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1 Problem

Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose the tangents at the points B(1,7) and D(4.-2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

Solution 2

The input parameters for this construction are

Symbol	Value	Description
r	5	Radius
Α	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Centre
В	$\begin{pmatrix} 1 \\ 7 \end{pmatrix}$	Point B
D	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	Point D

Circle equation : $x^2 + y^2 - 2x - 4y - 20 = 0$

Equations of tangents at B, D are given by

$$x + 7y - (x + 1) - 2(y + 7) - 20 = 0$$

$$4x - 2y - (x + 4) - 2(y - 2) - 20 = 0$$

The above equations result in the system

$$u = 7$$

$$3x - 4y = 20$$

From (3),(4) let

$$\mathbf{Z} = \begin{pmatrix} 0 & 1 \\ 3 & -4 \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} 7 \\ 20 \end{pmatrix}$$

Solve (5) and (6)

$$\therefore$$
 Coordinates of C is $\mathbf{C} = \begin{pmatrix} 16 \\ 7 \end{pmatrix}$

Length of BC is

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -15\\0 \end{pmatrix} \tag{7}$$

$$\|\mathbf{B} - \mathbf{C}\| = \left\| \begin{pmatrix} -15\\0 \end{pmatrix} \right\| \tag{8}$$

$$=\sqrt{\begin{pmatrix} -15 & 0 \end{pmatrix} \begin{pmatrix} -15 \\ 0 \end{pmatrix}} \tag{9}$$

$$=15 \tag{10}$$

Letting,

$$v1 = A - B \tag{11}$$

$$\mathbf{v2} = \mathbf{A} - \mathbf{C} \tag{12}$$

Area of the $\triangle ABC$ is given by

$$=\frac{1}{2}\|\mathbf{v1}\times\mathbf{v2}\|\tag{13}$$

Area of the of quadrilateral ABCD is given by

$$=2\times\frac{1}{2}\|\mathbf{v1}\times\mathbf{v2}\|\tag{14}$$

...The area of quadrilateral ABCD=75 sq.units

termux commands:

bash sh.sh

The below python code realizes the above construction:

https://github.com/velicharlagokulkumar/FWC_module1/ blob/main/matrices/circle/codes/matrix.py

3 Construction (1)

(2)

(3)

(4)

(5)

(6)

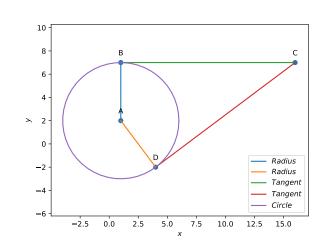


Figure of construction