Ej 1:

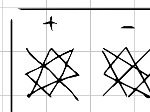
$a \in \mathbb{R}$

$$A = \begin{pmatrix} a+2 & 1 & 1 \\ 1 & a+2 & 1 \\ 1 & 1 & a+2 \end{pmatrix} \in \mathcal{M}_3(\mathbb{R}) \text{ simetrica}$$

$\lambda = ?$

$\det(A - \lambda I)$

SARRUS



$\det(A - \lambda I_3) \Rightarrow$ Restar a la diagonal λ

$$\begin{vmatrix} a+2-\lambda & 1 & 1 \\ 1 & a+2-\lambda & 1 \\ 1 & 1 & a+2-\lambda \end{vmatrix} = \left(1 + 1 + (a+2-\lambda)^3\right) - \left(a+2-\lambda + a+2-\lambda + a+2-\lambda\right)$$

$a - \lambda = x$

$c1 + c2 + c3$

$$(a+2-\lambda)^3 + (2-6) + (-3a) + (-3\lambda)$$

$$\begin{vmatrix} a+4-\lambda & 1 & 1 \\ a+4-\lambda & a+2-\lambda & 1 \\ a+4-\lambda & 1 & a+2-\lambda \end{vmatrix} = (a+4-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 1 & a+2-\lambda & 1 \\ 1 & 1 & a+2-\lambda \end{vmatrix}$$

$$= (a+4-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 0 & a+1-\lambda & 0 \\ 0 & 0 & a+1-\lambda \end{vmatrix} = \underbrace{(a+4-\lambda)(a+1-\lambda)^2}_{\text{Polinomio Caracteristico}}$$

$\lambda_1 = a+4$

$\lambda_2 = a+1$

$\lambda_1 = a+4$

$\lambda_2 = a+1$

$$\text{rang}(A - \lambda_1 I_3) = 1$$

$$\text{rang}(A - \lambda_2 I_3) = 2$$

$(a+2)-(a+4)$

$$\text{rang}(A - (a+4) I_3) = \text{rg}\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} = 2$$

$(a+2)-(a+1)$

$$\text{rg}(A - (a+1) I_3) = \text{rg}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = 1$$