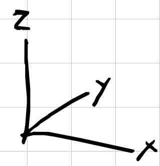
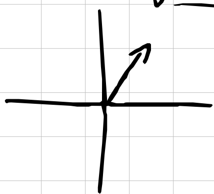
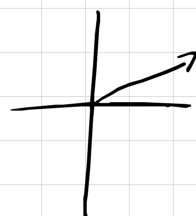
$$\begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



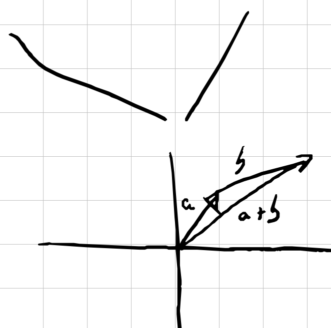
Sum of Vectors



$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad + $$



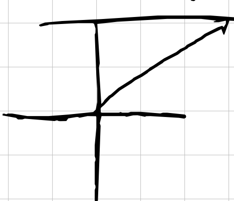
$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}$$

Mult (Scaling)



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot 2 = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$\hat{i} = 1_x$
$\hat{j} = 1_y$ $\Big\}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ | Unidades basicas de medida de vectores "basis vectors" |

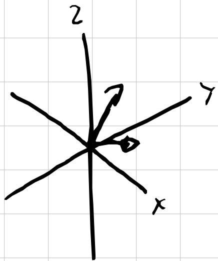
$(3)\hat{i} + (-2)\hat{l} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

Linear combination of $\vec{v}$ and $\vec{w}$

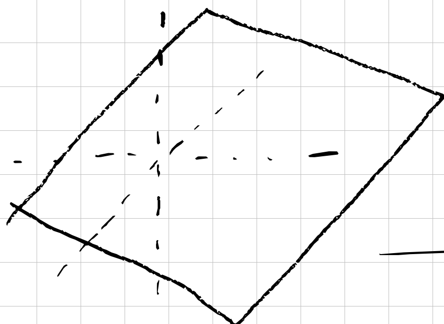
$\underbrace{a\vec{v} + b\vec{w}}_{\text{Scalars}}$

Si solo alteramos el scalar de uno de los vectores formaremos lineas rectas en el espacio.

$\boxed{\text{Span:}}$ conjunto de todas las combinaciones lineales de dos vectores, si ambos estan sobre la misma linea son todos los puntos de la misma, si estan en diferentes sera todo el espacio



El span de 2 vectores en 3 dimensiones visualiza como una pantalla, un espacio 2d en el espacio de 3d



Ya que todos sus combinaciones pasaron por ambos

3 Vectores

$a\vec{v} + b\vec{w} + c\vec{u}$

Si el 3° Vector esta en el Span de los otros 2 te sigues quedando en ese espacio 2d. En caso de no estarlo extiende el Span a todo el espacio 3d

Independencia y Dependencia Lineal

Cuando al añadir un vector no incrementa el Span se le llama "Linearly dependent).
En caso de añadir al span se les llama Linearly independent

The basis of a Vector space is a set of linearly independent vectors that span the full Space

Las 2 definiciones de Independencia lineal

- $a\vec{v} + b\vec{w} + c\vec{u} = \vec{0}$ Solo es posible si $a = b = c = 0$

$\vec{u} \neq a\vec{v} + b\vec{w}$

La interpretación mas intuitiva es el pensamiento de que todos estan fuera del span de los otros 2

Linear Transformation/function

Tratamos de transformación por la facilidad de imaginar ese movimiento

Reglas de la transformación lineal

1. Las lineas no deben curvarse

2. El origen debe ser fijo

Un pensamiento para imaginar esto es mantener los grid lines paralelos.

Descripcion Numerica

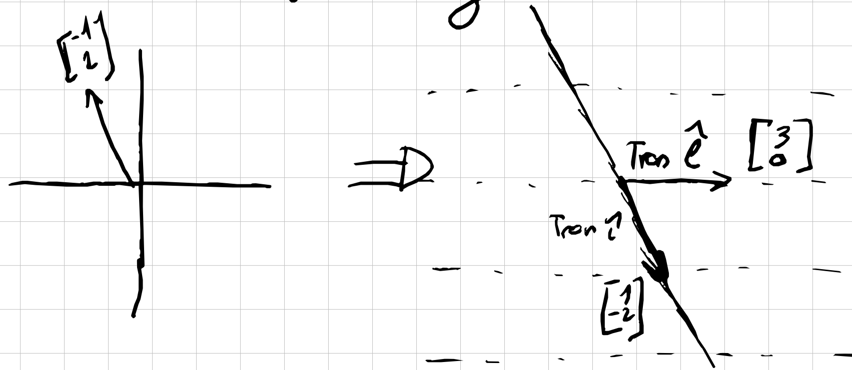
Eg $\begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow \vec{v} = -1\hat{i} + 2\hat{j}$

$$\Downarrow$$

$$\text{Transformed } \vec{v} = -1(\text{transformed } \hat{i}) + 2(\text{transformed } \hat{j})$$

Ejemplo $\Rightarrow$

Ejemplo Tranformacion con conocimiento de $\hat{i}$ y $\hat{e}$



$$\vec{v} = -1\hat{i} + 2\hat{e}$$

$$Tr\,\vec{v} = -1(Tr\,\hat{i}) + 2(Tr\,\hat{e})$$

$$Tr\,\vec{v} = -1\begin{bmatrix}1\\-2\end{bmatrix} + 2\begin{bmatrix}3\\0\end{bmatrix}$$

$$\vec{v} = \begin{bmatrix}-1\\2\end{bmatrix} + \begin{bmatrix}6\\0\end{bmatrix}$$

$$\vec{v} = \begin{bmatrix}5\\2\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix} \to x(\hat{i}) + y(\hat{e})$$

$$\begin{bmatrix}x\\y\end{bmatrix} \to x\begin{bmatrix}\hat{i}^{\hat{i}}\\ \hat{i}^{\hat{e}}\end{bmatrix} + y\begin{bmatrix}\hat{e}^{\hat{i}}\\ \hat{e}^{\hat{e}}\end{bmatrix} = \begin{bmatrix} x\cdot\hat{i}^{\hat{i}} + y\cdot\hat{e}^{\hat{i}} \\ x\cdot\hat{i}^{\hat{e}} + y\cdot\hat{e}^{\hat{e}}\end{bmatrix}$$

$$x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix} = \underline{\begin{bmatrix}ax+by\\cx+dy\end{bmatrix}}$$

Si los vectores $\hat{i}$ y $\hat{e}$ se transforman en linear dependent todos los resultados lo serán

Formula final transformación

L = Transformación

$$\begin{bmatrix}x\\y\end{bmatrix} \Rightarrow x\begin{bmatrix}\hat{i}^{\hat{i}}\\ \hat{i}^{\hat{e}}\end{bmatrix} + y\begin{bmatrix}\hat{e}^{\hat{i}}\\ \hat{e}^{\hat{e}}\end{bmatrix} = \begin{bmatrix} x\cdot\hat{i}^{\hat{i}} + y\cdot e^{\hat{i}} \\ x\cdot\hat{i}^{\hat{e}} + y\cdot\hat{e}^{\hat{e}}\end{bmatrix}$$

$$L\vec{v} = x\,L(\hat{i}) + y\,L(\hat{e}) \Rightarrow x\begin{bmatrix}a\\c\end{bmatrix}+y\begin{bmatrix}b\\d\end{bmatrix} = \begin{bmatrix}ax+by\\cx+dy\end{bmatrix}$$

Composition of Rotation or Shear

Eg 90° & Shear

$$\begin{bmatrix}x\\y\end{bmatrix}\underbrace{\begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}}_{\text{Composition}} = \underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\text{Shear}}\left(\underbrace{\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}}_{\text{Rotation}}\begin{bmatrix}x\\y\end{bmatrix}\right)$$

Read right to left $\leftarrow$ $f(g(x))$

$$\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\text{Shear}}\underbrace{\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}}_{\text{Rotation}} = \begin{bmatrix}1 & -1\\1 & 0\end{bmatrix}$$

$$x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix} = \begin{bmatrix}ax+by\\cx+dy\end{bmatrix}$$

$$\overbrace{\begin{bmatrix}0&2\\1&0\end{bmatrix}}^{M_2}\overbrace{\begin{bmatrix}1&-2\\1&0\end{bmatrix}}^{M_1} = \;?$$

$$\begin{bmatrix}x\\y\end{bmatrix} \to x\begin{bmatrix}\hat{\imath}\\\hat{\imath}\end{bmatrix} + y\begin{bmatrix}\hat{\ell}\\\hat{\ell}\end{bmatrix} = \begin{bmatrix}x\cdot\hat{\imath} + y\cdot\hat{\ell}\\x\cdot\hat{\imath} + y\cdot\hat{\ell}\end{bmatrix}$$

$$\hat{\imath} = \begin{bmatrix}0&2\\1&0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = 1\begin{bmatrix}0\\1\end{bmatrix} + 1\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}1\cdot 0 + 1\cdot 2\\1\cdot 1 + 1\cdot 0\end{bmatrix} = \begin{bmatrix}1+2\\1+0\end{bmatrix} = \begin{bmatrix}3\\1\end{bmatrix}$$

$$\hat{\ell} = \begin{bmatrix}0&2\\1&0\end{bmatrix}\begin{bmatrix}-2\\0\end{bmatrix} = -2\begin{bmatrix}0\\1\end{bmatrix} + 0\begin{bmatrix}2\\0\end{bmatrix} = \begin{bmatrix}0+0\\2\quad 0\end{bmatrix} = \begin{bmatrix}0\\2\end{bmatrix}$$

Formula Transformacion multiple

$$\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}e&f\\g&h\end{bmatrix} = \begin{bmatrix}ea+gb & af+bh\\ec+gd & cf+dh\end{bmatrix}$$

$$M_2M_1 \neq M_1M_2$$

EL ORDEN IMPORTA

$$\overbrace{\begin{bmatrix}0&2\\1&0\end{bmatrix}}^{M_2}\overbrace{\begin{bmatrix}1&-2\\1&0\end{bmatrix}}^{M_1} = \begin{bmatrix}2&2\\1&-2\end{bmatrix}$$

$$\begin{bmatrix}-3&1\\2&5\end{bmatrix}\begin{bmatrix}5&3\\7&-3\end{bmatrix} = \begin{bmatrix}(-3)(5)+(1)(7) & (-3)(3)+(-3)(1)\\(2)(5)+(5)(7) & (2)(3)+(-3)(5)\end{bmatrix}$$

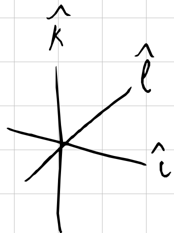
$$\begin{bmatrix}-15+7 & (-9)+(-3)\\10+35 & 6+(-15)\end{bmatrix}$$

$$\begin{bmatrix}-8&-12\\45&-9\end{bmatrix}$$

$$(AB)C = A(BC)$$

Matrix multiplication is associative

$$A(BC) = (AB)C$$



3D VECTORS

$$\begin{array}{ccc} \hat{i} & \hat{l} & \hat{k} \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x\hat{i} + y\hat{l} + z\hat{k}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xa + yb + zc \\ xd + ye + zf \\ xg + yh + zi \end{bmatrix}$$

$$\overbrace{\begin{bmatrix} 0 & -2 & 2 \\ 5 & 1 & 5 \\ 1 & 4 & -1 \end{bmatrix}}^{2^\circ \text{ trans}} \underbrace{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}}_{1^\circ \text{ Trans}}$$

$$\begin{bmatrix} (0)(0) + (3)(-2) + (6)(2) & (1)(0) + (4)(-2) + (7)(2) & (2)(0) + (5)(-2) + (8)(2) \\ (0)(5) + (3)(1) + (6)(5) & (1)(5) + (4)(1) + (7)(5) & (2)(5) + (5)(1) + (8)(5) \\ (0)(1) + (3)(4) + (6)(-1) & (1)(1) + (4)(4) + (7)(-1) & (2)(1) + (5)(4) + (8)(-1) \end{bmatrix}$$

$$\begin{bmatrix} 0 + (-6) + 12 & 0 + (-8) + 14 & 0 + (-10) + 16 \\ 0 + 3 + 30 & 5 + 4 + 35 & 10 + 5 + 40 \\ 0 + 12 + (-6) & 1 + 16 + (-7) & 2 + 20 + (-8) \end{bmatrix}$$

$$\begin{bmatrix} 6 & 6 & 6 \\ 33 & 44 & 55 \\ 6 & 10 & 14 \end{bmatrix}$$

$$x\begin{bmatrix}a\\c\end{bmatrix} + y\begin{bmatrix}b\\d\end{bmatrix} = \begin{bmatrix}ax + by\\cx + dy\end{bmatrix}$$

2D

$$\begin{bmatrix}x\\y\end{bmatrix} \rightarrow x\begin{bmatrix}\hat{\imath}\hat{\imath}\\\hat{\imath}\hat{\ell}\end{bmatrix} + y\begin{bmatrix}\hat{\ell}\hat{\imath}\\\hat{\ell}\hat{\ell}\end{bmatrix} = \begin{bmatrix}x\cdot\hat{\imath}\hat{\imath} & + y\cdot\hat{\ell}\hat{\imath}\\x\cdot\hat{\imath}\hat{\ell} & + y\cdot\hat{\ell}\hat{\ell}\end{bmatrix}$$

Formula Transformacion multiple

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}e & f\\g & h\end{bmatrix} = \begin{bmatrix}ea + gb & af + bh\\ec + gd & cf + dh\end{bmatrix}$$

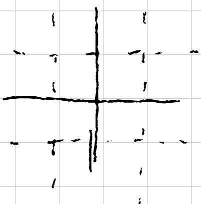
3D - BASICO

$$\begin{bmatrix}a & b & c\\d & e & f\\g & h & i\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}xa + yb + zc\\xd + ye + zf\\xg + yh + zi\end{bmatrix}$$

Determinante $\begin{bmatrix}a & b\\c & d\end{bmatrix} \rightarrow \underline{ad - bc}$

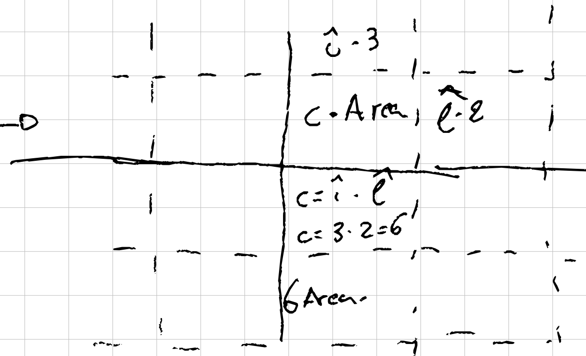
Calcutate of the area in the matrix

c· Area

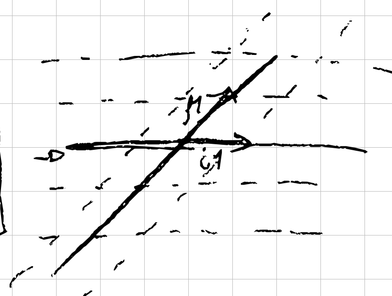


$\rightarrow \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \rightarrow$

$\hat{i}\cdot 3$
$\hat{e}\cdot 2$



$\rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow$



$c = 1\cdot 1 = 1$

se mantiene el area

DETERMINANT OF A TRANSFORMATION

$\det\left(\begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}\right) = 6$

Si el det es 0 la transforma en una dimension menos

Si durante una transformacion $\hat{i}$ y $\hat{e}$ cambian su referencia frente a la otra de izq a drch su orientación se a alterado. Cuando esto sucede el DET es negativo

En 3d

$$\det\left(\begin{bmatrix} 1 & 0 & 0.5 \\ 0.5 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\right) =$$ Volumen del parallelipiped (cubo deformado) que se genera

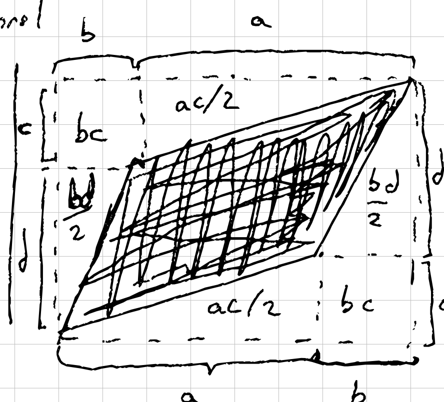
Si es 0 significa que es de 2d o 1d y por ello volumen 0

FORMULAS DET 2D

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \underbrace{ad}_{\text{incremento básico}} - \overbrace{bc}^{\text{deformación en diagonal}}$$

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+b)(c+d) - ac - bd - 2bc$$

$$= ad - bc$$



$$\det(M_1 M_2) = \det(M_1)\det(M_2)$$

FORMULAS DET 3D

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a\det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) - b\det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right) + c\det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$$

LINEAR SYSTEM OF EQUATIONS

$$2x + 5y + 3z = -3$$
$$4x + 0y + 8z = 0$$
$$1x + 3y + 0z = 2$$

$$\longrightarrow \underbrace{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}}_{\vec{v}}$$

$$A\vec{x} = \vec{v}$$

$$\vec{x} = A^{-1}\vec{v}$$

$$2x + 2y = -4$$

$$1x + 3y = -1$$

$$\underset{A}{\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}} \underset{\vec{x}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{\vec{v}}{\begin{bmatrix} -4 \\ -1 \end{bmatrix}} \longrightarrow ? \begin{cases} \det(A) = 0 & \text{"No solution"} \\ \det(A) \neq 0 \rightarrow \vec{x} = A^{-1}\vec{v} \end{cases}$$

Inverse transformation $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}^{-1}$ A^{-1}

Identity transformation $A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\vec{x} = A^{-1}\vec{v}$$

RANKS

number of dimensions in the output / Column ~~Space~~ span

Eg: 2d Matrix
independent → Rank 2
dependent → Rank 1

Eg 3d: independent Rank 3
dependent rank 2 or 1

Less rank more restrictive

Si es [illegible] Matriz "Full RANK"

NULL SPACE / KERNEL

Todos los vectores que acaban en el punto de origen