

MATRIX ANALYSIS USING PYTHON

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Assignment

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1 Problem

Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose the tangents at the points B(1,7) and D(4,-2) on the circle meet at the point C. Find the area of the quadrilateral ABCD.

2 Construction

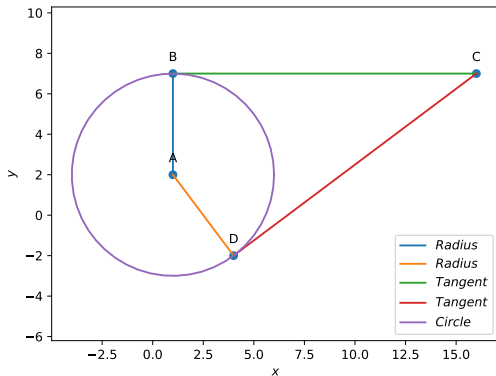


Figure of construction

3 Solution

Circle equation : $x^2 + y^2 - 2x - 4y - 20 = 0$
The standard equation of the conics is given as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$$

The given circle can be expressed as conics with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -\begin{pmatrix} 1 \\ 2 \end{pmatrix}, f = -20$$

Radius and Centre are

$$r = \sqrt{\mathbf{u}^T \mathbf{u} - f}, \mathbf{A} = -\mathbf{u}$$

The steps for constructing above figure are :

1. Generate a circle of radius r with centre \mathbf{A}

2. Locate \mathbf{B}, \mathbf{D} on the circle

3. Find the Normal vectors to \mathbf{AB}, \mathbf{AD} say $\mathbf{m}_1, \mathbf{m}_2$

4. Find the equations of the tangents and use them to find the intersection \mathbf{C}

The input parameters for this construction are

Symbol	Value	Description
\mathbf{A}	$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$	Centre
\mathbf{B}	$\begin{pmatrix} 1 \\ 7 \end{pmatrix}$	Point B
\mathbf{D}	$\begin{pmatrix} 4 \\ -2 \end{pmatrix}$	Point D

\mathbf{C} is obtained as the point of intersection of the tangents at \mathbf{B} and \mathbf{D} . The equation of both tangents are respectively

$$\begin{aligned} \mathbf{x} &= \mathbf{B} + \lambda_1 \mathbf{m}_1 \\ \mathbf{x} &= \mathbf{D} + \lambda_2 \mathbf{m}_2 \end{aligned} \quad (4)$$

and their intersection is given by

$$\mathbf{B} + \lambda_1 \mathbf{m}_1 = \mathbf{D} + \lambda_2 \mathbf{m}_2 \quad (5)$$

$$\Rightarrow (\mathbf{m}_1 \quad \mathbf{m}_2) \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \mathbf{D} - \mathbf{B} \quad (6)$$

which can be used to obtain λ_1, λ_2 and consequently \mathbf{C} , using (4)

$$\therefore \text{Coordinates of } \mathbf{C} \text{ is } \mathbf{C} = \begin{pmatrix} 16 \\ 7 \end{pmatrix}$$

Letting,

$$\mathbf{v}_1 = \mathbf{A} - \mathbf{B} \quad (7)$$

$$\mathbf{v}_2 = \mathbf{A} - \mathbf{C} \quad (8)$$

Area of the $\triangle ABC$ is given by

$$= \frac{1}{2} \|\mathbf{v}_1 \times \mathbf{v}_2\| \quad (9)$$

Area of the quadrilateral ABCD is given by

$$(1) \quad = 2 \times \frac{1}{2} \|\mathbf{v}_1 \times \mathbf{v}_2\| \quad (10)$$

\therefore The area of quadrilateral ABCD = 75 sq. units

termux commands :

(2) `bash sh2.sh.....using shell command`

Below python code realizes the above construction :

(3) https://github.com/velicharlagokulkumar/FWC_module1/blob/main/matrices/circle/codes/matrix.py