Week 6: Trading Costs

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Keim and Madhavan (1997)

- Explicit costs
  - Commissions
    - We don't worry about them too much
- Implicit costs
  - Quoted bid-ask spread
    - Market maker's compensation for providing liquidity; price of immediacy
  - Effective spread
    - "True" spread, since many traders trade within the quoted spread
  - Price impact
    - Larger trades move prices
  - Opportunity cost
    - Impossible to measure

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Roll (1984)

- The bid-ask spread complicates research, since we don't observe the true price.
  - We have three prices: bid, P_b, ask, P_a, and true price, P^*
  - The true price is often between P_a and P_b, although it need not be.
  - How do we define returns: From P_a to P_a , P_b to P_b, P_b to P_a ...?
  - How is $P_a - P_b$ determined?

- Roll (1984) provides a simple model of how the bid-ask spread might impact the time-series properties of returns.
  - Provides most of the intuition and the framework on how we think about the bid-ask spread.

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Roll (1984)

- The observed market price is

$$P_t = P_t^* + q_t \frac{s}{2}$$

  - P_t^* : fundamental price in a frictionless economy
  - s: bid-ask spread (independent of the P_t level)
  - q_t : i.i.d index variable - takes values of 1 with prob. 0.5 (buy)
    - takes value of -1 with prob. 0.5 (sell)
- q_t is unobservable. But, with the assumptions, $E[q_t] = 0$ and $Var(q_t) = 1$
- For simplicity assume that P_t^* does not change - $\text{Var}(\Delta P_t^*) = 0$
- The change in price is (define the *cost* $c = s/2$):

$$\Delta P_t = \Delta P_t^* + q_t \frac{s}{2} - q_{t-1} \frac{s}{2} = \Delta P_t^* + c\Delta q_t$$

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Roll (1984)

- Its variance and autocovariance are:
  - $Var(\Delta \mathrm{P_t}) = \mathrm{Var}(\Delta P_t^*) + c^2 Var(I_t) + c^2 Var(I_t) = 2c^2 (= s^2/2)$
  - $Cov(\Delta P_t, \Delta P_{t-1}) = -c^2$

- Note:
  - The fundamental value is fixed, but there is variation from c.
  - The bid-ask spread induces negative correlation in returns even in the absence of other fluctuations.
  - The variance and covariance depend on the magnitude of the bid-ask spread.
  - In this particular example, it induces a 1st-order serial correlation.

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Roll (1984)

- We can also express the cost (aka half-spread) as a function of the covariance:
  - $c = [-Cov(\Delta P_t, \Delta P_{t-1})]^{-1/2}$
- In practice, we can find $Cov(\Delta P_t, \Delta P_{t-1}) > 0$
- To avoid this problem, Roll (1984) defines the cost as

$$c = -[|Cov(\Delta P_t, \Delta P_{t-1})|]^{-1/2}$$

- Roll calls s$(= 2 \times c)$ the “effective spread,” which is estimable.

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Hasbrouck (2009)

- Takes the Roll (1984) model:

$$\Delta P_t = c\Delta q_t + \epsilon_t$$

- … generalizes it by adding a market factor:

$$\Delta P_t = c\Delta q_t + \beta_m r_{mt} + \epsilon_t$$

σ_ϵ^2

- … makes a few assumptions
  - $\epsilon_t \sim N(0, \sigma_\epsilon^2)\ \&\ i.i.d.$
  - Priors for the unknowns $\{c, \sigma_\epsilon^2, q_1, \dots, q_T\}$

- … and sequentially draws the parameter estimates using a Gibbs sampler to characterize the posterior densities

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Hasbrouck (2009)

- The Gibbs measure achieves the highest correlation (96.5%) with high-frequency TAQ data estimates

Table III

Correlations between Liquidity Measures for the Comparison Sample

The comparison sample consists of approximately 150 NASDAQ firms and 150 NYSE/Amex firms selected in a capitalization-stratified random draw in each of the years from 1993 to 2005. Definitions of the liquidity measures are given in Table I. Partial correlations are adjusted for log (end-of-year price) and log (market capitalization).

	c_{it}^{TAQ}	c_{it}^{Gibbs}	c_{it}^{Moment}	$PropZero_{it}$	λ_{it}	I_{it}
Pearson correlation						
c_{it}^{TAQ}	1.000	0.965	0.878	0.611	0.513	0.612
c_{it}^{Gibbs}	0.965	1.000	0.917	0.579	0.450	0.589
c_{it}^{Moment}	0.878	0.917	1.000	0.451	0.378	0.504
$PropZero_{it}$	0.611	0.579	0.451	1.000	0.311	0.252
λ_{it}	0.513	0.450	0.378	0.311	1.000	0.668
I_{it}	0.612	0.589	0.504	0.252	0.668	1.000

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Novy-Marx and Velikov (2016)

- Apply the Gibbs measure to 23 anomalies and create a taxonomy of anomalies
  - Useful rules of thumb for anomalies & whether they survive trading costs based on their turnover
  - Typical value-weighted anomaly ~50bps
  - If turnover more than 50% per month, net returns are negative

- Several new methods
  - “Generalized alpha”
  - Buy/hold spread trading cost mitigation technique

- Price impact estimation

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Novy-Marx and Velikov (2016)

- Trading cost calculation procedure
  - Track portfolio weights over time
  - Whenever a position is entered or exited, assume half of the effective spread (i.e., Hasbrouck's effective cost) is paid

- Interpretation: lower bound cost for average trader using market orders

- Note: lots of missing observations in Hasbrouck for small stocks
  - Creates look-ahead bias (excluding stocks with missing data)
  - Thus, we fill in 29% of stock-months (4% of market cap) in based on $\sqrt{(rankME_i - rankME_j)^2 + (rankME_i - rankME_j)^2}$

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Trading costs formulas from DNMV (2022)

- Net returns for month t are given by:
  - Long side:

$$f_t^{net} = f_t^{gross} - TC_t^f$$

  - Short side:

$$f_t^{S,net} = -f_t^{gross} - TC_t^f$$

- Trading cost calculation procedure
  - Turnover (TO) for a factor f in month t defined as:

$$TO_t^f = \frac{1}{2}\sum_{i=1}^{N_t} |w_{i,t} - \widetilde{w}_{i,t-}|$$

  - Trading costs (TC) defined as:

$$TC_t^f = \sum_{i=1}^{N_t} \left|w_{i,t} - \widetilde{w}_{i,t-}\right| \cdot c_{i,t}$$

where $c_{i,t}$ is the one-way transaction cost (i.e., effective half-spread)

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Novy-Marx and Velikov (2016)

- Problem with performance evaluation: we can't regress net anomaly strategy returns on net factor returns
  - If a loading is negative, the alpha from this regression effectively assumes you "earn" the trading costs

- Solution: Generalized alpha (α^*):

$$\frac{MVE_{\{X,y\}}}{w_{y,MVE\{X,y\}}} = \alpha^* + \beta^* MVE_X + \epsilon^*$$

  - Does the mean-variance efficient portfolio of the factors and the strategy under evaluation improve the investment opportunity set for an investor who has access to the MVE of the factors only
  - $w_{y,MVE\{X,y\}}$ terms ensures it's the same as alpha when there is no trading costs

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Novy-Marx and Velikov (2016): Taxonomy

Table 3
Value-weighted returns

Anomaly	$E[r^e_{gross}]$	α^{FF4}_{gross}	TO	T-costs	$E[r^e_{net}]$	α^{FF4}_{net}
Panel A: Low-turnover strategies						
Size	0.33	−0.14	1.23	0.04	0.28	
	[1.66]	[−1.77]			[1.44]	
Gross profitability	0.40	0.52	1.96	0.03	0.37	0.51
	[2.94]	[3.83]			[2.74]	[3.77]
Value	0.47	−0.17	2.91	0.05	0.42	−0.02
	[2.68]	[−1.76]			[2.39]	[−0.17]
ValProf	0.82	0.50	2.94	0.06	0.77	0.49
	[5.18]	[4.01]			[4.82]	[3.93]
Accruals	0.27	0.27	5.74	0.09	0.18	0.19
	[2.14]	[2.15]			[1.43]	[1.55]
Asset growth	0.37	0.07	6.37	0.11	0.26	0.03
	[2.52]	[0.58]			[1.75]	[0.21]
Investment	0.56	0.35	6.40	0.10	0.46	0.31
	[4.44]	[2.90]			[3.60]	[2.62]
Piotroski's F-score	0.20	0.31	7.24	0.11	0.09	0.24
	[1.04]	[1.75]			[0.45]	[1.37]
Panel B: Mid-turnover strategies						
Net issuance	0.57	0.58	14.4	0.20	0.37	0.41
	[3.70]	[4.10]			[2.43]	[2.93]
Return-on-book equity	0.71	0.84	22.3	0.38	0.33	0.59
	[2.96]	[4.41]			[1.38]	[3.18]
Failure probability	0.85	0.94	26.1	0.61	0.24	0.70
	[2.52]	[4.89]			[0.73]	[3.55]
ValMomProf	1.43	0.68	26.8	0.43	0.99	0.68
	[7.41]	[5.52]			[5.18]	[5.22]
ValMom	0.93	−0.12	28.7	0.41	0.51	
	[4.81]	[−1.31]			[2.67]	
Idiosyncratic volatility	0.63	0.83	24.6	0.52	0.11	0.41
	[2.13]	[5.14]			[0.37]	[2.57]
Momentum	1.33	0.35	34.5	0.65	0.68	0.40
	[4.80]	[3.04]			[2.45]	[3.12]
PEAD (SUE)	0.72	0.58	35.1	0.46	0.26	0.29
	[4.52]	[4.31]			[1.60]	[2.21]
PEAD (CAR3)	0.91	0.87	34.7	0.57	0.34	0.38
	[6.54]	[6.39]			[2.41]	[2.85]
Panel C: High-turnover strategies						
Industry momentum	0.93	0.83	90.1	1.22	−0.29	
	[3.97]	[3.52]			[−1.20]	
Industry relative reversals	0.98	1.05	90.3	1.78	−0.80	
	[5.72]	[6.66]			[−4.73]	
High-frequency combo	1.61	1.48	91.0	1.45	0.16	0.05
	[11.21]	[9.93]			[1.11]	[0.35]
Short-run reversals	0.37	0.45	90.9	1.65	−1.28	
	[1.71]	[2.22]			[−6.02]	
Seasonality	0.84	0.82	91.1	1.46	−0.62	
	[5.21]	[5.03]			[−3.88]	
Industry relative reversals (low volatility)	1.25	1.17	94.0	1.06	0.19	0.07
	[9.36]	[8.96]			[1.41]	[0.57]

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Novy-Marx and Velikov (2016)

- Mean-variance efficient weights with no costs given by:

$$\omega_{MVE} = \frac{\Sigma^{-1}\mu_e}{1\Sigma^{-1}\mu_e}$$

  - we can estimate with ex-post vector of average returns & variance-covariance matrix

- However, with costs, we can't just apply the formula with net returns
  - Need to do a numerical optimization

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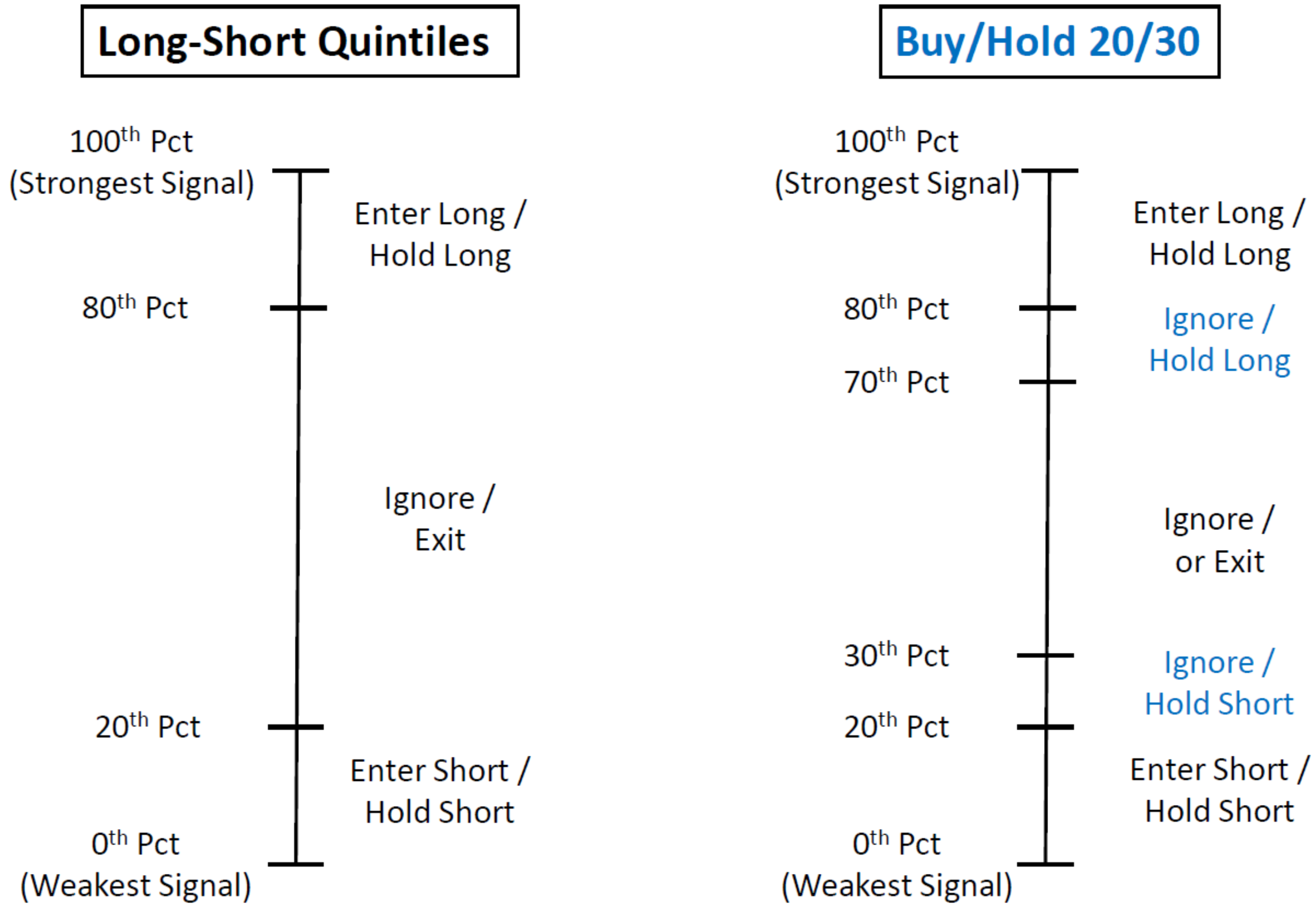
Novy-Marx and Velikov (2016)

- Buy/hold spreads: a simple trading cost mitigation technique



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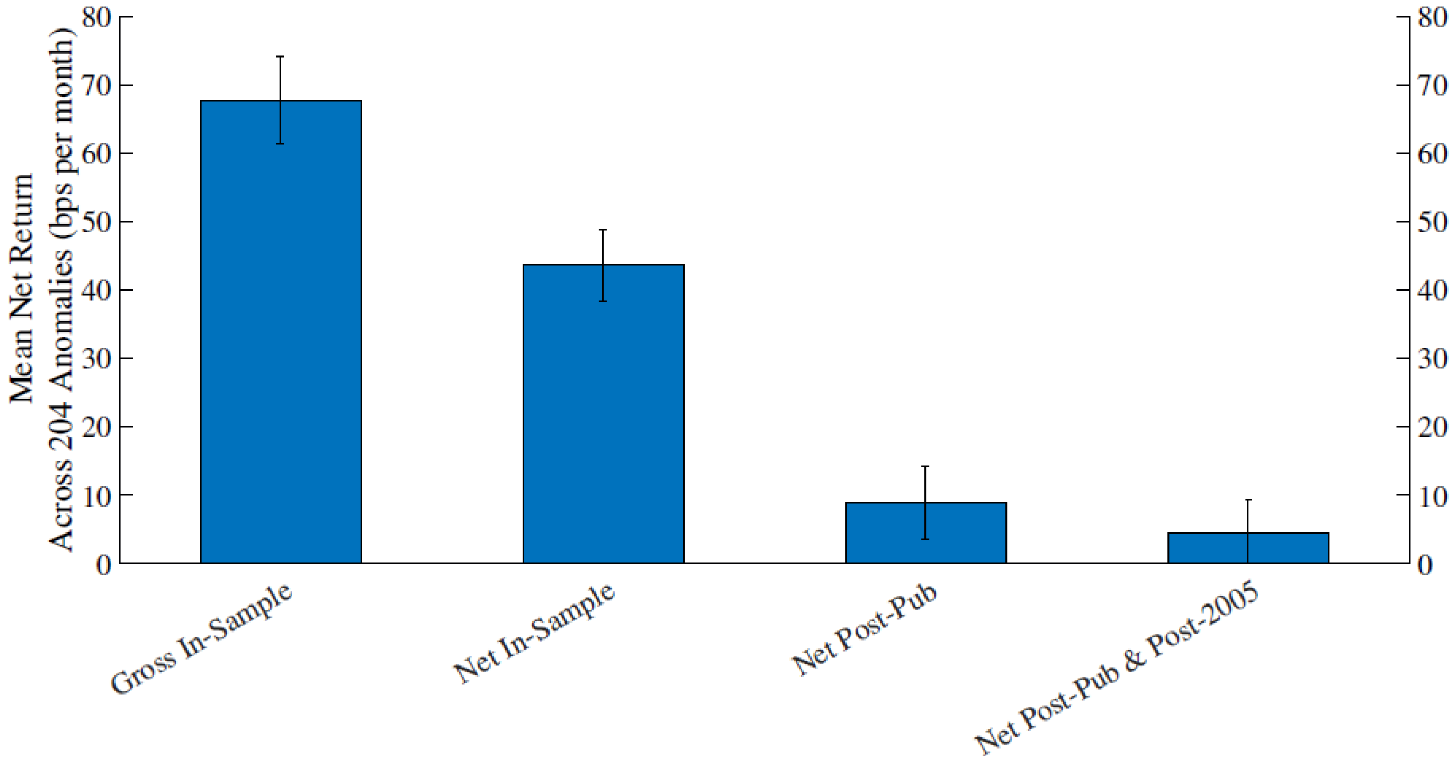
Chen and Velikov (2022)

- Main goal: "zero in" on the average anomaly

Figure 1: Anomaly Mean Long-Short Returns. Error bars show two standard errors.



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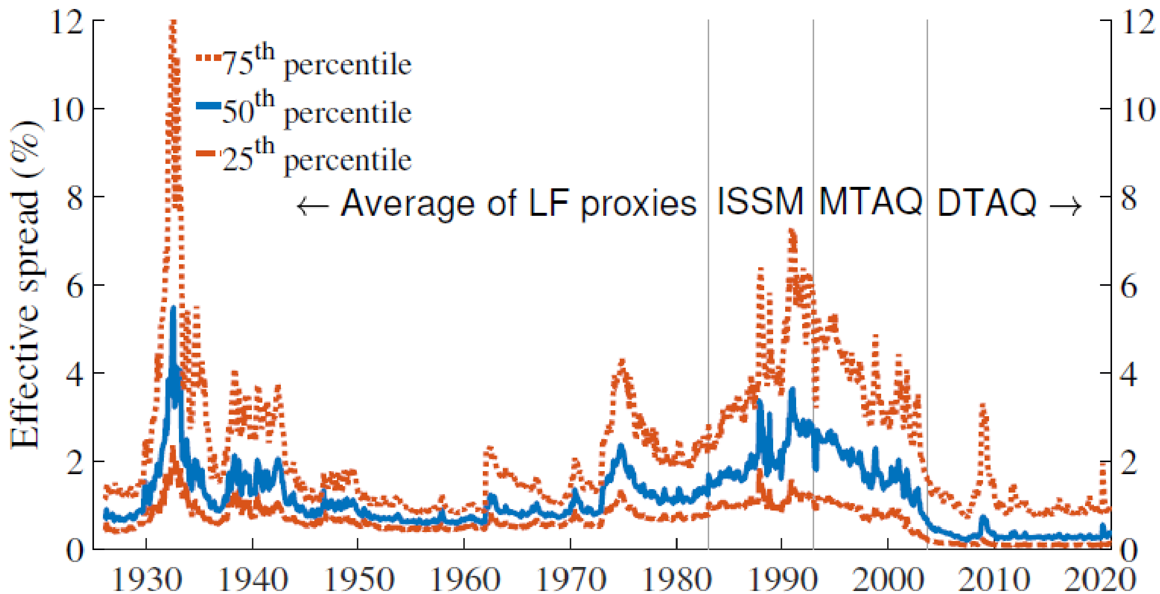
Chen and Velikov (2022)

- Main (in my opinion) contribution: a new trading cost measure that combines what's in the literature

Figure 3: Combined Effective Spreads Over Time. Spreads combine high-frequency and low-frequency data. We use high-frequency Daily TAQ (DTAQ), Monthly TAQ (MTAQ), and ISSM when available. Otherwise, we use the average of four low frequency proxies: Gibbs (Hasbrouck (2009)), HL (Corwin and Schultz (2012)), CHL (Abdi and Ranaldo (2017)), and VoV (Kyle and Obizhaeva (2016)). The combined spread tracks well-known structural changes like the entry of NASDAQ (early 1970s) and decimalization (early 2000s).



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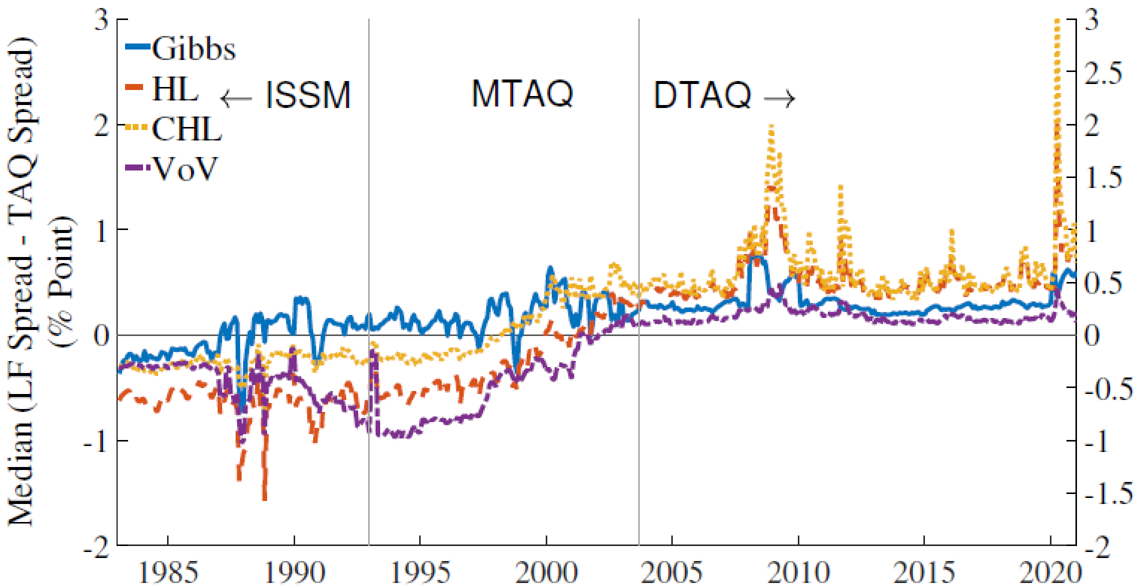
- Why do we need the new measure?
  - Severe bias in low-frequency measures post-decimalization
- Especially important for post-publication results

Figure 2: The Bias in Low-Frequency Effective Spread Proxies. We take the difference between low-frequency effective spreads and TAQ effective spreads at the firm-month level and then take the median across firms to calculate the median error in each month. Low-frequency spreads are from Hasbrouck (2009) (Gibbs), Corwin and Schultz (2012) (HL), Abdi and Ranaldo (2017) (CHL), and Kyle and Obizhaeva (2016) (VoV). Post-decimalization, low-frequency proxies are biased upward by roughly 25-50 bps. LF spread data are found at https://sites.google.com/site/chenandrewy/, HF spread code is at https://github.com/chenandrewy/hf-spreads-all, and replication code is at https://github.com/velikov-mihail/Chen-Velikov.



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- Combination "back-cast"
  - Stronger correlation of combined measure with high-frequency data
  - Simple average outperforms individual measures
  - Errors "average out"

Table 1: Correlations Between Low-Frequency Proxies and High-Frequency Effective Bid-Ask Spreads

Correlations are pooled. We examine four low frequency proxies: Gibbs is Hasbrouck's (2009) Gibbs estimate of the Roll model, HL is Corwin and Schultz's (2012) high-low spread, CHL is Abdi and Ranaldo's (2017) close-high-low, and VoV (volume-over-volatility) is Fong et al.'s (2017) implementation of Kyle and Obizhaeva (2016) microstructure invariance hypothesis. LF_Ave is the equal weighted average of the four low frequency proxies. TAQ and ISSM are computed from high-frequency data. The low frequency measures are imperfectly correlated, suggesting that they contain distinct information. LF_Ave has the highest correlation with high-frequency spreads. HF spread code is found at https://github.com/chenandrewy/hf-spreads-all and https://github.com/velikov-mihail/Chen-Velikov. LF spread data is at https://sites.google.com/site/chenandrewy/.

Panel A: LF spread correlations (1926-2020)

	Gibbs	HL	CHL	VoV
Gibbs	1.00			
HL	0.63	1.00		
CHL	0.74	0.86	1.00	
VoV	0.74	0.53	0.73	1.00

Panel B: Correlations with TAQ (1993-2020)

	TAQ	Gibbs	HL	CHL	VoV	LF_Ave
TAQ	1.00					
Gibbs	0.84	1.00				
HL	0.64	0.60	1.00			
CHL	0.79	0.72	0.85	1.00		
VoV	0.84	0.72	0.53	0.74	1.00	
LF_Ave	0.90	0.89	0.82	0.93	0.86	1.00

Panel C: Correlations with ISSM (1983-1992)

	ISSM	Gibbs	HL	CHL	VoV	LF_Ave
ISSM	1.00					
Gibbs	0.88	1.00				
HL	0.77	0.74	1.00			
CHL	0.83	0.78	0.88	1.00		
VoV	0.86	0.81	0.62	0.74	1.00	
LF_Ave	0.92	0.94	0.88	0.93	0.87	1.00

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- Depending on data availability, we either:
  - Calculate monthly realized spreads from high-frequency data

1. Calculate effective spreads for all eligible trades, k, in ISSM & TAQ

$$[Effective\ Spread]_k = 2|\log(P_k) - \log(M_k)|$$

where

- P_k is the price of the kth trade
- M_k is the prevailing midpoint of the matched NBBO quotes

2. For each stock-day, take a share-weighted average across all trades
3. For each stock-month, take an equal-weighted average across all days

- Use average of up to four low-frequency estimators from the literature
  - Gibbs (Hasbrouck, 2009)
  - High-low spread (Corwin and Schultz, 2012)
  - Close-high-low (Abdi and Ranaldo, 2017)
  - Volume-over-volatility (Kyle and Obizhaeva, 2016)

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- Trading cost hierarchy

1. Daily TAQ (2003 – 2020)
2. Monthly TAQ (1993 – 2003)
3. ISSM (1983 – 1992)
   - NASDAQ data missing prior to 1987 and sporadically between 1987 and 1991
4. Low-frequency average (1926 – 2020)
   - Require at least one of the four measures
5. Match based on closest distance in market capitalization and idiosyncratic volatility rank space (1926 – 2020)

$$d = \sqrt{\left(rankME_i - rankME_j\right)^2 + \left(rankIVOL_i - rankIVOL_j\right)^2}$$

6. Match based to nearest stock based on market capitalization (1926 – 2020)

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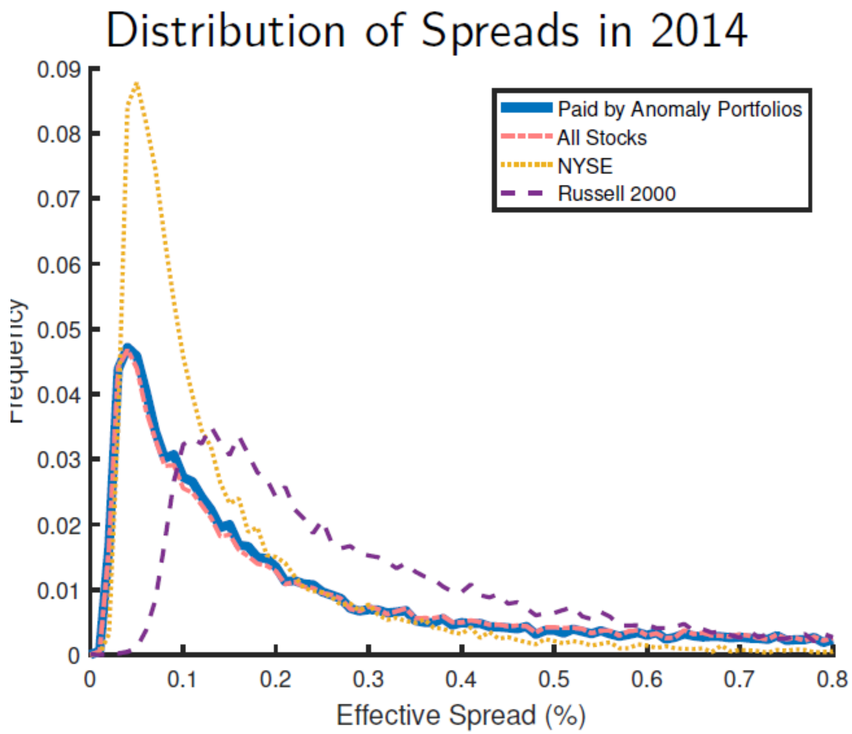
- Long right tail of the effective spread distribution



- Decimalization: spread ≈ \$0.01, price ≈ \$20 ⇒ spread ≈5 bps
  - But that's the mode! 20% of NYSE stocks have spreads > 20 bps

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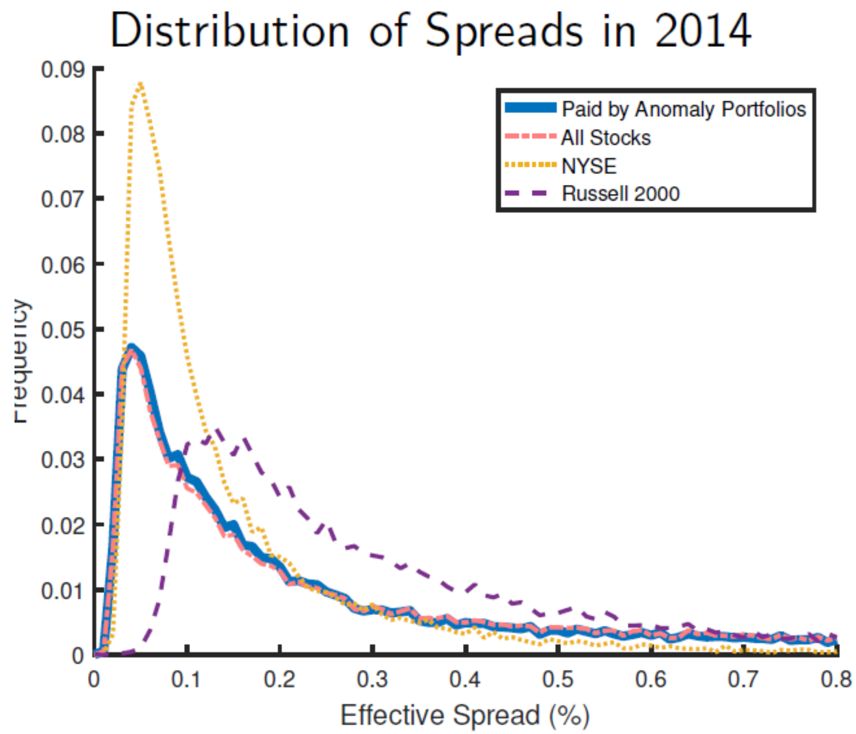

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