Week 4: Factor Wars

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Agenda for today

- Production-based asset pricing
  - Simple q-theory model
  - Main papers in this literature

- Investment and profitability

- Empirical factor models

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State of asset pricing

- Towards a general-equilibrium asset pricing model consistent with cross-sectional facts (Cochrane, 2011):

> *"Standard consumption-based asset pricing links asset price fluctuations to macroeconomics through consumer first-order conditions. One can and should also link asset prices to macroeconomic events through producer first order conditions as well. At a minimum, this step will have to be part of the larger goal, a general-equilibrium economic model that simultaneously generates quantity (business cycle) and asset pricing facts."*

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Production-based asset pricing

- Cochrane (1991, 1996)
- Zhang (2005)
- Liu, Whited, and Zhang (2009)
- Chen, Novy-Marx, and Zhang (2010)
- Zhang & Co.

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Neoclassical theory of investment history of thought

- Keynes: *I* determined by "animal spirits" which fluctuated strongly
- Samuelson (1939): Keynesian "Multiplier' model of investment
- Hall and Jorgensen (1967): neoclassical model with no adjustment costs
  - Firms optimize by choosing level of capital stock considering production function, depreciation, tax, etc.
- Tobin (1969): *q*-theory
  - Firms invest if $q = \frac{Market\ value\ of\ installed\ capital}{Replacement\ value} > 1$
  - I.e., invest until marginal $q = 1$
- Abel (1981) and Hayashi (1982): marginal *q* model with convex costs of adjustment
  - Hayashi (1982) derives conditions under which marginal q = average q

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Standard q-theory model

- Firms maximize the present value of expected profits:

$$D_t = \pi(K_t, Z_t) - I_t - \psi(I_t, K_t)$$

  - where:
    - K_t - capital
    - I_t - investment
    - Z_t - an exogenous shock process
    - $\pi(\cdot,\cdot)$- the maximized operating profit function
    - $\psi(\cdot,\cdot)$ - the adjustment cost function

- Denote:
  - r - discount rate
  - δ - depreciation rate

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Standard q-theory model

- Firm's problem:

$$V(K_0, Z_0) = \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} D_t \right]$$

$$s.t. \quad K_{t+1} = (1-\delta)K_t + I_t$$

$\{Z_t\}$ follows an exogenous stochastic process

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Standard q-theory model

- Formulating it recursively, we get the following Bellman equation:

$$V(K,Z) = \max_{\{I,K'\}} \left\{ \pi(K,Z) - I - \psi(I,K) + \frac{1}{(1+r)} E_{Z'|Z} V(K',Z') \right\}$$

$$s.t. \quad K' = (1-\delta)K + I$$

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Standard q-theory model

- The first order conditions w.r.t. I are given by:

$$\underbrace{1+\psi_1(I,K)}_{MC} = \underbrace{\frac{1}{1+r} E_{Z'|Z} V_1(K',Z')}_{MB=:q}$$

- And after applying the envelope condition, we get:

$$1+\psi_1(I,K) = \underbrace{\frac{1}{1+r} E(\pi_1(K',Z') - \psi_2(I',K') + (1-\delta)q')}_{q}$$

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Standard q-theory model

- Solving the previous expression forward, we get:

$$q_t = E_t\left[\sum_{j=1}^{\infty}\underbrace{\left(\frac{1}{1+r}\right)^j}_{\substack{Discount\\Factor}}\underbrace{\left[(1-\delta)^{j-1}\left(\pi_1\left(K_{t+j},Z_{t+j}\right)-\psi_2\left(I_{t+j},K_{t+j}\right)\right)\right]}_{Future\ Marginal\ Product\ of\ Capital}\right]$$

- Now back to the anomalies. Ceteris paribus,
  - Value: $\uparrow q \Rightarrow \downarrow r$
  - Investment: $\uparrow q \Leftrightarrow \uparrow I \Rightarrow \downarrow r$
  - Profitability: $\uparrow MPK \Rightarrow \uparrow r$
  - Net issuance: $\uparrow q \Leftrightarrow \uparrow I \Leftrightarrow \uparrow Issuance \Rightarrow \downarrow r$

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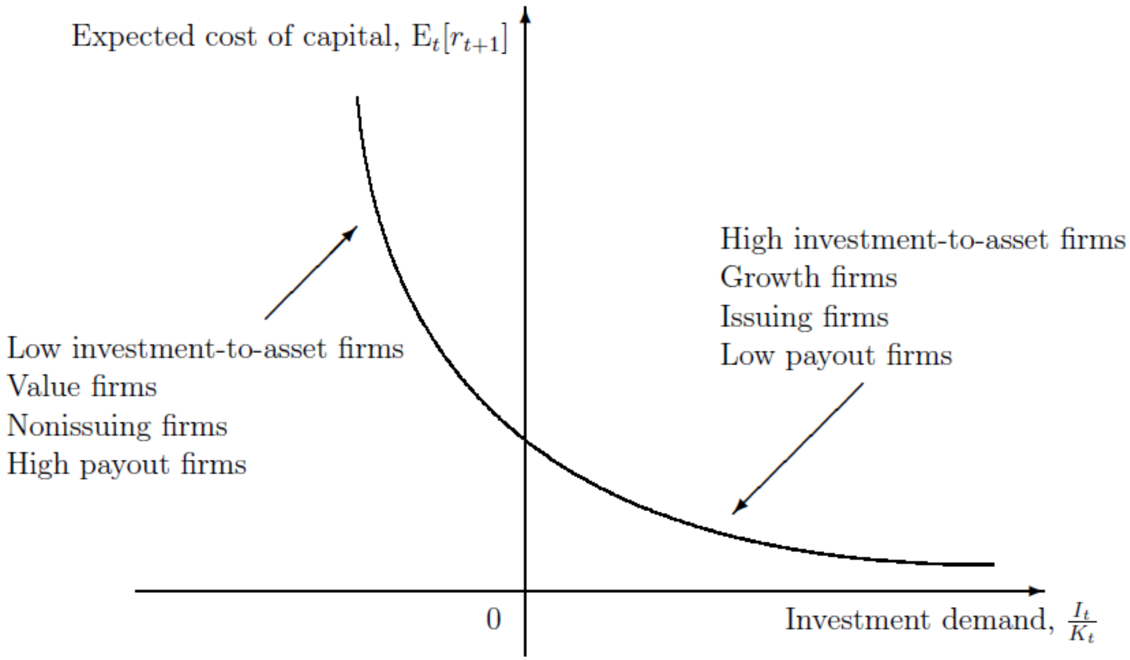
Standard *q*-theory model

- Picture from Zhang (NBER WP, 2005):

Figure 1. The Downward-Sloping Investment-Demand Function



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Standard q-theory model to production based asset pricing

- From standard q-theory:

$$1 + \psi_1(I, K) = \underbrace{\frac{1}{1+r} E(\pi_1(K', Z') - \psi_2(I', K') + (1-\delta)q')}_{q}$$

- Rearrange:

$$1 = E_t\left[\underbrace{\frac{1}{1+r}}_{\substack{Discount\\Factor}} \underbrace{\frac{\pi_1(K_{t+1}, Z_{t+1}) - \psi_2(I_{t+1}, K_{t+1}) + (1-\delta)\left(1 + \psi_1(I_{t+1}, K_{t+1})\right)}{1 + \psi_1(I_t, K_t)}}_{Return\ on\ investment\ (r^I_{t+1})}\right]$$

- Assume an exogenous stochastic discount factor and we get our familiar pricing equation:

$$1 = E_t\left[M_{t,t+1} \times r^I_{t+1}\right]$$

- Cochrane's (1991) insight is that a stochastic discount factor should also price investment returns!

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Testing production based asset pricing

- Model is typically used as a theoretical justification for anomalies
- Although you can test it explicitly
  - Liu, Whited, and Zhang (2009) add debt & get the following investment return:

$$r^I_{i,t+1} = \frac{(1+\tau_{t+1})\left(\pi_1(K_{i,t+1}, Z_{i,t+1}) - \psi_2(I_{i,t+1}, K_{i,t+1})\right) + \tau_{t+1}\delta_{i,t+1} + (1-\delta_{i,t+1})\left(1 + (1-\tau_{t+1})\psi_1(I_{i,t+1}, K_{i,t+1})\right)}{1 + (1-\tau_t)\psi_1(I_{i,t}, K_{i,t})}$$

  - where τ_t is the tax benefit of debt
- The model-implied stock return:

$$r^I_{i,t+1} = \omega_{it} r^D_{i,t+1} + (1-\omega_{it}) r^S_{i,t+1}$$

$$\Rightarrow r^{I\omega}_{i,t+1} \equiv r^S_{i,t+1} = \frac{r^I_{i,t+1} - \omega_{it} r^D_{i,t+1}}{(1-\omega_{it})}$$

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Liu, Whited, and Zhang (2009) Estimation

- Assuming:
  - Cobb-Douglas CRS production function (α – capital share)
  - Quadratic adjustment cost function (a – adjustment cost parameter)
- They run a one-step GMM with identity weighting matrix, optimizing over α and a with the following moment conditions:

$$E\left[r_{i,t+1}^{l\omega} - r_{i,t+1}^{S}\right] = 0$$

and

$$E\left[\left(r_{i,t+1}^{l\omega} - E\left[r_{i,t+1}^{l\omega}\right]\right)^2 - \left(r_{i,t+1}^{S} - E\left[r_{i,t+1}^{S}\right]\right)^2\right] = 0$$

- Using the following test asset portfolios:
  - 10 Standardized Unexpected Earnings (SUE) portfolios from Chan, Jegadeesh, and Lakonishok (1996)
  - 10 Book-to-market portfolio from Fama and French (1993)
  - 10 investment portfolios from Titman, Wei, and Xie (2004)

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Post Fama and French (1993)

- Post Fama and French (1993) world
  - Wide adoption of the three-factor model
  - Score of papers trying to explain SMB and HML

- A few challenger factors, but not much drama
  - Carhart (1997) UMD factor (FF3 + UMD)
  - Pastor-Stambaugh (2003) LIQ factor (FF3 + UMD + LIQ)

- Slowly, a bunch of anomaly papers start getting published
  - Two effects in particular seem robust and related to risk

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Investment and Profitability

- Investment
  - Titman, Wei, and Xie (JFQA, 2004) – CAPX / Moving average CAPX
  - Anderson and Garcia-Feijoo (JF, 2006) – CAPX growth
  - Daniel and Titman (JF, 2006) – composite issuance
  - Cooper, Gullen, and Schill (JF, 2008) – AT growth
  - Lyandres, Sun, and Zhang (RFS, 2008) – tangible investment ($\Delta PPEGT + \Delta INVT$)
  - Pontiff and Woodgate (JF, 2008) – net equity issuance

- Profitability
  - Haugen and Baker (JFE, 1996) – ROE as part of a composite measure
  - Fama and French (JFE, 2006) – expected profitability and expected growth of assets
  - Chen, Novy-Marx, and Zhang (WP, 2010) - ROE
  - Novy-Marx (JFE, 2012) – gross profitability

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Factor Wars

- Zhang (JF, 2005) is a huge success
  - Rational theory for value as a risk factor
  - Value firms are firms that have more assets in place, and costly irreversibility makes it difficult for them to liquidate assets in tough times

- Zhang (NBER WP, 2005) proposes rationalizing a bunch of the more prominent anomalies with a *q*-theoretic model
  - Follows up with multiple papers explaining one anomaly at a time

Factor Wars

- "A Better Three-Factor Model That Explains More Anomalies" by Chen and Zhang (2010) is forthcoming in the JF
  - Empirical factor model motivated by *q*-theory
  - Has MKT, Investment, and ROA factors
  - Seems like Fama and French (1993) is done

- However, Novy-Marx (2010) puts out a draft of "Can a Better Three-Factor Model Explain More Anomalies?"
  - Finds an error in the Chen and Zhang (2010) paper

- Chen and Zhang (2010) is pulled from the JF
  - Chen, Novy-Marx, and Zhang (2010) join forces and put out a paper titled "An Alternative Three-Factor Model"
  - Error is fixed, results are weaker
  - They get rejected from JF, paper is still a working paper

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Factor Wars: Fast forward to 2015

- Fama and French (JFE, 2015): "A five-factor asset pricing model"
    - They add an operating profitability and asset growth factors to the Fama and French (1993) model
- Hue, Xue, and Zhang (2015): "Digesting anomalies: An investment approach"
    - They add a size factor to Chen, Novy-Marx, and Zhang (2010) and propose a four-factor *q*-theory model
        - Also ROA become ROE
- All hell breaks loose from then on

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Factor Wars

- Fama and French
  - 2015 JFE: Incremental variables and the investment opportunity set
  - 2016 RFS: Dissecting anomalies with a five-factor model
  - 2017 JFE: International tests of a five-factor asset pricing model
  - 2018 JFE: Choosing factors
  - 2019 RFS: Comparing cross-section and time-series factor models

- Zhang & Co
  - 2019 RF: Which factors?
  - 2019 RFS: Replicating anomalies
  - 2020 RF: An augmented *q*-factor model with expected growth

- Others:
  - Barillas and Shanken (2017, RFS): Which alpha?
  - Barillas and Shanken (2018, JF): Comparing asset pricing models
  - Kozak, Nagel, and Santosh (2018, JF): Interpreting factor models
  - Bryzgalova (2022, R&R RFS): Spurious factors in linear asset pricing models
  - Detzel, Novy-Marx, and Velikov (2022, WP): Model selection with transaction costs

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Model Selection with Transaction Costs

- Hundreds of cross-sectional predictor of equity returns
  - Which asset-pricing (factor) model is the "right one"?
    - E.g., Fama and French, 1993-2018; Hou, Xue, and Zhang, 2015-present; Barillas and Shanken (2018)

- Factor models typically judged on how small α's are in:

$$E(r_{it}) = \alpha_i + \boldsymbol{\beta}_{\boldsymbol{i}}' E(\boldsymbol{f}_{\boldsymbol{t}})$$

- Arbitrageurs buy $\alpha > 0$ / sell $\alpha < 0$ until $\alpha = 0$ (e.g., Ross, JET 1976)
  - … but only if it's profitable!

- Correct (risk) model should price cross-section of **risk premia**
  - **… not** cross-section of unprofitable "mispricing"

Maximum squared Sharpe ratio $\Leftrightarrow$ smallest α

- Gibbons, Ross, and Shanken (1989): $E(r_{it}) = \alpha_i + \boldsymbol{\beta}_i' E(\boldsymbol{f}_t)$

  $\Rightarrow$ asset i improves mean-variance frontier relative to just $\boldsymbol{f}_t$:

$$SR^2(r_i, f) = SR^2(f) + \alpha_i^2/\sigma(\varepsilon_i)$$

- Barillas and Shanken (2017): Correct model has:

$$\text{Largest } SR^2(f) \Leftrightarrow \text{Smallest } \alpha$$

- But: low-volatility industry relative reversal (LVIRR) strategy has $SR^2 = 4.8$!
  - Fama-French 5-factor and Hou-Xue-Zhang 4-factor models have 1.2 and 2.0
  - Should we just have a one-factor model w/ LVIRR?
  - Of course not!
    - After transaction costs, LVIRR has essentially zero returns!

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DNMV (2022): What do we do?

- Correct factors for transaction costs following Novy-Marx and Velikov (2016)
  - Apply *effective spread* measure of Hasbrouck (2009)

- Horse-race models **after costs** via:
  - Maximum squared Sharpe ratios, $SR^2(f)$
  - Smallest α in 120 anomalies

- Models:
  - **FF5 / FF6:** Fama and French *JFE* (2015, 2018) 5- and 6-factor
  - **FF5C / FF6C:** Cash- instead of operating-profitability
  - **HXZ4:** Hou, Xue, and Zhang *RFS* (2015) 4-factor
  - **BS6:** Barillas and Shanken *JF* (2018) 6-factor
  - **Key difference:** Latter 2's factors update *monthly*

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Figure 1: The paper in a nutshell

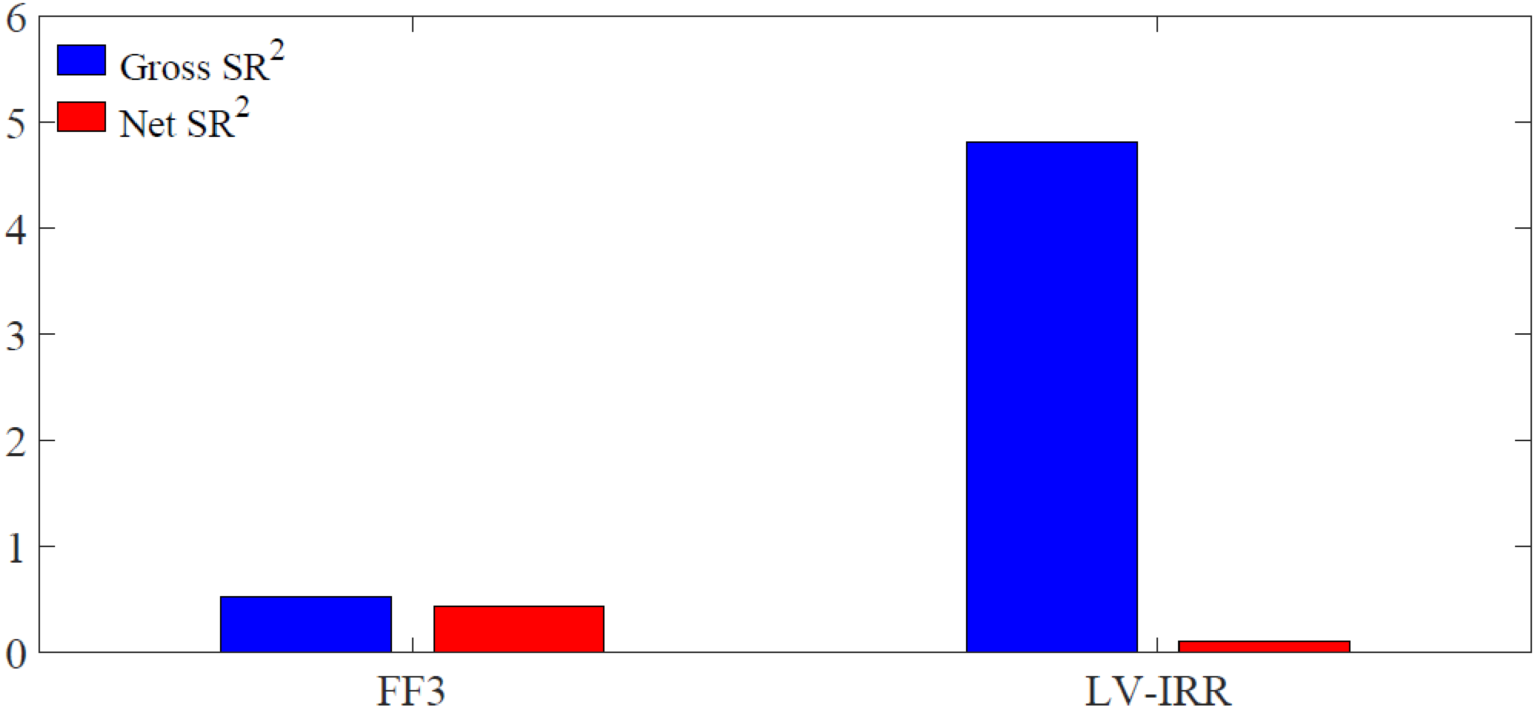
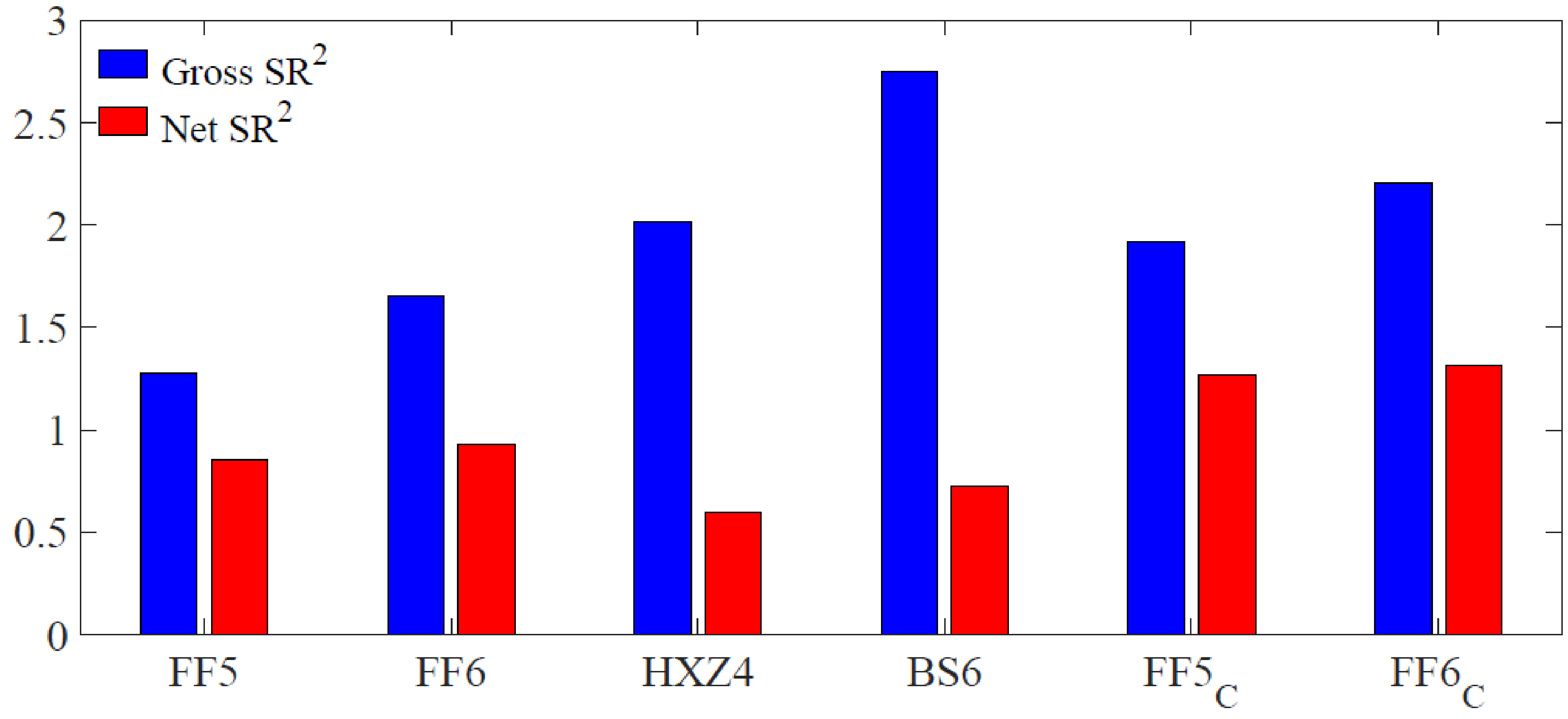


Figure 1: Max square Sharpe ratio before and after costs



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Monthly-rebalanced factors have much higher tcosts

	Average returns					
	Gross	t-statistic	Net	t-statistic	TO (%)	TC (%)
Panel A: Fama-French (mostly annual):						
MKT	0.56	2.95	0.56	2.95		
SMB	0.20	1.52	0.17	1.31	2.85	0.03
HML	0.36	2.89	0.31	2.45	5.10	0.05
RMW	0.28	2.79	0.22	2.23	5.27	0.05
CMA	0.31	3.75	0.22	2.61	9.76	0.09
RMW_C	0.36	4.52	0.29	3.58	7.36	0.07
MOM	0.66	3.56	0.19	1.03	51.40	0.47
Panel B: Hou, Xue, Zhang / AQR (monthly):						
ME	0.27	2.07	0.12	0.89	18.62	0.15
ROE	0.55	5.00	0.22	2.01	36.36	0.33
IA	0.38	4.82	0.16	2.03	24.59	0.22
HML(m)	0.34	2.21	0.14	0.90	19.37	0.20

Spanning tests

Base model ($M0$)	Supplementary model ($M1$)											
	Gross						Net					
	FF5	FF6	HXZ4	BS6	$FF5_C$	$FF6_C$	FF5	FF6	HXZ4	BS6	$FF5_C$	$FF6_C$
Panel A: $MVE_{M1 \cup M0,t} = \alpha + \beta MVE_{M0,t} + \varepsilon_t$												
FF5		1.10 [3.46]	1.62 [5.93]	2.14 [9.58]	1.31 [5.93]	1.67 [6.75]		0.28 [1.59]	0.36 [2.03]	0.40 [2.07]	1.04 [4.55]	1.12 [4.84]
FF6	0.00 [0.00]		0.99 [4.53]	1.75 [7.64]	1.11 [5.08]	1.11 [5.08]	0.00 [0.00]		0.14 [1.28]	0.14 [1.28]	0.94 [4.23]	0.94 [4.23]
HXZ4	0.12 [1.50]	0.18 [1.73]		1.18 [5.11]	0.75 [3.63]	0.88 [4.15]	0.93 [4.23]	0.97 [4.28]		0.48 [2.50]	1.78 [5.51]	1.84 [5.68]
BS6	0.60 [3.90]	0.60 [3.90]	0.04 [0.80]		0.81 [4.43]	0.81 [4.43]	0.61 [2.90]	0.61 [2.90]	0.00 [0.00]		1.55 [4.83]	1.55 [4.83]
$FF5_C$	0.25 [2.55]	0.64 [3.45]	0.90 [4.25]	1.58 [8.41]		0.59 [2.74]	0.00 [0.00]	0.12 [1.16]	0.04 [0.74]	0.12 [1.16]		0.12 [1.16]
$FF6_C$	0.22 [2.32]	0.22 [2.32]	0.48 [2.95]	1.31 [6.51]	0.00 [0.00]		0.00 [0.00]	0.00 [0.00]	0.00 [0.06]	0.00 [0.06]	0.00 [0.00]	

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100K out-of-sample bootstraps

Panel B: Out-of-sample bootstrap results								
		Probability (%) that the row model performs at least as well as the column model						
	Mean-SR^2	FF5	FF6	HXZ4	BS6	FF5_C	FF6_C	Best
FF5	0.65		47.9	78.6	73.5	2.8	7.7	1.2
FF6	0.66	52.1		80.5	80.1	7.6	3.1	1.6
HXZ4	0.45	21.4	19.5		35.8	4.6	4.8	2.5
BS6	0.49	26.5	19.9	64.2		5.4	3.6	2.2
FF5_C	1.03	97.2	92.4	95.4	94.6		58.7	54.9
FF6_C	1.01	92.3	96.8	95.2	96.4	41.3		37.6

- FF5 and FF6 tend to beat HXZ4 and BS6
- FF5C and FF6C tend to beat all

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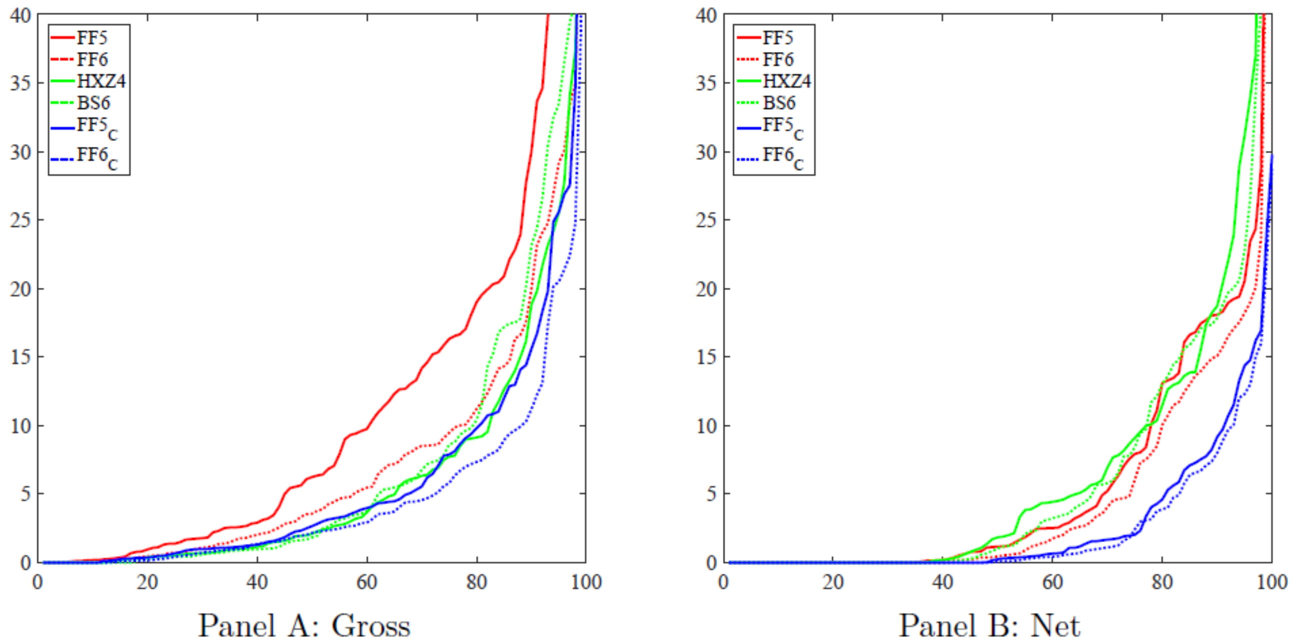
Pricing 120 anomalies of Chen and Zimmerman (2018)

- For each model, M, and anomaly, A,

$$\%\Delta \mathrm{SR}^2(\mathrm{M},\mathrm{A}) \triangleq \frac{\mathrm{SR}^2(\mathrm{M},\mathrm{A})}{\mathrm{SR}^2(\mathrm{M})} - 1$$

- Larger $\%\Delta \mathrm{SR}^2(\mathrm{M},\mathrm{A}) \Rightarrow M$ worse at pricing A
- Fig 2: Percentiles of $\%\Delta \mathrm{SR}^2(\mathrm{M},\mathrm{A})$ for each model



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