Week 5: Liquidity

Professor Mihail Velikov
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Liquidity

- Multi-dimensional concept. E.g., for stocks, less liquidity ⇒
  - More costly to trade
  - Higher bid-ask spread
  - Higher price impact
  - Slower speed of execution

- From Li, Novy-Marx, and Velikov (CFR, 2019):
  - "Given these subtleties, many adopt an attitude for liquidity akin to Supreme Court Justice Potter Stewart's "you know it when you see it" doctrine for recognizing pornography (*Jacobellis v. Ohio*, 1964)."

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Liquidity and asset pricing

- Do investors require compensation for individual stock liquidity?
  - Generally, positive relation between illiquidity and expected stock returns
    - Amihud and Mendelson (1986) – bid-ask spread
    - Brennan and Subrahmanyam (1996) – price impact
    - Easley et al. (2002) – PIN

- Time-series properties of aggregate liquidity measures
  - Existence of predictability and commonality in liquidity
    - Chordia, Roll, and Subrahmanyam (2004), Hasbrouck and Seppi (2001), Amihud (2002), Jones (2002)

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Liquidity and asset pricing

- Is the systematic component of liquidity priced?
  - Yes, with a positive price of risk
    - Pastor and Stambaugh (2003) – 7.5%/year premium
    - Acharya and Pederson (2005) – 1.1%/year premium

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Main issue: Liquidity is unobservable

- We try to proxy
  - Intuitively, a market with high transaction costs is illiquid
    - How do we measure them? Size of trade? Timing? Trading venue? Counterparties?
      - Data not available for many of these
- Trading volume (scaled or unscaled)
  - Intuition is simple: more trading, more liquidity
  - Indirect but widely used measure
  - Typically available
  - But: correlation with volatility, which is associated with illiquidity

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Main issue: Liquidity is unobservable

- Trading frequency
  - Number of trades executed within a specified interval, without regard to trade size
  - Same caveat about volatility
- Bid-ask spread (Level or %)
  - Measures the cost of executing a small trade immediately.
  - Calculated as the difference between bid/ask price and the bid-ask midpoint
  - Only good for limited quantities & time periods
- Quote size
  - Quantity of securities tradable at the bid & ask prices
  - Measures market depth

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Main issue: Liquidity is unobservable

- Order imbalance
  - Difference between buyer- and seller-initiated volume (either share or dollar) over some period of time
  - Measures informed trading

- Price impact
  - It considers the rise (fall) in price that is typically associated with a buyer-initiated (seller-initiated) trade
  - Impossible to estimate
    - Plagued by endogeneity issues
    - I.e., traders trade more price impact is low

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Amihud and Mendelson (1986)

- Microstructure equilibrium model
  - Negative relation between transaction costs (S, %) and stock prices

- In equilibrium
  - Investors choose asset depending on (S). If an investor is willing to hold stocks for a long period, they are willing to buy high S stocks (Liquidity clientele)
  - Model links:
    - Expected return and bid-ask spread
    - Expected return and turnover (reflecting holding period differences)

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Amihud and Mendelson (1986)

- Findings
  - Using 49 stock portfolios, a 7x7 sort on beta and S, they estimate a GLS panel regression:

Table 5

Generalized least squares estimates of the difference between the mean monthly excess return of the portfolio with the highest spread and beta – portfolio (7, 7), the 49th portfolio – and the mean monthly excess returns of each of the other 48 portfolios. These are the estimated coefficients of the 48 portfolio dummy variables DP_{ij} in the pooled cross-section and time-series regression model (8), using GLS, over the entire period 1961–1980.
t-statistics for all unmarked table entries are greater than 1.96, implying that the estimated coefficient is significant at better than the 2.5% level (one-tail test).

Spread group, i	Beta group, j 1 (low)	2	3	4	5	6	7 (high)	Mean
1 (low)	−0.0117	−0.0132	−0.0111	−0.0100	−0.0111	−0.0091	−0.0108	−0.0110
2	−0.0113	−0.0109	−0.0109	−0.0094	−0.0115	−0.0079	−0.0033[b]	−0.0093
3	−0.0127	−0.0113	−0.0078[a]	−0.0118	−0.0100	−0.0082	−0.0094	−0.0102
4	−0.0113	−0.0120	−0.0091	−0.0099	−0.0059[a]	−0.0064	−0.0052[b]	−0.0085
5	−0.0120	−0.0101	−0.0111	−0.0077	−0.0062[a]	−0.0030[b]	−0.0041[b]	−0.0077
6	−0.0108	−0.0074[a]	−0.0072	−0.0070	−0.0032[b]	−0.0035[b]	−0.0020[b]	−0.0059
7 (high)	−0.0080	−0.0049[b]	−0.0063	−0.0068	−0.0019[b]	−0.0013[b]	0.0000	−0.0042
Mean	−0.0111	−0.0100	−0.0091	−0.0089	−0.0071	−0.0056	−0.0050	

[a] $1.645 < t < 1.96$, implying significance at better than the 5% level (one-tail test).
[b] $t < 1.645$, insignificant at the 5% level (one-tail test).

  - and find that returns are increasing in the spread

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Eleswarapu (1997)

- Effects of β, bid-ask spread and size in Nasdaq, 1976-90. Spread portfolios are updated at the beginning of the year.

- Findings: Results are stronger than for the NYSE, where many transactions take place within the spread; on Nasdaq, transactions are more likely at the quotes.

Table II

Estimates of Coefficients (Standard Errors) for Fama-Macbeth Type Regressions of Excess Returns for the 49 Portfolios of Nasdaq Firms with Annually Updated Spreads, 1976–1990

Assignment of a stock to a particular beta/spread portfolio in a given test year depends on two criteria: 1) the average spread in the previous year, and 2) a stock's beta estimated with 36 months of preceding returns. In the cross-sectional regression, the portfolio spread (S_{pt}) is computed from the average of the firm's spread in January and December months of the preceding year. The Size (equity value) is the value in December in the year preceding each test year. The portfolio beta (β_{pt}) is the unconditional beta; that is, it is computed using the monthly portfolio returns from all the test period years. The cross-sectional regression is fit in each month, t, of the test-period years. The coefficients are the time-series means with corresponding standard errors.

(A): $R_{pt} = a_0 + a_1\beta_{pt} + e_{pt}$

(B): $R_{pt} = b_0 + b_1 S_{pt} + e_{pt}$

(C): $R_{pt} = c_0 + c_1\beta_{pt} + c_2 S_{pt} + e_{pt}$

(D): $R_{pt} = d_0 + d_1 \text{Log(Size)} + e_{pt}$

(E): $R_{pt} = e_0 + e_1\beta_{pt} + e_2 S_{pt} + e_3 \text{Log(Size)} + e_{pt}$

Variable	(A)	(B)	(C)	(D)	(E)
All Months					
Beta	−0.0021		−0.0031		−0.0026
	(0.0047)		(0.0047)		(0.0049)
Avg. Spread		0.0256	0.0258		0.0394
		(0.0129)	(0.0133)		(0.0129)
Log(Size)				−0.0013	0.0004
				(0.0007)	(0.0007)
N	180	180	180	180	180
January					
Beta	0.0739		0.0679		0.0633
	(0.0204)		(0.0208)		(0.0215)
Avg. Spread		0.2435	0.2263		0.1749
		(0.0641)	(0.0657)		(0.0822)
Log(Size)				−0.0166	−0.0048
				(0.0029)	(0.0029)
N	15	15	15	15	15
Non-January					
Beta	−0.0090		−0.0095		−0.0086
	(0.0045)		(0.0045)		(0.0047)
Avg. Spread		0.0058	0.0076		0.0271
		(0.0117)	(0.0123)		(0.0116)
Log(Size)				0.0001	0.0008
				(0.0007)	(0.0007)
N	165	165	165	165	165

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Datar, Naik and Radcliffe (1998):

- Liquidity is measured by turnover = volume / number of shares.
- Results persist over subperiods.

Table 2

Average slopes of monthly cross-sectional regressions of returns on turnover, book-to-market, log of size and beta. From July 1963 to December 1991 All months Including January.

Returns are regressed each month on the explanatory variables. The GLS estimates of average slopes and associated t-statistics (in parentheses) are calculated using Eqs. (2) and (3). The *Panel A* shows the results for the *complete dataset* while the *panel B* shows the results for the *trimmed dataset*.

Constant	Turnover	Book-to-market	Log of size	Beta
Panel A				
0.73	– 0.04			
(32.51)	(– 8.86)			
0.83	– 0.04	0.21		
(34.19)	(– 9.07)	(10.15)		
2.39	– 0.05		– 0.07	
(12.32)	(– 10.58)		(– 7.48)	
1.69	– 0.05	0.16	– 0.04	
(8.09)	(– 10.56)	(6.99)	(– 3.68)	
2.30	– 0.04	0.14	– 0.05	– 0.37
(9.70)	(– 8.58)	(5.97)	(– 4.65)	(– 5.76)

- Findings: Liquidity (higher turnover) is associated with lower expected returns

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Brennan and Subrahmanyam (1996)

- Illiquidity is estimated following Glosten and Harris (1988):

$$\Delta p_t = \lambda q_t + \psi[D_t - D_{t-1}] + y_t$$

  - λ – price impact coefficient
  - q_t - order flow
  - D_t - sign of order
  - ψ – fixed cost component

- Use ISSM data for the estimation of λ
  - Intraday quotes & trades

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Brennan and Subrahmanyam (1996)

- Asset pricing implication of price impact
  - Pooled time series & cross section model estimated by GLS
  - Estimated on 25 portfolios (5x5 sort on size and λ):

$$R_{it} = \alpha + \sum_{k=2}^{5} \gamma_k L_{ik} + \beta_i MKT_t + \delta_i SMB_t + \kappa_i HML_t + \epsilon_{it}$$

  - L_{ik} = Indicator variables for the 5 λ portfolios
- Findings: Returns increasing & concave in λ

	Estimate$*10^3$ (t-statistic)	Average size ($ millions)	$\lambda*100$	C_q*10^3	C_n
Constant	0.01 (0.01)				
L1	Base case	1,735.79	0.0042	0.085	1.25
L2	1.63 (2.05)	1,773.08	0.0132	0.156	2.06
L3	2.82 (2.88)	1,955.04	0.0249	0.208	2.40
L4	4.37 (3.86)	1,298.49	0.0452	0.302	3.34
L5	5.52 (5.26)	1,324.33	0.1478	0.479	5.04

Amihud (2002)

- A proxy for illiquidity:

$$ILLIQ_{i,y} = \frac{1}{D_{iy}} \sum_{t=1}^{D_{iy}} \frac{|R_{iyd}|}{VOLD_{iyd}}$$

- Intuition: proxy for market impact
  - What is the effect on return given trading volume?
  - Difference is it uses total volume, not signed volume (order imbalance)
  - Nevertheless, empirically it has strong positive correlation with the Glosten and Harris (1988) estimated lambda

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Amihud (2002)

- Results: illiquidity strong predictor of returns

Variable	All months	Excl. January	1964–1980	1981–1997	All months	Excl. January	1964–1980	1981–1997
Constant	−0.364	−0.235	−0.904	0.177	1.922	1.568	2.074	1.770
	(0.76)	(0.50)	(1.39)	(0.25)	(4.06)	(3.32)	(2.63)	(3.35)
BETA	1.183	0.816	1.450	0.917	0.217	0.260	0.297	0.137
	(2.45)	(1.75)	(1.83)	(1.66)	(0.64)	(0.79)	(0.59)	(0.30)
ILLIQMA	0.162	0.126	0.216	0.108	0.112	0.103	0.135	0.088
	(6.55)	(5.30)	(4.87)	(5.05)	(5.39)	(4.91)	(3.69)	(4.56)
R100	1.023	1.514	0.974	1.082	0.888	1.335	0.813	0.962
	(3.83)	(6.17)	(2.47)	(2.96)	(3.70)	(6.19)	(2.33)	(2.92)
R100YR	0.382	0.475	0.485	0.279	0.359	0.439	0.324	0.395
	(2.98)	(3.70)	(2.55)	(1.59)	(3.40)	(4.27)	(2.04)	(2.82)
Ln *SIZE*					−0.134	−0.073	−0.217	−0.051
					(3.50)	(2.00)	(3.51)	(1.14)
SDRET					−0.179	−0.274	−0.136	−0.223
					(1.90)	(2.89)	(0.96)	(1.77)
DIVYLD					−0.048	−0.063	−0.075	−0.021
					(3.36)	(4.28)	(2.81)	(2.11)

[a] *t*-statistics in parentheses.

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Other measures

- Tons of papers proposing illiquidity measures:
  - Stoll and Whaley (1983): Spread + Commission – direct and observable data.
  - Roll’s (1984) effective spread
  - Glosten and Harris’s (1988) λ and Ψ
  - Frequency of zero return days: Lesmond, Ogden, and Trzcinka (1999)
  - Frequency of non-trading days: used in Rabinovitch et al. (2003)
  - Hasbrouk’s (2009) effective spread
  - Corwin and Schultz’es (2012) effective spread
  - Abdi and Ranaldo’s (2017) effective spread
- See the horse race in Goyenko, Holden, and Trczinka (2009)

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Liquidity risk: Pastor and Stambaugh (2003)

- Greater exposure to liquidity risk ⇒ higher expected returns

- Develop a market-wide, systematic liquidity factor
  - Innovations in aggregate liquidity

- Start by estimating individual stock liquidity, γ_{it} from:

$$\mathrm{r}^{\mathrm{e}}_{\mathrm{i,d+1,t}} = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} sign\left(r^{e}_{i,d,t}\right) \cdot \nu_{i,d,t} + \epsilon_{i,d+1,t}$$

  - $\mathrm{r}^{\mathrm{e}}_{\mathrm{i,d+1,t}} \equiv r_{i,d,t} - r_{m,d,t}$

- Intuitively, it's a price impact measure
  - Although assumes all volume is buy/sell

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Pastor and Stambaugh (2003)

- Aggregate individual stock liquidity for each month:

$$\hat{\gamma}_t = \frac{1}{N}\sum_{i=1}^{N}\hat{\gamma}_{i,t}$$

- Get its innovations as the unexpected changes in scaled aggregate liquidity (i.e., L_t from the regression):

$$\Delta\hat{\gamma}_t = a + b\Delta\hat{\gamma}_{t-1} + c\left(\frac{m_{t-1}}{m_t}\right)\hat{\gamma}_{t-1} + 100L_t$$

  - $\Delta\hat{\gamma}_t = \left(\frac{m_t}{m_1}\right)\frac{1}{N_t}\sum_{i=1}^{N_t}(\hat{\gamma}_{i,t} - \hat{\gamma}_{i,t-1})$
  - m_t is market cap of all eligible stocks in month t-1

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Pastor and Stambaugh (2003)

- Innovations are non-traded, so to get a traded factor they estimate exposures and form long/short portfolios

- Two estimations:
  - Historical betas:

$$r_{i,t}^{e} = \beta_i^0 + \beta_i^L L_t + \beta_i^M MKT_t + \beta_i^S SMB_t + \beta_i^H HML_t + \epsilon_{i,t}$$

  - Predicted betas:

$$\beta_{i,t-1}^{L} = \psi_{1,i} + \psi_{2,i}' Z_{i,t-1}$$

    - Z includes the historical beta, average individual liquidity, average volume, past performance, return volatility, price, and shares outstanding

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Replication notes from Li, Novy-Marx, and Velikov (2019)

- We were able to replicate the results for the historical beta factor
  - Although we spent quite some time to figure out they were dropping zero volume days (not noted in the paper)
  - We were only able to figure that out because they were posting their historical factor

- We couldn't replicate the predicted liquidity factor

- Raised a few issues with the robustness

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Replication notes from Li, Novy-Marx, and Velikov (2019)

- High correlations of our replication with PS posted innovations in liquidity

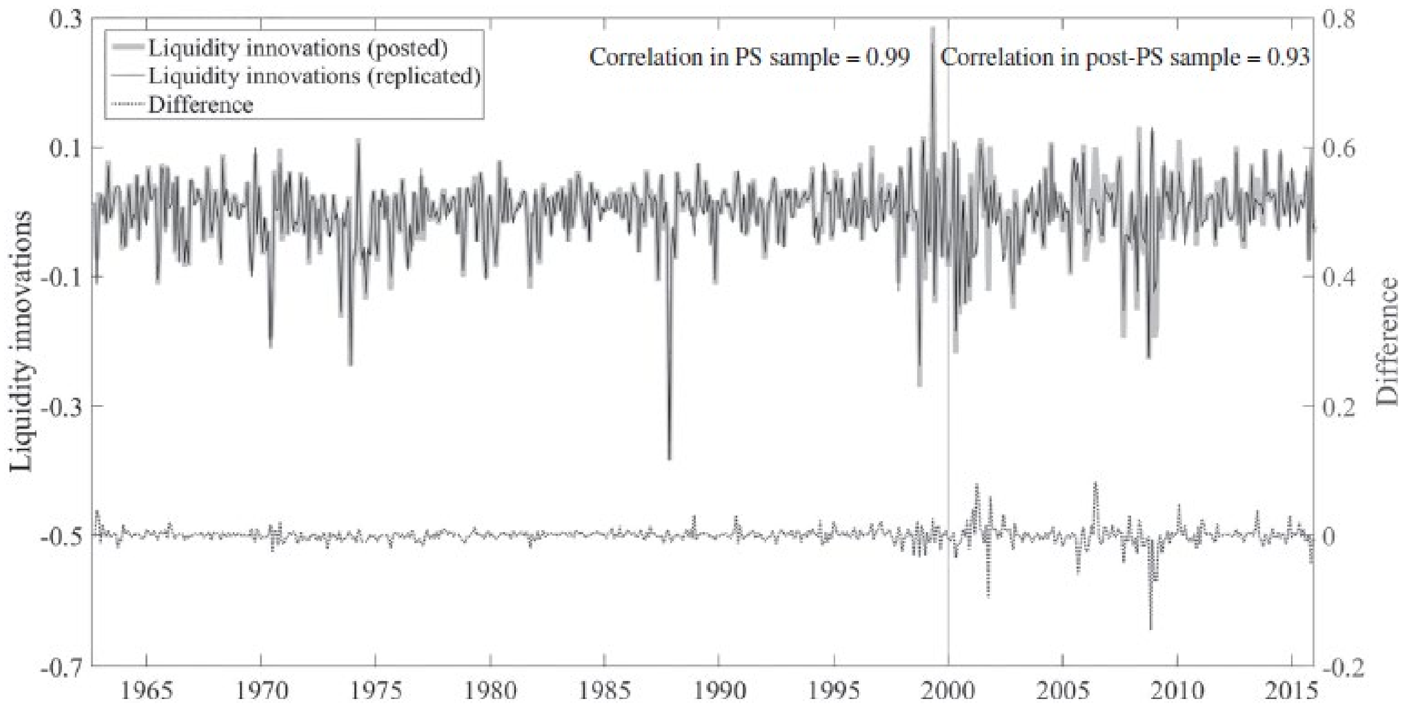

Figure 1: Aggregate Liquidity Innovations.

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Replication notes from Li, Novy-Marx, and Velikov (2019)

- And with the posted historical factor

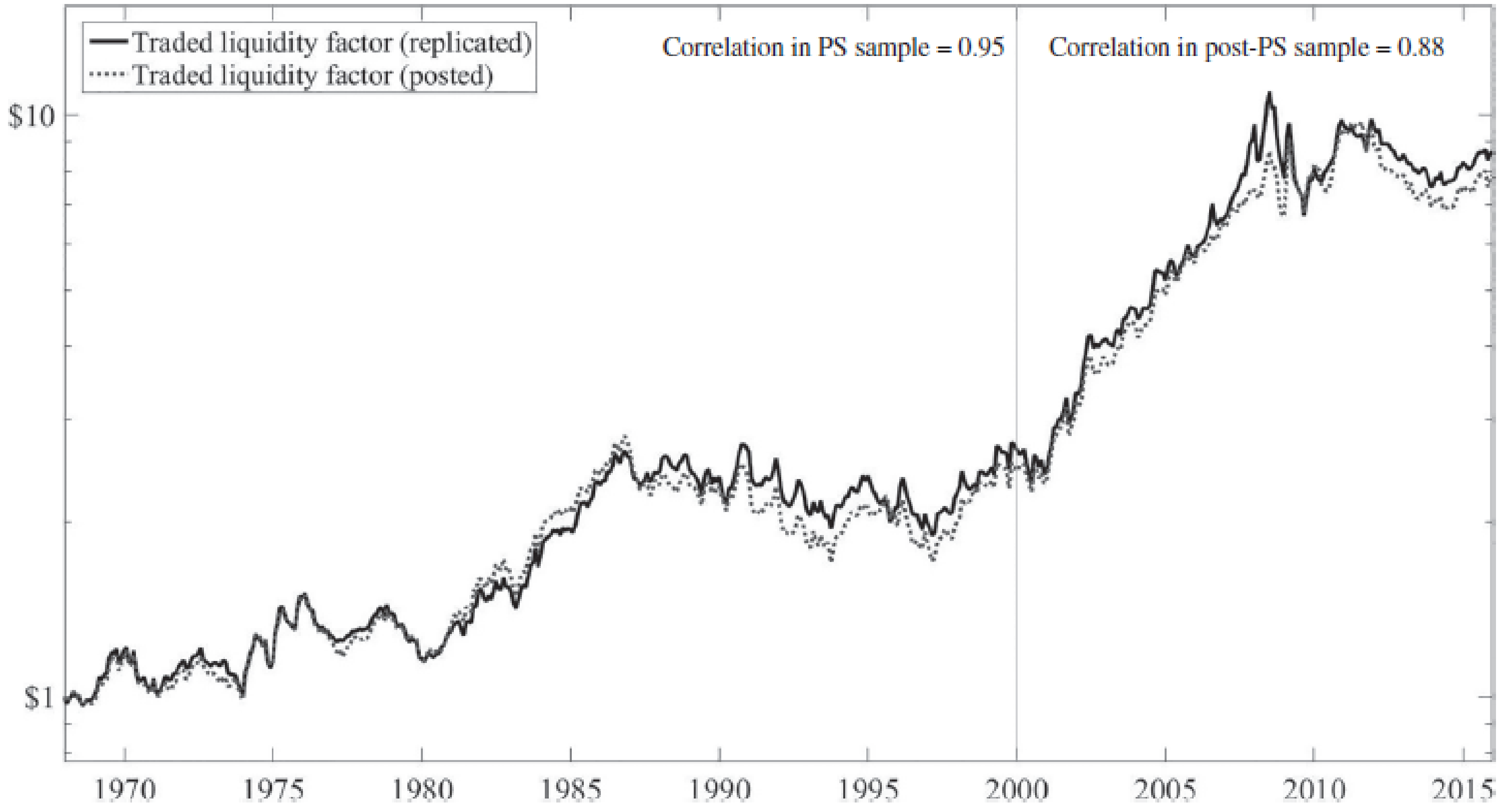

Figure 2: Traded Liquidity Factor Performance.

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Robustness notes from Li, Novy-Marx, and Velikov (2019)

- Crucial for results to construct portfolios with December rebalancing

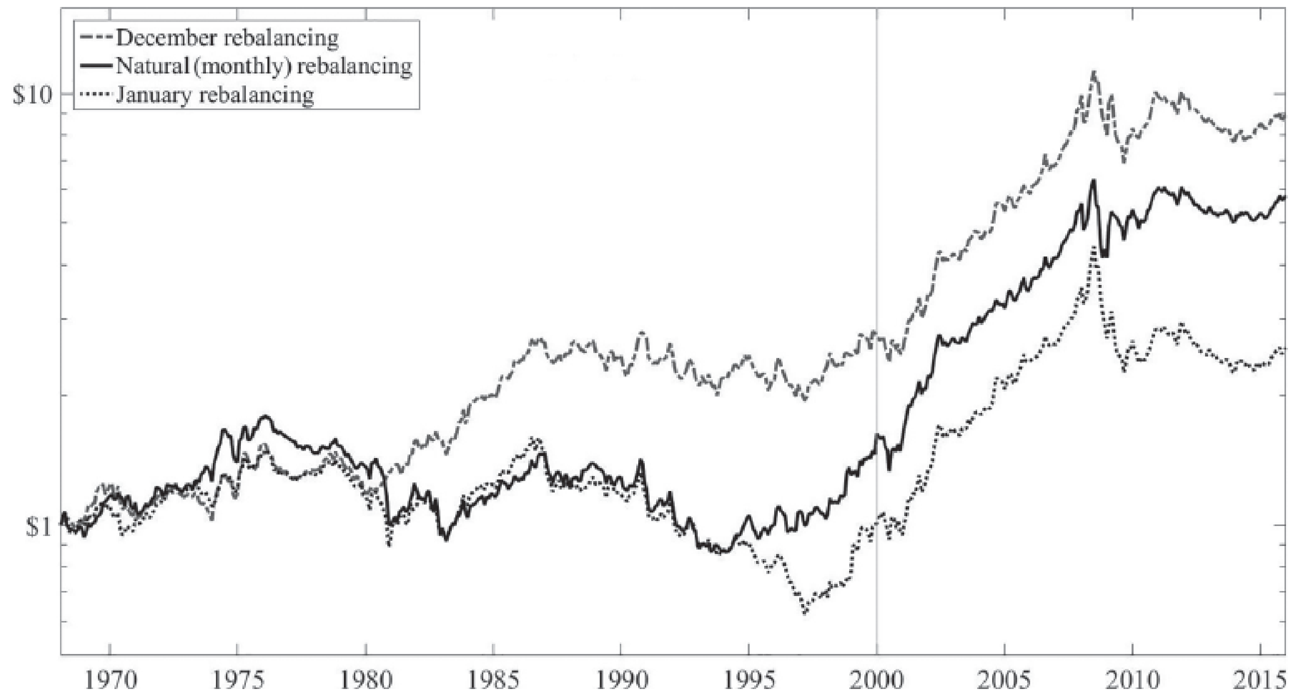

Figure 3: Cumulative Performance of Liquidity Factors with Alternative Rebalancing.

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Robustness notes from Li, Novy-Marx, and Velikov (2019)

- Similar for the decile sort & the name breaks

$n^{\text{portfolio}}$	Name Break	NYSE Break	Market Cap Break
Panel A: t-Statistics on Average Excess Returns			
2	2.08	2.07	1.85
3	2.02	1.88	2.06
5	1.94	1.51	1.54
10	2.99	2.63	1.87
20	0.08	1.33	1.25
Panel B: t-Statistics on Alphas Relative to Fama and French (1993) Three-Factor Model			
2	1.77	1.76	1.54
3	1.73	1.55	1.82
5	1.94	1.48	1.53
10	3.04	2.68	1.81
20	0.08	1.40	1.27
Panel C: t-Statistics on Alphas Relative to Carhart (1997) Four-Factor Model			
2	1.95	1.99	1.82
3	2.14	1.99	2.26
5	2.19	1.74	1.77
10	2.78	2.64	1.78
20	0.34	1.57	1.22
Panel D: t-Statistics on Alphas Relative to Fama and French (2015) Five-Factor Model			
2	1.93	1.88	1.68
3	2.19	2.13	2.09
5	2.33	2.03	1.85
10	2.82	2.55	1.63
20	−0.09	1.29	0.68

Table 4: Performance of Liquidity Risk Factors Constructed Using Alternative Sorts.

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Replication notes from Li, Novy-Marx, and Velikov (2019)

- Impossible to replicate the performance of the predicted betas factor

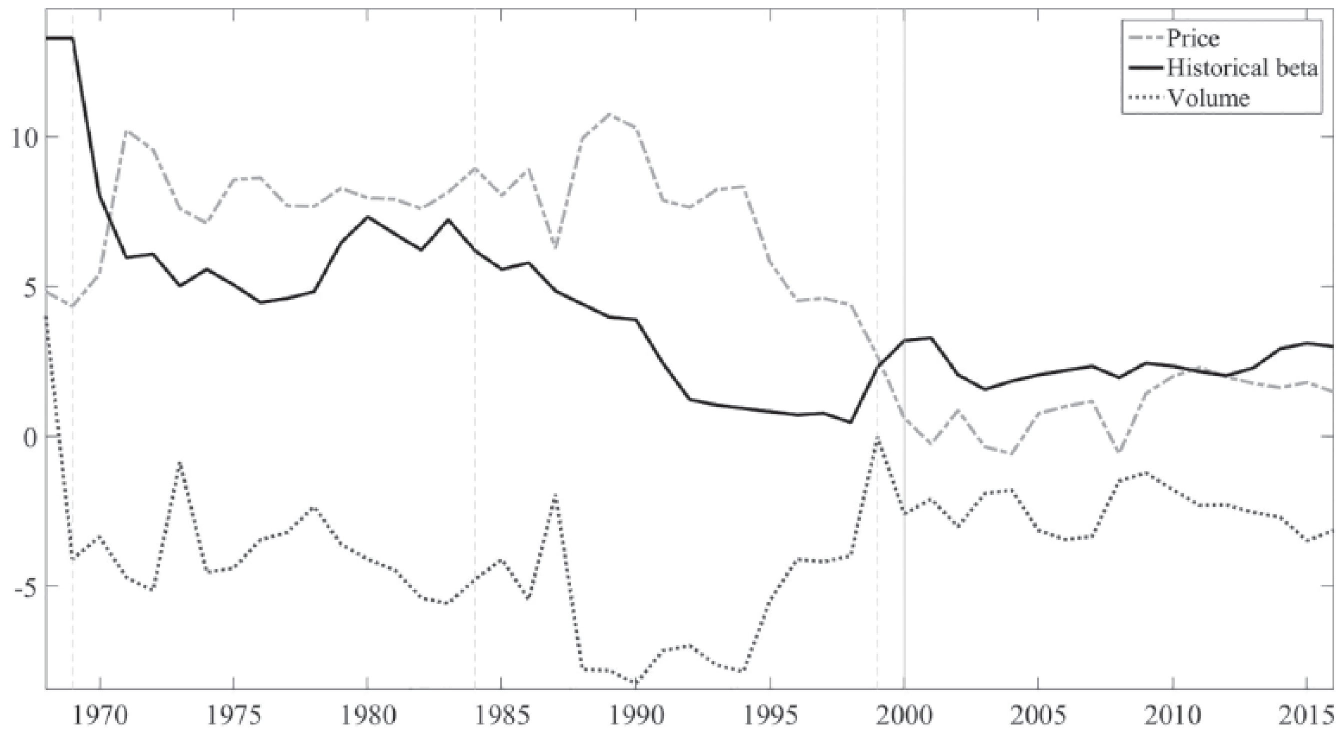

Figure 4: Importance of Most Significant Variables Predicting Liquidity Risk.

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Replication notes from Li, Novy-Marx, and Velikov (2019)

- Although to their credit, the historical factor works really well out-of-sample

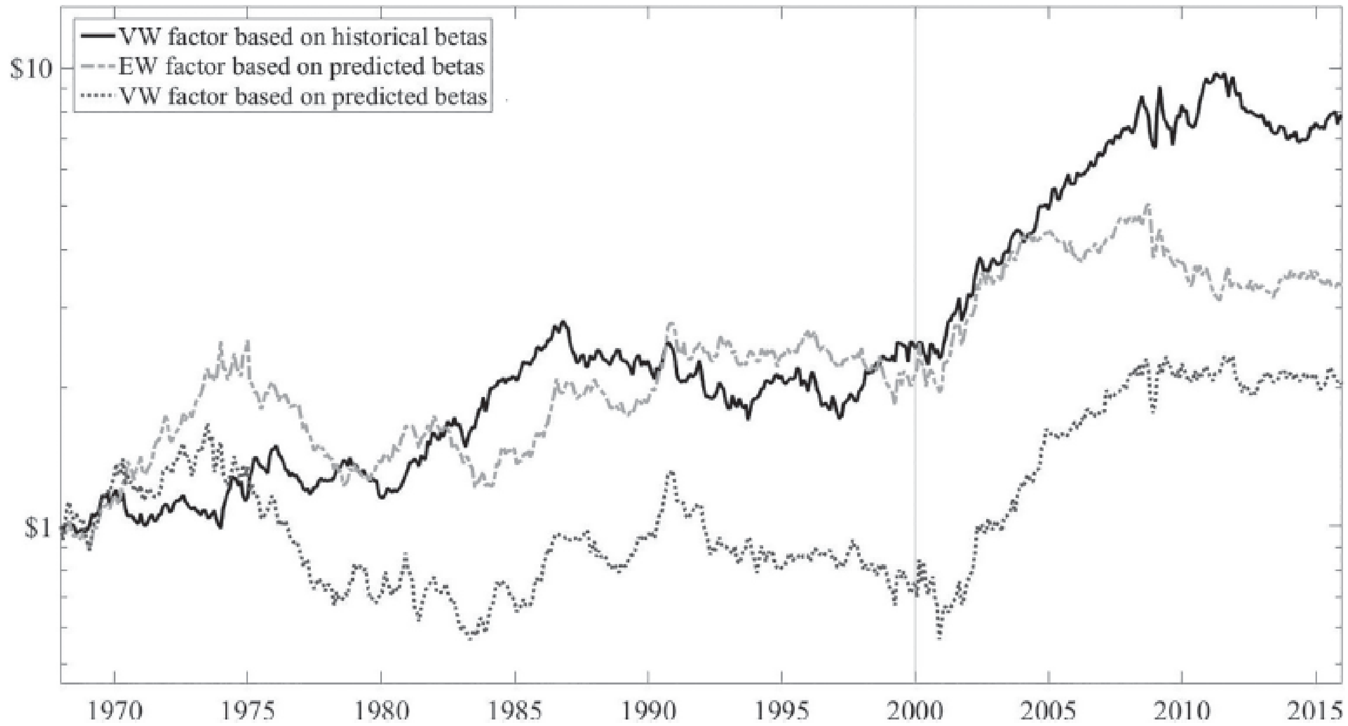

Figure 5: Cumulative Performance of Predicted Liquidity Risk Factors.

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Robustness notes from Li, Novy-Marx, and Velikov (2019)

- Claim that the predicted beta factor prices momentum, however, is completely bogus

	Factor Constructed Using Momentum Variables When Predicting Liquidity				Factor Constructed Excluding Momentum Variables When Predicting Liquidity			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Pastor–Stambaugh Sample, January 1966 to December 1999								
alpha	0.45	0.71	0.44	0.59	0.12	0.22	0.17	0.17
	[2.57]	[4.90]	[3.03]	[4.04]	[0.90]	[1.76]	[1.28]	[1.32]
MKT		−0.28	−0.28	−0.27		−0.14	−0.14	−0.13
		[−7.89]	[−8.16]	[−7.45]		[−4.62]	[−4.59]	[−4.24]
SMB		−0.44	−0.40	−0.38		−0.17	−0.16	−0.15
		[−8.81]	[−8.27]	[−7.50]		[−3.91]	[−3.68]	[−3.23]
HML		−0.11	−0.02	0.02		0.02	0.04	0.05
		[−1.85]	[−0.41]	[0.24]		[0.39]	[0.70]	[0.67]
UMD			0.27				0.05	
			[6.64]				[1.42]	
RMW				0.46				0.17
				[4.82]				[1.97]
CMA				−0.03				0.02
				[−0.25]				[0.22]