Hypotesis Testing

Hypotesis: Idea que puede ser testeada

Tipos de hypotesis

- Null Hypotesis: H_0
  | One to be tested
- Alternative Hypotesis: H_1 or H_A
  | Everything else

Ej: null hypotesis:

Medium salary = 113.000 $

Alternative Hypotesis

Medium salary ≠ 113.000

Si es lo suficiente cerca de la mean aceptas la hipotesis

* Null hypotesis = inocente until proven guilty

* Los resultados de las hypotesis hacen referencia al population parameter en lugar de al sample statistic

* El objetivo de un investigador es tratar de rechazar la null hypotesis para demostrar que es solida

Significance Level & Rejection Región

α = probability of rejecting the null hypotesis when its true (we only want to reject it when is false)

<u>Typical values</u>: 0.01, 0.05, 0.1 been 0.05 the most common one

Ejemplo test.

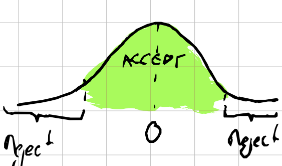
$H_0: \mu_0 =$ grades average are 70%

$H_1: \mu_1 \neq$ grades " " "

1. Perform Z test

$$Z = \frac{\overset{\text{s-de mean}}{\bar{X}} - \overset{\text{hypothesized mean}}{\mu}}{\underbrace{s/\sqrt{n}}_{\text{standard error}}}$$

2. Esta formula escala o estandariza y a más parecidos sean $\bar{x}$ de μ más se acercera el resultado a 0



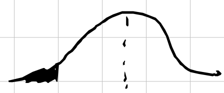
<u>Rejection región</u>: conjunto de valores simetricos respecto a 0 que se encuentran fuera de el rango de aceptación.

Para esto haremos uso de la Z table y el valor de $\alpha/2$. para un significance level de 0.05 sería el valor 0.025 equivalente a 1.96 en la Z-table por lo que el rango de aceptación sería $[-1.96, 1.96]$ siendo todo lo de fuera rechazado

Ej ONE-SIDED test

$H_0: \mu_0 \geq 125\,000$ \$

$H_1: \mu_1 < 125\,000$ \$



En casos de solo one sided la rejection region seran solo los valores de una de las tails y usara el valor de α en lugar de $\alpha/2$ para coger los valores de la Z-table en caso de 0.05 seria 1.645 que para este ejemplo de menor que seria negativo

Types of errors in testing

* Names are literally type I and type II

- Type I: reject a true null hypotesis also called false positive. The probability of havin this error is α (level of significant)

- Type II: false negative. Accept a false null hypotesis the probability of this error is β.
  β, depends on n and σ and the probability of maken the error is $1-\beta$ also called power of the test

	H_0 true	H_0 False
Accept	✓	False Negative (Type II)
Reject	False Positive (Type I)	✓

Test for single population: Known variance

Ej. Salarios again $$

$\bar{x} = 100\,200$ \$ $\qquad H_0: \mu_0 = 113000$ \$ $\qquad \alpha = 0.05$

Population std $= 15\,200$ \$ $\qquad H_1: \mu_1 \neq 113000$ \$

Standard error $= 2739$ \$ $\qquad n = 30$

1° Paso Estandarizamos para poder trabajar y calculamos critical value

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$$

Z = Z-score = standariced variable associated with the test

z = critical value = value obtained from Z-table

$Z \sim N(0,1)$ $\quad Z \sim N(\bar{x} - \mu_0, 1)$

$$Z = \frac{100200 - 113000}{2739} = -4.67 \qquad z_{0.05/2} = \text{pick value closer to } 1 - 0{,}025 = 0{,}975 \text{ and sum col and row in this case} = 1.96$$

2. Paso Reject if |Z-score| > |critical value|

$|Z| > |z_{0.025}|$?

$|-4.67| > |1.96|$ ✓ true

3. We reject the hypothesis since $|Z| > |z_{\alpha/2}|$

P- VALUE

Valor más pequeño para α en el que aun podemos rechazar la null hypotesis

Siguiendo el ejemplo anterior

$\bar{x} = 100\,200$ \$ $\qquad H_0: \mu_0 = 113000$ \$ $\qquad \alpha = 0.05$

Population std = 15 200 \$ $\qquad H_1: \mu_1 \neq 113.000$ \$

Standard error = 2 739 \$ $\qquad n = 30$

Z-score = -4.67

1. Buscamos el valor en la Z-table = |Z-score|
   Si es demasiado grande y no esta cogemos el mas grande. Ej. 1-0.000 1 para 3.9 en este caso.

2. Ejemplo con Z = 2.12
   - En este caso podriamos rechazarla en 0.05 pero no en 0.01.
   - Al buscar en la tabla vemos que para 2.1, 0.02 [2.12] el z-value es 0.09830 por lo que podemos ver que el p-value
     - One tailed (1 - 0.983) = 0.017
     - Two tailed (1 - 0.983) × 2 = 0.034

The closer to 0 p-value is the better results you get

$\boxed{\text{Si P-Value < Significance Level} \Rightarrow \text{Reject Hypotesis}}$

Single Population: variance unknown

Ej. % de correos leidos de la empresa

$H_0: \mu_0 \leq 40\%$ $\quad \bar{x} = 37.7\%$ $\quad n = 10$ $\quad$ degrees of freedom $= n - 1 = 9$

Standard deviation 13.74% $\quad \alpha = 0.05$

$H_1: \mu_1 > 40\%$ $\quad$ Standard error 4.34%

Usaremos T-score $= \dfrac{\bar{x} - \mu}{s / \sqrt{n}} = \dfrac{37.7\% - 40\%}{4.34\%} = -0.53$

$t_{\alpha,k} = 1.833$

$\overset{T}{|-0.53|} < \overset{t}{1.83} \rightarrow$ no podemos rechazar la hipotesis

Accept if $|T\text{-score}| <$ critical value (t)

Reject if $|T\text{-score}| >$ critical value (t)

p-value $= 0.30 > 0.05$ (significant value)

We cannot reject the null hypothesis

Decision rule

p-val $> \alpha$ = accept null

p-val $< \alpha$ = reject null

Multiple Popolation: Dependant Sample

Ej. La pildora de magnesio en sangre

Hypotesis:

$$\begin{cases} H_0: \mu_{Before} \geq \mu_{After} = \mu_B - \mu_A \geq 0 = D_0 \geq 0 \\ H_1: \mu_B < \mu_A = \mu_B - \mu_A < 0 = D_0 < 0 \\ H_0: D_0 \geq 0 \\ H_1: D_0 < 0 \end{cases}$$

1. Calculamos los datos de la diferencia

$\bar{d} = -0.33$

Standar deviation $= 0.45$

Standard Error $= 0.14$

$$T = \frac{\bar{d} - \mu_0}{St.error} = \frac{-0.33 - 0}{0.14} = -2.29$$

p-value $= 0.024$

α.

if $\alpha = 0.05$ $\quad p < \alpha \Rightarrow$ reject hypotesis

$\alpha = 0.01$ $\quad p > \alpha \Rightarrow$ accep hypotesis

Jest independent Samples : known variance
Multiple Population

Ej : grades engineering vs manadgment

$H_0 : \mu_E - \mu_M = -4\%$

$H_1 = \quad " \quad \neq -4\%$

Two sidez since we want to know id is not exactly -4%

	E	M	Diff
n	100	70	?
$\bar{x}$	58%	65%	-7%
Pop StD	10%	6%	1.23%

$$\sqrt{\frac{\sigma_e^2}{n_e} + \frac{\sigma_m^2}{n_m}}$$ Standard error

Big Sample (>30) so we use Z

$$Z = \frac{\bar{x} - \mu_0}{\text{StD error}} = \frac{(-7\%) - (-4\%)}{1.23\%} = -2.44$$

p-value = 0.015

α		
0.05	> p	- reject
0.01	< p	- accept

SAMPLE SIZE

VARIANCE	Big	Small
known	Z-Statistic	t-statistic
Unknown	Depend on researcher but is good to go Z-Statistic	t-statistic

Clarification on

Ej:

α	
$p < 0.5$ - reject	Estoy a un 95% seguro de que la null hypotesis es falsa pero no a un 99% seguro
$p > 0.1$ - accept	

Testing $\begin{cases} \text{INDEPENDENT SAMPLES} \\ \text{UNKNOWN VARIANCE - ASSUMED EQUAL} \end{cases}$

5. diferencia precio manzanas en NY y LA

$H_0: \mu_{NY} - \mu_{LA} = 0$

$H_1: \quad '' \quad \neq 0$

	NY	LA
Mean	3.94	3.28
Std dev	0.18	0.27
n	10	8

1. Use pool variance

$$s_p^2 = \frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2} =$$

$$= \frac{(10-1)0.18^2 + (8-1)0.27^2}{10+8-2} = 0.05 = s_p$$

Pooled variance: 0.05

Pooled std = 0.11

Degrees of freedom = 10+8-2 = 16

$$T = \frac{\bar{d} - \mu_0}{\text{St error}} = 6.53$$

p-value = 0.0000001

Normally with $\underset{\substack{\uparrow \\ 2}}{T}$ or $\underset{\substack{\uparrow \\ 4}}{Z}$ scores higher than 2 or 4 respectively we will reject the hypotesis.

Also if p value have more than 3 zeros in decimals we reject it