

MATRIX ANALYSIS USING PYTHON

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Assignment

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1 Problem

The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse

$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \quad (1)$$

2 Construction

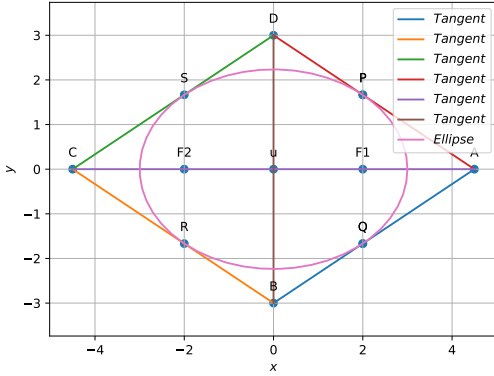


Figure of construction

3 Solution

Ellipse equation :

$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \quad (2)$$

The standard equation of the conics is given as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (3)$$

Below python code realizes the above construction :

https://github.com/velicharlagokulkumar/FWC_module1/blob/main/matrices/conics/codes/matrix.py

The steps for constructing above figure are :

1. Generate a Ellipse with vertex \mathbf{u}

2. Find the ends of latus rectum

3. Find the equations of the tangent at one end of latus rectum and use that to find the X-intercept and Y-intercept

4. Find the Area of the quadrilateral formed by all the tangents

The given ellipse can be expressed as conics with parameters

$$\lambda_1 = 5, \lambda_2 = 9 \quad (4)$$

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -45 \quad (5)$$

Eccentricity:

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \quad (6)$$

$$\Rightarrow e = 2/3 \quad (7)$$

Focus:

$$f_0 = -f, \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (8)$$

$$\mathbf{F} = \pm e \sqrt{\frac{|f_0|}{\lambda_2 (1 - e^2)}} \quad (9)$$

$$\mathbf{F1} = e \sqrt{\frac{|f_0|}{\lambda_2 (1 - e^2)}} \mathbf{e}_1 \quad (10)$$

$$\mathbf{F2} = -e \sqrt{\frac{|f_0|}{\lambda_2 (1 - e^2)}} \mathbf{e}_1 \quad (11)$$

$$\Rightarrow \mathbf{F1} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{F2} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (12)$$

Length of latus rectum:

$$l = 2 \frac{\sqrt{|f_0 \lambda_1|}}{\lambda_2} \quad (13)$$

$$l/2 = \frac{\sqrt{|f_0 \lambda_1|}}{\lambda_2} \Rightarrow 5/3 \quad (14)$$

End points of latus rectum:

$$\mathbf{P} = \begin{pmatrix} F \\ l/2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} F \\ -l/2 \end{pmatrix} \quad (15)$$

$$\mathbf{R} = \begin{pmatrix} -F \\ l/2 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -F \\ -l/2 \end{pmatrix} \quad (16)$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ 5/3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ -5/3 \end{pmatrix} \quad (17)$$

$$\mathbf{R} = \begin{pmatrix} -2 \\ 5/3 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -2 \\ -5/3 \end{pmatrix} \quad (18)$$

Equation of tangent to (3) at the point of contact \mathbf{p} , is

$$(\mathbf{V}\mathbf{p} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{p} + f = 0 \quad (19)$$

$$\left(\begin{pmatrix} 5 & 0 \\ 0 & 9 \end{pmatrix} \begin{pmatrix} 2 \\ 5/3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right)^\top \begin{pmatrix} x \\ y \end{pmatrix} + (0 \ 0) \begin{pmatrix} 2 \\ 5/3 \end{pmatrix} - 45 = 0 \quad (20)$$

$$\begin{pmatrix} 10 \\ 15 \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} - 45 = 0 \quad (21)$$

let

$$\mathbf{n}^\top = \begin{pmatrix} 10 \\ 15 \end{pmatrix}, c = 45 \quad (22)$$

X-intercept :

$$\frac{c}{n^\top e1} \quad (23)$$

Y-intercept :

$$\frac{c}{n^\top e2} \quad (24)$$

$$\mathbf{A} = \begin{pmatrix} 9/2 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \quad (25)$$

$$\mathbf{C} = \begin{pmatrix} -9/2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (26)$$

$$(27)$$

Letting,

$$\mathbf{v1} = \mathbf{u} - \mathbf{D} \quad (28)$$

$$\mathbf{v2} = \mathbf{u} - \mathbf{A} \quad (29)$$

Area of the ΔDuA is given by

$$= \frac{1}{2} \|\mathbf{v1} \times \mathbf{v2}\| \quad (30)$$

Area of the of quadrilateral ABCD is given by

$$= 4 \times \frac{1}{2} \|\mathbf{v1} \times \mathbf{v2}\| \quad (31)$$

∴ The area of quadrilateral ABCD=27 sq.units

termux commands :

bash sh3.sh.....using shell command