Topic 4: Trading Costs

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Keim and Madhavan (1997)

- Explicit costs
  - Commissions
    - We don't worry about them too much
- Implicit costs
  - Quoted bid-ask spread
    - Market maker's compensation for providing liquidity; price of immediacy
  - Effective spread
    - "True" spread, since many traders trade within the quoted spread
  - Price impact
    - Larger trades move prices
  - Opportunity cost
    - Impossible to measure

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Roll (1984)

- The bid-ask spread complicates research, since we don't observe the true price.
  - We have three prices: bid, P_b, ask, P_a, and true price, P^*
  - The true price is often between P_a and P_b, although it need not be.
  - How do we define returns: From P_a to P_a , P_b to P_b, P_b to P_a ...?
  - How is $P_a – P_b$ determined?

- Roll (1984) provides a simple model of how the bid-ask spread might impact the time-series properties of returns.
  - Provides most of the intuition and the framework on how we think about the bid-ask spread.

Roll (1984)

- The observed market price is

$$P_t = P_t^* + q_t \frac{s}{2}$$

  - P_t^* : fundamental price in a frictionless economy
  - s: bid-ask spread (independent of the P_t level)
  - q_t : i.i.d index variable - takes values of 1 with prob. 0.5 (buy)
    - takes value of -1 with prob. 0.5 (sell)
- q_t is unobservable. But, with the assumptions, $E[q_t] = 0$ and $Var(q_t) = 1$
- For simplicity assume that P_t^* does not change - Var$(\Delta P_t^*) = 0$
- The change in price is (define the *cost* $c = s/2$):

$$\Delta P_t = \Delta P_t^* + q_t \frac{s}{2} - q_{t-1} \frac{s}{2} = \Delta P_t^* + c\Delta q_t$$

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Roll (1984)

- Its variance and autocovariance are:
  - $Var(\Delta \mathrm{P_t}) = \mathrm{Var}(\Delta P_t^*) + c^2 Var(I_t) + c^2 Var(I_t) = 2c^2 (= s^2/2)$
  - $Cov(\Delta P_t, \Delta P_{t-1}) = -c^2$

- Note:
  - The fundamental value is fixed, but there is variation from c.
  - The bid-ask spread induces negative correlation in returns even in the absence of other fluctuations.
  - The variance and covariance depend on the magnitude of the bid-ask spread.
  - In this particular example, it induces a 1st-order serial correlation.

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Roll (1984)

- We can also express the cost (aka half-spread) as a function of the covariance:
  - $c = [-Cov(\Delta P_t, \Delta P_{t-1})]^{-1/2}$

- In practice, we can find $Cov(\Delta P_t, \Delta P_{t-1}) > 0$

- To avoid this problem, Roll (1984) defines the cost as

$$c = -[|Cov(\Delta P_t, \Delta P_{t-1})|]^{-1/2}$$

- Roll calls s$(= 2 \times c)$ the "effective spread," which is estimable.

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Hasbrouck (2009)

- Takes the Roll (1984) model:

$$\Delta P_t = c\Delta q_t + \epsilon_t$$

- … generalizes it by adding a market factor:

$$\Delta P_t = c\Delta q_t + \beta_m r_{mt} + \epsilon_t$$

σ_ϵ^2

- … makes a few assumptions
  - $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$ & $i.i.d.$
  - Priors for the unknowns $\{c, \sigma_\epsilon^2, q_1, \dots, q_T\}$

- … and sequentially draws the parameter estimates using a Gibbs sampler to characterize the posterior densities

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Hasbrouck (2009)

- The Gibbs measure achieves the highest <u>correlation</u> (96.5%) with high-frequency TAQ data estimates

Table III

Correlations between Liquidity Measures for the Comparison Sample

The comparison sample consists of approximately 150 NASDAQ firms and 150 NYSE/Amex firms selected in a capitalization-stratified random draw in each of the years from 1993 to 2005. Definitions of the liquidity measures are given in Table I. Partial correlations are adjusted for log (end-of-year price) and log (market capitalization).

	c_{it}^{TAQ}	c_{it}^{Gibbs}	c_{it}^{Moment}	$PropZero_{it}$	λ_{it}	I_{it}
Pearson correlation						
c_{it}^{TAQ}	1.000	0.965	0.878	0.611	0.513	0.612
c_{it}^{Gibbs}	0.965	1.000	0.917	0.579	0.450	0.589
c_{it}^{Moment}	0.878	0.917	1.000	0.451	0.378	0.504
$PropZero_{it}$	0.611	0.579	0.451	1.000	0.311	0.252
λ_{it}	0.513	0.450	0.378	0.311	1.000	0.668
I_{it}	0.612	0.589	0.504	0.252	0.668	1.000

Novy-Marx and Velikov (2016)

- Apply the Gibbs measure to 23 anomalies and create a taxonomy of anomalies
  - Useful rules of thumb for anomalies & whether they survive trading costs based on their turnover
  - Typical value-weighted anomaly ~50bps
  - If turnover more than 50% per month, net returns are negative

- Several new methods
  - "Generalized alpha"
  - Buy/hold spread trading cost mitigation technique

- Price impact estimation

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Novy-Marx and Velikov (2016)

- Trading cost calculation procedure
  - Track portfolio weights over time
  - Whenever a position is entered or exited, assume half of the effective spread (i.e., Hasbrouck's effective cost) is paid

- Interpretation: lower bound cost for average trader using market orders

- Note: lots of missing observations in Hasbrouck for small stocks
  - Creates look-ahead bias (excluding stocks with missing data)
  - Thus, we fill in 29% of stock-months (4% of market cap) in based on $\sqrt{(rankME_i - rankME_j)^2 + (rankME_i - rankME_j)^2}$

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Trading costs formulas from DNMV (JF, 2023)

- Net returns for month t are given by:
  - Long side:

$$f_t^{net} = f_t^{gross} - TC_t^f$$

  - Short side:

$$f_t^{S,net} = -f_t^{gross} - TC_t^f$$

- Trading cost calculation procedure
  - Turnover (TO) for a factor f in month t defined as:

$$TO_t^f = \frac{1}{2}\sum_{i=1}^{N_t} |w_{i,t} - \widetilde{w}_{i,t-}|$$

  - Trading costs (TC) defined as:

$$TC_t^f = \sum_{i=1}^{N_t} \left|w_{i,t} - \widetilde{w}_{i,t-}\right| \cdot c_{i,t}$$

where $c_{i,t}$ is the one-way transaction cost (i.e., effective half-spread)

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Novy-Marx and Velikov (2016)

- Problem with performance evaluation: we can't regress net anomaly strategy returns on net factor returns
  - If a loading is negative, the alpha from this regression effectively assumes you "earn" the trading costs

- Solution: Generalized alpha (α^*):

$$\frac{MVE_{\{X,y\}}}{w_{y,MVE\{X,y\}}} = \alpha^* + \beta^* MVE_X + \epsilon^*$$

  - Does the mean-variance efficient portfolio of the factors and the strategy under evaluation improve the investment opportunity set for an investor who has access to the MVE of the factors only
  - $w_{y,MVE\{X,y\}}$ terms ensures it's the same as alpha when there is no trading costs

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Novy-Marx and Velikov (2016): Taxonomy

Table 3
Value-weighted returns

Anomaly	$E[r^e_{gross}]$	α^{FF4}_{gross}	TO	T-costs	$E[r^e_{net}]$	α^{FF4}_{net}
Panel A: Low-turnover strategies						
Size	0.33	−0.14	1.23	0.04	0.28	
	[1.66]	[−1.77]			[1.44]	
Gross profitability	0.40	0.52	1.96	0.03	0.37	0.51
	[2.94]	[3.83]			[2.74]	[3.77]
Value	0.47	−0.17	2.91	0.05	0.42	−0.02
	[2.68]	[−1.76]			[2.39]	[−0.17]
ValProf	0.82	0.50	2.94	0.06	0.77	0.49
	[5.18]	[4.01]			[4.82]	[3.93]
Accruals	0.27	0.27	5.74	0.09	0.18	0.19
	[2.14]	[2.15]			[1.43]	[1.55]
Asset growth	0.37	0.07	6.37	0.11	0.26	0.03
	[2.52]	[0.58]			[1.75]	[0.21]
Investment	0.56	0.35	6.40	0.10	0.46	0.31
	[4.44]	[2.90]			[3.60]	[2.62]
Piotroski's F-score	0.20	0.31	7.24	0.11	0.09	0.24
	[1.04]	[1.75]			[0.45]	[1.37]
Panel B: Mid-turnover strategies						
Net issuance	0.57	0.58	14.4	0.20	0.37	0.41
	[3.70]	[4.10]			[2.43]	[2.93]
Return-on-book equity	0.71	0.84	22.3	0.38	0.33	0.59
	[2.96]	[4.41]			[1.38]	[3.18]
Failure probability	0.85	0.94	26.1	0.61	0.24	0.70
	[2.52]	[4.89]			[0.73]	[3.55]
ValMomProf	1.43	0.68	26.8	0.43	0.99	0.68
	[7.41]	[5.52]			[5.18]	[5.22]
ValMom	0.93	−0.12	28.7	0.41	0.51	
	[4.81]	[−1.31]			[2.67]	
Idiosyncratic volatility	0.63	0.83	24.6	0.52	0.11	0.41
	[2.13]	[5.14]			[0.37]	[2.57]
Momentum	1.33	0.35	34.5	0.65	0.68	0.40
	[4.80]	[3.04]			[2.45]	[3.12]
PEAD (SUE)	0.72	0.58	35.1	0.46	0.26	0.29
	[4.52]	[4.31]			[1.60]	[2.21]
PEAD (CAR3)	0.91	0.87	34.7	0.57	0.34	0.38
	[6.54]	[6.39]			[2.41]	[2.85]
Panel C: High-turnover strategies						
Industry momentum	0.93	0.83	90.1	1.22	−0.29	
	[3.97]	[3.52]			[−1.20]	
Industry relative reversals	0.98	1.05	90.3	1.78	−0.80	
	[5.72]	[6.66]			[−4.73]	
High-frequency combo	1.61	1.48	91.0	1.45	0.16	0.05
	[11.21]	[9.93]			[1.11]	[0.35]
Short-run reversals	0.37	0.45	90.9	1.65	−1.28	
	[1.71]	[2.22]			[−6.02]	
Seasonality	0.84	0.82	91.1	1.46	−0.62	
	[5.21]	[5.03]			[−3.88]	
Industry relative reversals (low volatility)	1.25	1.17	94.0	1.06	0.19	0.07
	[9.36]	[8.96]			[1.41]	[0.57]

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Novy-Marx and Velikov (2016)

- Mean-variance efficient weights with no costs given by:

$$\omega_{MVE} = \frac{\Sigma^{-1}\mu_e}{1\Sigma^{-1}\mu_e}$$

  - we can estimate with ex-post vector of average returns & variance-covariance matrix

- However, with costs, we can't just apply the formula with net returns
  - Need to do a numerical optimization

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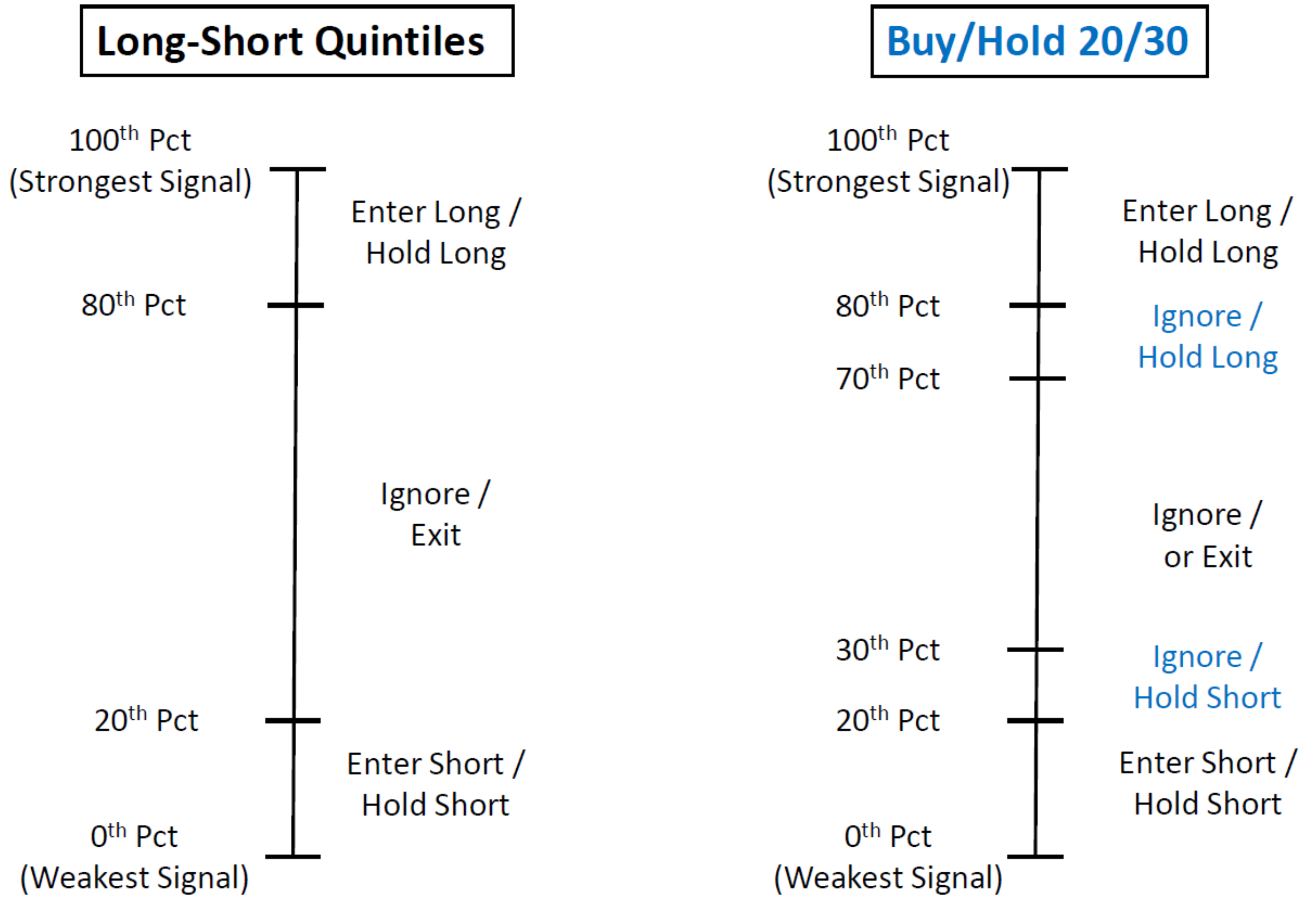
Novy-Marx and Velikov (2016)

- Buy/hold spreads: a simple trading cost mitigation technique



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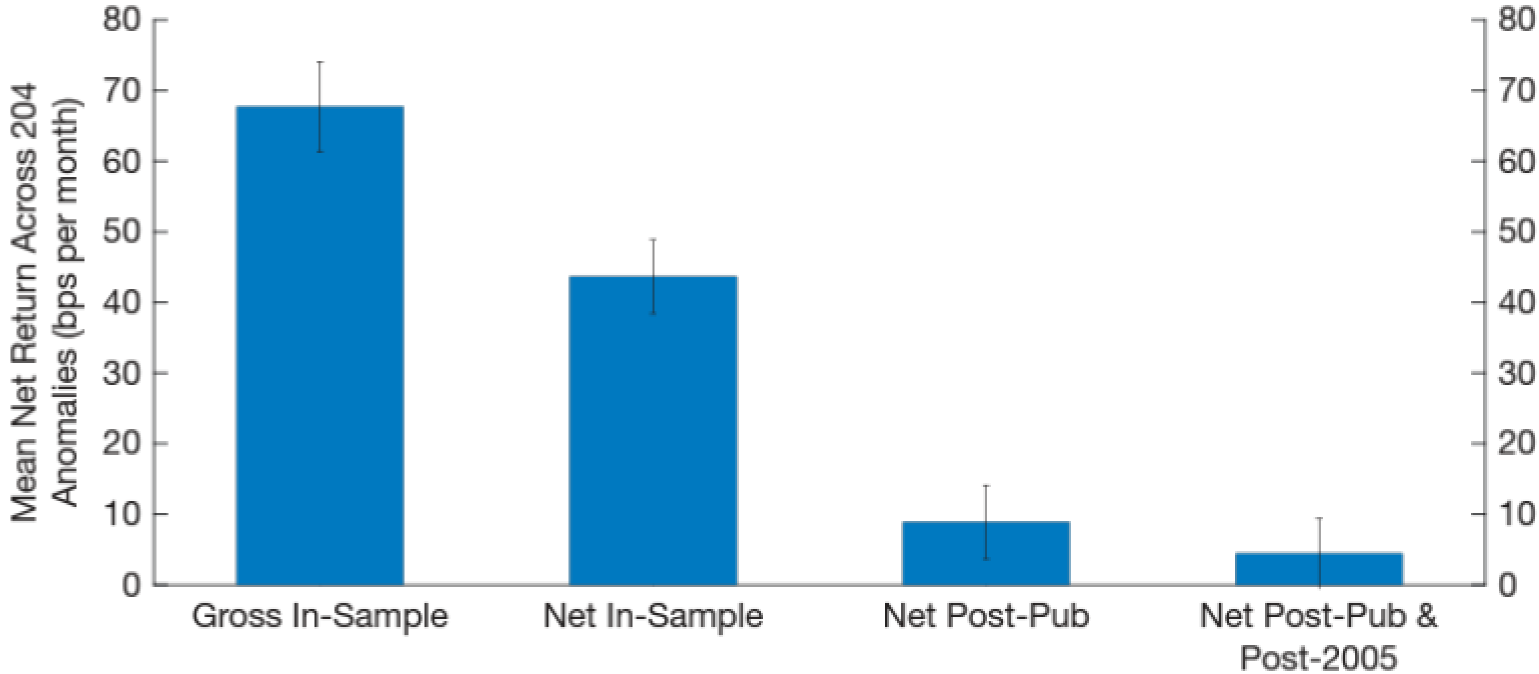
Chen and Velikov (JFQA, 2023)

- Main goal: "zero in" on the average anomaly

FIGURE 1

Anomaly Mean Long–Short Returns

The error bars in Figure 1 show 2 standard errors.



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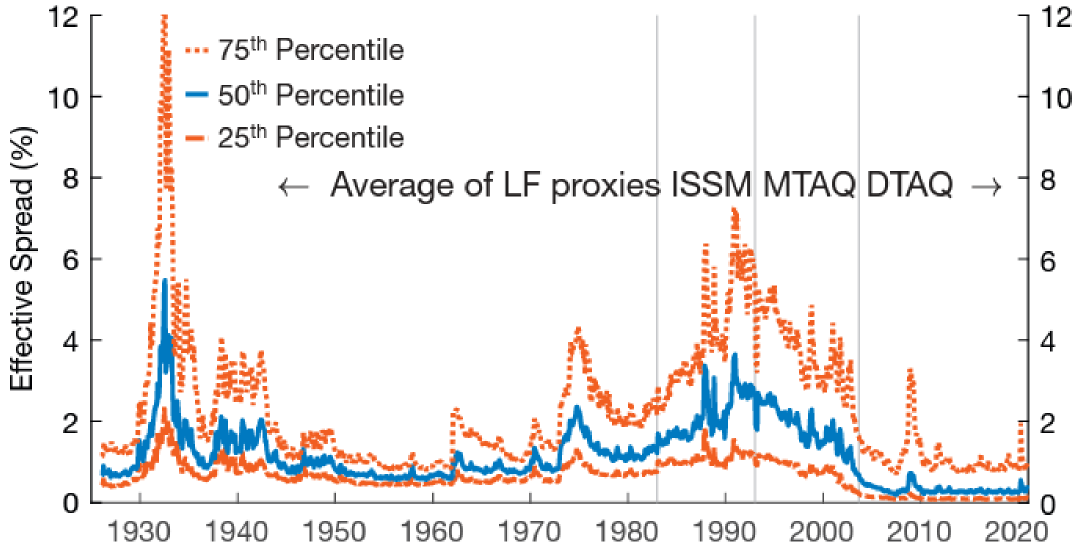
Chen and Velikov (JFQA, 2023)

- Main (in my opinion) contribution: a new trading cost measure that combines what's in the literature

FIGURE 3

Combined Effective Spreads over Time

Spreads in Figure 3 combine high-frequency and low-frequency data. We use high-frequency Daily TAQ (DTAQ), Monthly TAQ (MTAQ), and ISSM when available. Otherwise, we use the average of four low frequency proxies: Gibbs (Hasbrouck (2009)), HL (Corwin and Schultz (2012)), CHL (Abdi and Ranaldo (2017)), and VoV (Kyle and Obizhaeva (2016)). The combined spread tracks well-known structural changes like the entry of NASDAQ (early 1970s) and decimalization (early 2000s).



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Chen and Velikov (JFQA, 2023)

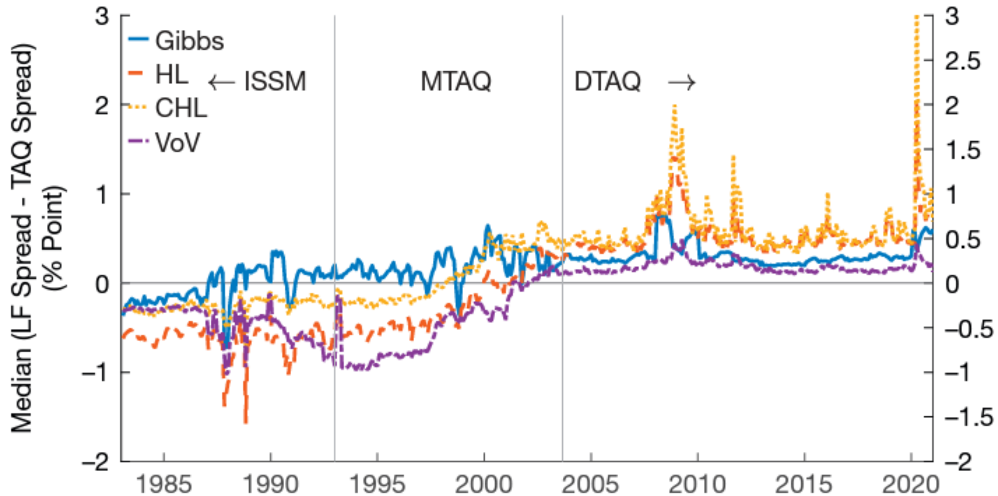
- Why do we need the new measure?
  - Severe bias in low-frequency measures post-decimalization

- Especially important for post-publication results

FIGURE 2

The Bias in Low-Frequency Effective Spread Proxies

In Figure 2, we take the difference between low-frequency effective spreads and TAQ effective spreads at the firm-month level and then take the median across firms to calculate the median error in each month. Low-frequency spreads are from Hasbrouck (2009) (Gibbs), Corwin and Schultz (2012) (HL), Abdi and Ranaldo (2017) (CHL), and Kyle and Obizhaeva (2016) (VoV). Post-decimalization, low-frequency proxies are biased upward by roughly 25–50 bps. LF spread data are found at https://sites.google.com/site/chenandrewy/, HF spread code is at https://github.com/chenandrewy/hf-spreads-all, and replication code is at https://github.com/velikov-mihail/Chen-Velikov.



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- Combination "back-cast"
  - Stronger correlation of combined measure with high-frequency data
  - Simple average outperforms individual measures
  - Errors "average out"

TABLE 1

Correlations Between Low-Frequency Proxies and High-Frequency Effective Bid–Ask Spreads

Table 1 examines four low-frequency proxies: Gibbs is Hasbrouck's (2009) Gibbs estimate of the Roll model, HL is Corwin and Schultz's (2012) high-low spread, CHL is Abdi and Ranaldo's (2017) close-high-low, and VoV (volume-over-volatility) is Fong et al.'s (2017) implementation of Kyle and Obizhaeva (2016) microstructure invariance hypothesis. Correlations are pooled. LF_AVE is the equal-weighted average of the four low-frequency proxies. TAQ and ISSM are computed from high-frequency data. The low-frequency measures are imperfectly correlated, suggesting that they contain distinct information. LF_AVE has the highest correlation with high-frequency spreads. Code is found at https://github.com/chenandrewy/hf-spreads-all and https://github.com/velikov-mihail/Chen-Velikov. LF spread data is at https://sites.google.com/site/chenandrewy/.

Panel A. LF Spread Correlations (1926–2020)

	Gibbs	HL	CHL	VoV
Gibbs	1.00			
HL	0.63	1.00		
CHL	0.74	0.86	1.00	
VoV	0.74	0.53	0.73	1.00

Panel B. Correlations with TAQ (1993–2020)

	TAQ	Gibbs	HL	CHL	VoV	LF_AVE
TAQ	1.00					
Gibbs	0.84	1.00				
HL	0.64	0.60	1.00			
CHL	0.79	0.72	0.85	1.00		
VoV	0.84	0.72	0.53	0.74	1.00	
LF_AVE	0.90	0.89	0.82	0.93	0.86	1.00

Panel C. Correlations with ISSM (1983–1992)

	ISSM	Gibbs	HL	CHL	VoV	LF_AVE
ISSM	1.00					
Gibbs	0.88	1.00				
HL	0.77	0.74	1.00			
CHL	0.83	0.78	0.88	1.00		
VoV	0.86	0.81	0.62	0.74	1.00	
LF_AVE	0.92	0.94	0.88	0.93	0.87	1.00

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- Depending on data availability, we either:
  - Calculate monthly realized spreads from high-frequency data

1. Calculate effective spreads for all eligible trades, k, in ISSM & TAQ

$$[Effective\ Spread]_k = 2|\log(P_k) - \log(M_k)|$$

where

- P_k is the price of the kth trade
- M_k is the prevailing midpoint of the matched NBBO quotes

2. For each stock-day, take a share-weighted average across all trades
3. For each stock-month, take an equal-weighted average across all days

- Use average of up to four low-frequency estimators from the literature
  - Gibbs (Hasbrouck, 2009)
  - High-low spread (Corwin and Schultz, 2012)
  - Close-high-low (Abdi and Ranaldo, 2017)
  - Volume-over-volatility (Kyle and Obizhaeva, 2016)

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- Trading cost hierarchy

1. Daily TAQ (2003 – 2020)
2. Monthly TAQ (1993 – 2003)
3. ISSM (1983 – 1992)
   - NASDAQ data missing prior to 1987 and sporadically between 1987 and 1991
4. Low-frequency average (1926 – 2020)
   - Require at least one of the four measures
5. Match based on closest distance in market capitalization and idiosyncratic volatility rank space (1926 – 2020)

$$d = \sqrt{\left(rankME_i - rankME_j\right)^2 + \left(rankIVOL_i - rankIVOL_j\right)^2}$$

6. Match based to nearest stock based on market capitalization (1926 – 2020)

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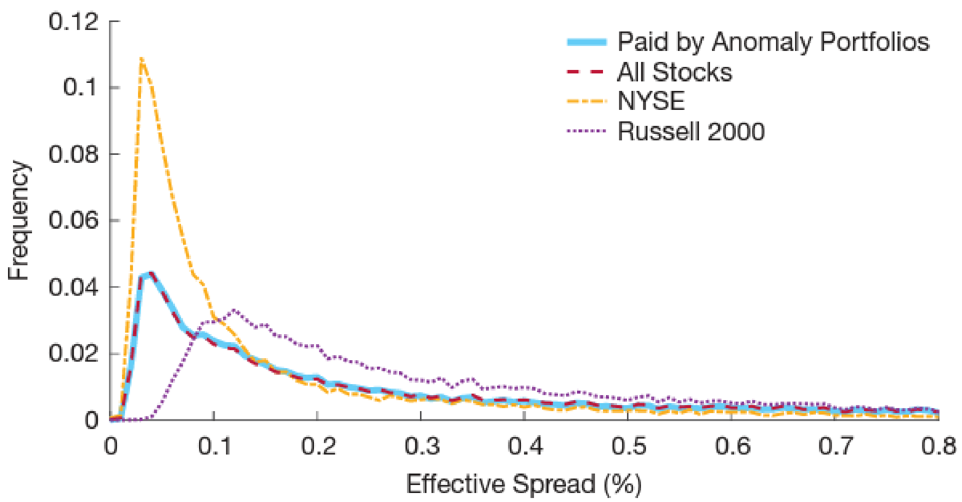
- Long right tail of the effective spread distribution

FIGURE 4

Distribution of Spreads Paid by Academic Implementations in 2014

In Figure 4, we compare the effective spreads paid by academic implementations with those of all stocks, NYSE stocks, and Russell 2000 stocks. “Paid by anomaly portfolios” pools across all trades implied by 204 academic implementations in 2014. Other distributions are pooled across all stock months in 2014. Academic implementations trade stocks across the entire liquidity spectrum, resulting in large trading costs despite the near-zero modal spreads of recent years.



- Decimalization: spread ≈ \$0.01, price ≈ \$20 ⇒ spread ≈5 bps
  - But that’s the mode! 20% of NYSE stocks have spreads > 20 bps

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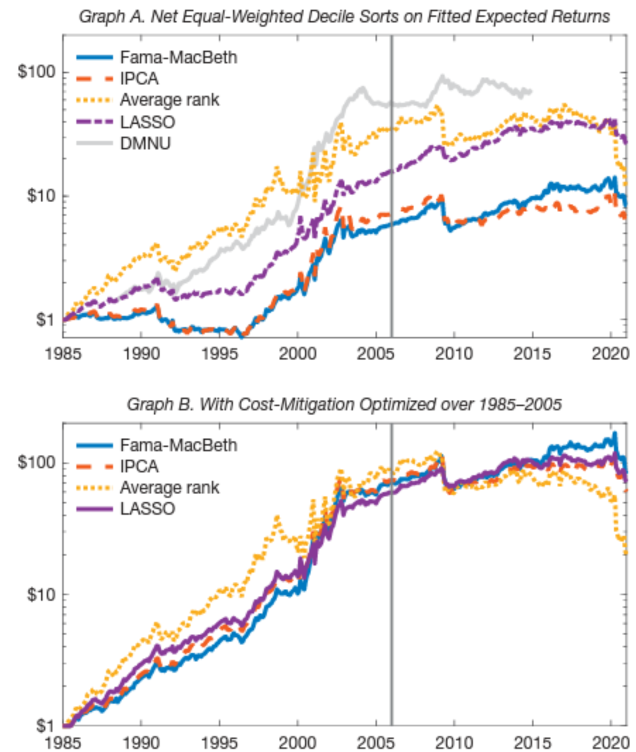
Chen and Velikov (JFQA, 2023)

- We also looked at combination strategies
- Found that their performance significantly deteriorates after 2003
- Will have to do something along these lines for your next data exercise

FIGURE 8

Cumulative Return of $1 Invested in Combination Strategies

Figure 8 sorts stocks on the expected gross return implied by various models using 58 predictors that are published pre-2006 and satisfy availability and continuity conditions. Fits use the past 120 months of data and stocks below the 20th percentile market cap are dropped. Graph A shows results without cost mitigation. Graph B optimizes costs using data from 1985–2005. For comparison, "DMNU" shows the market-neutral component of DeMiguel et al.'s (2020) regularized out-of-sample portfolio, scaled to have the same volatility as our Fama–Macbeth strategy. For all strategies, impressive gains flatten out around 2003, and the 2005 cutoff matters little.



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