MATRIX ANALYSIS USING PYTHON

V.GOKULKUMAR
velicharlagokulkumar@gmail.com
FWC22034 IITH Future Wireless Communication (FWC) Assignment

September 27, 2022

Contents

1 Problem

The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse

$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \tag{1}$$

2 Construction

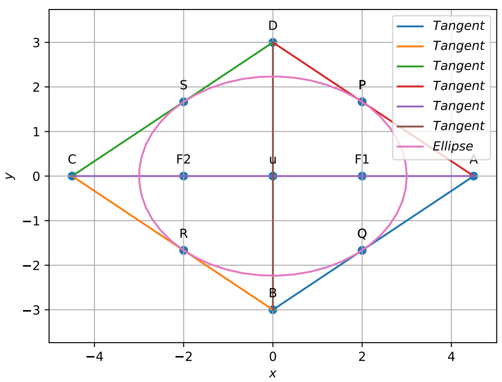

Figure of construction

3 Solution

Ellipse equation :

$$\frac{x^2}{9} + \frac{y^2}{5} = 1 \tag{2}$$

The standard equation of the conics is given as :

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3}$$

Below python code realizes the above construction :

https://github.com/velicharlagokulkumar/FWC_module1/blob/main/matrices/conics/codes/matrix.py

The steps for constructing above figure are :

1. Generate a Ellipse with vertex $\mathbf{u}$
2. Find the ends of latus rectum
3. Find the equations of the tangent at one end of latus rectum and use that to find the X-intercept and Y-intercept
4. Find the Area of the quadrilateral formed by all the tangents

The given circle can be expressed as conics with parameters

$$\lambda_1 = 5, \lambda_2 = 9 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -45 \tag{5}$$

Eccentricity:

$$e = \sqrt{1 - \frac{\lambda_1}{\lambda_2}} \tag{6}$$

$$\implies e = 2/3 \tag{7}$$

Focus:

$$f0 = -f, \mathbf{e1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{8}$$

$$\mathbf{F} = \pm e \sqrt{\frac{|f_0|}{\lambda_2 \left(1 - e^2\right)}} \tag{9}$$

$$\mathbf{F1} = e \sqrt{\frac{|f_0|}{\lambda_2 \left(1 - e^2\right)}} \mathbf{e}_1 \tag{10}$$

$$\mathbf{F2} = -e \sqrt{\frac{|f_0|}{\lambda_2 \left(1 - e^2\right)}} \mathbf{e}_1 \tag{11}$$

$$\implies \mathbf{F1} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{F2} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{12}$$

Length of latus rectum:

$$l = 2 \frac{\sqrt{|f_0 \lambda_1|}}{\lambda_2} \tag{13}$$

$$l/2 = \frac{\sqrt{|f_0 \lambda_1|}}{\lambda_2} \implies 5/3 \tag{14}$$

End points of latus rectum:

$$\mathbf{P} = \begin{pmatrix} F \\ l/2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} F \\ -l/2 \end{pmatrix} \tag{15}$$

$$\mathbf{R} = \begin{pmatrix} -F \\ l/2 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -F \\ -l/2 \end{pmatrix} \tag{16}$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ 5/3 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ -5/3 \end{pmatrix} \tag{17}$$

$$\mathbf{R} = \begin{pmatrix} -2 \\ 5/3 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -2 \\ -5/3 \end{pmatrix} \tag{18}$$

Equation of tangent to (3) at the point of contact $\mathbf{p}$, is

$$(\mathbf{V}\mathbf{p} + \mathbf{u})^\top \mathbf{x} + \mathbf{u}^\top \mathbf{p} + f = 0 \tag{19}$$

$$\left(\begin{pmatrix} 5 & 0 \\ 0 & 9 \end{pmatrix} \begin{pmatrix} 2 \\ \frac{5}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)^\top \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ \frac{5}{3} \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} - 45 = 0 \tag{20}$$

$$\begin{pmatrix} 10 \\ 15 \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} - 45 = 0 \tag{21}$$

let

$$\mathbf{n} = \begin{pmatrix} 10 \\ 15 \end{pmatrix}, c = 45 \tag{22}$$

X-intercept :

$$\frac{c}{n^\top e1} \tag{23}$$

Y-intercept :

$$\frac{c}{n^\top e2} \tag{24}$$

$$\mathbf{A} = \begin{pmatrix} 9/2 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{25}$$

$$\mathbf{C} = \begin{pmatrix} -9/2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{26}$$

$$\tag{27}$$

Letting,

$$\mathbf{v1} = \mathbf{u} - \mathbf{D} \tag{28}$$

$$\mathbf{v2} = \mathbf{u} - \mathbf{A} \tag{29}$$

Area of the ΔDuA is given by

$$= \frac{1}{2}\|\mathbf{v1} \times \mathbf{v2}\| \tag{30}$$

Area of the of quadrilateral ABCD is given by

$$= 4 \times \frac{1}{2}\|\mathbf{v1} \times \mathbf{v2}\| \tag{31}$$

$\therefore$The area of quadrilateral ABCD=27 sq.units

termux commands :

bash sh3.sh............using shell command