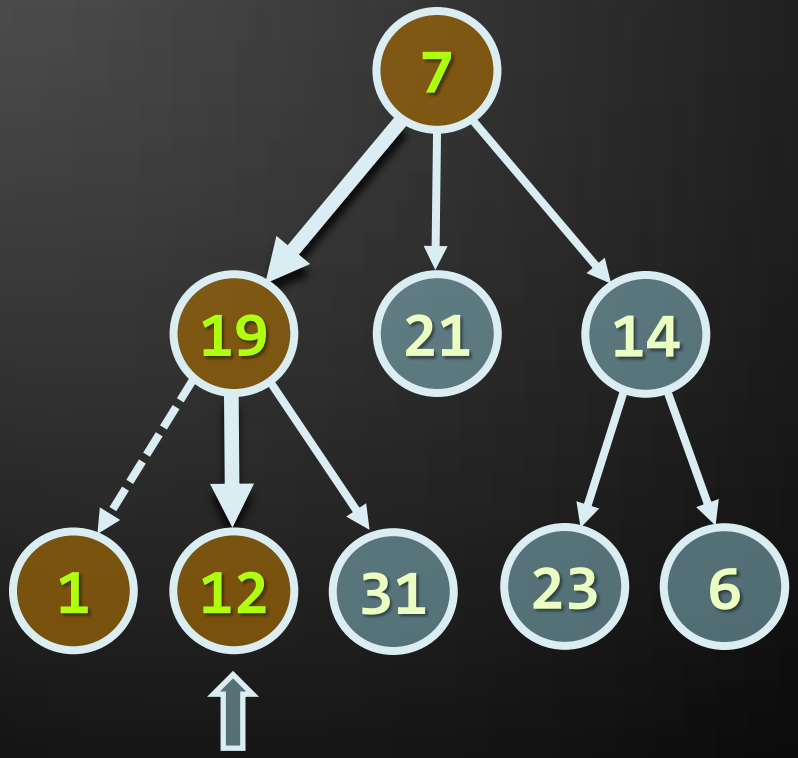
DFS in Action (Step 4)

- Stack: 7, 19
- Output: 1



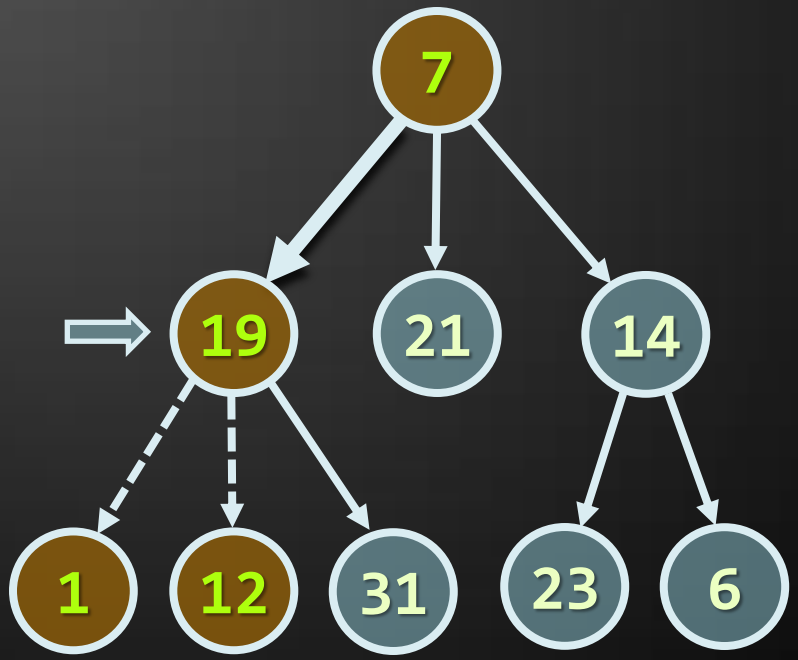
DFS in Action (Step 5)

- Stack: 7, 19, 12
- Output: 1



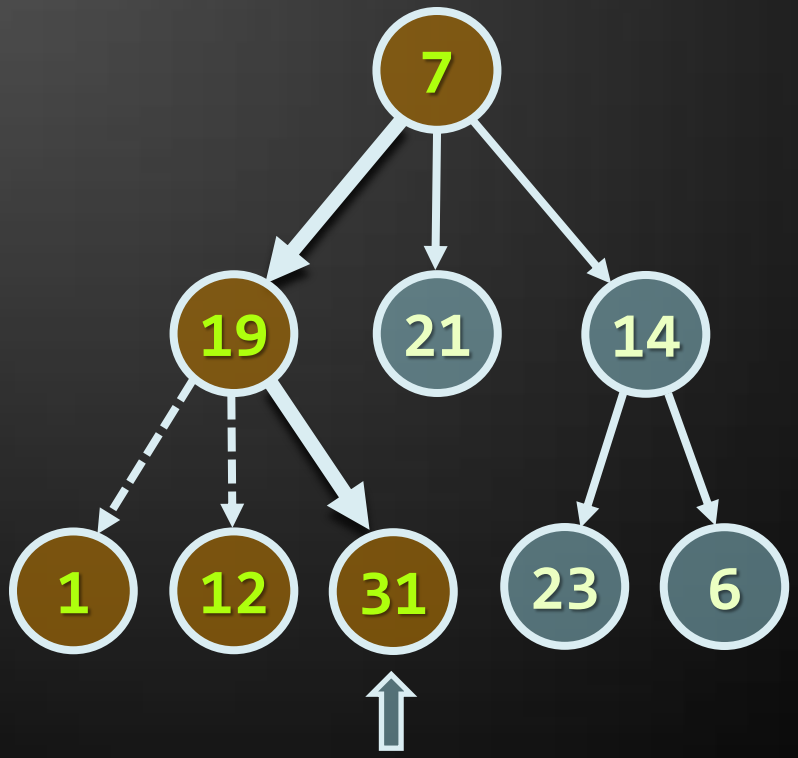
DFS in Action (Step 6)

- Stack: 7, 19
- Output: 1, 12



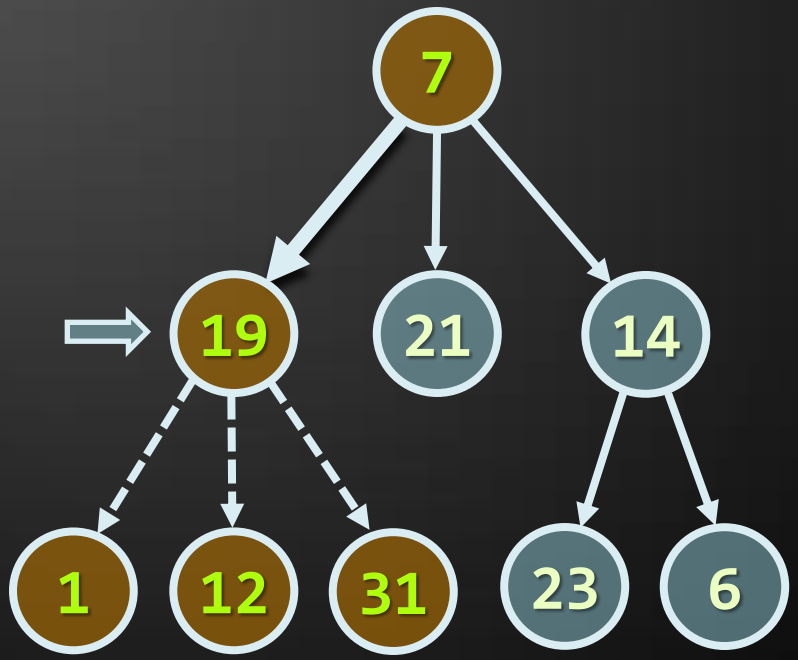
DFS in Action (Step 7)

- Stack: 7, 19, 31
- Output: 1, 12



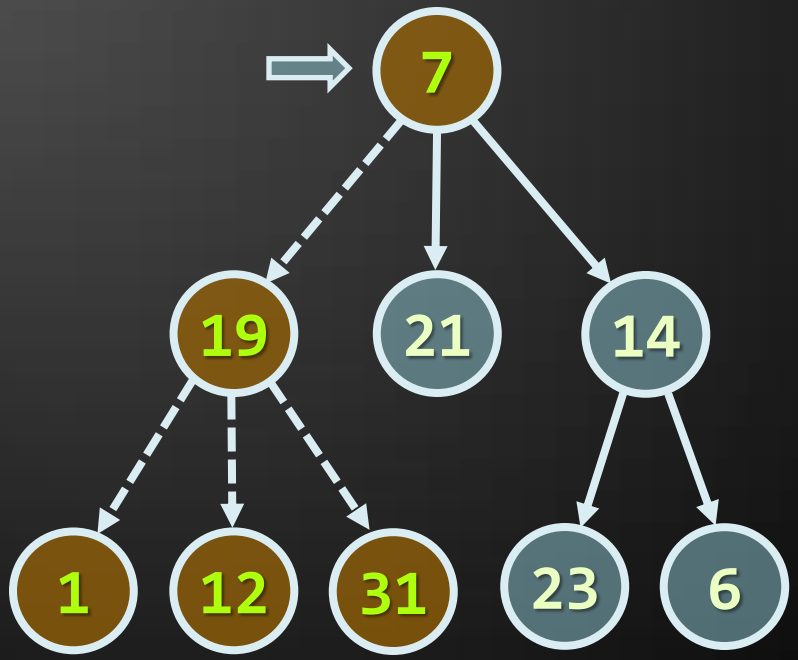
DFS in Action (Step 8)

- Stack: 7, 19
- Output: 1, 12, 31



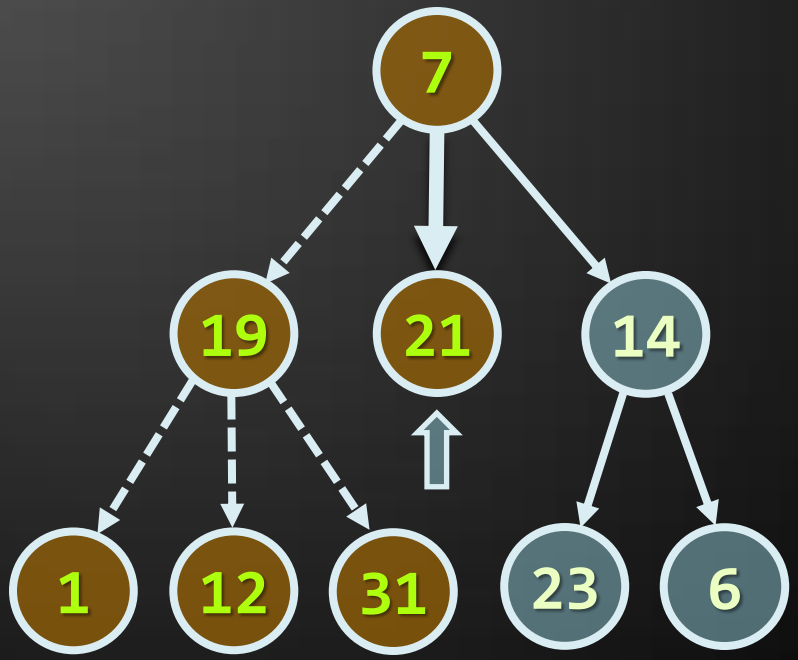
DFS in Action (Step 9)

- Stack: 7
- Output: 1, 12, 31, 19



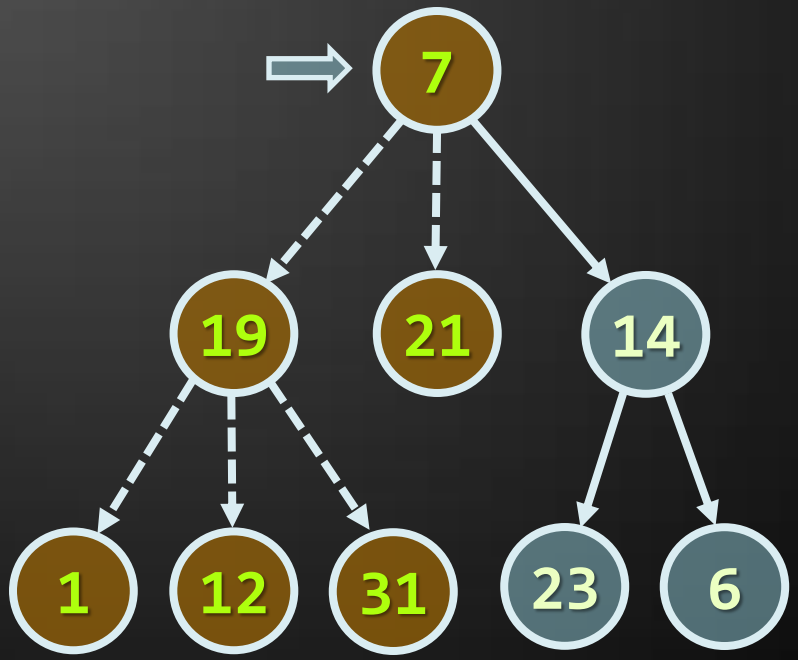
DFS in Action (Step 10)

- Stack: 7, 21
- Output: 1, 12, 31, 19



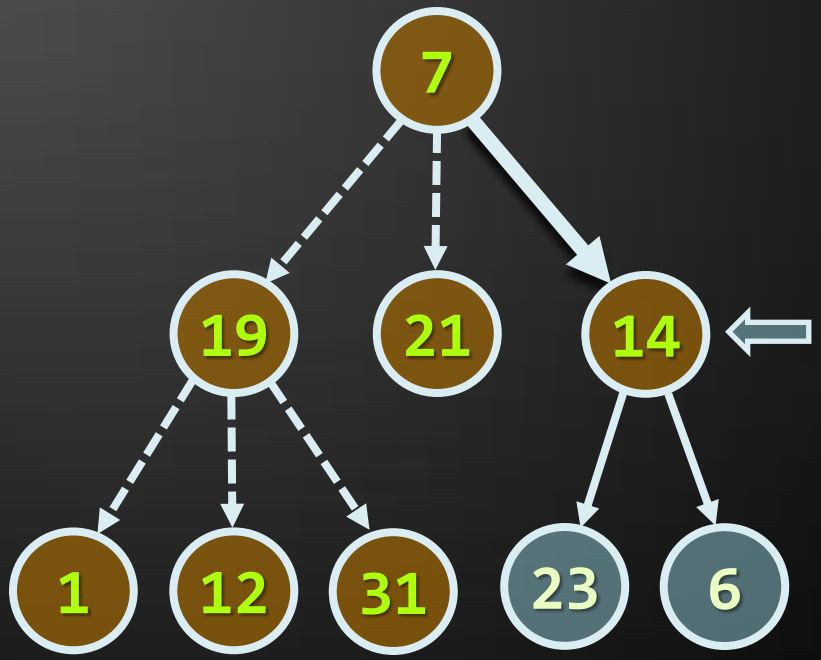
DFS in Action (Step 11)

- Stack: 7
- Output: 1, 12, 31, 19, 21



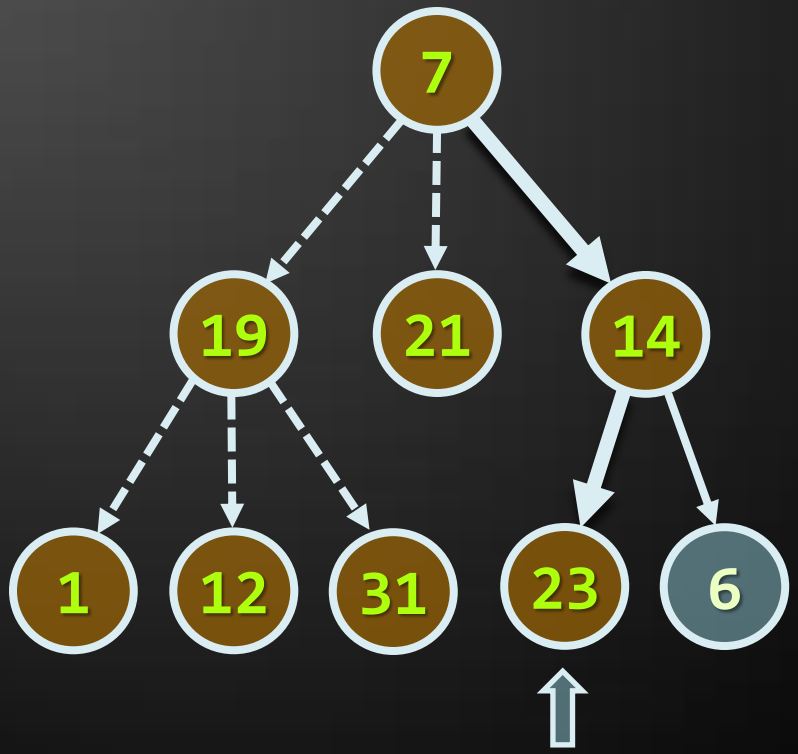
DFS in Action (Step 12)

- Stack: 7, 14
- Output: 1, 12, 31, 19, 21



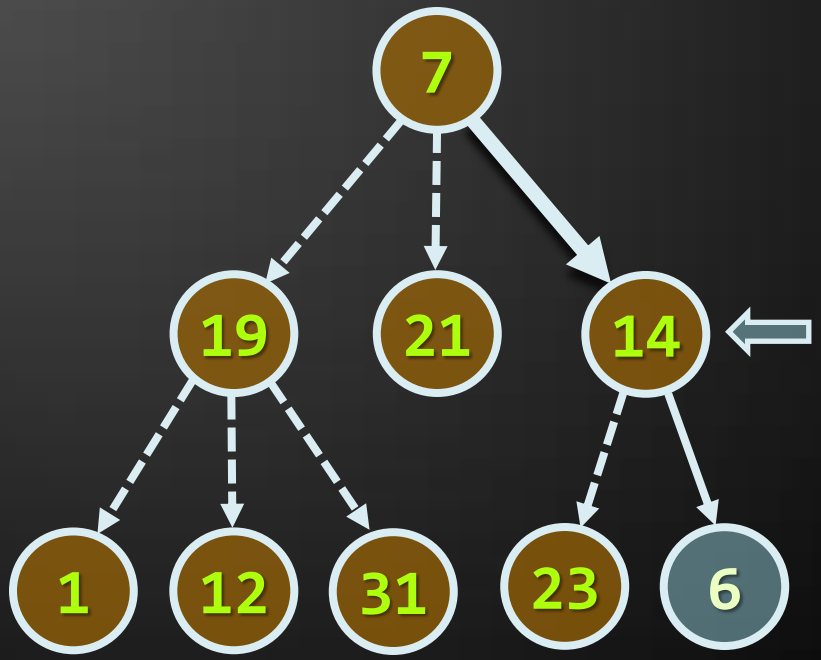
DFS in Action (Step 13)

- Stack: 7, 14, 23
- Output: 1, 12, 31, 19, 21



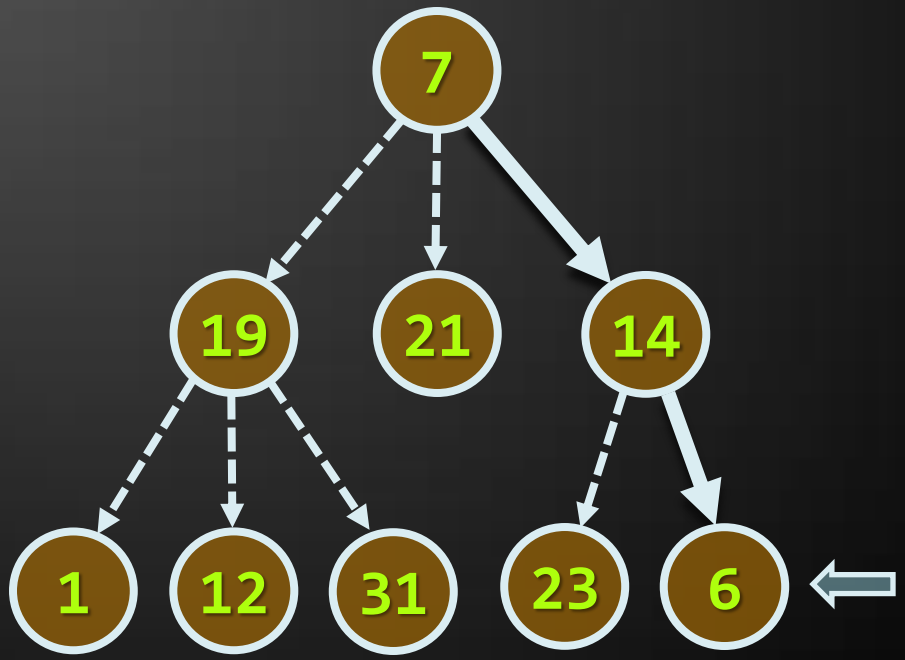
DFS in Action (Step 14)

- Stack: 7, 14
- Output: 1, 12, 31, 19, 21, 23



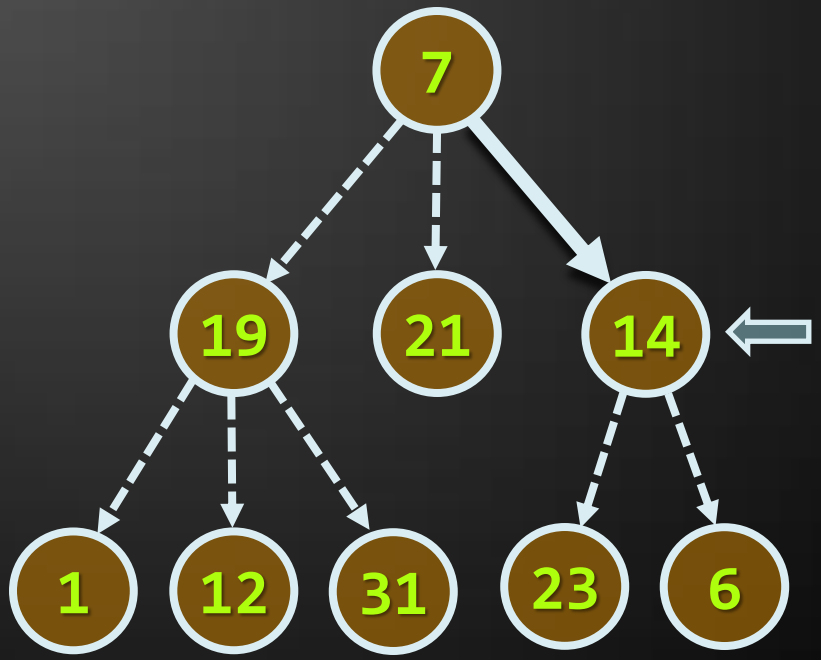
DFS in Action (Step 15)

- Stack: 7, 14, 6
- Output: 1, 12, 31, 19, 21, 23



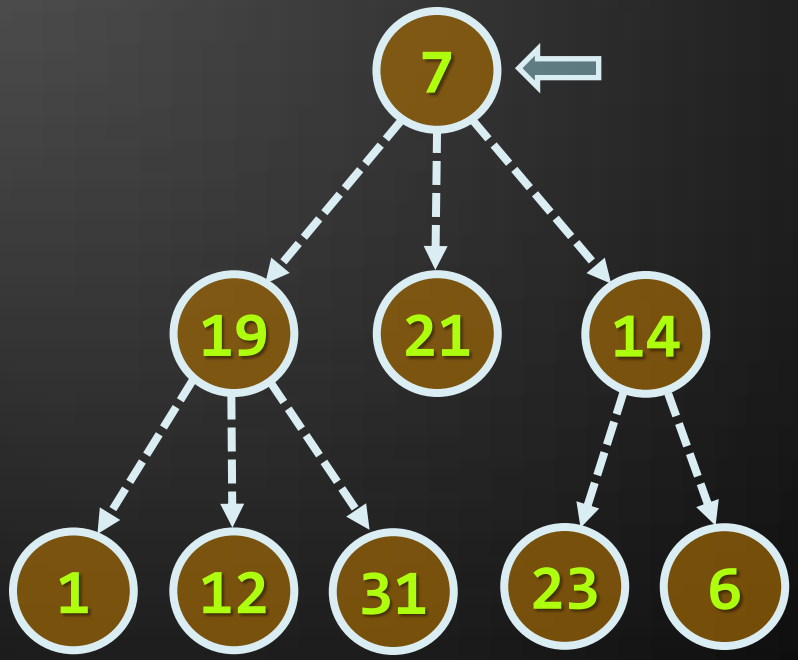
DFS in Action (Step 16)

- Stack: 7, 14
- Output: 1, 12, 31, 19, 21, 23, 6



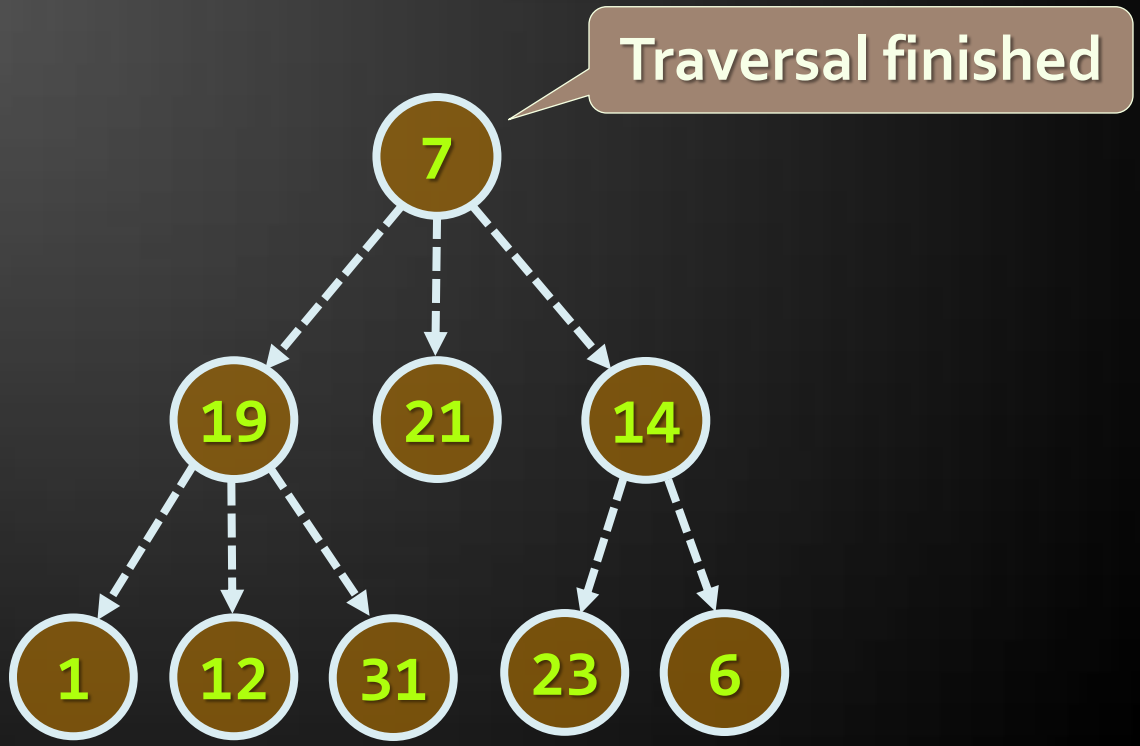
DFS in Action (Step 17)

- Stack: 7
- Output: 1, 12, 31, 19, 21, 23, 6, 14



DFS in Action (Step 18)

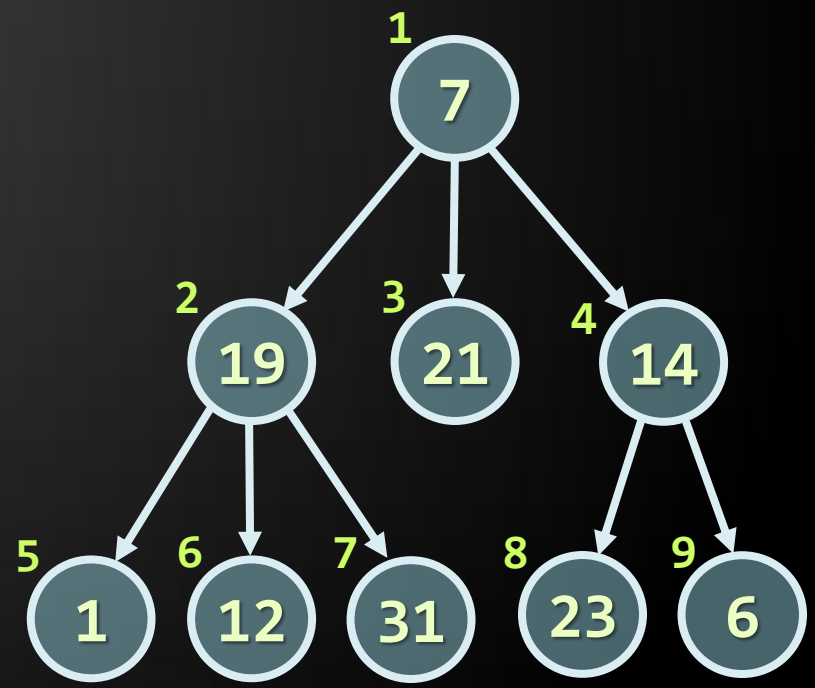
- Stack: (empty)
- Output: 1, 12, 31, 19, 21, 23, 6, 14, 7



Breadth-First Search (BFS)

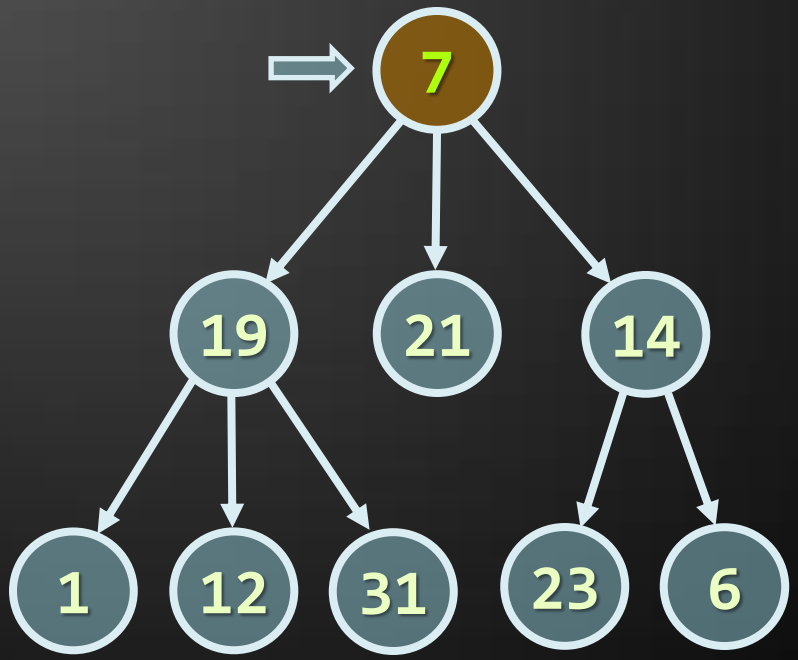
- Breadth-First Search first visits the neighbor nodes, later their neighbors, etc.
- BFS algorithm pseudo code

```
BFS(node)
{
  queue ← node
  while queue not empty
    v ← queue
    print v
    for each child c of v
      queue ← c
}
```



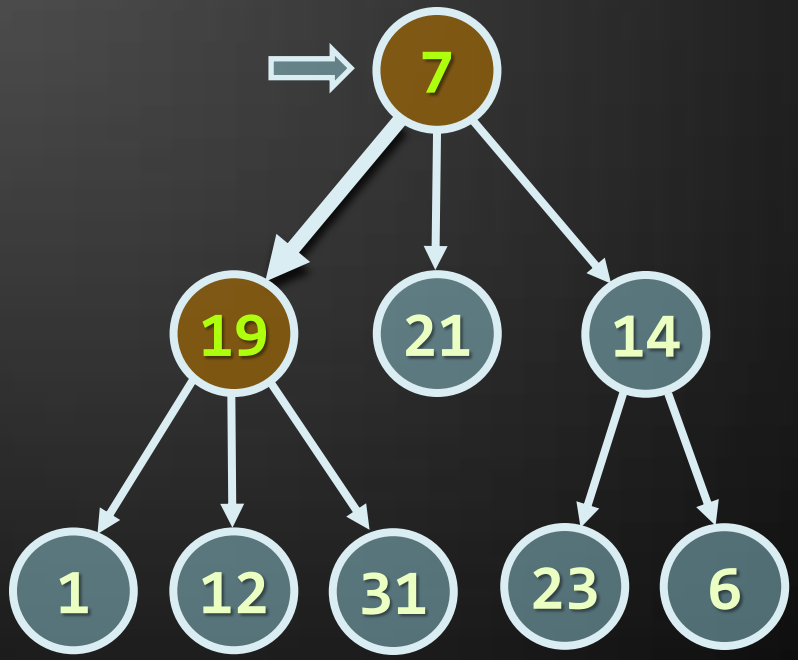
BFS in Action (Step 1)

- Queue: 7
- Output: 7



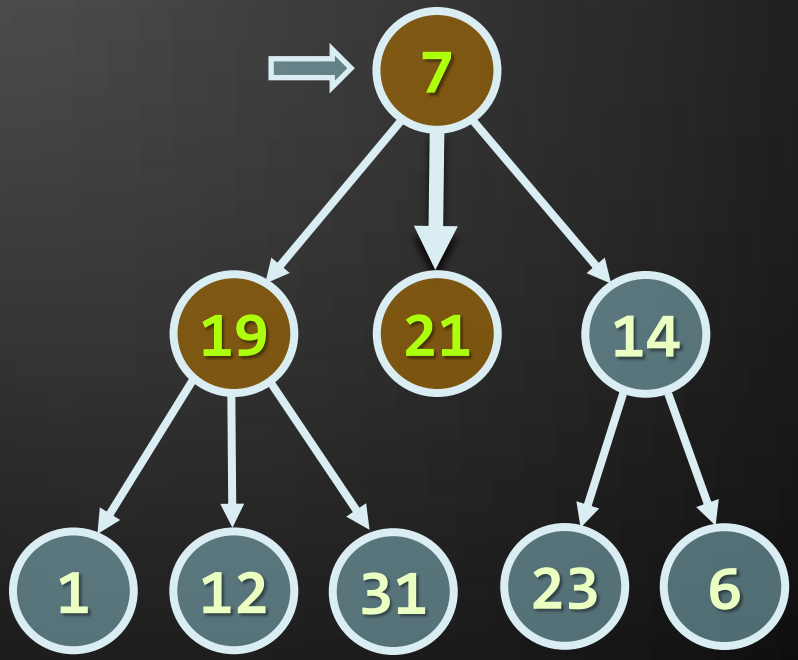
BFS in Action (Step 2)

- Queue: 7, 19
- Output: 7



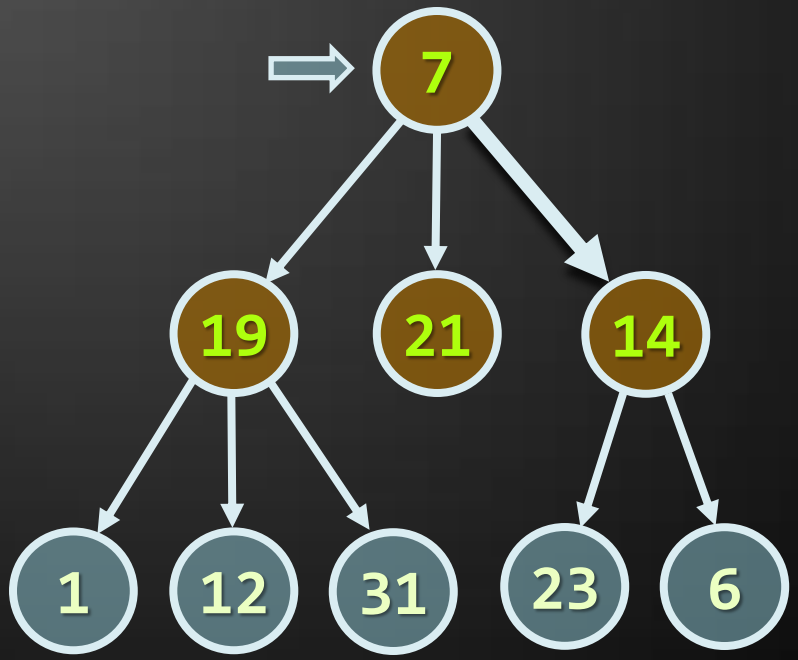
BFS in Action (Step 3)

- Queue: 7, 19, 21
- Output: 7



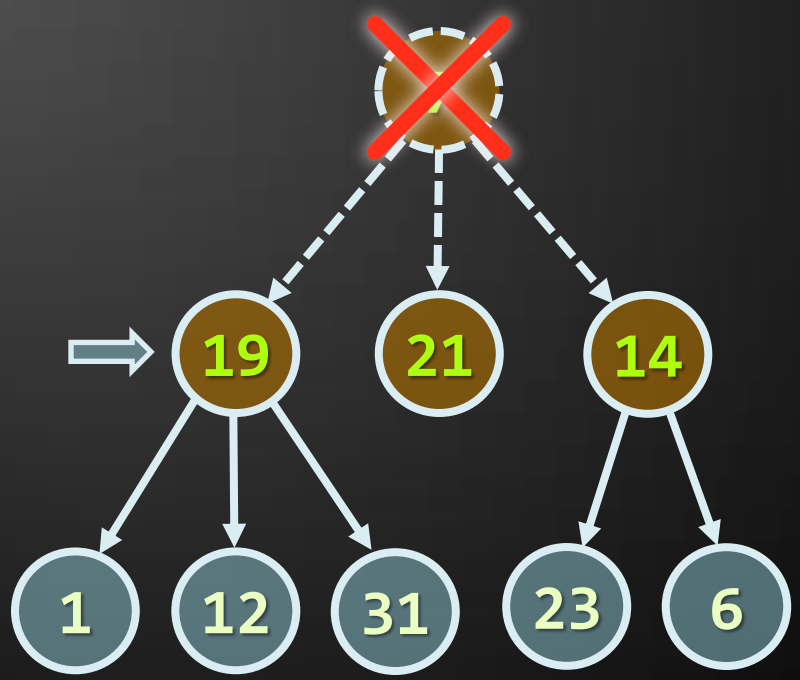
BFS in Action (Step 4)

- Queue: 7, 19, 21, 14
- Output: 7



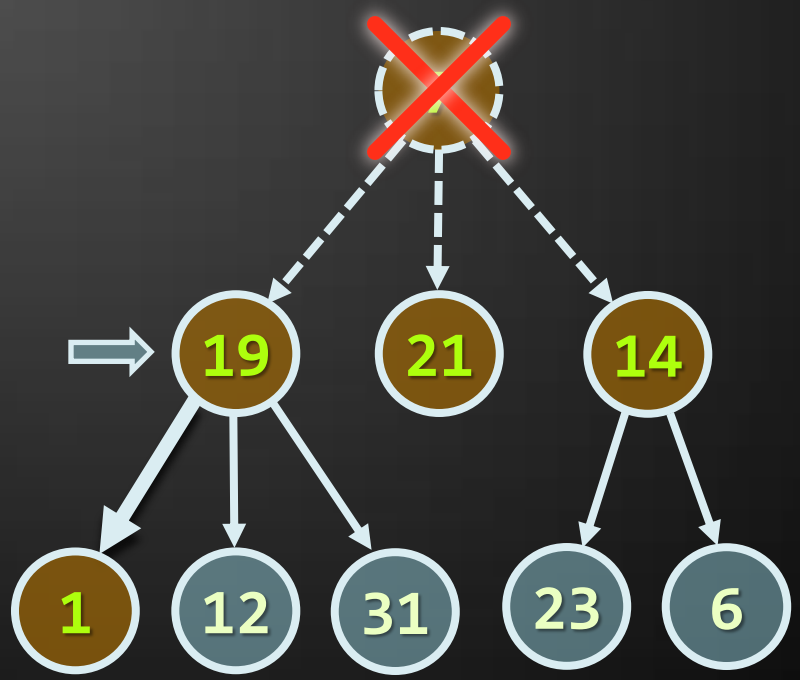
BFS in Action (Step 5)

- Queue: ~~7~~, 19, 21, 14
- Output: 7, 19



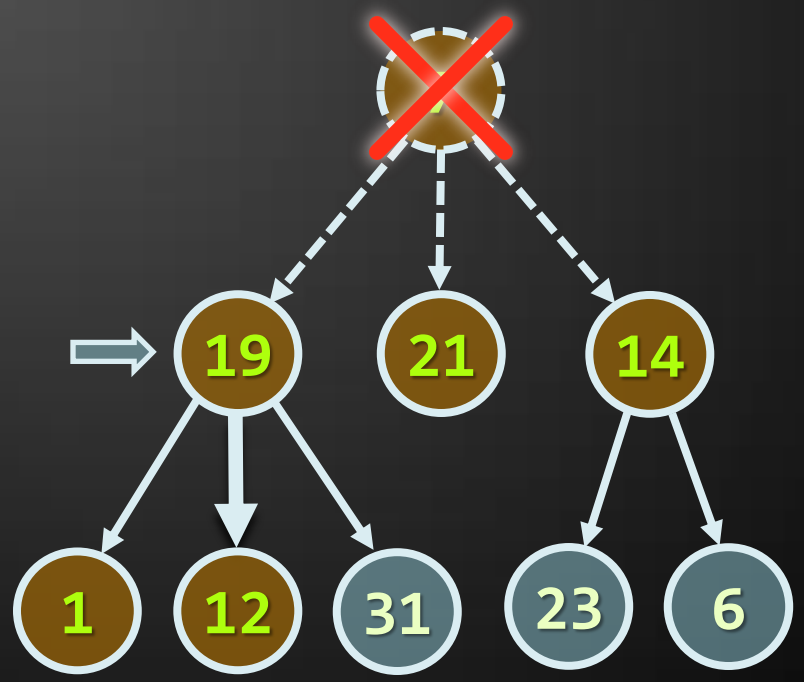
BFS in Action (Step 6)

- Queue: ~~7~~, 19, 21, 14, 1
- Output: 7, 19



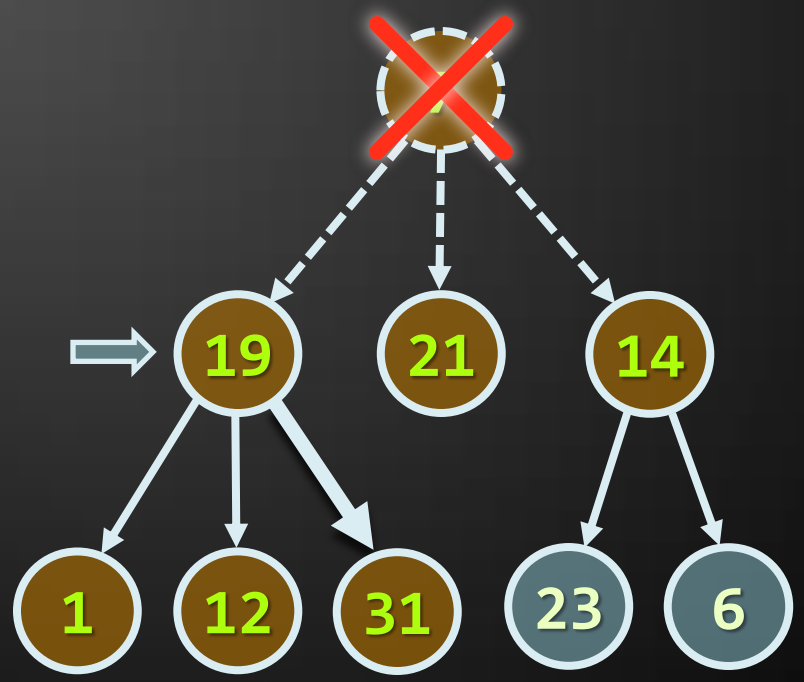
BFS in Action (Step 7)

- Queue: ~~7~~, 19, 21, 14, 1, 12
- Output: 7, 19



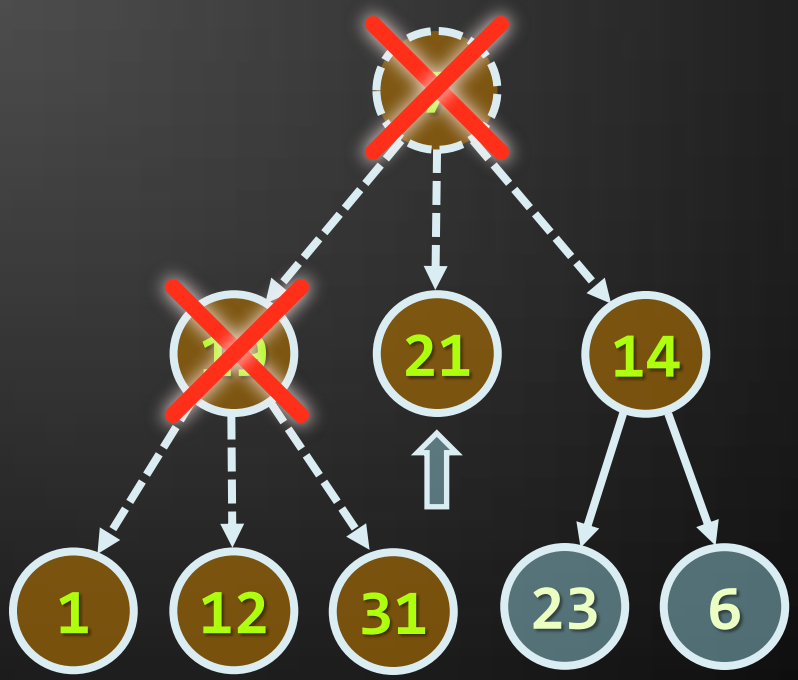
BFS in Action (Step 8)

- Queue: ~~7~~, 19, 21, 14, 1, 12, 31
- Output: 7, 19



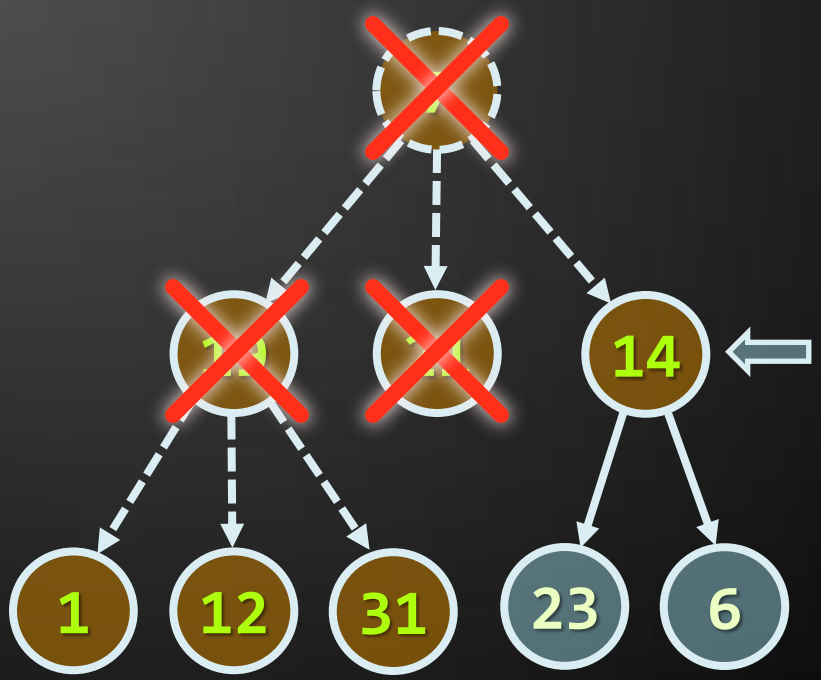
BFS in Action (Step 9)

- Queue: ~~7~~, ~~19~~, 21, 14, 1, 12, 31
- Output: 7, 19, 21



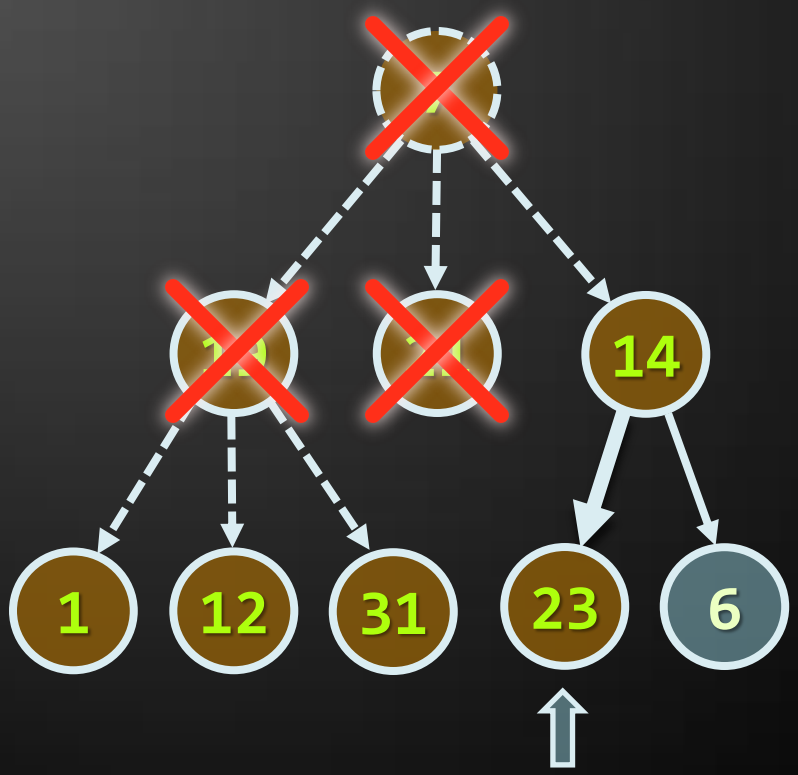
BFS in Action (Step 10)

- Queue: ~~7~~, ~~19~~, ~~21~~, 14, 1, 12, 31
- Output: 7, 19, 21, 14



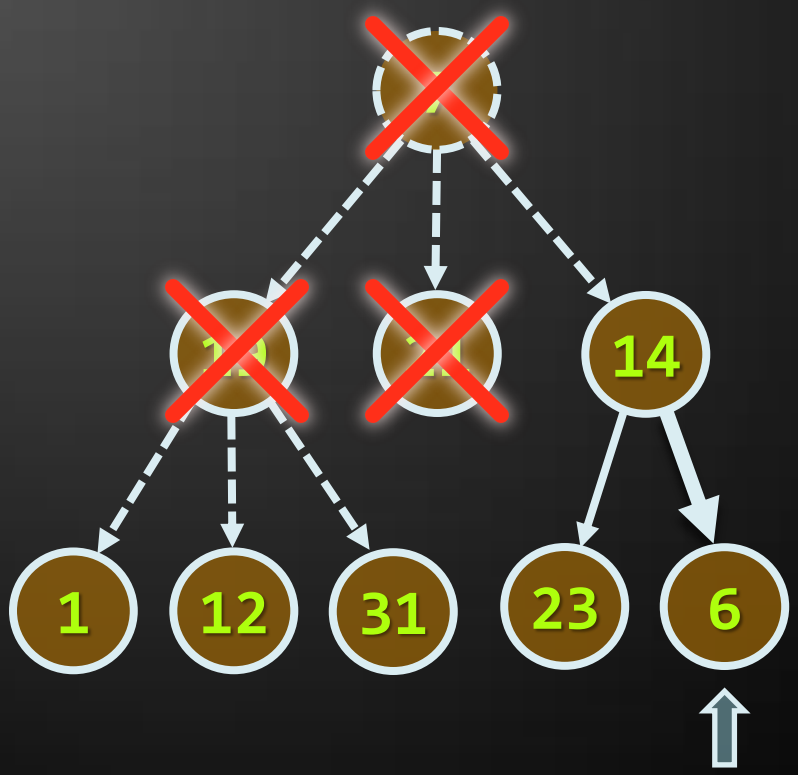
BFS in Action (Step 11)

- Queue: ~~7~~, ~~19~~, ~~21~~, 14, 1, 12, 31, 23
- Output: 7, 19, 21, 14



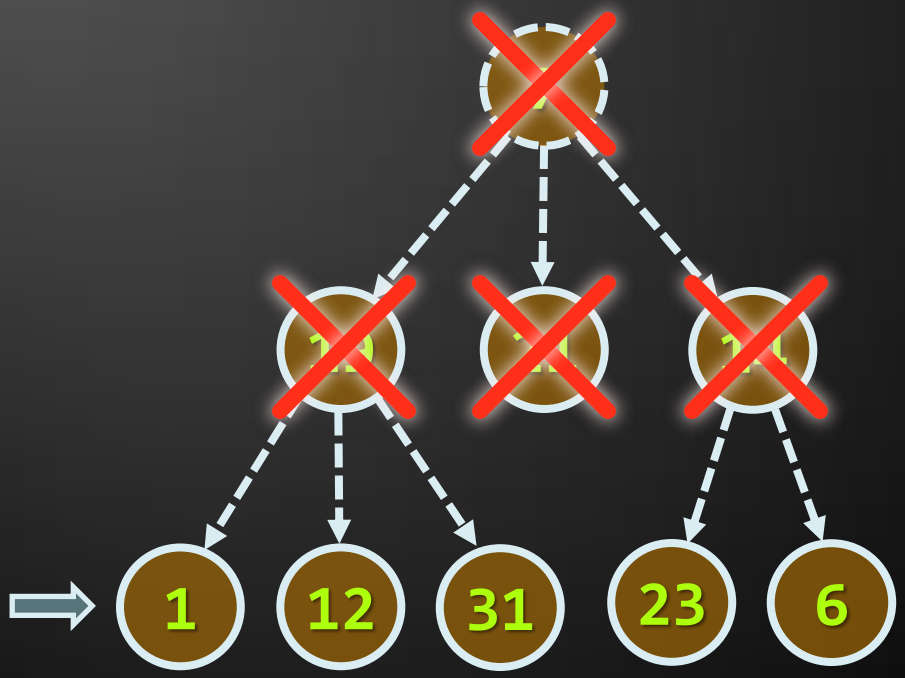
BFS in Action (Step 12)

- Queue: ~~7~~, ~~19~~, ~~21~~, 14, 1, 12, 31, 23, 6
- Output: 7, 19, 21, 14



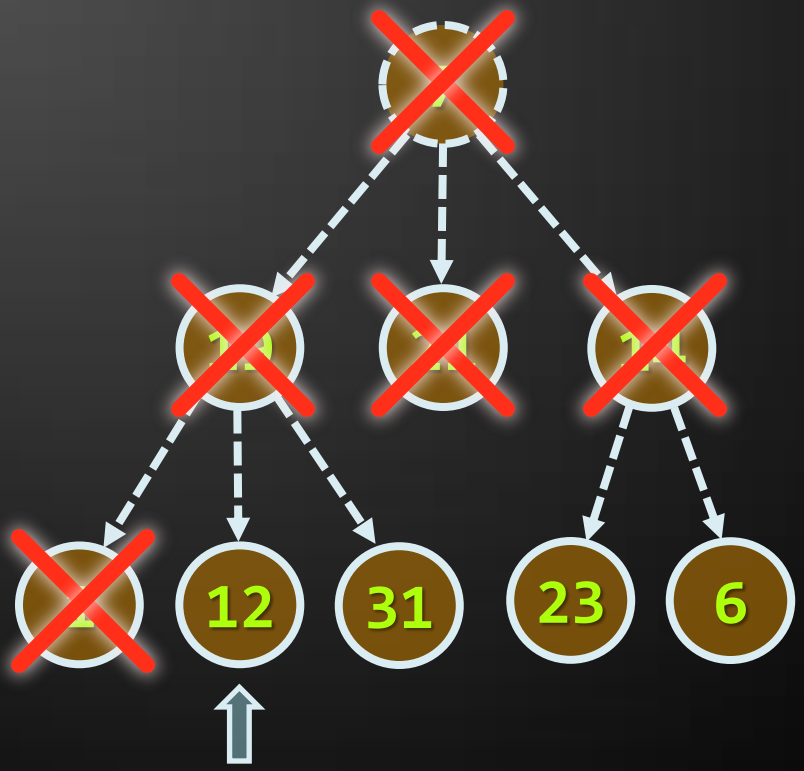
BFS in Action (Step 13)

- Queue: ~~7~~, ~~19~~, ~~21~~, ~~14~~, 1, 12, 31, 23, 6
- Output: 7, 19, 21, 14, 1



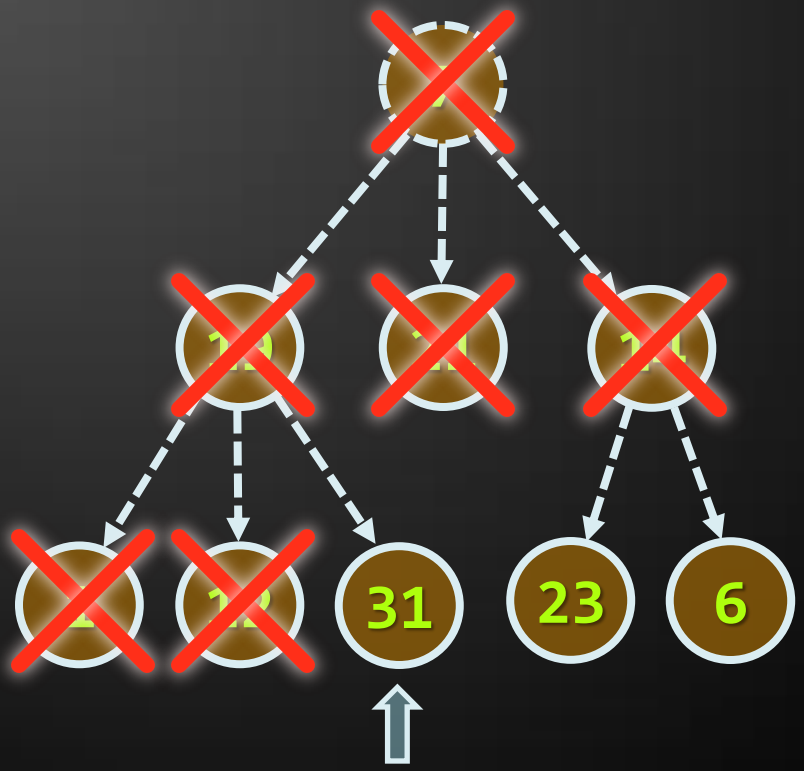
BFS in Action (Step 14)

- Queue: ~~7~~, ~~19~~, ~~21~~, ~~14~~, ~~1~~, 12, 31, 23, 6
- Output: 7, 19, 21, 14, 1, 12



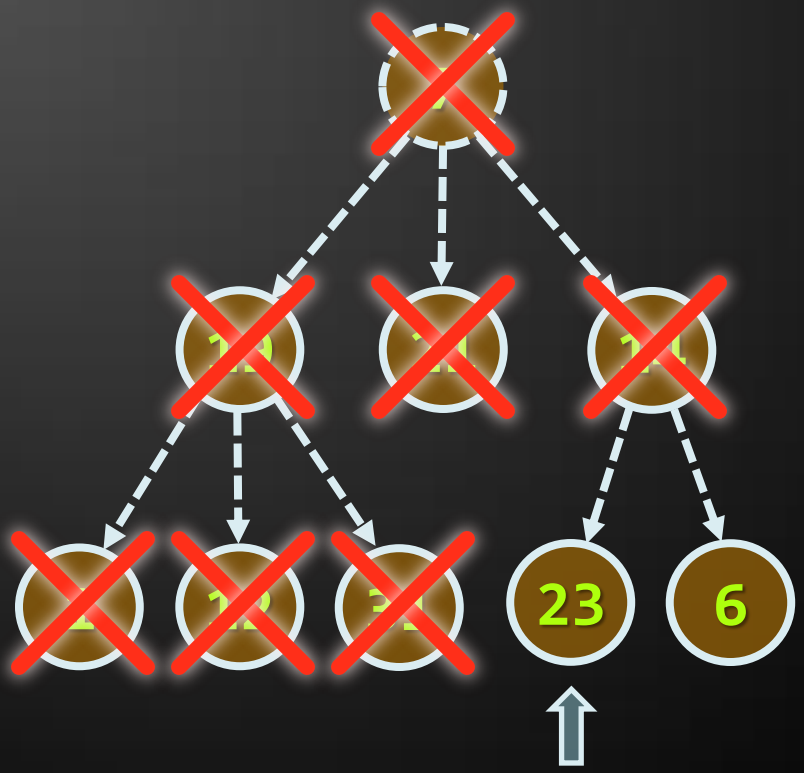
BFS in Action (Step 15)

- Queue: ~~7~~, ~~19~~, ~~21~~, ~~14~~, ~~1~~, ~~12~~, 31, 23, 6
- Output: 7, 19, 21, 14, 1, 12, 31



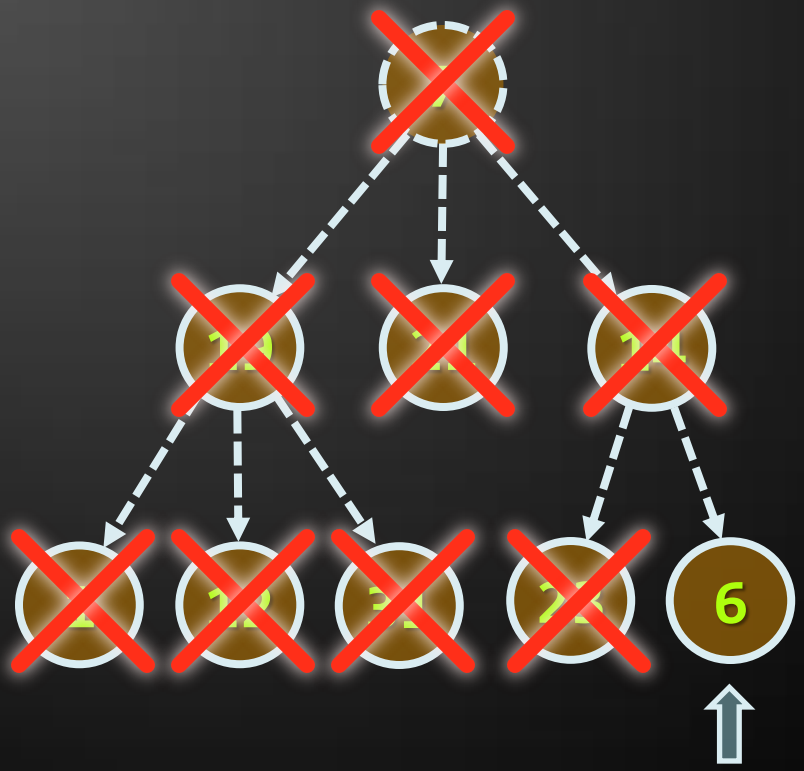
BFS in Action (Step 16)

- Queue: ~~7~~, ~~19~~, ~~21~~, ~~14~~, ~~1~~, ~~12~~, ~~31~~, 23, 6
- Output: 7, 19, 21, 14, 1, 12, 31, 23



BFS in Action (Step 16)

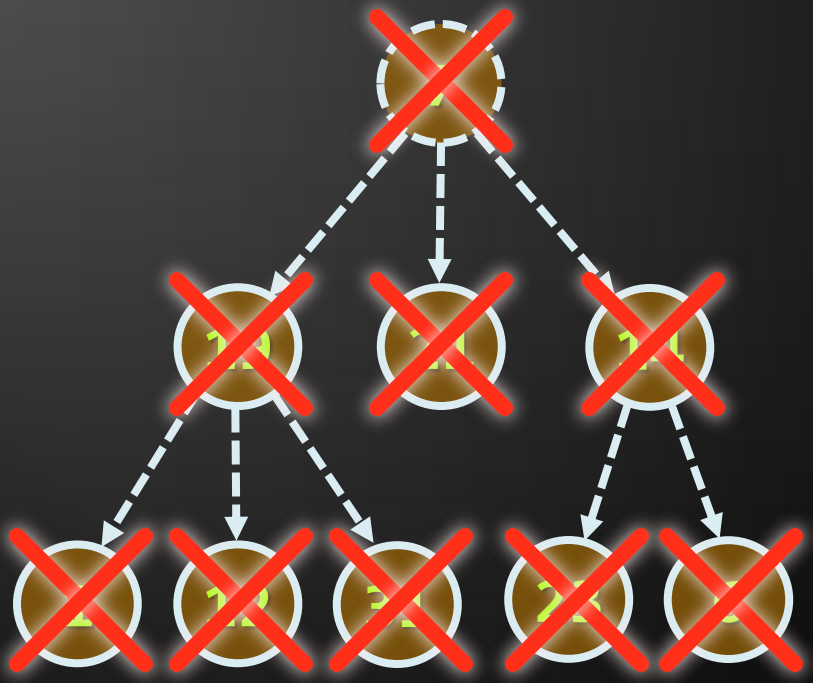
- Queue: ~~7~~, ~~19~~, ~~21~~, ~~14~~, ~~1~~, ~~12~~, ~~31~~, ~~23~~, 6
- Output: 7, 19, 21, 14, 1, 12, 31, 23, 6



BFS in Action (Step 17)

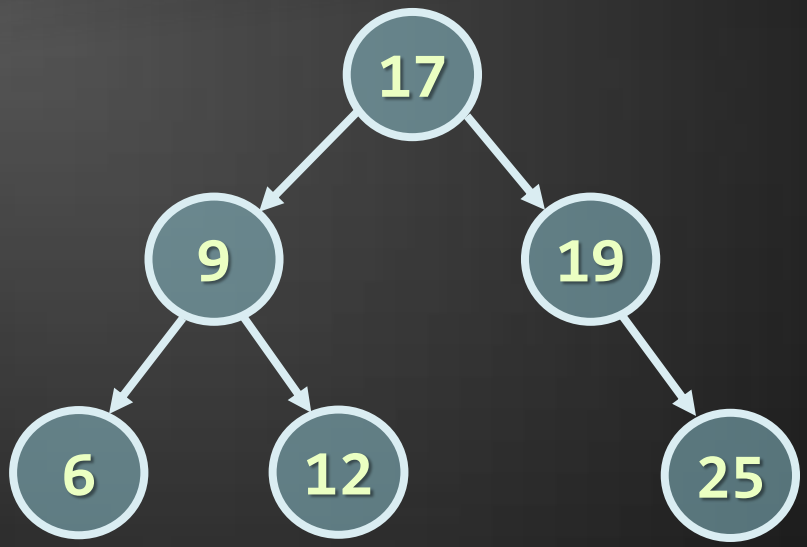
- Queue: ~~7, 19, 21, 14, 1, 12, 31, 23, 6~~
- Output: 7, 19, 21, 14, 1, 12, 31, 23, 6

The queue is empty → stop



Binary Trees DFS Traversals

- DFS traversal of binary trees can be done in pre-order, in-order and post-order



  - Pre-order: root, left, right → 17, 9, 6, 12, 19, 25
  - In-order: left, root, right → 6, 9, 12, 17, 19, 25
  - Post-order: left, right, root → 6, 12, 9, 25, 19, 17

Iterative DFS and BFS

- What will happen if in the Breadth-First Search (BFS) algorithm a stack is used instead of queue?
  - An iterative Depth-First Search (DFS) – in-order

```
BFS(node)
{
  queue ← node
  while queue not empty
    v ← queue
    print v
    for each child c of v
      queue ← c
}
```

```
DFS(node)
{
  stack ← node
  while stack not empty
    v ← stack
    print v
    for each child c of v
      stack ← c
}
```

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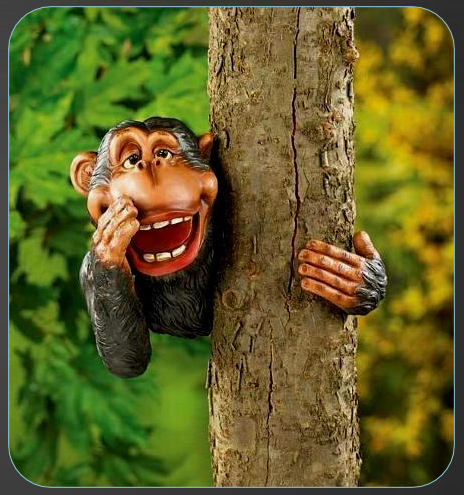

Trees and Traversals

Live Demo

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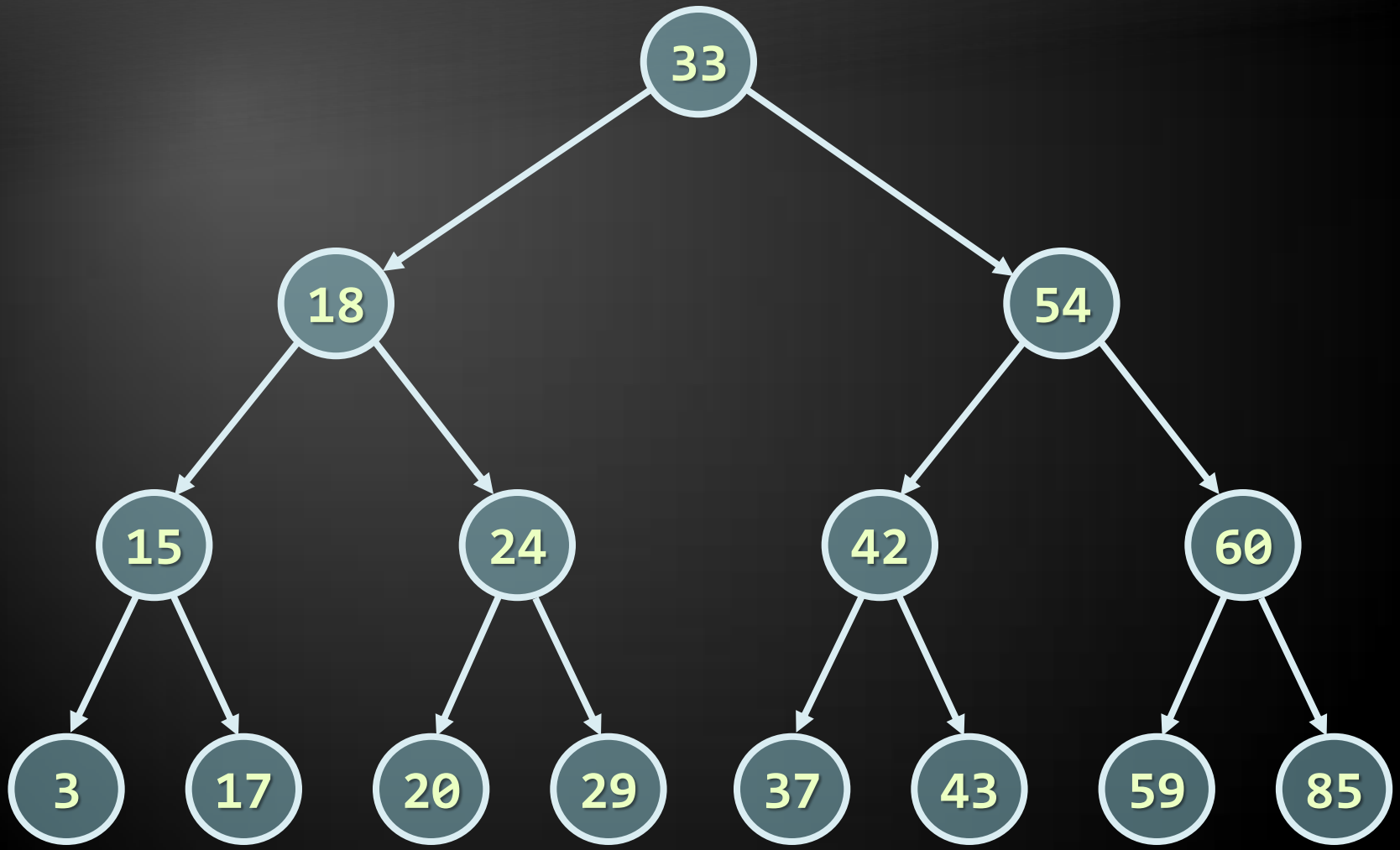
Balanced Search Trees

AVL Trees, B-Trees, Red-Black Trees, AA-Trees

Balanced Binary Search Trees

- Ordered Binary Trees (Binary Search Trees)
  - For each node x the left subtree has values ≤ x and the right subtree has values > x
- Balanced Trees
  - For each node its subtrees contain nearly equal number of nodes → nearly the same height
- Balanced Binary Search Trees
  - Ordered binary search trees that have height of $\log_2(n)$ where n is the number of their nodes
  - Searching costs about $\log_2(n)$ comparisons

Balanced Binary Search Tree – Example



Balanced Binary Search Trees

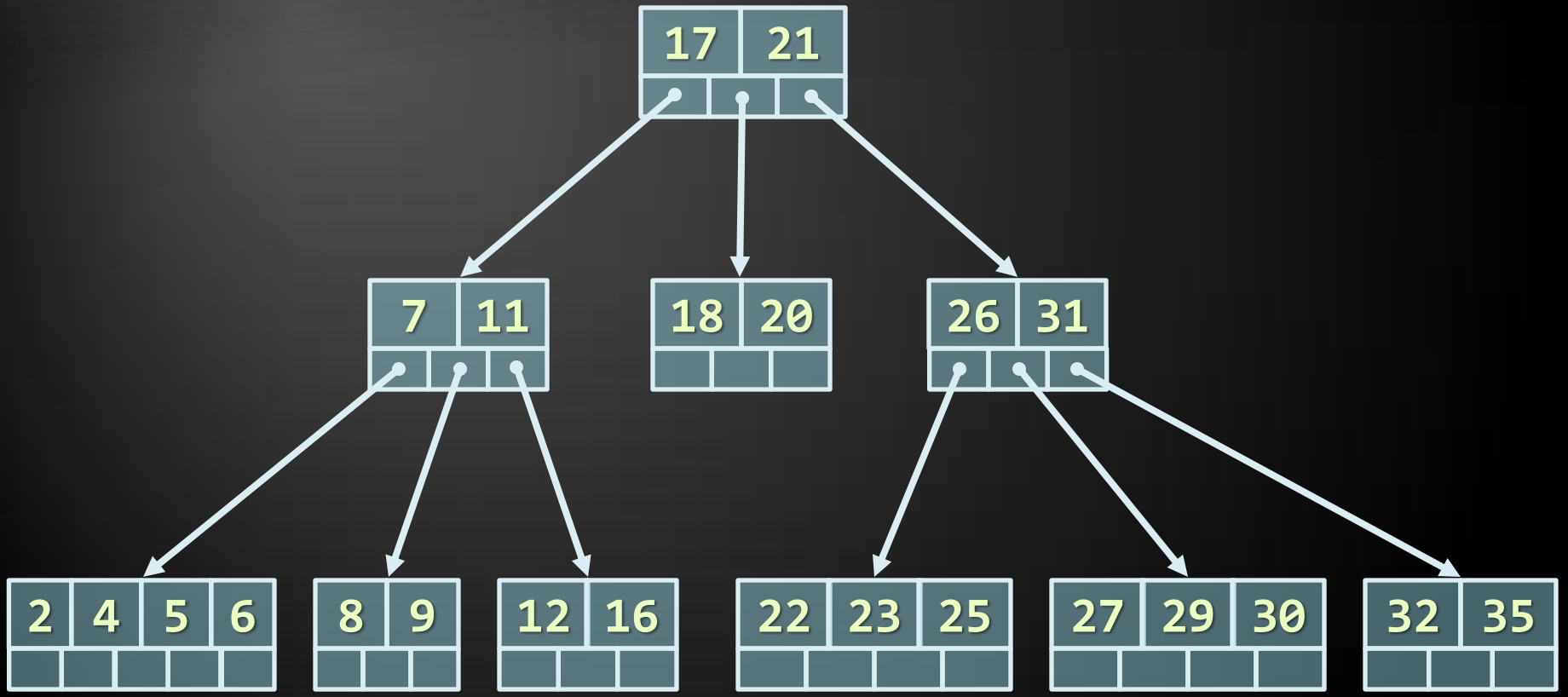
- Balanced binary search trees are hard to implement
  - Rebalancing the tree after insert / delete is complex
- Well known implementations of balanced binary search trees
  - AVL trees – ideally balanced, very complex
  - Red-black trees – roughly balanced, more simple
  - AA-Trees – relatively simple to implement
- Find / insert / delete operations need log(n) steps

B-Trees

- B-trees are generalization of the concept of ordered binary search trees
  - B-tree of order d has between d and 2*d keys in a node and between d+1 and 2*d+1 child nodes
  - The keys in each node are ordered increasingly
  - All keys in a child node have values between their left and right parent keys
- If the b-tree is balanced, its search / insert / add operations take about log(n) steps
- B-trees can be efficiently stored on the disk

B-Tree – Example

- B-Tree of order 2 (also known as 2-3-4-tree):



Balanced Trees in .NET

- .NET Framework has several built-in implementations of balanced search trees:
  - `SortedDictionary<K,V>`
    - Red-black tree based map of key-value pairs
  - `OrderedSet<T>`
    - Red-black tree based set of elements
- External libraries like "Wintellect Power Collections for .NET" are more flexible
  - http://powercollections.codeplex.com

Summary

- Trees are recursive data structure – node with set of children which are also nodes
- Binary Search Trees are ordered binary trees
- Balanced trees have weight of log(n)
- Graphs are sets of nodes with many-to-many relationship between them
  - Can be directed/undirected, weighted / unweighted, connected / not connected, etc.
- Tree / graph traversals can be done by Depth-First Search (DFS) and Breadth-First Search (BFS)

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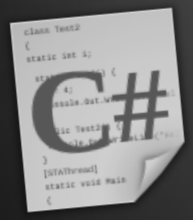
Trees and Traversals



Questions?

```
DFS(node)
{
  stack ← node
  visited[node] = true
  while stack not empty
    v ← stack
    print v
    for each child c of v
      if not visited[c]
        stack ← c
        visited[c] = true
}
```

```
DFS(node)
{
  for each child c of node
    DFS(c);
  print the current node;
}
```

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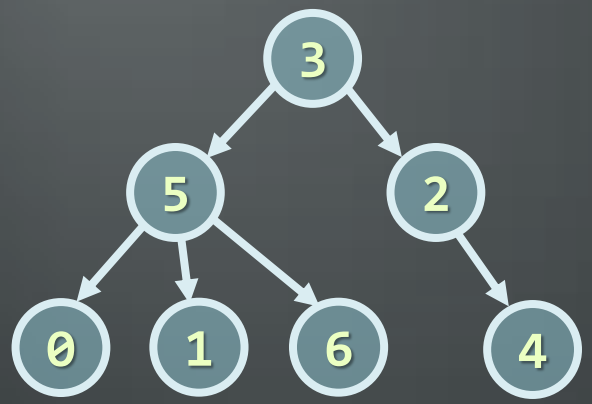
Exercises

1. You are given a tree of N nodes represented as a set of N-1 pairs of nodes (parent node, child node), each in the range (0..N-1). Example:

```
7
2 4
3 2
5 0
3 5
5 6
5 1
```



Write a program to read the tree and find:

a) the root node

b) all leaf nodes

c) all middle nodes

d) the longest path in the tree

e) * all paths in the tree with given sum S of their nodes

f) * all subtrees with given sum S of their nodes

Exercises (2)

2. Write a program to traverse the directory `C:\WINDOWS` and all its subdirectories recursively and to display all files matching the mask `*.exe`. Use the class `System.IO.Directory`.
3. Define classes `File { string name, int size }` and `Folder { string name, File[] files, Folder[] childFolders }` and using them build a tree keeping all files and folders on the hard drive starting from `C:\WINDOWS`. Implement a method that calculates the sum of the file sizes in given subtree of the tree and test it accordingly. Use recursive DFS traversal.

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