OPTIMIZATION USING PYTHON

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FWC22034 IITH Future Wireless Communication (FWC) Assignment

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1 Problem

Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:

$$f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right] \tag{1}$$

2 Solution

A function f(x) is said to be convex if following inequality is true for $\lambda \in [0, 1]$:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \tag{2}$$

Checking convexity of $f(x)$:

$$\begin{aligned} &\lambda\left(4x_1 - \frac{1}{2}x_1^2\right) + (1 - \lambda)\left(4x_2 - \frac{1}{2}x_2^2\right) \geq \\ &4\left(\lambda x_1 + (1 - \lambda)x_2\right) - \frac{1}{2}\left(\lambda x_1 + (1 - \lambda)x_2\right)^2 \end{aligned} \tag{3}$$

$$x_1^2\left(\frac{\lambda^2 - \lambda}{2}\right) + x_2^2\left(\frac{\lambda^2 - \lambda}{2}\right) + 2x_1x_2\left(\frac{\lambda - \lambda^2}{2}\right) \geq 0 \tag{4}$$

$$\left(\frac{\lambda^2 - \lambda}{2}\right)\left(x_1^2 + x_2^2 - 2x_1x_2\right) \geq 0 \tag{5}$$

$$-\frac{1}{2}\lambda\left(1 - \lambda\right)\left(x_1 - x_2\right)^2 \geq 0 \tag{6}$$

$$\implies \frac{1}{2}\lambda\left(1 - \lambda\right)\left(x_1 - x_2\right)^2 \leq 0 \tag{7}$$

Equation (7) holds true for all $\lambda \in (0, 1)$. Hence the given function f(x) is concave For a general quadratic equation

$$f(x) = ax^2 + bx + c \tag{8}$$

1. For Maxima :
   Using gradient ascent method,

$$x_n = x_{n-1} + \mu\frac{df(x)}{dx} \tag{9}$$

$$\frac{df(x)}{dx} = 4 - x \tag{10}$$

After substituting 10 in 9 we get:

$$x_n = x_{n-1} + \mu(4 - x_{n-1}) \tag{11}$$

In equation (9), μ is a variable parameter known as step size. x_{n+1} is the next position. The plus sign refers to the maximization part of gradient ascent. Assume, $\mu = 0.001$, Taking $x_0 = -2$,precision= 0.00000001 and following the above method, we keep doing iterations until $x_{n+1} - x_n$ becomes less than the value of precision we have chosen.

(a) The absolute maxima occurs at 3.9999900196756437

$$x_n = 4 \tag{12}$$

(b) The value of f(x) at maxima is 7.999999999950196

$$\boxed{\text{Maxima} = 7.999999999950196 \approx 8} \tag{13}$$

$$\boxed{\text{Maxima Point} = 3.9999900196756437 \approx 4} \tag{14}$$

Hence, The maxima value of $f(x)$ at $x = 4$ is 8.

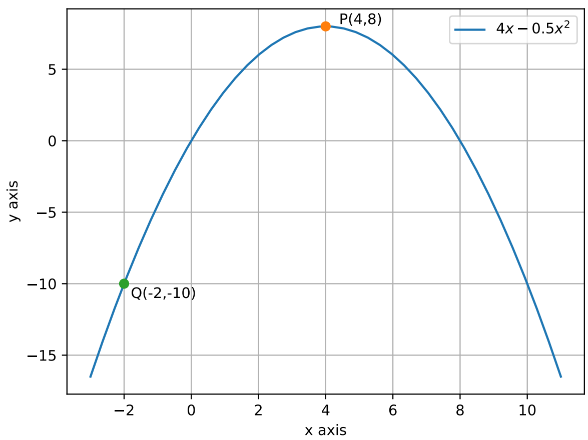

Figure 1: $f(x) = 4x - 0.5x^2$

2. For Minima : Critical point is given by

$$\frac{df(x)}{dx} = 0 \tag{15}$$

$$\implies x = 4 \tag{16}$$

and,end points are $x = -2$ and $x = 4.5$. Using table1,

x	$f(x)$
-2	-10
4	8
4.5	7.875

Table 1: Value of $f(x)$

$$\boxed{\text{Minima} = -10} \tag{17}$$

$$\boxed{\text{Minima Point} = -2} \tag{18}$$

Hence, The minima value of $f(x)$ at $x = -2$ is -10.

The following python code computes the maxima,minima value as plotted in Fig. 1.

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https://github.com/velicharlagokulkumar/FWC_module1/blob/main/optimization/advanced/codes/optimize2.py
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