Hypotesis Testing

Hypotesis. Una que pude ser testada

Tipos de hypotesis

- Null Hypotesis: H_0
  - One to be tested
- Alternative Hypotesis: H_1 or H_a
  - Everything else

Ej: null hypotesis:

Medium salary = 113.000 $

Alternative Hypotesis

Medium salary ≠ 113.000

Si es lo suficiente cerca de la mean aceptemos la hipotesis

* Null hypotesis = inocente until proven guilty
* Los resultados de las hypotesis hacen referencia al population parameter en lugar de al sample statistic
* El objetivo de un investigador es tratar de rechazar la null hypotesis para ser de-alter que es solida

Significance Level & Rejection Region

α = probability of rejecting the null hypotesis when its true (we only want to reject it when is false)

Typical values: 0.01, 0.05, 0.1 been 0.05 the most common one

Ejemplo test.

H_0: μ_0 = grades average are 70%

H_1: $\mu_1 \neq$ grades " " "

1. Perform Z test

$$Z = \frac{\bar{X} - \mu}{|s/\sqrt{n}|}$$

($\bar{X}$: sample mean; μ: hypothesized mean; $s/\sqrt{n}$: standard error)

2. Esta formula escala o estandariza y a más parecidas sea $\bar{X}$ de μ más se acercara el resultado a 0

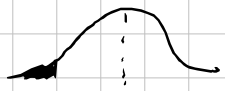
Rejection region: conjunto de valores simetricos respecto a 0 que se encuentran fuera de el rango de aceptación.

Para esto hacemos uso de la Z table y el valor de $\alpha/2$. para un significance level de 0.05 sería el valor 0.025 equivalente a 1.96 en la Z-table por lo que el rango de aceptacion seria $[-1.96, 1.96]$ siendo todo lo de fuera rechazado

Ej ONE-SIDED Test

$H_0: \mu_0 \geq 125000\$$

$H_1: \mu_1 < 125000\$$



En casos de solo one sided la rejection region serán solo los valores de una de las tails y usará el valor de α en lugar de $\alpha/2$ para coger los valores de la Z-table en caso de 0.05 sería -1.645 que para este ejemplo de menor que sería negativo

Types of errors in testing

* Names are literally type I and type II

- Type I: reject a true null hypotesis also called false positive. The probability of having this error is α (level of significant)

- Type II: false negative. Accept a false null hypotesis the probability of this error is β.

  β depends on n and σ and the probability of making the error is $1-\beta$ also called power of the test

	H_0 true	H_0 False
Accept	✓	False Negative (Type II)
Reject	False Positive (Type I)	✓

Test for single population: Known variance

Ej. Salaries again $$

$\bar{x} = 100200\$$ $H_0: \mu_0 = 113000\$$ $\alpha = 0.05$

Population std = 15200 $\$$ $H_1: \mu_1 \neq 113000\$$

Standard error = 2739 $\$$ $n = 30$

1° Paso Estandarizamos para poder trabajar y calcular critical value

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$$

Z: Z-score = standarized variable associated with the test

z = critical value = value obtained from Z-table

$z \sim N(0,1)$ $Z \sim N(\bar{x} - \mu_0, 1)$

$$Z = \frac{100200 - 113000}{2739} = -4.67$$

$z_{\alpha 0.5/2}$ = pick value closer to $1 - 0.025 = 0.975$ and sum col and row in this case $= 1.96$

2. Paso Reject if $|Z\text{-score}| > |\text{critical value}|$

$|Z| > |z_{0.025}|$?

$|-4.67| > |1.96|$ ✓ true

3. We reject the hypothesis since $|Z| > |z_{\alpha/2}|$

P-VALUE

Valor más pequeño para α en el que aun podemos rechazar la null hypotesis

Siguiendo el gemplo anterior

$\bar{x} = 100\,200\,\$$ $H_0: \mu_0 = 113000\,\$$ $\alpha = 0.05$

Population std = 15 100 $\$$ $H_1: \mu_1 \neq 113000\,\$$

Standard error = 2 739 $\$$ $n = 30$

Z-score = -4.67

1. Buscamos el valor en la Z-table = |Z-score|
   si es demasiado grande y no está cogemos el más grande. Ej. 1-0.0000 para 3.9 en esta caso.

2. Ejemplo con $Z = 2.12$
   - En este caso podríamos rechazarla en 0.05 pero no en 0.01
   - Al buscar en la table vemos vemos que para 2.1, 0.02 (2.12) el z-value es 0.09830 por lo que podemos ver que el p-value
     - One tailed $(1 - 0.983) = 0.017$
     - Two tailed $(1 - 0.983) \times 2 = 0.034$

The closer to 0 p-value is the better results you get

Si P-Value < Significance Level → Reject Hypotesis

Single Population: variance unknown

Ej. % de correos leídos de la empresa

$H_0: \mu_0 \leq 40\%$ $\quad \bar{x} = 37.7\%$ $\quad n = 10$ $\quad$ degrees of freedom $= n - 1 = 9$

Standard deviation 13.74% $\quad \alpha = 0.05$

$H_1: \mu_1 > 40\%$ $\quad$ Standard error 4.34%

Usaremos T-score $= \dfrac{\bar{x} - \mu}{s / \sqrt{n}} = \dfrac{37.7\% - 40\%}{4.34\%} = -0.53$

$t_{\alpha, k} = 1.833$

T $\quad$ t

▷ $|-0.53| < 1.83 \rightarrow$ no podemos rechazar la hipotesis

Accept if |T-score| < critical value (t)

Reject if |T-score| > critical value (t)

p-value = 0.30 > 0.05 (significant value)

We cannot reject the null hipothesis

Decision rule

p-val $> \alpha$ = accept null

p-val $< \alpha$ = reject null

Multiple Popolation: Dependant Sample

Ej. La pildora de magnesio en sangre

Hypotesis:

$$\begin{cases} H_0: \mu_{Before} \geq \mu_{After} = \mu_B - \mu_A \geq 0 = D_0 \geq 0 \\ H_1: \mu_0 < \mu_A = \mu_B - \mu_A < 0 = D_0 < 0 \\ H_0: D_0 \geq 0 \\ H_1: D_0 < 0 \end{cases}$$

1. Calculamos los datos de la diferencia

$\bar{d} = -0.33$
Standar deviation $= 0.45$
Standard Error $= 0.14$

$$T = \frac{\bar{d} - \mu_0}{\text{St. error}} = \frac{-0.33 - 0}{0.14} = -2.29$$

p-value $= 0.024$

α.

if $\alpha = 0.05$ $\quad p < \alpha \Rightarrow$ reject hypotesis

$\alpha = 0.01$ $\quad p > \alpha \Rightarrow$ accep hypotesis

Test independent Samples : known variance

Multiple Population

Ej : grades engineering vs management

$H_0 : \mu_E - \mu_M = -4\%$
$H_1 = \quad " \quad \neq -4\%$

Two sides since we want to know if is not exactly -4%

	E	M	Diff
n	100	70	?
$\bar{x}$	58%	65%	-7%
Pop Std	10%	6%	1.23%

$\sqrt{\frac{\sigma_e^2}{n_e} + \frac{\sigma_m^2}{n_m}}$ Standard error

Big Sample (>30) so we use Z

$$Z = \frac{\bar{x} - \mu_0}{\text{Std error}} = \frac{(-7\%) - (-4\%)}{1.23\%} = -2.44$$

p-value = 0.015

α
$0.05 > p$ - reject
$0.01 < p$ - accept

SAMPLE SIZE

VARIANCE	Big	Small
known	Z-Statistic	t-statistic
Unknown	depend on researcher but good to go Z-Statistic	t-statistic

Clarification on α

Ej:

$p < 0.05$ - reject
$p > 0.01$ - accept

Estoy a un 95% seguro de que la null hypotesis es falsa pero no a un 99% seguro

Testing { INDEPENDENT SAMPLES
UNKNOWN VARIANCE - ASSUMED EQUAL

5. diferencia precio mazanas en NY y LA

$H_0: \mu_{NY} - \mu_{LA} = 0$

$H_1: \quad " \quad \neq 0$

	NY	LA
Mean	3.94	3.23
Std dv	0.18	0.27
n	10	8

1. Use pool variance

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} =$$

$$= \frac{(10-1)0.18^2 + (8-1)0.27^2}{10+8-2} = 0.05 = s_p$$

Pooled variance: 0.05

Pooled std = 0.22

Degrees of freedom = 10+8-2 = 16

$$T = \frac{\bar{d} - \mu_0}{\text{St error}} = 6.53$$

p-value = 0.0000001

Normally with T or Z scores higher than 2 or 4 respectively we will reject the hypotesis.
Also if p value have more than 3 zeros in decimals we reject it