

# Linear Regression

It is linear approximation of a casual relationship between two or more variables.

process

GET SAMPLE  
DATA

→

Design model  
that works for  
that sample

→ Make predictions  
for whole population

Types of variable - Dependent (predicted) -  $Y = F(x_1, x_2, \dots, x_k)$   
Independent  $x_1, x_2, \dots, x_k$

## TYPES OF REGRESSIONS - SIMPLE REGRESSION

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$

\*  $y$  = dependent variable

$x$  = independent variable

$\beta_1$  = quantify the dependent effect

$\beta_0$  = constant base

$\epsilon$  = error (on average is 0)

{ Ej: income ( $y$ ) depends on years of education ( $x$ )

{ In USA  $\beta_1 \approx 5000$   $\beta_0$  = minimum wage

Equation

$$\hat{y} = b_0 + b_1 x_1$$

\* when we had a

hat  $\hat{y}$  it means  
is an estimate or predicted  
value

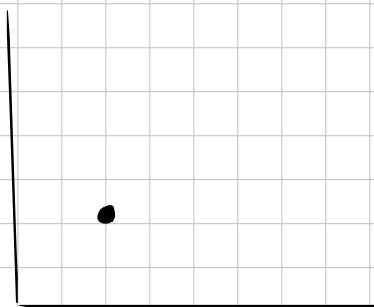
# CORRELATION vs REGRESSION

□ Correlation do not imply causation □

## CORRELATION

- ▷ Imply relationship
- ▷ Variables move together
- ▷  $p(x, y) = p(y, x)$

### SINGLE POINT

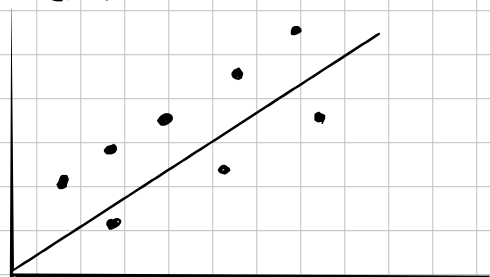


GRAPH  
IN  
ECONOMICS  
+  
STATISTICS

## REGRESSION

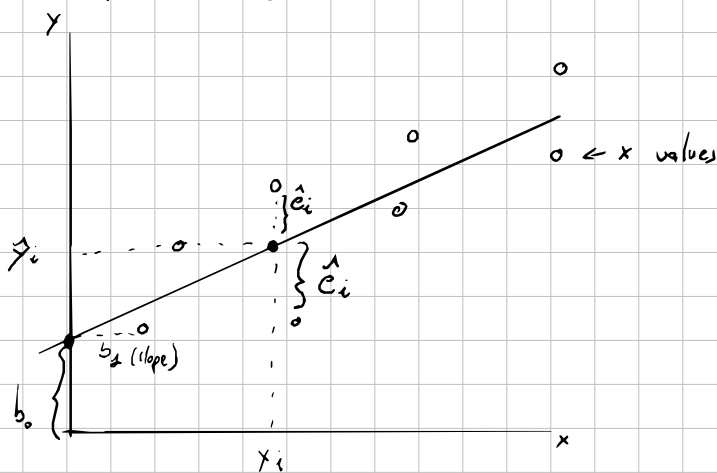
- ▷ One variable affect the other
- ▷ Instead of move together is more a cause and effect situation
- ▷  $p(x, y) \neq p(y, x)$  E.g. Income and education  
education usually mean more income but not the other way

### LINE



## GEOMETRICAL REPRESENTATION OF LINEAR REGRESSION

$$\hat{y} = b_0 + b_1 x_1$$



# Decomposition of Variability

## ▷ SST / TSS (Sum of Squares Total)

- Sum of the total variability of the dataset

$$\sum_{i=1}^n (y_i - \bar{y})^2$$

## ▷ SSR - Sum of Squares Regression - ESS

- Measure of how well the line fit the data

$$\sum_{i=1}^n (\hat{y}_i - \bar{y})^2$$

\* If  $SSR = SST$  it means your regression model is perfect and capture all the observed variability

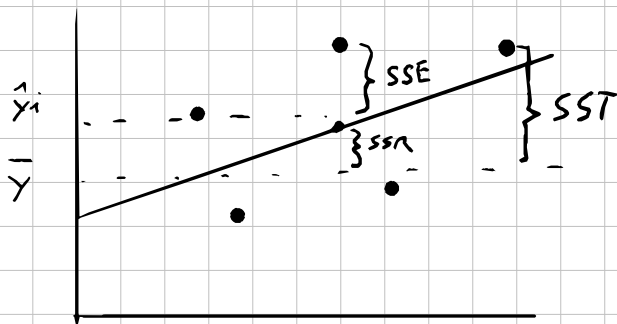
## ▷ SSE - Sum of Squares Error - RSS

- Measure the unexplained variability by the regression

$$\sum_{i=1}^n e_i^2$$

Total Variability = Explained Variability + Unexplained Variability

$$\underline{\underline{SST = SSR + SSE}}$$



What is the OLS