

Crossnumbers

Quarterly

Issue



Jan 2020



Editorial

Welcome to 2020 and our fourth year of publication. Doesn't time just fly by when you're having fun? Don't answer that!

This issue has the first of a four part 'whodunnit' mystery and at the end of the year there will be a prize of £100 for the first correct entry drawn from those correct.

The final quarter of 2019 for The Magpie had *Hexaflex* by EP, *Follow the Diagonals* by Googly and *Leave Nothing Out* by Arden.

Oyler had the pleasure of test solving *Hexaflex* for the setters. It was a puzzle set in hexadecimal with an interesting set of clues that were mainly based around cubes. The grid was a strange shape and once filled had to be converted to letters from a code given. Then the fun part (or not so fun part if you have arthritic fingers) as solvers had to assemble a flexagon or infinity cube then flex it in order to read the information it contained leading to A Night at the Opera, Bohemian Rhapsody and 5min 55secs. A most inventive puzzle.

Next up was the master setter Googly with *Follow the Diagonals* which was a letter/number assignment puzzle. The diagonals when coded back to letters gave instructions as to what had to happen to four of the entries. The puzzle was expertly plotted and gave up its secrets in a steady fashion with some lovely deductions. The only quibble was some 4-digit entries with 3 unchecked cells which is a no-no. Only Googly could get away with that!

The final Magpie mathematical of the year was Arden's *Leave Nothing Out* which bore more than a passing resemblance to a couple of puzzles that appeared in CQ! We were given nine 3x3 grids with the six clues satisfying all nine grids. Each grid contained nine distinct digits with each grid having a different digit from 1 to 9 absent. Once filled solvers then had to submit a single 3x3 grid correctly numbered and barred. A terrific logical challenge.

The final Listener mathematical of the year was *Full Steam Ahead* by Hedgehog which was a letter/number assignment puzzle using the numbers from 1 to 11 inclusive. Hedgehog certainly used his skill as a crossword setter here as each answer had to have a number from 1 to 11 inserted to form the entry and is a technique normally seen in crossword puzzles where a letter has to be inserted to form the entry. The inserted numbers when converted back to letters and taken in clue order spelt out an instruction for solvers to follow which was relevant to the title. Great puzzle and in some ways it was similar to his puzzle a few months ago in The Magpie.

This year the sixteen puzzles in The Listener and The Magpie have been set by 12 different setters. This is two more than last year thanks in part to the new setting duo of EP.

This quarter's puzzles are listed below with the easier ones at the start and the hard ones at the end.

Puzzle Title	Setter
One More	Oyler
Mistransmissions	Rad
Pentomino Square	Moog
Cosy	Zag
Magnificent Seven	MatriX
Ternary II	Michael Peake
Prime Reversal pairs	Oyler
Square Dance	Arden
Kluedo	The Wizards of OZ

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One More by Oyer

All the entries are one more than their clue answer. References in clues refer to the entry and not the answer. No entry starts with zero and all are distinct.

1	2	3	4	5
6		7	8	
9		10		11
	12			

Across

- 3 equals 5dn [2]
- 6 triangular [3]
- 8 prime [2]
- 9 square [2]
- 10 prime [3]
- 12 equals 9dn [2]

Down

- 1 palindrome [2]
- 2 9ac x 9dn [4]
- 4 8ac x 5dn [4]
- 5 square [2]
- 7 square [2]
- 9 triangular [2]
- 11 cube [2]

Mistransmissions by Rad

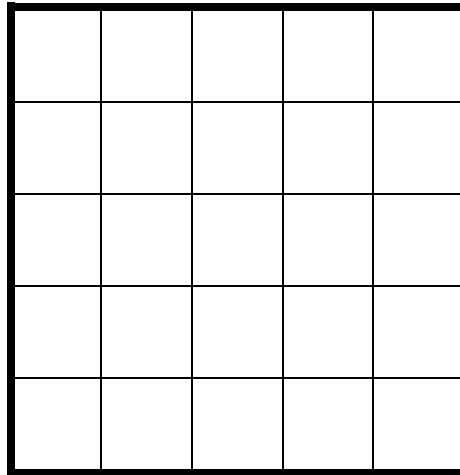
A common error in the transmission of numerical data is the replacement of AAB by ABB or vice versa.

Eight distinct 3-digit primes have been transmitted in this way and the resulting numbers are entered in the grid. Four of the entries are themselves prime and no two of these are mistransmissions of each other. The other four at 1ac, 2dn, 3dn and 8ac are each 13 times a prime. The digits in the four corner cells are distinct.

1	2		3
4	5	6	
7			
	8		

Pentomino Square by Moog

Five different pentominoes have each been assigned to a distinct non-zero digit with that digit appearing five times in each shape. The letters in the clues are all prime numbers with the same letter representing the same prime throughout and are in ascending order. The clues are products of the primes which give a 5-digit entry. There are no zeros and all entries are distinct. Solvers should highlight the pentominoes used in the final grid.



Row Top to bottom

Column L → R

CFK

AAQ

BCN

ADO

BBBBCL

AIL

AAAAP

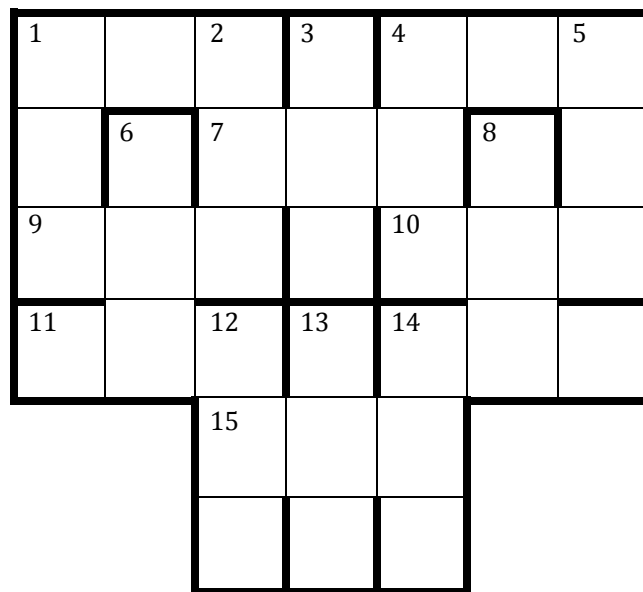
AABBG I

ABHM

AEFJ

Cosy by Zag

All entries are distinct 3-digit 'cosy' numbers. A 'cosy' number is one where the largest digit is the sum of the two smaller ones and allows the case where the largest digit is repeated so 990 is an example of a 'cosy' triangular number. A 'friendly' number is divisible by its digit sum. To dispel any final uncertainty the sum of the 34 entered digits is also a 'cosy' number. No entry starts with zero.



Across

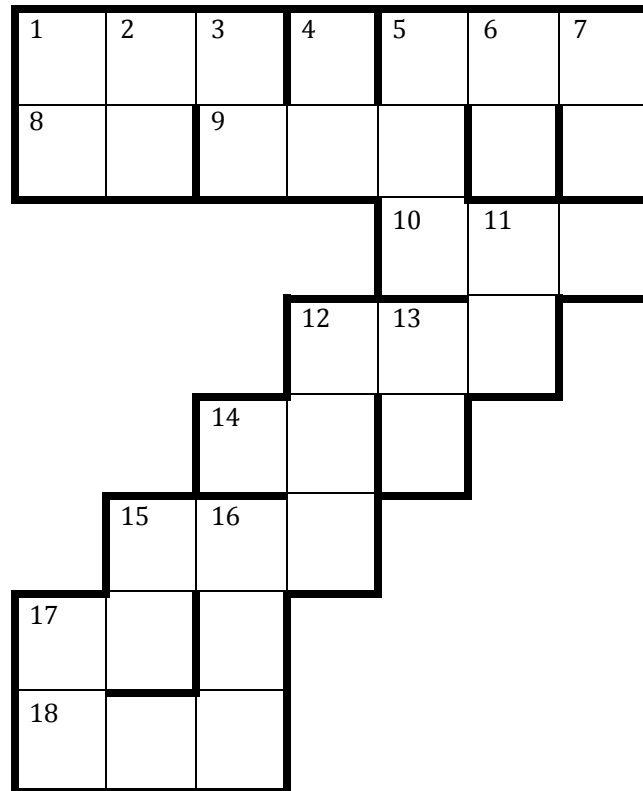
- 1 square
- 4 square
- 7 triangular
- 9 prime
- 10 jumble of 7ac
- 11 prime
- 14 prime
- 15 friendly

Down

- 1 jumble of 5dn
- 2 prime
- 3 square
- 4 reverse of a prime
- 5 see 1dn
- 6 friendly
- 8 friendly
- 12 prime
- 13 prime
- 14 prime

Magnificent Seven by MatriX

Only seven of the digits from 0 to 9 inclusive appear. The 2-digit entries are all prime numbers and only those with additional properties are clued. A friendly number is divisible by its digit sum. Entries are distinct and have ***no repeated digits***. No entry starts with zero.



Across

- 1** jumble of 16dn [3]
- 5** square [3]
- 9** jumble of 18ac [3]
- 10** triangular [3]
- 12** square [3]
- 15** friendly [3]
- 18** see 9ac [3]

Down

- 1** digit sum of 13dn [2]
- 5** triangular [3]
- 11** sum of the 7 digits used [2]
- 12** triangular [3]
- 16** composite [3]

Ternary II by Michael Peake

For each entry in Grid A, find the digit sum, convert to base 3, and place in Grid B at the same place. The entries in Grid A are all distinct, but that need not apply to Grid B. Clues apply to both grids. No entry begins with zero.

Grid A (Base 10)

1	2	3	4	5	6
7		8		9	
10	11		12	13	
14		15	16		17
18	19	20		21	
22			23		

Grid B (Base 3)

1	2	3	4	5	6
7		8		9	
10	11		12	13	
14		15	16		17
18	19	20		21	
22			23		

Across

- 1** square [3]
- 4** prime [3]
- 9** triangular [2]
- 12** prime [3]
- 16** triangular [3]
- 18** prime [2]
- 21** twice a triangular number [2]
- 22** prime [3]

Down

- 3** triangular [3]
- 6** prime [3]
- 15** prime [3]
- 17** square [3]
- 19** triangular [2]

Prime Reversal Pairs by Oyler

Some non-palindromic prime numbers when reversed remain prime numbers and as such form a 'prime reversal pair'. Each letter in the clues refers to a 'prime reversal pair'. Within a clue a letter can take either of its two possible values. For example if X was the 347/743 pair then a clue $X + X$ could be $347 + 347$, $743 + 743$ or $347 + 743$ giving an entry of 694, 1486 or 1090 respectively. No entry starts with zero and all are distinct.

1	2	3	4	5	6	7
	8	9		10		
11				12		
	13				14	15
16			17		18	

Across

- 1 A [2]
- 3 B [2]
- 5 $BB + B - F$ [3]
- 8 BD [5]
- 11 $A + A$ [3]
- 12 G [3]
- 13 CE [5]
- 16 $H + H$ [3]
- 17 B [2]
- 18 $B - B$ [2]

Down

- 1 F [2]
- 2 $A + B$ [2]
- 4 BG [5]
- 6 $A + F - F$ [2]
- 7 $J = \text{sum of entered digits}$ [3]
- 9 D [3]
- 10 $G - A - F$ [3]
- 11 C [3]
- 13 E [2]
- 14 E [2]
- 15 $E + E$ [2]

Square Dance by Arden

Answers must perform a square dance. Clues describe a property of their answer, all of which are distinct as are all entries. Some entries have leading zeros.

1		2	3	4	5	
6	7	8				9
	10		11	12		
13		14			15	
16		17	18	19		20
	21	22	23		24	
25				26		

Across

- 1** Fibonacci [3]
- 4** triangular [3]
- 6** cube [2]
- 8** palindrome [3]
- 10** factorial [3]
- 12** has 3 distinct prime factors [3]
- 14** cube [3]
- 16** prime [3]
- 18** composite [3]
- 22** fourth power [3]
- 24** perfect [2]
- 25** prime [3]
- 26** prime [3]

Down

- 1** composite [3]
- 2** perfect [3]
- 3** prime [2]
- 4** abundant [3]
- 5** square [2]
- 7** equals digit sum of its cube [3]
- 9** prime [3]
- 11** square [3]
- 13** triangular [3]
- 15** prime [3]
- 17** Fibonacci [3]
- 19** prime [3]
- 20** composite [3]
- 21** square [2]
- 23** prime [2]

Kluedo by The Wizards of OZ

Introduction

Four famous fictional detectives; Sherlock Holmes, Jane Marple, Hercule Poirot and John Rebus have been invited to a house in Oxford. They all arrive and the following morning one of them is found dead. Beside the body are four logic grids that will allow solvers to work out who died, how they died, in which room they died and who did it.

Your guides through the puzzle are The Wizards of OZ who are Oyler and Zag. Thanks go to Arden for coming up with this idea for a pseudonym a couple of years ago.

The first grid is the victim and method of dispatch.

The victim can be Holmes (Ho), Marple (Ma), Poirot (Po) or Rebus (Re).

The method of dispatch can be cord (Co) suitable for strangulation/hanging, an electrocution risk (El), a knife (Kn) or poison (Pn).

Grid 1 : Victim / Method

Some entries have clues with multiple parts which are separated by a colon (:). In these cases solvers must decide which part has to be used and reject the red herring. An even digit in a cell gives a positive response in that that character could have died by that method. Whilst an odd digit means that it didn't happen. No entry starts with zero and all entries are distinct.

v / m	Co	El	Kn	Pn
Ho	A a		B b	c
Ma	C	d		
Po	e	D		f
Re	E		F	

Across

- A** 3F : 7F
- B** triangular
- C** prime : palindrome
- D** Fe – e : Ff – f
- E** triangular
- F** prime

Down

- a** triangular
- b** multiple of B : multiple of C
- c** square
- d** multiple of A : multiple of B
- e** B + c : B – c
- f** prime

Solutions for Issue 12

Doubles by Chalicea

¹ 2	1	² 2	2	³ 1	5
⁴ 7	2	⁵ 9	⁶ 1	8	⁷ 1
⁸ 2	⁹ 1	0	¹⁰ 4	2	0
2	3	¹¹ 6	5	¹² 3	6
¹³ 3	0	¹⁴ 1	3	6	1

11ac and 12ac are the only possible solutions and fixed as 65 and 36 (so 130 and either 18 or 72 must appear elsewhere in the grid).

3ac could be 15 or 10 but its digits have to sum to the same as the digits of 10ac.

100 is not a triangle number so 100 is excluded at 10ac and 3ac must be 15.

3ac is a factor of 13ac, which can only be 30 (as $45 - 4 + 5$ - does not give a prime).

This fixes the third digit of 8ac and 10ac as 0.

See 1d: only one factor (13) is shared by 5ac and 11ac so 5ac must be 91

1ac must be one of 1222, 2122, 2212 or 2221. 2d can only, therefore be 2906.

(1906 does not sum to a prime).

1d has to add to 13, its first digit can only be 2 (it is some form of 2/2/2/7)

4ac is the only remaining 2 digit answer so must be $2 \times 12ac = 72$, fixing 1d as 2722

3d must be the double of 5ac = 182

As 8ac and 10ac are multiples of 13ac, they are 210 and 420

Only 130 fits 9d (as a multiple of 13 and double 65)

1061, 2122 and 1361 have to fill the empty cells (as the only possible entries fulfilling the doubles requirement).

Five by Five by Peter Chamberlain

¹ 1	² 5	³ 3	⁴ 7	⁵ 6
⁶ 2	5	⁷ 6	7	6
⁸ 5	⁹ 3		¹⁰ 8	¹¹ 1
¹² 2	3	¹³ 3	¹⁴ 1	6
¹⁵ 2	1	¹⁶ 7	3	9

10ac/14ac are 16 or 81 in some order. From 11dn though 10ac must be 81 and 14ac 16 to give 11dn as 169. 14dn can only be 13 so 16ac can only be 739. 12ac can be 144 or 233 and 15ac can be 21 or 89. If 12ac is 144 then 12dn is 12 or 18 which doesn't fit the clue so 12ac is 233. Hence 12dn is 22, 15ac 21 and 13dn is 37. 9dn can be 131 or 331. There are 5 possible entries for 1dn, 125, 216, 343, 512 and 729. We can eliminate 729 from 8ac and 216 and 512 from 6ac. If 1dn is 343 then 6ac is 49 and 1ac can be 325, 351 or 378. All of these fail on the 2dn clue. So 1dn is 125, 6ac is 25 and 8ac is 53. 1ac can be 136, 153 or 171. The latter fails on 2dn. Regardless of whether 1ac is 136 or 153 3dn ends in 6 so 7ac can be 625 or 676. If it's 625 then 5dn can be 15, 45 or 55. Only 15 would fit with 4ac as 11 however 4dn would be 128 which fails the clue. So 6ac is 676 with 5dn as 36 or 66. However 36 has no fit for 4ac but 66 does. So 5dn is 66 and 4ac 76. Hence 1ac must be 153 and 3dn 36.

Six by Three by Oyler

¹ 2	² 3	³ 1	⁴ 9	⁵ 0	⁵ 5
⁶ 4	4	9	⁷ 6	⁸ 2	5
0	⁹ 8	6	1	¹⁰ 8	3

Consideration of the terminal digits of primes, squares and triangular numbers is one way to proceed. 6ac is prime so its last digit is odd but not 5. This is the penultimate digit of 3dn, a square, so the square must end in a 6. Possible entries for 9ac are 561, 861 or 465 and for 3dn are 196, 576 or 676. Therefore 7ac must start with an even digit. If 3dn is 676 then 3ac would be forced to be 630 and 4dn 361 but that grid would contain too many 6s. If 9ac is 465 then 4dn is either 225 or 625. If it's 225 then 7ac is forced to be 256 with 8dn 55 which has too many 5s. If it's 625 then 7ac is forced to be 225 however 3ac would then be 861 which fails on 3dn. So 9ac/4dn ends in 1. 3dn 576 fails as 3ac would be 595 and 7ac 625 so too many 5s. So 3dn is 196. 3ac can only be from 136 or 190. The former fails in that 4dn would be 361 so too many 6s. So 3ac is 190 and 4dn 961. 7ac is 625, 8dn is 28 and 10ac is 83. 2dn can't be 13 so must be 34. The digit not used is 7. 6ac can only be 449. So 5dn is 55 and 9ac is 861. 1ac is 23 and 6dn is 40 which is the sum of the entered digits 76 – square (36).

Fibonacci Bridges by Zag

¹ 6	² 1	³ 6		⁴ 1	⁵ 4	⁶ 9
⁷ 4	⁸ 4	⁸ 2	3	3	⁹ 2	2
¹⁰ 3	¹¹ 1	0		¹² 8	¹³ 3	4
	4				7	
¹⁴ 8	¹⁵ 4	¹⁵ 3		¹⁶ 6	7	3
¹⁷ 2	¹⁸ 3	¹⁹ 6	1	0	²⁰ 1	0
8	6	1		²¹ 1	1	0

All four connections are Fibonacci and they must be 144, 233, 377, 610 or 987.

Consider elements like 1ac in the four quadrants each the product of two overlapping 2-digit numbers. The top left quadrant is one of the 1ac,2dn,7ac combinations for some p&q: 1p1, p1&q1 or p9&q9; 4p4, p2&q2 or p8&q8; 5p5, p5&q5; 6p6, p4&q4 or p6&q6; 9p9, p3&q3 or p7&q7. Size considerations limit the options to just one; 1ac=616, 2dn=14, 7ac=44. The bottom left quadrant 18dn=16, 25, 36, 49, 64 or 81 with corresponding 14dn: 6p6, 0p0 (invalid), 8p8, 6p6, 4p4, 8p8 respectively. Size considerations limit this to 14dn=828, 17ac=23, 18dn=36. The top right quadrant 9ac can be 11, 22, 33, 44, 55, 66 (higher amounts produce 4-digit 3dn). The only solution is 6dn=924, 9ac=22, 9dn=42. The bottom right quadrant 20dn is one of 11, 22 (duplicate) or 33 with 21ac,20dn,20ac solution 110,11,10.

12ac is an anagram of 14ac. 13dn cannot start 8 so 12ac is 8p4 for some p and 14ac=84p fixing 11dn=144. 3dn starting 62, 63, 66 or 69 is a multiple of 10ac having 1 as a middle digit allowing 3dn,10ac solutions: 620,310; 630,210; 660,110 (duplicate 110); 696,116. If 3dn=630 or 696 then 8ac is 377 or 987 and 4dn=178. However any possible fill falls short of that confirming 3dn=620, 10ac=310 with 8ac=233.

12ac is one of 834, 864, 894 with corresponding 14ac of 843, 846, 849. The 15dn,19ac,13dn candidates are: 361,610,377; 961,610,987. The latter's digits already total 142 exceeding the 138 at 4dn confirming 14ac=843, 15dn=361, 19ac=610, 12ac=834, 13dn=377. The total of the digits is now 129 requiring 9 for the outside digits of 16ac. Only prime 16ac=673 allows a prime 601 at 16dn.

Factor Sums by Oyler

¹ 2	² 5	³ 6	⁴ 5	⁵ 6	⁶ 6
⁶ 2	⁷ 0	⁸ 8	⁹ 1	¹⁰ 7	¹¹ 2
¹² 6	¹³ 8	¹⁴ 6	¹⁵ 5	¹⁶ 7	¹⁷ 5
¹⁸ 4	¹⁹ 2	²⁰ 8	²¹ 7	²² 9	²³ 4
²⁴ 4	²⁵ 3	²⁶ 9	²⁷ 4	²⁸ 2	²⁹ 9

Each entry must be less than its clue. The formula given which was $\prod_{i=1}^k \frac{p_i^{\alpha_i+1}-1}{p_i-1}$ allows you to find the sum of all of the factors if you know the prime factors of the number. For example 84 is $2^2 \times 3 \times 7$ and inputting these gives $\frac{2^3-1}{2-1} \cdot \frac{3^2-1}{3-1} \cdot \frac{7^2-1}{7-1} = 7 \cdot 4 \cdot 8 = 224$. It is easiest to tackle those whose sum of the factors have one or two factors. So the factor sum of 2dn 93 is just 3×31 and you need to find two prime numbers that will give a 3 and a 31. Now 3 is just $\frac{2^2-1}{2-1}$ so we have 2 as a factor of our entry. 31 is $\frac{5^3-1}{5-1}$ so it's 5^2 and our number is $2 \times 5^2 = 50$. 6ac can be 10, 20 30 or 40. Of these the first is obviously too small and the last two too large hence the entry is 20. Knowing part of a 2-digit entry allows you, by trial and error, to calculate the possible factor sums from the formula and aids to the solving. The only real stumbling blocks are at 4ac, 11ac and the bottom two rows. However careful consideration of the possible entries and given that all entries are distinct leads to the grid above.

Prime Coding by Zag

¹ 9	² 2	³ 4	⁴ 3	⁵ 8	⁶ 7	⁷ 7
⁷ 6	⁸ 5	⁹ 5	¹⁰ 3	¹¹ 8	¹² 6	¹³ 7
¹⁴ 2	¹⁵ 2	¹⁶ 8	¹⁷ 3	¹⁸ 8	¹⁹ 2	²⁰ 9

For the coding 3 primes form a pandigital set using the digits 1 to 9. The sum of the 3 primes is 3-digit which means the primes themselves are 3-digit otherwise there would be a 4-digit prime. The endings are from 1,3,7,9. If 1 is an ending then the total of the 3 primes must exceed 1000 as the first digits are at least 2,3,4; the end digits 1,7,9; the middle digits 5,6,8 for a total of 1100. The endings are thus 3,7,9. The lead digits can be 1,2,4 or 1,2,5. The former has middle digits of 5,6,8 for a total of 909 which has an ineligible zero. The latter has middle digits of 4,6,8 for a total of 999.

Primes with a lead digit of 1,2,5 with a middle digit of 4,6,8 and a final digit of 3,7,9 are:
 149, 163, 167, 263, 269, 283, 547, 563, 569, 587. Either 149 or 547 is present combined with either 283 or 587 for 4&8 to appear. This allows:
 149+283 leaving 5,6,7 which cannot form a prime,
 149+587 leaving 2,3,6 and 263 is prime,
 547+283 leaving 1,6,9 and with 1 fixed in the lead position no prime is possible.
 The 3 primes are 149, 263 and 587. No digit codes to itself so 456 must code to 149. That forces 123 to code to 587, 789 to 263 and the full coding is 123456789 to 587149263.

With italics for answers and regular font for entries, $4dn=999,333$. The palindrome at $12ac$ must form a digit product with middle digit 9 at $8ac$. This forces $12ac=838$, $8ac=192,538$. $5dn=12,58$ or $22,88$ allowing $5ac$ entries of: 516, 525, 534, 543, 552, 561, 859, 868, 877, 886, 895. The corresponding pre-entry values are: *148, 171, 195, 159, 117, 184, 261, 228, 233, 282, 216*. Of these only *233* is prime confirming the $5ac=877$, $5dn=88$. $6dn$ starting 3 dividing the $6dn$ entry starting 7 permits $6dn$ =entries of 36,72; 37,74; 38,76 or 39,78 with only 38,76 a consistent pairing. If $9ac=89$, there is no $10dn$ so $9ac=83,67$ and $10nd=36,79$.

The $11ac$ ending plus the $10dn$ ending of 9 codes to 5 in $3dn$ so $11ac$ ends 2. The first digit of the $7ac$ answer encodes to a digit less than itself limiting $7ac$ to *49,13* or *81,65*. The only combination that work for $7ac,11ac=65,22$; $7dn=65+22=87,62$.

The $2dn+13ac$ square is 100 so the $13ac,2dn,3dn$ possible answers are: *19,81,41* ($2dn$ =duplicate 65); *29,71,51* (allows *576* as $1ac$ reverse square); *39,61,61* (duplicate 61); *49,51,71* (no $1ac$ reverse square); *59,41,81* ($3dn$ =duplicate 65); *69,31,91* (no $1ac$ reverse square). This confirms $13ac=29$, $2dn=71,25$, $3dn=51,45$, $1ac=675,924$.

Frequency for Primes by MatriX

¹ 1	² 3			³ 4	7
⁴ 2	3	⁵ 3	⁶ 1	3	⁷ 1
5	1	⁸ 1	1	1	2
⁹ 7	2	7	¹⁰ 3	4	7

Suppose no digit occurs with zero frequency. The grid requires 22 digits. 3dn&4dn need 6 digits between them which must be 1,2,3,4,5,7. Consider the 3 digit combinations including a 1 together with the complementary combinations using the other 3 digits. Combinations like 1,2,3 can be excluded as any permutation is divisible by 3. 137 can be eliminated as there is no prime ending left for the partner. This leaves 3dn&4dn combinations: 251 or 521 & 347 or 743; 431 & 257.

1ac is prime so 2dn must start 1,3 or 7. This allows 3ac,2dn pairings of: 23,113 or 757; 41,337; 47,331 or 733 or 773; 53, 353 or 727. The 1ac,6ac pairings are: 11,313 or 773; 13,131 or 311; 23,233; 31,113; 41,443; 47,277 or 353.

If 3dn=251 or 257, 6ac=353 but no variation fits 2dn. If 3dn=521, no 6ac is possible. If 3dn=347 or 743, 6ac=443 and no variation fits 2dn. If 3dn=431, 4dn=257, 6ac can be 131 or 233;

If 6ac=233, 3ac=starts 4 and there is no valid 2dn. If 6ac=131, 1ac=13, 3ac starts 4 and then 3ac=47, 2dn=331; 4ac=233 is forced.

So far 1,2,3,4,5,7 occur with frequencies 5,1,4,1,1,2. Of the 8 unfilled cells, 4 require a prime ending. 2,4,5 require at least 3 more to ensure they have different frequencies. With the 11 prime digits present either 15 or 16 prime endings are possible which means they either use the frequencies 3,5,7 or 4,5,7.

With frequencies 3,5,7 the four prime digits can be 1,1,3,7 or 3,3,3,7. If 8ac ends 1, 6dn=113, 8ac=41, 5dn=347, 10ac ends 1 but none of 321, 341 or 351 are prime. If 8ac ends 3, 6dn=131 or 137, 8ac=23 (forms square with 13), 43 (forms a triangular with 233) or 53 (forms a square with 3ac=47) so 8ac cannot end 3 either. If 8ac ends 7, 6dn=173, 8ac=47, 5dn has to be 347 but no more 7's are available. This establishes 1,3,7 use the frequencies 4,5,7.

The additional prime digits can be 1,1,7,7,7 or 1,1,3,7,7 or 3,3,3,7,7. If there is no additional 3 then 6dn starting 11 or 17 has no valid entry. If there is no 1 then if 8ac ends 7, 6dn=173, 8ac=37 (forms triangular with 173) or 47 (duplicates 3ac). If 8ac ends 3 then 8ac is one of 23

(forms square with 13), 43 (forms triangular with 233), 53 (forms square with 47), 73 (forms a triangular with 47). This means the additional prime digits are 1,1,3,7,7 confirming 1 has frequency 9, 3 frequency 5 and 7 frequency 4.

Possible 8ac answers are 11, 37, 41, 71. All others form illegal squares or triangulars in combination with another answer. If 8ac=37, 6dn=113 requiring two 3's with only one available. If 8ac=71, 5dn=373, 6dn=113 again requiring two 3's. If 8ac=11, 6dn=113, so 9ac&10ac end 7. 5dn=317, 10an=347, 9an=727 or 757 (forms triangular with 233), 7dn=127 or 157 and frequency considerations for 2 dictate 7dn must have middle digit 2. This yields a solution with 3 palindromes 11, 131, 727.

If 8ac=41, 5dn=347, 6dn=113, 9ac=727. Another 2 is required forcing 7dn=127 and only 2 palindromes 131&727 appear so this case is eliminated.

Square Enough by Nod

¹ 1	² 7	³ 2	⁴ 1	⁵ 3	⁶ 4	⁷ 5	⁸ 2	⁹ 2
⁹ 9	9	2	2	6	¹⁰ 4	3	5	7
¹¹ 9	2	5	¹² 9	7	2	1	9	7
¹³ 2	2	6	7	5	¹⁴ 1	¹⁵ 5	2	2
¹⁶ 5	¹⁷ 1	2	5	2	4	3	2	2
¹⁸ 3	3	6	5	7	¹⁹ 6	²⁰ 8	²¹ 4	2
²² 7	4	1	²³ 3	2	2	7	6	6
²⁴ 6	5	6	2	²⁵ 4	4	5	2	2
5	7	²⁶ 1	5	2	2	7	5	7

Using the notation A1 to mean the answer to clue 1ac and EA1 to mean the entry at 1ac, etc, a possible solution starts as follows:

D6 = SumDigits = 3 \Rightarrow (D6, ED6) \in {(12, 145), (21, 442)} \Rightarrow 2[ED6] = 4 = 1[EA10]
A10 = palindrome and EA10 (4) starts with 4 \Rightarrow **A10 = 66 & EA10 = 4357**

D23 = divisible by its digit sum \in {10, 12, 18, 20, 21, 24, 27, 30}
 \Rightarrow ED23 \in {~~101~~, 145, 325, ~~401~~, 442, 577, ~~730~~, ~~901~~}
 \Rightarrow (D23, ED23) \in {(12, 145), (18, 325), (21, 442), (24, 577)}
D4 = 2 * D23 \Rightarrow (D4, ED4, D23, ED23) \in {(~~24~~, ~~577~~, ~~12~~, ~~145~~), (36, 1297, 18, 325), (42, 1765, 21, 442), (~~48~~, ~~2305~~, ~~24~~, ~~577~~)}
 \Rightarrow (D4, ED4, D23, ED23) \in {(36, 1297, 18, 325), (42, 1765, 21, 442)}

A14 = D6 + D23, eliminating entries with zero leaves:
D6 = 21 \Rightarrow D23 $<$ 21 \Rightarrow A14 = 39, D23 = 18, D4 = 36
ED6 = 442, EA14 = 1522, ED23 = 325, ED4 = 1297

A20 = prime, can't end 3 or 7 else EA20 ends 0, so must end 1 or 9 and EA20 ends in 2
ED8 = .7.222. = D8 ^ 2 + 1
 \Rightarrow D8 \in {1335, 1339, 1665, 1668, 1932} but D8 has 12 factors \Rightarrow D8 \in {1665, 1668}
 \Rightarrow (D8, ED8) \in {(1665, 2772226), (1668, 2782225)}
Now A12 can't end in 8 \Rightarrow **D8 = 1665, ED8 = 2772226**

D7 = palindrome; ED7 = .5.2. \Rightarrow **D7 = 161, ED7 = 25922**

EA12 = 9.2.97 \Rightarrow A12 * A12 = 9.2.96 \Rightarrow **A12 = 986, EA12 = 972197**
A12 has strictly decreasing digits

The Terrible Thirties by Moog

A ^a	1	9	3	2	5 ^d
6	B ^e	1	0	C	7
D ^f	1	9	E ^g	3	7
1	F	3	8	6	2

The start to all these puzzles is to find the prime factorisation for each year as shown below. The clues split into four sets of two. A cannot be 1930 as c would start with 0 so A is either a misprint or refers to another year. If it's a misprint then it would have to be F however the clue $F/(h - c)$ must have a misprint in the bracket and would lead to either a duplicated year or a year less than 1000. So A is not a misprint but refers to a different year so the first three digits of A are 193. Thus the clue $c(a - b)$ must have a misprint in the bracket. There are 3 candidates for both clue and year correct; 1932, 1936 and 1938. With, admittedly, a fair bit of effort using the factor pairs for 1932 which don't contain a single digit as they are impossible to get from the terms in the brackets the remainder can be eliminated. There are a few close calls though that have both B and f staring with 1 but they ultimately fail. So the years 1936 and 1938 must have the correct clues with no typos. Thus $a = 16$ and $f = 11$. This tells us that the clue for 1937 must have a misprint as $a > f$. If $f > a$ then it could still be valid. There are two pairs for dg 34/57 or 38/51 in some order. The year 1931 is correct but the clue contains a typo and possible entries for F are 3862, 5793 or 7724. There is only one possibility for the second in the set which is the year 1939 with $c(a - b)$ and the typo in the bracket so c is 277 and the typo is a. Thus A is the year 1932. If Be is correct then it can only be 1930 as 1937 would have F starting with 9 which it can't. So B is 10 and e is 193 with $F = 3862$, $g = 38$ and $d = 51$. So h must be 32. This fills the grid and accounts for the remaining clues. See below. The following table details the ordering with the misprints in red italics.

Year	Factorisation	Clue	Correct
1930	2.5.193	A	Be
1931	1931	$F/(h - c)$	$F/(h - \textcolor{red}{b})$
1932	2 ² .3.7.23	$(e - D)(Bf + C - h)$	A
1934	2.967	Be	$h(\textcolor{red}{A} - e(a - f))/a$
1936	2 ⁴ .11 ²	aff	aff
1937	13.149	$h(B - e(a - f))/a$	$(\textcolor{red}{h} - D)(Bf + C - h)$
1938	2.3.17.19	dg	dg
1939	7.277	$c(a - b)$	$c(\textcolor{red}{E} - b)$

Table of square numbers

1	121	441	961	1681	2601	3721	5041	6561	8281
4	144	484	1024	1764	2704	3844	5184	6724	8464
9	169	529	1089	1849	2809	3969	5329	6889	8649
16	196	576	1156	1936	2916	4096	5476	7056	8836
25	225	625	1225	2025	3025	4225	5625	7225	9025
36	256	676	1296	2116	3136	4356	5776	7396	9216
49	289	729	1369	2209	3249	4489	5929	7569	9409
64	324	784	1444	2304	3364	4624	6084	7744	9604
81	361	841	1521	2401	3481	4761	6241	7921	9801
100	400	900	1600	2500	3600	4900	6400	8100	10000

Table of triangular numbers

1	231	861	1891	3321	5151	7381
3	253	903	1953	3403	5253	7503
6	276	946	2016	3486	5356	7626
10	300	990	2080	3570	5460	7750
15	325	1035	2145	3655	5565	7875
21	351	1081	2211	3741	5671	8001
28	378	1128	2278	3828	5778	8128
36	406	1176	2346	3916	5886	8256
45	435	1225	2415	4005	5995	8385
55	465	1275	2485	4095	6105	8515
66	496	1326	2556	4186	6216	8646
78	528	1378	2628	4278	6328	8778
91	561	1431	2701	4371	6441	8911
105	595	1485	2775	4465	6555	9045
120	630	1540	2850	4560	6670	9180
136	666	1596	2926	4656	6786	9316
153	703	1653	3003	4753	6903	9453
171	741	1711	3081	4851	7021	9591
190	780	1770	3160	4950	7140	9730
210	820	1830	3240	5050	7260	9870

Table of Fibonacci and Lucas numbers

Fibonacci				Lucas			
1	8	89	987	1	18	199	2207
1	13	144	1597	3	29	322	3571
2	21	233	2584	4	47	521	5778
3	34	377	4181	7	76	843	9349
5	55	610	6765	11	123	1364	15127

Table of cubes

1	64	343	1000	2197	4096	6859
8	125	512	1331	2744	4913	8000
27	216	729	1728	3375	5832	9261

Harshad or Niven or Zag's Friendly numbers

1	10	100	200	300	400	500	600	700	800	900
2	12	102	201	306	402	504	603	702	801	902
3	18	108	204	308	405	506	605	704	803	910
4	20	110	207	312	407	510	612	711	804	912
5	21	111	209	315	408	511	621	715	810	915
6	24	112	210	320	410	512	624	720	820	918
7	27	114	216	322	414	513	629	730	825	935
8	30	117	220	324	420	516	630	732	828	936
9	36	120	222	330	423	518	640	735	832	954
	40	126	224	333	432	522	644	736	840	960
	42	132	225	336	440	531	645	738	846	966
	45	133	228	342	441	540	648	756	864	972
	48	135	230	351	444	550	660	770	870	990
	50	140	234	360	448	552	666	774	874	999
	54	144	240	364	450	555	684	777	880	
	60	150	243	370	460	558	690	780	882	
	63	152	247	372	465	576		782	888	
	70	153	252	375	468	588		792		
	72	156	261	378	476	592				
	80	162	264	392	480	594				
	81	171	266	396	481					
	84	180	270	399	486					
	90	190	280							
		192	285							
		195	288							
		198								

Table of primes < 1000

2	101			401		601	701		
3	103				503				
5	107		307			607			907
7	109			409	509		709	809	
11		211	311					811	911
13	113		313			613			
17			317			617			
19				419		619	719		919
				421	521			821	
23		223			523			823	
	127	227					727	827	
29		229						829	929
31	131		331	431		631			
		233		433			733		
37	137		337						937
	139	239		439			739	839	
41		241			541	641			941
43				443		643	743		
47			347		547	647			947
	149		349	449					
	151	251					751		
53			353			653		853	953
	157	257		457	557		757	857	
59			359			659		859	
61				461		661	761		
	163	263		463	563			863	
67	167		367	467					967
		269			569		769		
71		271			571				971
73	173		373			673	773		
		277			577	677		877	977
79	179		379	479					
	181	281						881	
83		283	383			683		883	983
				487	587		787	887	
89			389						
	191			491		691			991
	193	293			593				
97	197		397				797		997
	199			499	599				

4-digit primes

1009	1367	1723	2113	2521	2897	3329	3719	4133	4567	4999
1013	1373	1733	2129	2531	2903	3331	3727	4139	4583	5003
1019	1381	1741	2131	2539	2909	3343	3733	4153	4591	5009
1021	1399	1747	2137	2543	2917	3347	3739	4157	4597	5011
1031	1409	1753	2141	2549	2927	3359	3761	4159	4603	5021
1033	1423	1759	2143	2551	2939	3361	3767	4177	4621	5023
1039	1427	1777	2153	2557	2953	3371	3769	4201	4637	5039
1049	1429	1783	2161	2579	2957	3373	3779	4211	4639	5051
1051	1433	1787	2179	2591	2963	3389	3793	4217	4643	5059
1061	1439	1789	2203	2593	2969	3391	3797	4219	4649	5077
1063	1447	1801	2207	2609	2971	3407	3803	4229	4651	5081
1069	1451	1811	2213	2617	2999	3413	3821	4231	4657	5087
1087	1453	1823	2221	2621	3001	3433	3823	4241	4663	5099
1091	1459	1831	2237	2633	3011	3449	3833	4243	4673	5101
1093	1471	1847	2239	2647	3019	3457	3847	4253	4679	5107
1097	1481	1861	2243	2657	3023	3461	3851	4259	4691	5113
1103	1483	1867	2251	2659	3037	3463	3853	4261	4703	5119
1109	1487	1871	2267	2663	3041	3467	3863	4271	4721	5147
1117	1489	1873	2269	2671	3049	3469	3877	4273	4723	5153
1123	1493	1877	2273	2677	3061	3491	3881	4283	4729	5167
1129	1499	1879	2281	2683	3067	3499	3889	4289	4733	5171
1151	1511	1889	2287	2687	3079	3511	3907	4297	4751	5179
1153	1523	1901	2293	2689	3083	3517	3911	4327	4759	5189
1163	1531	1907	2297	2693	3089	3527	3917	4337	4783	5197
1171	1543	1913	2309	2699	3109	3529	3919	4339	4787	5209
1181	1549	1931	2311	2707	3119	3533	3923	4349	4789	5227
1187	1553	1933	2333	2711	3121	3539	3929	4357	4793	5231
1193	1559	1949	2339	2713	3137	3541	3931	4363	4799	5233
1201	1567	1951	2341	2719	3163	3547	3943	4373	4801	5237
1213	1571	1973	2347	2729	3167	3557	3947	4391	4813	5261
1217	1579	1979	2351	2731	3169	3559	3967	4397	4817	5273
1223	1583	1987	2357	2741	3181	3571	3989	4409	4831	5279
1229	1597	1993	2371	2749	3187	3581	4001	4421	4861	5281
1231	1601	1997	2377	2753	3191	3583	4003	4423	4871	5297
1237	1607	1999	2381	2767	3203	3593	4007	4441	4877	5303
1249	1609	2003	2383	2777	3209	3607	4013	4447	4889	5309
1259	1613	2011	2389	2789	3217	3613	4019	4451	4903	5323
1277	1619	2017	2393	2791	3221	3617	4021	4457	4909	5333
1279	1621	2027	2399	2797	3229	3623	4027	4463	4919	5347
1283	1627	2029	2411	2801	3251	3631	4049	4481	4931	5351
1289	1637	2039	2417	2803	3253	3637	4051	4483	4933	5381
1291	1657	2053	2423	2819	3257	3643	4057	4493	4937	5387
1297	1663	2063	2437	2833	3259	3659	4073	4507	4943	5393
1301	1667	2069	2441	2837	3271	3671	4079	4513	4951	5399
1303	1669	2081	2447	2843	3299	3673	4091	4517	4957	5407
1307	1693	2083	2459	2851	3301	3677	4093	4519	4967	5413
1319	1697	2087	2467	2857	3307	3691	4099	4523	4969	5417
1321	1699	2089	2473	2861	3313	3697	4111	4547	4973	5419
1327	1709	2099	2477	2879	3319	3701	4127	4549	4987	5431
1361	1721	2111	2503	2887	3323	3709	4129	4561	4993	5437

5441	5851	6299	6737	7193	7639	8101	8581	9007	9437	9883
5443	5857	6301	6761	7207	7643	8111	8597	9011	9439	9887
5449	5861	6311	6763	7211	7649	8117	8599	9013	9461	9901
5471	5867	6317	6779	7213	7669	8123	8609	9029	9463	9907
5477	5869	6323	6781	7219	7673	8147	8623	9041	9467	9923
5479	5879	6329	6791	7229	7681	8161	8627	9043	9473	9929
5483	5881	6337	6793	7237	7687	8167	8629	9049	9479	9931
5501	5897	6343	6803	7243	7691	8171	8641	9059	9491	9941
5503	5903	6353	6823	7247	7699	8179	8647	9067	9497	9949
5507	5923	6359	6827	7253	7703	8191	8663	9091	9511	9967
5519	5927	6361	6829	7283	7717	8209	8669	9103	9521	9973
5521	5939	6367	6833	7297	7723	8219	8677	9109	9533	
5527	5953	6373	6841	7307	7727	8221	8681	9127	9539	
5531	5981	6379	6857	7309	7741	8231	8689	9133	9547	
5557	5987	6389	6863	7321	7753	8233	8693	9137	9551	
5563	6007	6397	6869	7331	7757	8237	8699	9151	9587	
5569	6011	6421	6871	7333	7759	8243	8707	9157	9601	
5573	6029	6427	6883	7349	7789	8263	8713	9161	9613	
5581	6037	6449	6899	7351	7793	8269	8719	9173	9619	
5591	6043	6451	6907	7369	7817	8273	8731	9181	9623	
5623	6047	6469	6911	7393	7823	8287	8737	9187	9629	
5639	6053	6473	6917	7411	7829	8291	8741	9199	9631	
5641	6067	6481	6947	7417	7841	8293	8747	9203	9643	
5647	6073	6491	6949	7433	7853	8297	8753	9209	9649	
5651	6079	6521	6959	7451	7867	8311	8761	9221	9661	
5653	6089	6529	6961	7457	7873	8317	8779	9227	9677	
5657	6091	6547	6967	7459	7877	8329	8783	9239	9679	
5659	6101	6551	6971	7477	7879	8353	8803	9241	9689	
5669	6113	6553	6977	7481	7883	8363	8807	9257	9697	
5683	6121	6563	6983	7487	7901	8369	8819	9277	9719	
5689	6131	6569	6991	7489	7907	8377	8821	9281	9721	
5693	6133	6571	6997	7499	7919	8387	8831	9283	9733	
5701	6143	6577	7001	7507	7927	8389	8837	9293	9739	
5711	6151	6581	7013	7517	7933	8419	8839	9311	9743	
5717	6163	6599	7019	7523	7937	8423	8849	9319	9749	
5737	6173	6607	7027	7529	7949	8429	8861	9323	9767	
5741	6197	6619	7039	7537	7951	8431	8863	9337	9769	
5743	6199	6637	7043	7541	7963	8443	8867	9341	9781	
5749	6203	6653	7057	7547	7993	8447	8887	9343	9787	
5779	6211	6659	7069	7549	8009	8461	8893	9349	9791	
5783	6217	6661	7079	7559	8011	8467	8923	9371	9803	
5791	6221	6673	7103	7561	8017	8501	8929	9377	9811	
5801	6229	6679	7109	7573	8039	8513	8933	9391	9817	
5807	6247	6689	7121	7577	8053	8521	8941	9397	9829	
5813	6257	6691	7127	7583	8059	8527	8951	9403	9833	
5821	6263	6701	7129	7589	8069	8537	8963	9413	9839	
5827	6269	6703	7151	7591	8081	8539	8969	9419	9851	
5839	6271	6709	7159	7603	8087	8543	8971	9421	9857	
5843	6277	6719	7177	7607	8089	8563	8999	9431	9859	
5849	6287	6733	7187	7621	8093	8573	9001	9433	9871	