Topic 3: Factor Wars

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SAFE PhD Course on Anomalies – June, 2024

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Agenda for today

- Production-based asset pricing
  - Simple *q*-theory model
  - Main papers in this literature
- Investment and profitability
- Empirical factor models

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State of asset pricing

- Towards a general-equilibrium asset pricing model consistent with cross-sectional facts (Cochrane, 2011):

> *"Standard consumption-based asset pricing links asset price fluctuations to macroeconomics through consumer first-order conditions. One can and should also link asset prices to macroeconomic events through producer first order conditions as well. At a minimum, this step will have to be part of the larger goal, a general-equilibrium economic model that simultaneously generates quantity (business cycle) and asset pricing facts."*

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Production-based asset pricing

- Cochrane (1991, 1996)
- Zhang (2005)
- Liu, Whited, and Zhang (2009)
- Chen, Novy-Marx, and Zhang (2010)
- Zhang & Co.

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Standard q-theory model

- Firms maximize the present value of expected profits:
$$D_t = \pi(K_t, Z_t) - I_t - \psi(I_t, K_t)$$
  - where:
    - K_t - capital
    - I_t - investment
    - Z_t - an exogenous shock process
    - $\pi(\cdot,\cdot)$- the maximized operating profit function
    - $\psi(\cdot,\cdot)$ - the adjustment cost function

- Denote:
  - r - discount rate
  - δ - depreciation rate

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Standard q-theory model

- Firm's problem:

$$V(K_0, Z_0) = \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} D_t \right]$$

$$s.t. \quad K_{t+1} = (1-\delta)K_t + I_t$$

$\{Z_t\}$ follows an exogenous stochastic process

Standard q-theory model

- Formulating it recursively, we get the following Bellman equation:

$$V(K,Z) = \max_{\{I,K'\}} \left\{ \pi(K,Z) - I - \psi(I,K) + \frac{1}{(1+r)} E_{Z'|Z} V(K',Z') \right\}$$

$$s.t. \quad K' = (1-\delta)K + I$$

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Standard q-theory model

- The first order conditions w.r.t. I are given by:

$$\underbrace{1+\psi_1(I,K)}_{MC} = \underbrace{\frac{1}{1+r} E_{Z'|Z} V_1(K',Z')}_{MB=:q}$$

- And after applying the envelope condition, we get:

$$1+\psi_1(I,K) = \underbrace{\frac{1}{1+r} E(\pi_1(K',Z') - \psi_2(I',K') + (1-\delta)q')}_{q}$$

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Standard q-theory model

- Solving the previous expression forward, we get:

$$q_t = E_t\left[\sum_{j=1}^{\infty}\underbrace{\left(\frac{1}{1+r}\right)^j}_{\substack{\mathit{Discount}\\ \mathit{Factor}}}\underbrace{\left[(1-\delta)^{j-1}\left(\pi_1\left(K_{t+j},Z_{t+j}\right)-\psi_2\left(I_{t+j},K_{t+j}\right)\right)\right]}_{\mathit{Future\ Marginal\ Product\ of\ Capital}}\right]$$

- Now back to the anomalies. Ceteris paribus,
  - Value: $\uparrow q \Rightarrow \downarrow r$
  - Investment: $\uparrow q \Leftrightarrow \uparrow I \Rightarrow \downarrow r$
  - Profitability: $\uparrow MPK \Rightarrow \uparrow r$
  - Net issuance: $\uparrow q \Leftrightarrow \uparrow I \Leftrightarrow \uparrow Issuance \Rightarrow \downarrow r$

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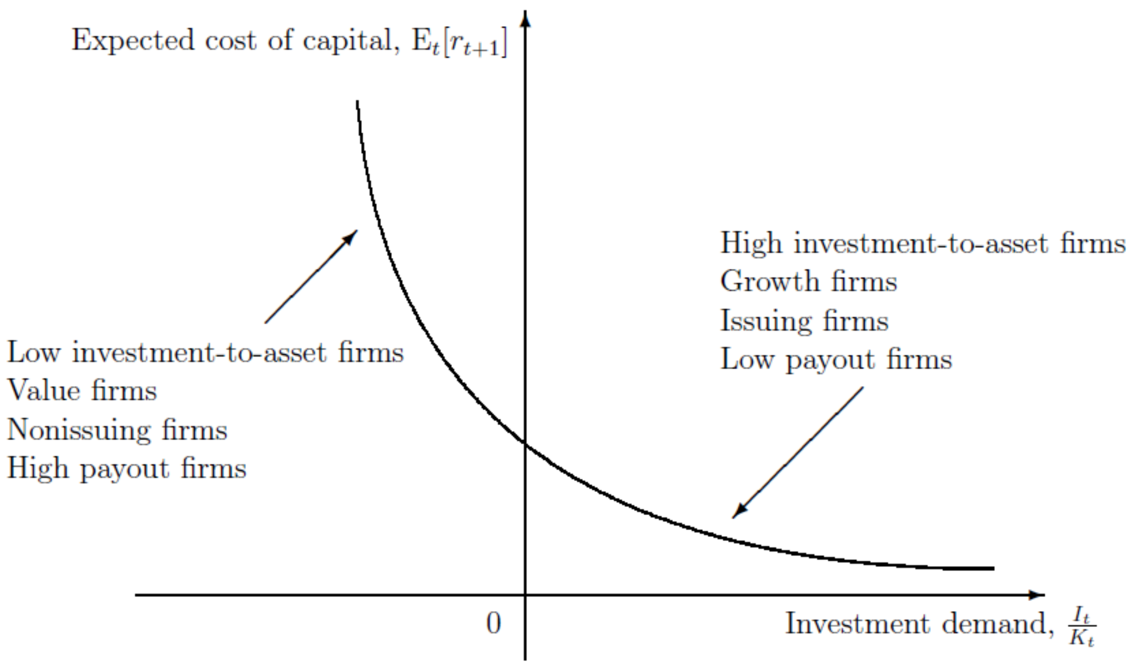
Standard *q*-theory model

- Picture from Zhang (NBER WP, 2005):

Figure 1. The Downward-Sloping Investment-Demand Function



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Standard q-theory model to production based asset pricing

- From standard q-theory:

$$1+\psi_1(I,K)=\underbrace{\frac{1}{1+r}E(\pi_1(K',Z')-\psi_2(I',K')+(1-\delta)q')}_{q}$$

- Rearrange:

$$1=E_t\left[\underbrace{\frac{1}{1+r}}_{\substack{Discount\\Factor}}\underbrace{\frac{\pi_1(K_{t+1},Z_{t+1})-\psi_2(I_{t+1},K_{t+1})+(1-\delta)\big(1+\psi_1(I_{t+1},K_{t+1})\big)}{1+\psi_1(I_t,K_t)}}_{Return\ on\ investment\ (r^I_{t+1})}\right]$$

- Assume an exogenous stochastic discount factor and we get our familiar pricing equation:

$$1=E_t\left[M_{t,t+1}\times r^I_{t+1}\right]$$

- Cochrane's (1991) insight is that a stochastic discount factor should also price investment returns!

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Testing production based asset pricing

- Model is typically used as a theoretical justification for anomalies

- Although you can test it explicitly
  - Liu, Whited, and Zhang (2009) add debt & get the following investment return:

$$r_{i,t+1}^{I} = \frac{(1+\tau_{t+1})\left(\pi_1(K_{i,t+1}, Z_{i,t+1}) - \psi_2(I_{i,t+1}, K_{i,t+1})\right) + \tau_{t+1}\delta_{i,t+1} + (1-\delta_{i,t+1})\left(1 + (1-\tau_{t+1})\psi_1(I_{i,t+1}, K_{i,t+1})\right)}{1 + (1-\tau_t)\psi_1(I_{i,t}, K_{i,t})}$$

  - where τ_t is the tax benefit of debt

- The model-implied stock return:

$$r_{i,t+1}^{I} = \omega_{it} r_{i,t+1}^{D} + (1-\omega_{it}) r_{i,t+1}^{S}$$

$$\Rightarrow r_{i,t+1}^{I\omega} \equiv r_{i,t+1}^{S} = \frac{r_{i,t+1}^{I} - \omega_{it} r_{i,t+1}^{D}}{(1-\omega_{it})}$$

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Liu, Whited, and Zhang (2009) Estimation

- Assuming:
  - Cobb-Douglas CRS production function (α – capital share)
  - Quadratic adjustment cost function (a – adjustment cost parameter)
- They run a one-step GMM with identity weighting matrix, optimizing over α and a with the following moment conditions:

$$E\left[r_{i,t+1}^{l\omega} - r_{i,t+1}^{S}\right] = 0$$

and

$$E\left[\left(r_{i,t+1}^{l\omega} - E\left[r_{i,t+1}^{l\omega}\right]\right)^2 - \left(r_{i,t+1}^{S} - E\left[r_{i,t+1}^{S}\right]\right)^2\right] = 0$$

- Using the following test asset portfolios:
  - 10 Standardized Unexpected Earnings (SUE) portfolios from Chan, Jegadeesh, and Lakonishok (1996)
  - 10 Book-to-market portfolio from Fama and French (1993)
  - 10 investment portfolios from Titman, Wei, and Xie (2004)

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Liu, Whited, and Zhang (2009) Estimation

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  - Cobb-Douglas CRS production function (α – capital share)
  - Quadratic adjustment cost function (a – adjustment cost parameter)
- They run a one-step GMM with identity weighting matrix, optimizing over α and a with the following moment conditions:

$$E\left[r^{l\omega}_{i,t+1} - r^{S}_{i,t+1}\right] = 0$$

and

$$E\left[\left(r^{l\omega}_{i,t+1} - E\left[r^{l\omega}_{i,t+1}\right]\right)^2 - \left(r^{S}_{i,t+1} - E\left[r^{S}_{i,t+1}\right]\right)^2\right] = 0$$

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Post Fama and French (1993)

- Post Fama and French (1993) world
  - Wide adoption of the three-factor model
  - Score of papers trying to explain SMB and HML

- A few challenger factors, but not much drama for a bit
  - Carhart (1997) UMD factor (FF3 + UMD)
  - Pastor-Stambaugh (2003) LIQ factor (FF3 + UMD + LIQ)

- Slowly, a bunch of anomaly papers start getting published
  - Two effects in particular seem robust and related to risk

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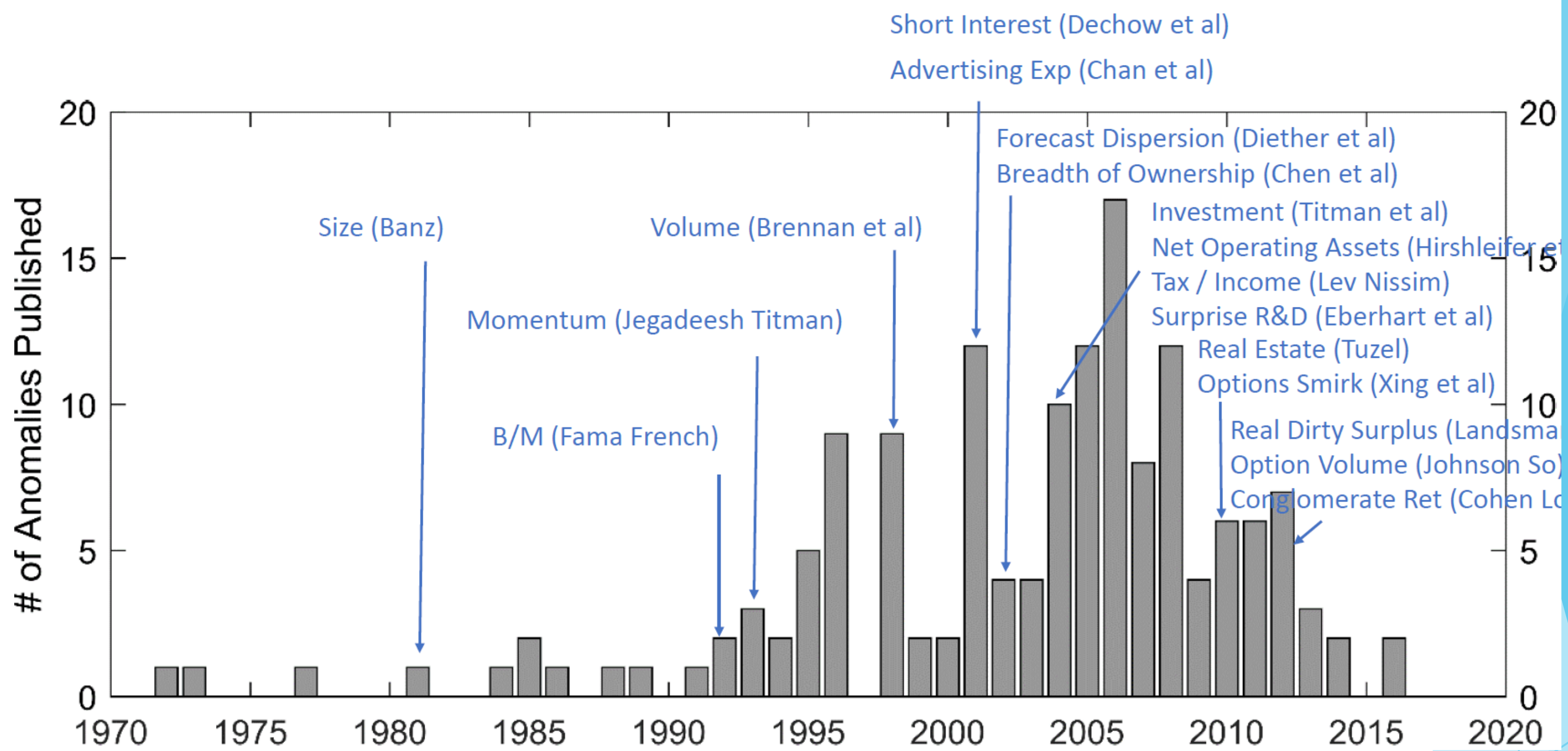
Post Fama and French (1993)



- Anomaly timeline from Chen and Velikov (JFQA, 2023)

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Investment and Profitability

- Investment
  - Titman, Wei, and Xie (JFQA, 2004) – CAPX / Moving average CAPX
  - Anderson and Garcia-Feijoo (JF, 2006) – CAPX growth
  - Daniel and Titman (JF, 2006) – composite issuance
  - Cooper, Gullen, and Schill (JF, 2008) – AT growth
  - Lyandres, Sun, and Zhang (RFS, 2008) – tangible investment ($\Delta PPEGT + \Delta INVT$)
  - Pontiff and Woodgate (JF, 2008) – net equity issuance

- Profitability
  - Haugen and Baker (JFE, 1996) – ROE as part of a composite measure
  - Fama and French (JFE, 2006) – expected profitability and expected growth of assets
  - Chen, Novy-Marx, and Zhang (WP, 2010) - ROE
  - Novy-Marx (JFE, 2012) – gross profitability

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Factor Wars

- Zhang (JF, 2005) is a huge success
  - Rational theory for value as a risk factor
  - Value firms are firms that have more assets in place, and costly irreversibility makes it difficult for them to liquidate assets in tough times

- Zhang (NBER WP, 2005) proposes rationalizing a bunch of the more prominent anomalies with a *q*-theoretic model
  - Follows up with multiple papers explaining one anomaly at a time

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Factor Wars

- "A Better Three-Factor Model That Explains More Anomalies" by Chen and Zhang (2010) is forthcoming in the JF
  - Empirical factor model motivated by *q*-theory
  - Has MKT, Investment, and ROA factors
  - Seems like Fama and French (1993) is done

- However, Novy-Marx (2010) puts out a draft of "Can a Better Three-Factor Model Explain More Anomalies?"
  - Finds an error in the Chen and Zhang (2010) paper

- Chen and Zhang (2010) is pulled from the JF
  - Chen, Novy-Marx, and Zhang (2010) join forces and put out a paper titled "An Alternative Three-Factor Model"
  - Error is fixed, results are weaker
  - They get rejected from JF, paper is still a working paper

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Factor Wars: Fast forward to 2015

- Fama and French (JFE, 2015): "A five-factor asset pricing model"
  - They add an operating profitability and asset growth factors to the Fama and French (1993) model

- Hue, Xue, and Zhang (2015): "Digesting anomalies: An investment approach"
  - They add a size factor to Chen, Novy-Marx, and Zhang (2010) and propose a four-factor *q*-theory model
    - Also ROA become ROE

- All hell breaks loose from then on

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Factor Wars

- Fama and French
  - 2015 JFE: Incremental variables and the investment opportunity set
  - 2016 RFS: Dissecting anomalies with a five-factor model
  - 2017 JFE: International tests of a five-factor asset pricing model
  - 2018 JFE: Choosing factors
  - 2019 RFS: Comparing cross-section and time-series factor models

- Zhang & Co
  - 2019 RF: Which factors?
  - 2019 RFS: Replicating anomalies
  - 2021 RF: An augmented q-factor model with expected growth

- Others:
  - Barillas and Shanken (2017, RFS): Which alpha?
  - Barillas and Shanken (2018, JF): Comparing asset pricing models
  - Kozak, Nagel, and Santosh (2018, JF): Interpreting factor models
  - Bryzgalova (2022, R&R RFS): Spurious factors in linear asset pricing models
  - Detzel, Novy-Marx, and Velikov (2023, JF): Model selection with transaction costs

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Model Selection with Transaction Costs

- Hundreds of cross-sectional predictor of equity returns
  - Which asset-pricing (factor) model is the "right one"?
    - E.g., Fama and French, 1993-2018; Hou, Xue, and Zhang, 2015-present; Barillas and Shanken (2018)

- Factor models typically judged on how small α's are in:

$$E(r_{it}) = \alpha_i + \boldsymbol{\beta}_i' E(\boldsymbol{f}_t)$$

- Arbitrageurs buy $\alpha > 0$ / sell $\alpha < 0$ until $\alpha = 0$ (e.g., Ross, JET 1976)
  - … but only if it's profitable!

- Correct (risk) model should price cross-section of **risk premia**
  - **… not** cross-section of unprofitable "mispricing"

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Maximum squared Sharpe ratio ⇔ smallest α

- Gibbons, Ross, and Shanken (1989): $E(r_{it}) = \alpha_i + \boldsymbol{\beta}_i' E(\boldsymbol{f}_t)$

  ⇒ asset i improves mean-variance frontier relative to just $\boldsymbol{f}_t$:

$$SR^2(r_i, f) = SR^2(f) + \alpha_i^2/\sigma(\varepsilon_i)$$

- Barillas and Shanken (2017): Correct model has:

  Largest $SR^2(f)$ ⇔ Smallest α

- But: low-volatility industry relative reversal (LVIRR) strategy has $SR^2 = 4.4$!
  - Fama-French 5-factor and Hou-Xue-Zhang 4-factor models have 1.3 and 1.8
  - Should we just have a one-factor model w/ LVIRR?
  - Of course not!
    - After transaction costs, LVIRR has essentially zero returns!

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DNMV (2022): What do we do?

- Correct factors for transaction costs following Novy-Marx and Velikov (2016)
  - Apply *effective spread* measure of Hasbrouck (2009)

- Horse-race models **after costs** via:
  - Maximum squared Sharpe ratios, $SR^2(f)$
  - Smallest α in 120 anomalies

- Models:
  - **FF5 / FF6:** Fama and French *JFE* (2015, 2018) 5- and 6-factor
  - **FF5C / FF6C:** Cash- instead of operating-profitability
  - **HXZ4:** Hou, Xue, and Zhang *RFS* (2015) 4-factor
  - **BS6:** Barillas and Shanken *JF* (2018) 6-factor
  - **Key difference:** Latter 2's factors update *monthly*

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Figure 1: The paper in a nutshell

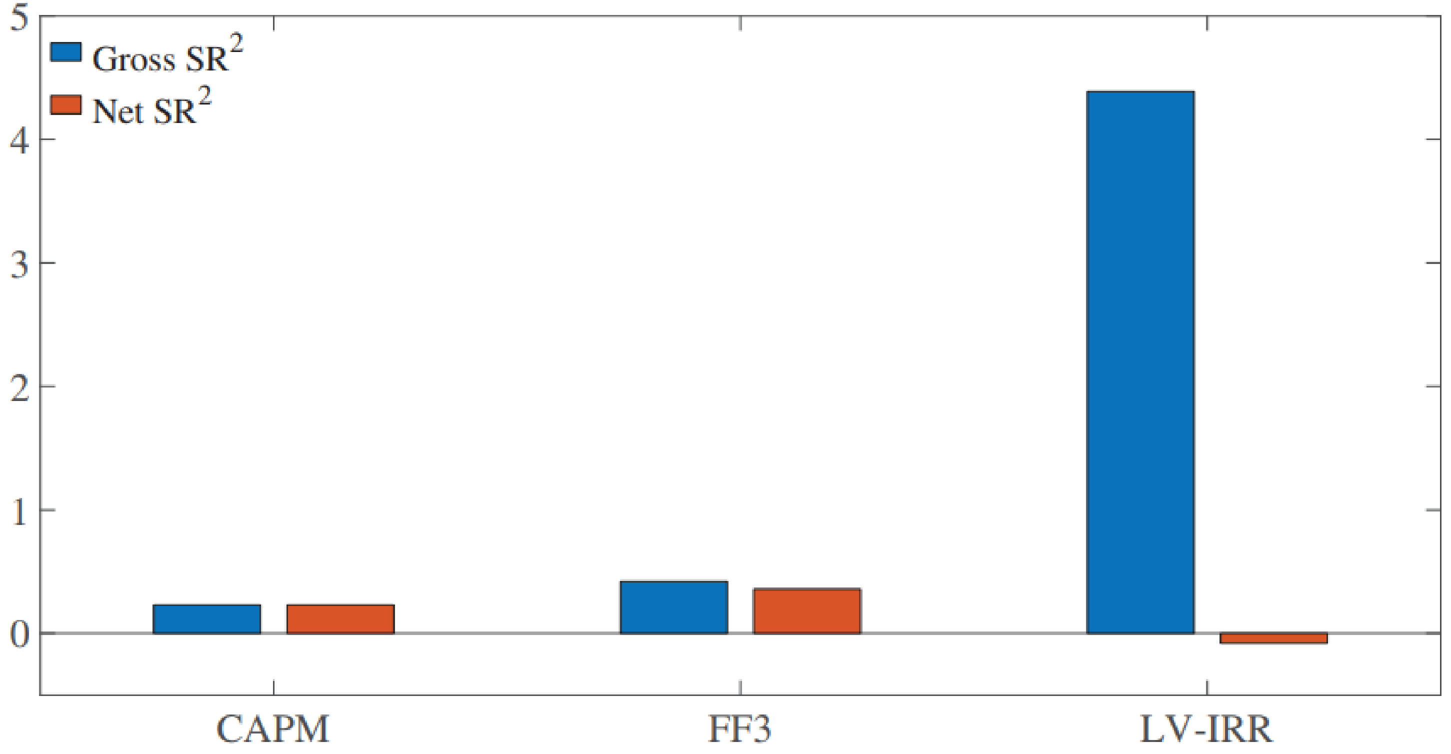

Figure 1. Maximum squared Sharpe ratios of the CAPM, the Fama-French three-factor (FF3) model, and the low-volatility industry-relative-reversal factor (LV-IRR). The blue (left) bars, "Gross SR^2," use factor returns that ignore transaction costs. The red (right) bars, "Net SR^2," use factor returns that account for transaction costs. A negative squared Sharpe ratio indicates that the corresponding Sharpe ratio is negative before squaring. The sample period is January 1972 to December 2021. (Color figure can be viewed at wileyonlinelibrary.com)

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Figure 2: Max square Sharpe ratio before and after costs

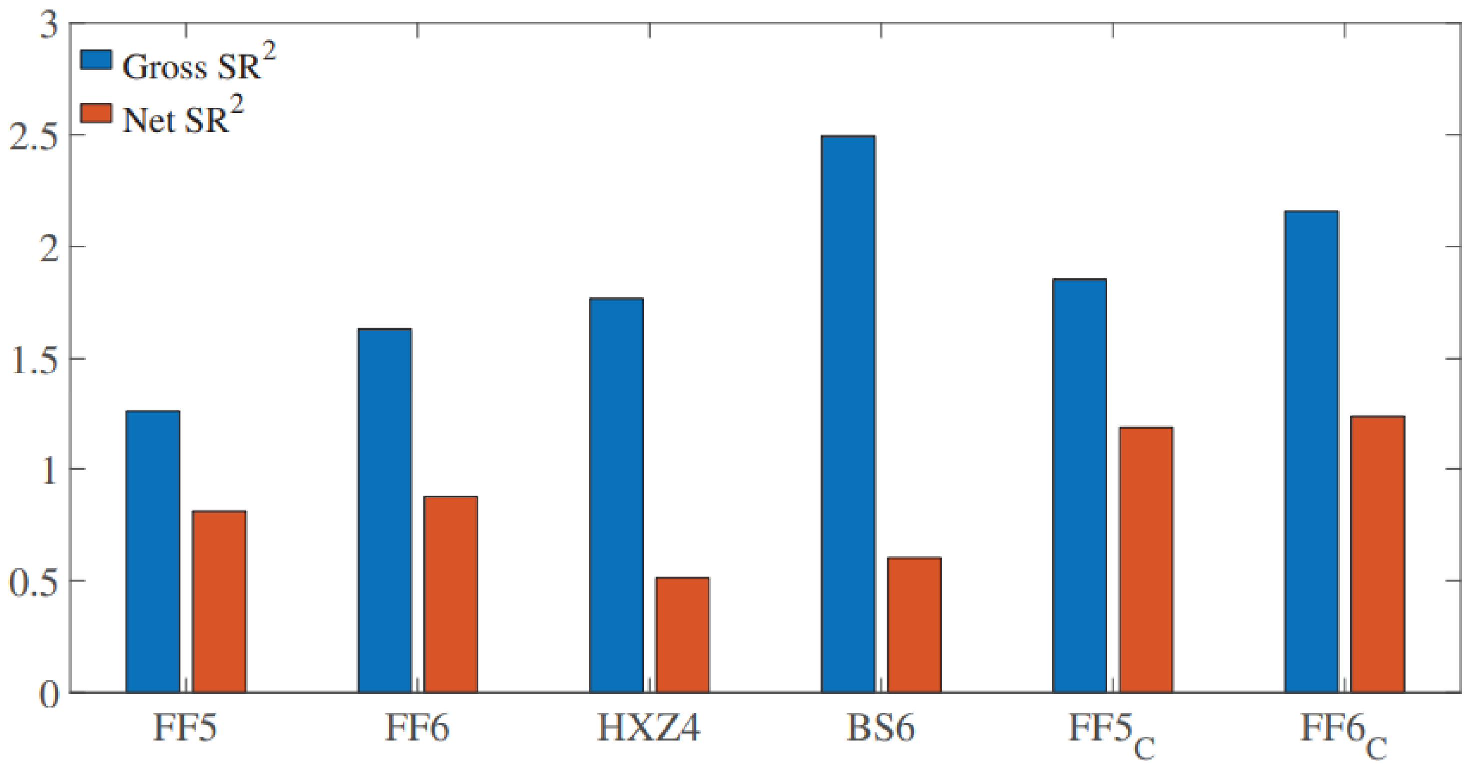

Figure 2. Maximum squared Sharpe ratios of factor models. This figure presents maximum squared Sharpe ratios from the factors in the models listed on the x-axis. The blue (left) columns, "Gross SR^2," use factor returns that ignore transaction costs. The red (right) columns, "Net SR^2," use factor returns that account for transaction costs. The sample period is January 1972 to December 2021. (Color figure can be viewed at wileyonlinelibrary.com)

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Monthly-rebalanced factors have much higher tcosts

Table II

Factor Summary Statistics

For each candidate asset pricing factor, this table presents average monthly returns and t-statistics, both gross and net of transaction costs, along with average monthly turnover, TO, and transaction costs, TC. MKT, SMB, HML, RMW, and CMA denote the Fama and French (2015) market, size, value, profitability, and investment factors, respectively. MOM denotes the Fama and French (2018) momentum factor. RMW_C denotes the Fama and French (2018) cash profitability factor, which is constructed similarly to RMW but using cash-based, not accruals-based, operating profitability. ME, ROE, and IA denote the Hou, Xue, and Zhang (2015) size, profitability, and investment factors, respectively. HML(m) denotes the monthly updated value factor of Asness and Frazzini (2013). The units for average returns, TO, and TC are % per month. The sample period is January 1972 to December 2021.

	Average Monthly Excess Return (%)					
	Gross	t-Statistic	Net	t-Statistic	TO	TC
MKT	0.63	3.38	0.63	3.38	0.0	0.00
SMB	0.14	1.15	0.11	0.89	3.6	0.03
HML	0.27	2.23	0.22	1.74	6.0	0.06
MOM	0.62	3.48	0.15	0.86	52.4	0.47
RMW	0.30	3.19	0.24	2.56	6.0	0.06
CMA	0.29	3.63	0.19	2.39	10.6	0.09
ME	0.24	1.93	0.08	0.61	20.2	0.17
ROE	0.53	4.85	0.19	1.75	38.2	0.33
IA	0.34	4.33	0.11	1.41	26.2	0.23
HML(m)	0.29	1.96	0.08	0.57	20.9	0.21
RMW_C	0.37	4.84	0.30	3.81	8.1	0.08

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Ex-post MVEs

Table III

Ex Post Mean-Variance Efficient Portfolios

For each of the asset pricing models specified by the row headings, this table presents the weights of each factor in the portfolio consisting of the model's factors that maximize the ex post squared Sharpe ratio, SR^2. Panel A uses factor returns that ignore transaction costs. Panel B uses factor returns that account for transaction costs. The sample period spans January 1972 through December 2021.

Panel A: Results Ignoring Transaction Costs

	Optimal Factor Weight (% of Holdings in the Ex Post MVE Portfolio)											
	MKT	SMB	HML	MOM	RMW	CMA	ME	ROE	IA	HML(m)	RMW_C	SR^2
FF5	18	11	−6		31	47						1.26
FF6	17	9	1	12	25	36						1.63
HXZ4	16						12	31	40			1.77
BS6	13	11		14				29	9	24		2.49
$FF5_C$	16	13	−5			31					45	1.85
$FF6_C$	16	11	0	8		25					39	2.16

Panel B: Results Accounting for Transaction Costs

	Optimal Factor Weight (% of Holdings in the Ex Post MVE Portfolio)											
	MKT	SMB	HML	MOM	RMW	CMA	ME	ROE	IA	HML(m)	RMW_C	SR^2
FF5	21	10	0		32	37						0.81
FF6	20	9	0	7	28	35						0.88
HXZ4	28						5	28	39			0.51
BS6	20	11		9				27	17	16		0.60
$FF5_C$	18	13	0			23					46	1.19
$FF6_C$	18	12	0	4		23					43	1.23

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Ex-ante MVEs net of costs

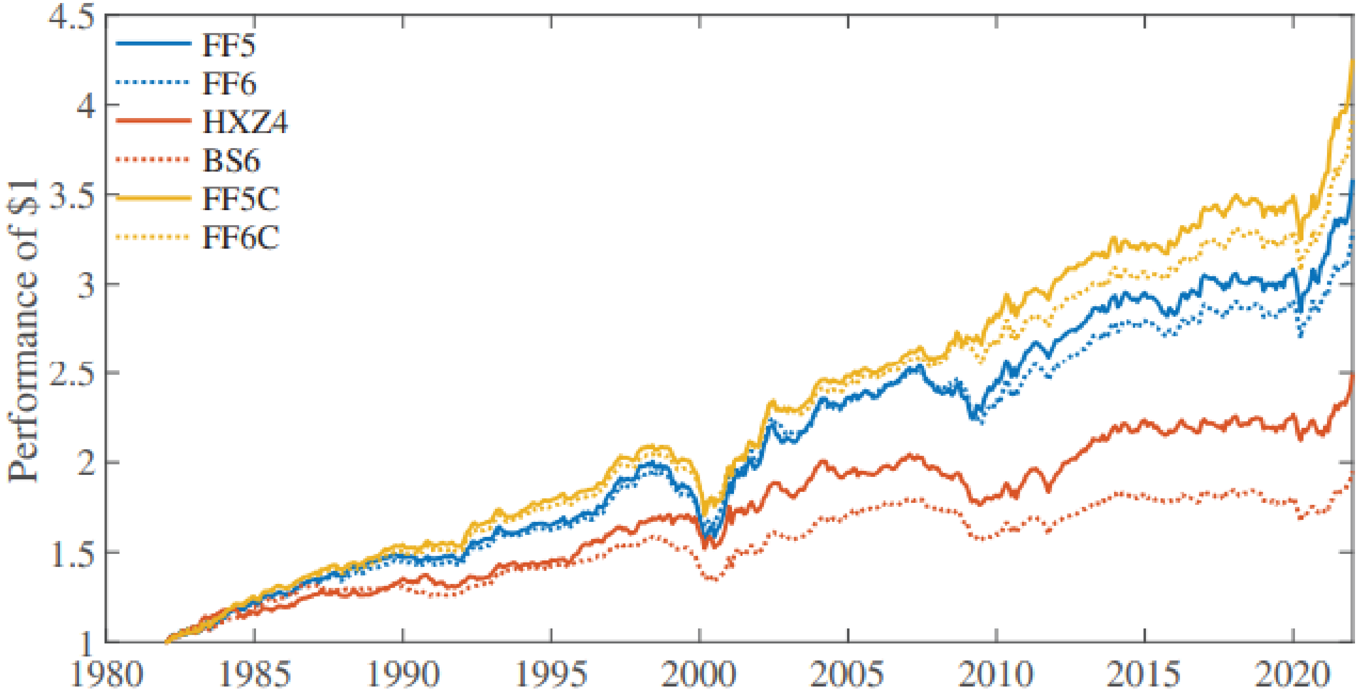

Figure 3. Performance of ex ante optimal portfolios of candidate model factors. The figure shows the performance over time, net of transaction costs, of $1 invested in portfolios constructed using factors from each of the six candidate models (FF5, FF6, HXZ4, BS6, $FF5_C$, and $FF6_C$). Each month, a portfolio holds each factor in proportion to the optimal weights estimated using only returns prior to portfolio formation in expanding windows. The data cover January 1972 through December 2021, but we require a minimum of 10 years to estimate portfolio weights, so the returns begin in January 1982. Appendix B provides details of our calculation of cumulative returns. (Color figure can be viewed at wileyonlinelibrary.com)

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Bootstraps

Table IV

Bootstrapped Net-of-Costs Maximum Squared Sharpe Ratios

For each model we consider, the first column presents average net-of-costs maximum squared Sharpe ratios, SR^2, from 100,000 in-sample (IS) or out-of-sample (OS) simulation runs. IS and OS simulations split the 600 sample months of our sample period, January 1972 through December 2021, into 300 adjacent pairs: months (1, 2), (3, 4), … (599, 600). A simulation run draws a random sample with replacement of 300 pairs. The IS simulation run chooses a month randomly from each pair in the run, reusing the same month if the pair is drawn more than once. We calculate IS SR^2 for all models on that sample of months using equation (2) and then apply the corresponding portfolio weights in the unused months of the simulation pairs to produce the corresponding OS estimate of the Sharpe ratio for the IS tangency portfolio. The six columns labeled by model names, "FF5" to "$FF6_C$," present the percentage of bootstrap simulations in which the squared Sharpe ratio of the model defined by the row heading is greater than that of the model defined by the column heading. The last column, "Best," presents the percentage of bootstrap simulation runs in which the model specified by the row heading has the highest squared Sharpe ratio among all models in the run. Panel A presents IS results and Panel B presents OS results.

Panel A: In-Sample Bootstrap Results

		Probability (%) that the Row Model Performs Better than the Column Model						
	Mean-SR^2	FF5	FF6	HXZ4	BS6	$FF5_C$	$FF6_C$	Best
CAPM	0.30	0.0	0.0	0.1	0.0	0.0	0.0	0.0
FF5	1.13		16.0	90.3	68.3	2.5	1.4	0.5
FF6	1.29	84.0		97.5	87.6	19.8	3.2	2.4
HXZ4	0.77	9.7	2.5		0.4	1.9	0.4	0.0
BS6	1.00	31.7	12.4	99.6		9.2	2.3	2.1
$FF5_C$	1.53	97.5	80.2	98.1	90.8		20.6	19.6
$FF6_C$	1.67	98.6	96.8	99.6	97.7	79.4		75.3

Panel B: Out-of-Sample Bootstrap Results

		Probability (%) that the Row Model Performs Better than the Column Model						
	Mean-SR^2	FF5	FF6	HXZ4	BS6	$FF5_C$	$FF6_C$	Best
CAPM	0.30	17.7	18.1	31.9	33.9	6.7	7.5	5.3
FF5	0.60		46.1	82.1	81.3	2.8	7.7	1.2
FF6	0.61	53.9		83.8	87.5	7.3	3.1	1.5
HXZ4	0.39	17.9	16.2		43.0	4.1	4.1	1.9
BS6	0.39	18.7	12.5	57.0		3.6	2.2	1.1
$FF5_C$	0.94	97.1	92.7	95.9	96.4		54.2	48.6
$FF6_C$	0.93	92.3	96.9	95.9	97.8	45.8		40.4

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Pricing 205 anomalies of Chen and Zimmerman (2022)

- For each model, M, and anomaly, A,

$$\%\Delta \mathrm{SR}^2(\mathrm{M},\mathrm{A}) \triangleq \frac{\mathrm{SR}^2(\mathrm{M},\mathrm{A})}{\mathrm{SR}^2(\mathrm{M})} - 1$$

- Larger $\%\Delta \mathrm{SR}^2(\mathrm{M},\mathrm{A}) \Rightarrow M$ worse at pricing A
- Fig 4: Percentiles of $\%\Delta \mathrm{SR}^2(\mathrm{M},\mathrm{A})$ for each model

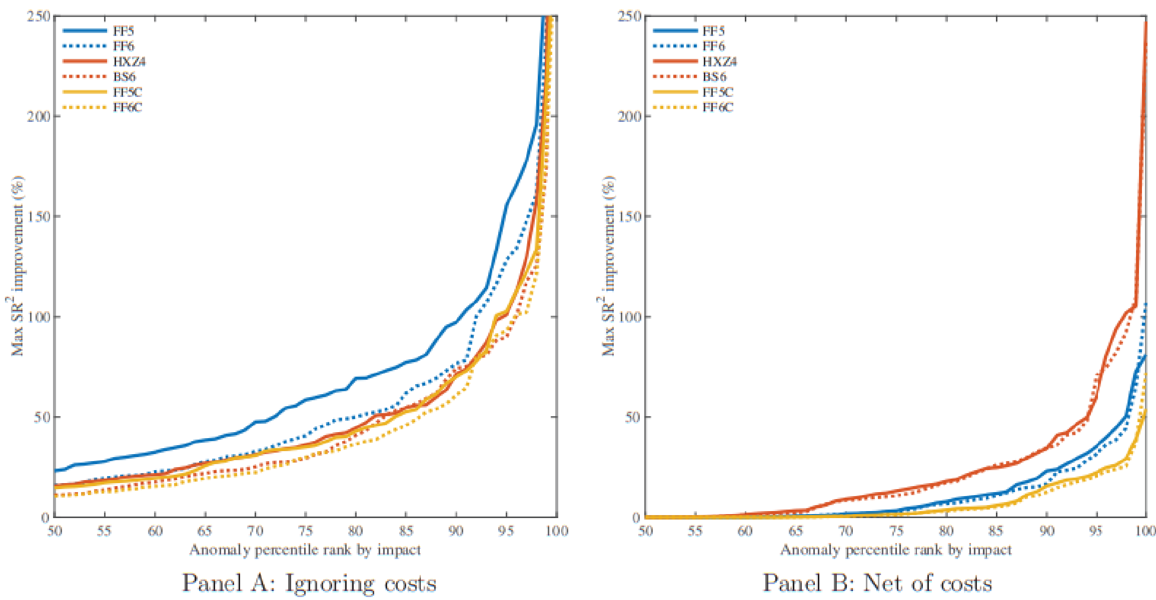

Figure 4. Frontier expansion (squared Sharpe ratio improvement) from adding anomalies to asset pricing models. For each asset pricing model we consider, M, and each of the 205 anomalies, A, described in Section IV, we compute the maximum ex post squared Sharpe ratio attainable from the model's factors, $\mathrm{SR}^2(M)$, along with the maximum squared Sharpe ratio attainable from the model's factors and the anomaly, $\mathrm{SR}^2(M,A)$. We then compute the squared Sharpe ratio improvement, $\%\Delta\mathrm{SR}^2(M,A) = \mathrm{SR}^2(M,A)/\mathrm{SR}^2(M) - 1$. This figure depicts plots of the percentiles from the distribution of the 120 $\%\Delta\mathrm{SR}^2(M,A)$ for each model specified by the plot legend. Panel A (B) ignores (accounts for) transaction costs. For ease of visualization, the y-axis is truncated at 250%. (Color figure can be viewed at wileyonlinelibrary.com)