Ejercicio 1

determinante orden $n \in \mathbb{Z}^+$ con $\alpha \in \mathbb{R}$

$$|A| = \begin{vmatrix} \alpha^2 & n & n & \cdots & n \\ n & \alpha^2 & n & \cdots & n \\ n & n & \alpha^2 & \cdots & n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n & n & n & \cdots & \alpha^2 \end{vmatrix} = \begin{vmatrix} \alpha^2/n & 1 & \cdots & 1 \\ 1 & \alpha^2/n & \cdots & 1 \\ \vdots & \vdots & \ddots & \\ 1 & 1 & & \alpha^2/n \end{vmatrix}$$

a) Calcular determinante

$$\det(A) = |A| = \begin{vmatrix} \alpha^2 + (n-1)n & n & \cdots & n \\ \alpha^2 + (n-1)n & \alpha^2 & \cdots & n \\ \vdots & \vdots & \ddots & \\ \alpha^2 + (n-1)n & n & & \alpha^2 \end{vmatrix}$$

$$(\alpha^2 + (n-1)n) \begin{vmatrix} 1 & n & n & n & \cdots & n \\ 1 & \alpha^2 & n & n & \vdots & n \\ 1 & n & \alpha^2 & n & \vdots & n \\ 1 & n & n & \alpha^2 & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 1 & n & n & n & & \alpha^2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & \cancel{n \quad n \quad \cdots \quad n} \\ 0 & \alpha^2 - n & 0 & \cdots & 0 \\ 0 & 0 & \alpha^2 - n & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & \alpha^2 - n \end{vmatrix} = (\alpha^2 + (n-1)n)(\alpha^2 - n)^{n-1}$$

$$(\alpha^2 + (n-1)n)(\alpha^2 - n)^{n-1}$$

b) Valores α el determinante vale 0

$$\pm\sqrt{n} \to (\sqrt{n}^2 + (n-1)n)\underbrace{(\sqrt{n}^2 - n)}_{0}^{n-1}$$

$$(n + (n-1)n)\overbrace{(n - n)}^{0}{}^{n-1}$$

$$0$$

$$0$$

c) $\to \pm 3$

$$(\alpha^2 + (9-1)9)(\alpha^2 - 9)^8$$

$$\underbrace{(\alpha^2 + 72)}_{\neq 0}(\alpha^2 - 9)^8$$

Ejercicio 2

a) $AX=B$ es compatible solo si

$$rg(A) = rg(A|B) = r$$

$r = n$ sistema determinado

$r < n$ sistema indeterminado

$rg(A) < rg(A|B)$ sería incompatible

b) $$\begin{cases} (a+1)x + 2y + z = 0 \\ x + ay + z = a-1 \\ x + y + z = a \end{cases}$$

$$\left(\begin{array}{ccc|c} a+1 & 2 & 1 & 0 \\ 1 & a & 1 & a-1 \\ 1 & 1 & 1 & a \end{array}\right) \Rightarrow \begin{pmatrix} 1 & 1 & 1 & a \\ 1 & a & 1 & a-1 \\ a+1 & 2 & 1 & 0 \end{pmatrix}$$

$$f_2 - f_1 \begin{pmatrix} 1 & 1 & 1 & a \\ 0 & a-1 & 0 & -1 \\ a+1 & 2 & 1 & 0 \end{pmatrix} \quad f_3 - (a+1)f_1 \begin{pmatrix} 1 & 1 & 1 & a \\ 0 & a-1 & 0 & -1 \\ 0 & -a+1 & -a & -(a+1) \end{pmatrix}$$

$-1f_1 \quad -(a+1)f_1$

$$\begin{bmatrix} 1 & 1 & 1 & a \\ 0 & a-1 & 0 & -1 \\ 0 & -a+1 & -a & -(a+1) \end{bmatrix} \quad f_3 - f_2 \quad \begin{bmatrix} 1 & 1 & 1 & a \\ 0 & a-1 & 0 & -1 \\ 0 & 0 & -a & -a-2 \end{bmatrix}$$

$$\begin{cases} x + y + z = a \\ 0 + (a-1)y + 0 = -1 \\ 0 \quad 0 \quad -az = -a-2 \end{cases}$$

$$(a-1)y = -1 \Rightarrow y = \frac{-1}{-a-1}$$

$$-az = -a-2 \qquad z = \frac{-a-2}{-a}$$

$$x + \left(\frac{-1}{-a-1}\right) + \left(\frac{-a-2}{-a}\right) = a$$

$$-x = \left(\frac{-1}{-a-1}\right) + \left(\frac{-a-2}{-a}\right) - a$$

$$rg(A) = \begin{pmatrix} a+1 & 2 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Calculamos determinante

$$rg(A) \qquad \begin{vmatrix} a+1 & 2 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Big((a+1)\cdot(a)\cdot 1\Big) + (2) + (1)$$

$$-$$

$$\Big((a) + (2) + (a+1)\Big)$$

$$\Big((a+1)(a)\Big) + 2 + 1 - a - 2 - a - 1$$

$$\Big((a+1)(a) + (-2a)\Big)$$

$$\vdots$$

rg

$$A = \begin{pmatrix} a+1 & 2 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ a-1 \\ a \end{pmatrix}$$

$rg(A) \Rightarrow$

$$|A| \begin{vmatrix} a+1 & 2 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix} \underset{C_2 - C_3}{\overset{C_1 - C_3}{=}} \begin{vmatrix} a+1 & 1 & 1 \\ 0 & a-1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$