Ejercicio 2

a) $AX = B$ a compatible solo si

$$rg(A) = rg(A|B) = r$$

$r = n$ sistema determinado

$r < n$ sistema indeterminado

$rg(A) < rg(A|B)$ seria incompatible

b) $\begin{pmatrix} a+1 & 2 & 1 & 0 \\ 1 & a & 1 & a-1 \\ 1 & 1 & 1 & a \end{pmatrix}$

$$\begin{vmatrix} a+1 & 2 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix} \xrightarrow{c_1 - c_3} \begin{vmatrix} a & 2 & 1 \\ 0 & a & 1 \\ 0 & 1 & 1 \end{vmatrix} \begin{pmatrix} a & 1 & 1 \\ 0 & a-1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$rg(A) = a(a-1) \rightarrow \underline{a = 0,1} \rightarrow r = 2$

$rg(A) = 2-1 = -1$ $a > 1 \rightarrow r = 3$

$$\begin{pmatrix} a+1 & 2 & 1 & 0 \\ 1 & a & 1 & a-1 \\ 1 & 1 & 1 & a \end{pmatrix} \xrightarrow{} \begin{vmatrix} 1 & 0 & a+1 \\ 1 & a-1 & 1 \\ 1 & a & 1 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & a & a+1 \\ 1 & a-1 & 1 \\ 1 & 0 & a+1 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & -1 & a \\ 1 & a-1 & 1 \\ 1 & 0 & a+1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & a-1 & 1 \\ 0 & -1 & a \\ 1 & 0 & a+1 \end{vmatrix} = \begin{vmatrix} 1 & a-1 & 1 \\ 0 & -1 & a \\ 1 & -1 & 1 \end{vmatrix}$$

$$\begin{pmatrix} 1 & a-1 & 1 \\ 0 & -1 & a \\ 1 & -1 & 1 \end{pmatrix} \Rightarrow \begin{vmatrix} 1 & -1 & 1 \\ 0 & -1 & a \\ 1 & a-1 & 1 \end{vmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & -1 & a \\ 0 & a- & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 & \cdot 1 \\ 0 & a & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & a & 0 \\ -1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 1 \\ 0 & a & 0 \\ -2 & -1 & 0 \end{vmatrix} \quad \begin{matrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & a & 0 \end{matrix} \quad \begin{matrix} 1 & -1 & 1 \\ 0 & -1 & 2 \\ 0 & a & 0 \end{matrix}$$

$$\begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & a \end{vmatrix} = 2a$$

$$\begin{vmatrix} a+1 & 2 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix} = a(a-1)$$

$$\begin{matrix} 0 \\ a-1 \\ a \end{matrix}$$

$$x_1 = \frac{\begin{vmatrix} 0 & 2 & 1 \\ a-1 & a & 1 \\ a & 1 & 1 \end{vmatrix}}{a(a-1)}$$

$$x_2 = \frac{\begin{vmatrix} a+1 & 0 & 1 \\ 1 & a-1 & 1 \\ 1 & a & 1 \end{vmatrix}}{a(a-1)}$$

$$x_3 = \frac{\begin{vmatrix} a+1 & 2 & 0 \\ 1 & a & a-1 \\ 1 & 1 & a \end{vmatrix}}{a(a-1)}$$