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OUTPUT DEVICES, COMPUTATION, AND THE FUTURE OF MATHEMATICAL CRAFTS

ABSTRACT. This paper argues that the advent of powerful, affordable output devices offers the potential for a vastly expanded landscape of computationally-enriched mathematical craft activities in education. While mathematical crafts have a venerable history in classrooms, they have also suffered from a reputation of being both intellectually marginal and technologically retrograde. Nonetheless, this paper argues that craft activities have both intellectual and emotional affordances that are relatively lacking in 'traditional' computer-based education; and that the combination of crafts and computation (facilitated by novel output devices and materials) can render such activities still more valuable. As springboards for discussion, the paper describes three software applications geared toward computational crafts (HyperGami, HyperSpider, and MachineShop). Each of these systems highlights its own particular set of issues relevant to the development of output technologies. Using these three systems as objects-to-think-with, the paper describes a wide variety of possible craft activities that could be invented or pursued in the near future with the aid of appropriately designed output devices.

KEY WORDS: computational crafts, HyperGami, HyperSpider, MachineShop, mathematical crafts, output devices

INTRODUCTION

As I write this sentence, I am glancing over at the color printer sitting beside my screen. In the popular jargon of the computer industry, that printer is called a 'peripheral' – which, upon reflection, is a rather odd way to describe it. What, precisely, is it peripheral to? If the ultimate goal of my activity is to produce a physical artifact, then one would have to conclude that the printer is a central – maybe *the* central – technological device in sight. Possibly, the designation of the printer as a peripheral device (and for that matter, the similar designation of the screen, keyboard, and scanner also on the desk) derives from a metaphor that has acted as a leitmotif in the literature of computer architecture since the days of EDVAC – namely, the metaphor of the central processing unit as a 'brain,' equipped with an internal memory, and directing the actions of the various surrounding limbs and senses that interact with the physical world.[1]

Any metaphor potentially traceable to John Von Neumann should, on principle, be treated with the utmost respect and caution. Nonetheless,

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from time to time, it is worth re-examining such unconscious structures of language and thought, if for no other reason than to spark our imaginations and question our own biases as educational technologists. What if 'peripherals' in fact aren't? What if we take the view that the production of physical artifacts is the central creative goal of computation – that the 'hand' is as important as the 'brain,' and as worthy of our attention as computer scientists?

This paper is devoted to exploring the ramifications of this view for the domain of mathematics education – partly as an exercise, partly out of the author's genuine conviction. Undoubtedly there are a vast number of situations in which the traditional metaphor is appropriate. When we are discussing the nature of educational simulations, for example, the truly interesting portion of the activity is in fact located within the abstract realm of the program – and thus, in some sense, within the 'brain' of the central processing unit. But there are other situations in which the purpose of a computational activity is to produce a physical object. In these situations, our attention as designers should be more equitably divided between the technological potential of both the 'computer' as traditionally understood and the output device.

This refocusing of our attention on the potential of output devices is anticipated by the fascinating discussion in Neil Gershenfeld's book *When Things Start to Think* (1999). Gershenfeld describes a late-night discussion-and-design session with Mitchel Resnick and other colleagues at the MIT Media Lab; in Gershenfeld's account, the researchers played with a variety of construction kits (ranging in spirit from educational kits to industrial-design kits). Their goal was to brainstorm a design for an 'Any Thing' construction kit that would be both more flexible and creative than those that currently exist. Gershenfeld writes:

> The more we talked, the more we realized that the most interesting part of Any Thing was not our proposed design, it was the 3D printer we were using to prototype it. . . . The real question posed by our Any Thing design review was whether 3D printing could be extended as Lego had been to incorporate sensors and actuators, computing and communications. If we could do this, then instead of forcing people to use their infinitely flexible and personal computers to browse through catalogs reflecting someone else's guesses at what they want, they could directly output it. . . . This is the dream of the personal fabricator, the PF, the missing mate to the PC (pp. 70–71).

For those of us interested in mathematics and science education, Gershenfeld's forward-looking suggestions for exploring the enhanced role of the printer in construction should have special resonance. Nor need we delay this exploration until the arrival of the hypothetical PF. Indeed, this paper will argue that there is a vast and gorgeous landscape of educational mathematical crafts – some of them traditional, some still to be invented – that can be facilitated, extended, created, enriched, or resuscitated by devoting renewed attention to computational output; and this paper will further argue that the techniques needed for this exploration range from the futuristic-but-plausible to the currently available. Perhaps most important, a renewed attention to output and fabrication can awaken us to the fact that crafts have a unique and powerful educational role to play – a role that has been given unfairly scant attention by a technological community hypnotized by the charm (and 'brainpower') of the central processing unit and consequently convinced of the 'peripheral' nature of all other devices.

The remainder of this paper is devoted to unpacking this argument. The following section – focusing on the educational power of mathematical crafts – provides a rationale for what follows. The third section then briefly presents three specific, illustrative examples of computational design tools (created at the University of Colorado) for several types of mathematically rich craft activities. In Section 4, these three examples are further used as springboards for a discussion of technological and (in some cases) cultural issues relevant to the place of output devices in mathematical education. Section 5 is more speculative and futuristic, and more in the spirit of Gershenfeld's text: here the discussion goes well beyond the boundaries suggested by our current examples. This section describes the potential landscape of both new and traditional mathematical crafts, and how these crafts could burgeon in an imagined nutritive climate of novel output devices. Finally, the paper concludes in the sixth section with a discussion of current and ongoing work, related work, and a number of open issues for future research.

MATHEMATICAL CRAFTS AND THE POWER OF PHYSICAL ARTIFACTS IN MATHEMATICS EDUCATION

Traditionally, mathematicians (or at least a notable subset of the profession) have demonstrated a certain aversion to the idea that physical objects can be sources of mathematical understanding. Indeed, there is something of an ancient pedigree to this aversion. In the classical Platonic view of geometry, physical approximations to geometric notions are only poor imitations of the true abstract forms: a 'circle' drawn in the sand is the crudest sort of approximation to the true, perfect, and eternal geometric circle that it is meant to stand for (see Shapiro, 2000, for an excellent introductory discussion). From the pedagogical standpoint, a Platonist might then argue that physical objects have a problematic role in mathematics education: at best, they might serve as hints, or highly flawed metaphors, for the true mathematical concepts that they represent, while at worst, they can be downright misleading, drawing students' attention away from the realm of abstraction to the worldly realm of the senses.

Even for non-Platonists, there is something to be said for this argument; abstraction, after all, is the hallmark of mathematical thinking. Thus, even for those mathematicians who remain unconvinced by (or uninterested in) the notion of the reality or 'higher existence' of mathematical concepts such as numbers, there is still a conviction that these concepts, and not their physical stand-ins, are the true objects of mathematical reasoning. This view resonates, moreover, with an ambivalence toward the use of physical objects as 'mathematical manipulatives,' particularly for more advanced branches of the subject.

This is not the occasion for a thorough discussion of the role of sensory (and especially tactile) experience in mathematical thinking (for an interesting recent discussion, see (Lakoff and Núñez, 2000)). Still, for the purposes of this paper, two points deserve mention. First, the undeniable importance of abstraction in mathematics does not preclude a central role for physical objects in the *development* of mathematical expertise. In other words, there is a plausible argument to be made that it is only through suitably complex and enriched experiences with the physical world that one can possibly come to a deep understanding of mathematical abstraction. This indeed is the more-or-less accepted pedagogical philosophy behind the use of manipulatives such as number rods and balance beams in early grades.[2] In general, objects such as number rods are seen as physicalized 'stepping stones' that assist the student in forming analogies between their own experiences and the world of mathematical abstractions. This, in a nutshell, constitutes the traditional intellectual component of the role of physical artifacts in mathematics education.[3]

There is a second point that deserves mention, however – another aspect of mathematical objects that goes beyond (or exists alongside) the purely intellectual aspect of the previous paragraph. There are important social and emotional affordances of physical objects – affordances, that is, beyond the realm of the purely intellectual – that uniquely enrich the mathematical experiences that students have. This is especially true of those mathematical activities that employ craft materials – everyday stuff such as paper, string, yarn, clay, and wire. Here, physical objects act not merely as cognitive supports (in the spirit of number rods), but rather as personally significant creations.

This is not by any means a new or original view. Indeed, there seems to be a hardy subculture in mathematics education of employing craft materials as the basis of mathematically rich activities. The materials employed are marvelously varied, including soap (for carving models (Papert, 1991)), wax (for polyhedral candles (Laffan, 1980), clay (for finding cross sections of solids (Carroll, 1988)), toothpicks (as a medium for mathematical puzzles (Welchman-Tischler, 1992, Ch. 9)), fiber (incorporating golden-section-based patterns in woven baskets (Johnson, 1999)), and even pumpkins (employing geometric constraints in Halloween carving (Angerame, 1999)). By far the richest traditions are in the use of string (e.g., for modelling surfaces) and, to an even greater extent, paper. We will return to these two popular materials a bit later, but for the present, we can take a broader view of the entire space of mathematical crafts. In particular, it should be noted that throughout the descriptions of these activities in the literature, there is a subtext of motivation: creating mathematical objects in these materials is enjoyable for students, and the resulting products are often attractive, whimsical, even beautiful.

The social affordances of craft objects in students' mathematical lives are powerful and pervasive, though rarely the subject of much concerted attention in the educational literature. It may be that these affordances are either so subtle as to escape notice, or – more likely – that they are so obvious that they seem beneath the attention of pedagogical theory. Nonetheless, they are worth listing briefly here, if only to inform the discussion to follow. To proceed, then: craft objects are things that students create for themselves, and thus have a personalized dimension lacking in, e.g., manufactured objects such as number rods. This means in turn that craft objects can play a special role as 'social currency': they can be given as gifts (to teachers, boy- or girl-friends, parents, etc.), traded, collected, kept as souvenirs of a class, and so forth. The material, real-world, physical manifestation of craft objects means – in some strange way that is hard to put one's finger on – that they play a different social role than 'virtual' self-created objects such as simulations or programs. One might trade or collect simulations, but typically one does not give a simulation as a Valentine's day gift or keep it as a personal memento (Eisenberg and Eisenberg, 1999; see also Csikszentmihalyi, 1993 for a wonderful discussion of the affective role of objects in people's lives).

Still other points deserve at least brief mention. (i) Craft objects take some time to create, and thus represent an exercise – sometimes an insurmountable challenge – for students' patience, skills, and attention span. (ii) At the same time, these objects, once created, often last longer than more 'virtual' creations: a mathematical model in paper or string may be kept by a student for months or years, and thus may be the focus of an educational experience that plays out over a protracted period of time. Similarly, a teacher may keep the previous year's creations and put them on display as spurs to the ambitions and inspiration of the following year's crop of students. (iii) The task of creating a craft object may be quite solitary, but (for more elaborate projects) it may also be the focus of a group activity in which groups of students participate, converse, share insights, or simply chat.

Finally, there are interesting absences, or biases, in the literature surrounding mathematical craft activities. First, such activities are often classified in ways that signify their noncentrality in a mathematical curriculum – i.e., as 'enrichment' or after-school activities. Second, there appears to be a substantial separation between the worlds of educational technology and the worlds of mathematical crafting. That is, materials such as paper, string, and clay are seen as 'low-tech' and thus – for better or worse – unaffected by the more technological culture epitomized by mathematical software. This disconnect is apparent from both sides: educational technologists have historically paid scant attention to crafts, while craft activities are portrayed as things to do in the absence (perhaps due to economic necessity) of computers and other high-tech materials and artifacts. Third, there is – as with the use of manipulatives – a notable absence of 'adult,' or even especially dignified, educational craft activities. The great majority of such activities in the literature are aimed at younger students, and thus focus on elementary mathematics. While this is certainly a boon to those younger students, it reflects an arguably unfortunate side to the culture of professional mathematics. At the risk of some caricature, it seems that the view of the professional community holds that older or more advanced students have already proven their motivation the hard way, by running the gauntlet of mathematics education to this point; and thus there is no need to provide them with enjoyable activities.[4]

The subjects already touched upon in this section could, and should, be the center of a much longer and more detailed discussion. Still, the scope of this paper – our focus on mathematical crafts, and on the educational and technological possibilities of output devices – precludes the sort of attention that these more foundational matters deserve. As a prelude to the remainder of the paper, we can summarize the main elements of the discussion so far:

- Despite the undeniable centrality of abstraction in mathematical thinking, physical objects play at the very least an important role as a source of experience and analogy in the development of mathematical understanding. This role has traditionally been at the heart of the intuition (and experimental literature) supporting the importance of mathematical manipulatives, particularly in the education of younger children.

- Mathematical crafts likewise have a venerable place in mathematics education. In some respects, these craft activities may be seen as variants of the use of more straightforward manipulatives. Craft creations, however, are much more personalized and expressive than objects such as number rods; though at the same time their intellectual link to mathematics may be seen as more oblique and subtle, and they thus may be seen as less intellectually direct (and less educationally efficient) than manipulatives. The educational literature surrounding craft activities often focuses on their affective and motivational role. While this is completely appropriate, a highly unfortunate corollary is that older, more advanced students – who are deemed unworthy of motivation – are often deprived of both the intellectual and affective benefits of mathematical craft activities.
- Despite their potential intellectual and social benefits, mathematical craft activities have historically been a focus of only slight attention from educational technologists, whose focus is much more likely to be on 'virtual' or screen-based creations such as simulations, animations, and intelligent tutoring systems. In this context, craft activities are tacitly assumed to be technologically retrograde, or slightly quaint.

The following section represents a first step in an argument to bridge this divide. Here, three illustrative software systems are presented in which the central purpose is to enrich the practice of mathematical crafting.

COMPUTATIONAL CRAFTS FOR MATHEMATICS EDUCATION: THREE ILLUSTRATIVE EXAMPLES

HyperGami: A Design Application for Mathematical Paper Sculptures

Of the three applications to be described in this section, HyperGami is by far the oldest and most developed, stemming from work begun over seven years ago. The current application (and its simpler Java-based version, JavaGami) has to date been downloaded free of charge by over 400 users (representing more than 30 of the United States and more than 20 foreign countries).[5]

HyperGami has been described at length in several previous publications (Eisenberg and Eisenberg, 1998, 1999; Eisenberg and Nishioka, 1997a, 1997b); here, only a very brief description will be provided to motivate the discussion of the subsequent sections. Essentially, the program is a software design tool with which students can create polyhedral models and sculptures in paper. The HyperGami user begins by designing, on the computer screen, a three-dimensional model of the polyhedron to be created; the software then 'unfolds' the solid into a two-dimensional folding net pattern which can be decorated by a wide variety of techniques. Finally, the decorated folding net is sent as output to a color printer, cut out, and folded into the desired polyhedral model.

The functionality of the program is suggested by the screen shot shown in Figure 1. Here, the user has created a new polyhedron – a variant of a truncated cube visible in the ThreeD window of the figure. The software has unfolded this shape to produce the net visible in the TwoD window; and the user now proceeds to decorate that folding net using pattern- and texture-filling tools and geometric-design painting tools. (Many other techniques, not shown in the scenario of Figure 1, are also available to the user.) Once the decoration phase has been completed, the folding net may be printed out to construct the physical model.

There are many more aspects to the HyperGami system than this skeletal description can touch upon (again, for further discussion, the earlier references are recommended). Still, several points deserve at least brief mention here:

- The HyperGami user is provided at the outset with a large set of classical polyhedra (and their associated folding nets): these include the Platonic and Archimedean solids and their duals. Crucially, however, the user is not limited in her construction to this preselected set; she may use a variety of customization operations to create a limitless set of never-before-seen shapes. These customization operations include (among others) functions for extending the faces of polyhedra with new vertices; truncating vertices; 'slicing' a polyhedron into two distinct shapes on either side of an arbitrary plane; taking the convex hull of a nonconvex shape; 'gluing' two shapes together at a congruent face; and many others. Figure 2 depicts how several of these functions have been composed to produce the shape under construction in Figure 1. In HyperGami (though not in JavaGami), the expert user is provided with an application-specific Scheme dialect through which she can go beyond even these functions and extend the software in arbitrarily complex directions.
- A HyperGami folding net may be decorated not only via the tools within the program itself, but also may be sent (as a Macintosh PICT file) to commercial graphics applications such as Photoshop or Illustrator for further decoration. In this sense, the HyperGami user – simply by virtue of creating an original folding net design on a computer – is automatically provided, 'for free,' with the ability to exploit a powerful range of commercial applications. Indeed, one can

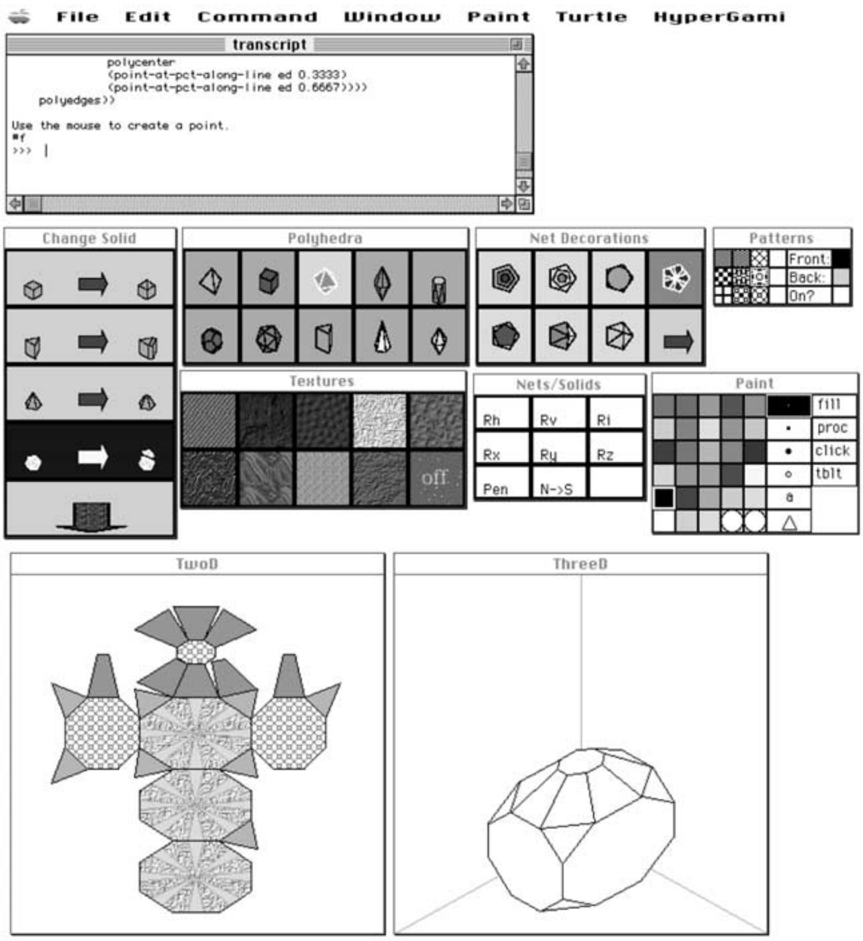

Figure 1. A screenshot of the HyperGami program in the course of a typical scenario. Briefly, the transcript window provides an interface to an application-enriched MacScheme (P1) interpreter. The various windows beneath the transcript window provide tools for viewing and altering polyhedra and for decorating folding nets. The two windows at bottom show a customized polyhedron (in the ThreeD window at right) and its system-generated folding net (in the TwoD window at left). Here, the user is in the process of decorating the folding net, which will later be printed out and assembled into a tangible model of the solid at right.

pursue this thought to note that still other computer-based techniques are available ‘for free’ – for example, a folding net may be sent by email, shared with others on the World Wide Web, and so forth.

- Creating an application such as HyperGami poses a number of distinct challenges in software and interface design. In the case of HyperGami, there are also interesting problems in computational

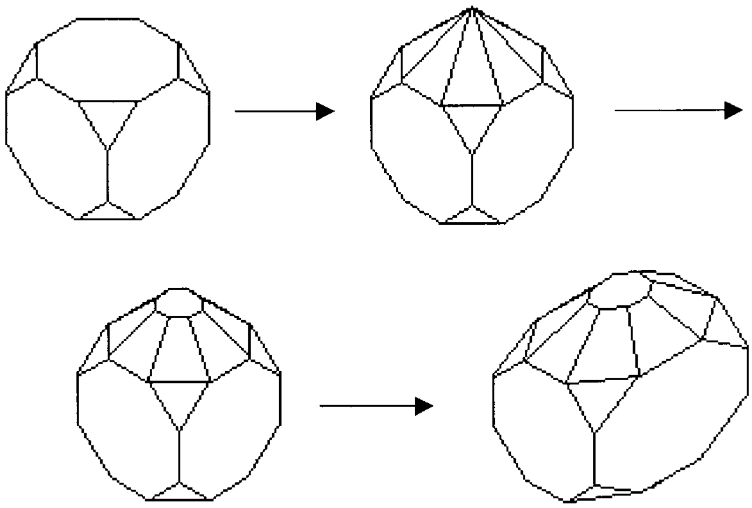

Figure 2. The steps by which the polyhedron in Figure 1 was designed. A truncated cube is first extended with an additional vertex on the top octagonal face; that vertex is then truncated; and the resulting solid is stretched along one axis.

geometry. In particular, the ‘unfolding algorithm’ is not a guaranteed method for unfolding arbitrary polyhedra into a single folding net; indeed, no such algorithm exists (Croft et al., 1991). For certain difficult polyhedra, the HyperGami algorithm may simply fail to produce a net. This point is noted here not simply for the sake of truth-in-advertising (though that impulse is also present), but to suggest that mathematical crafts, and the software tools that accompany them, are also sources of challenging mathematical and algorithmic research problems. (Further discussion of the ‘unfolding problem’ and its implications for interface design can be found in (Eisenberg, 1996)).

- Finally, it would be remiss not to include at least a few examples of HyperGami constructions, if only to indicate the expressive range of the system, and of the craft of polyhedral modelling. Figure 3 depicts several models made with the program, including complex polyhedral forms and representative sculpture (‘orihedra’ (Eisenberg and Nishioka, 1997b)).

HyperSpider: A Design Application for Mathematical Forms in String

HyperSpider is a system similar in spirit to HyperGami – that is, it is a design application for a mathematical craft, but in this case the craft material is string rather than paper. HyperSpider, created by Ted Chen (with contributions from the author and Andee Rubin) (Chen, 1999; Eisen-

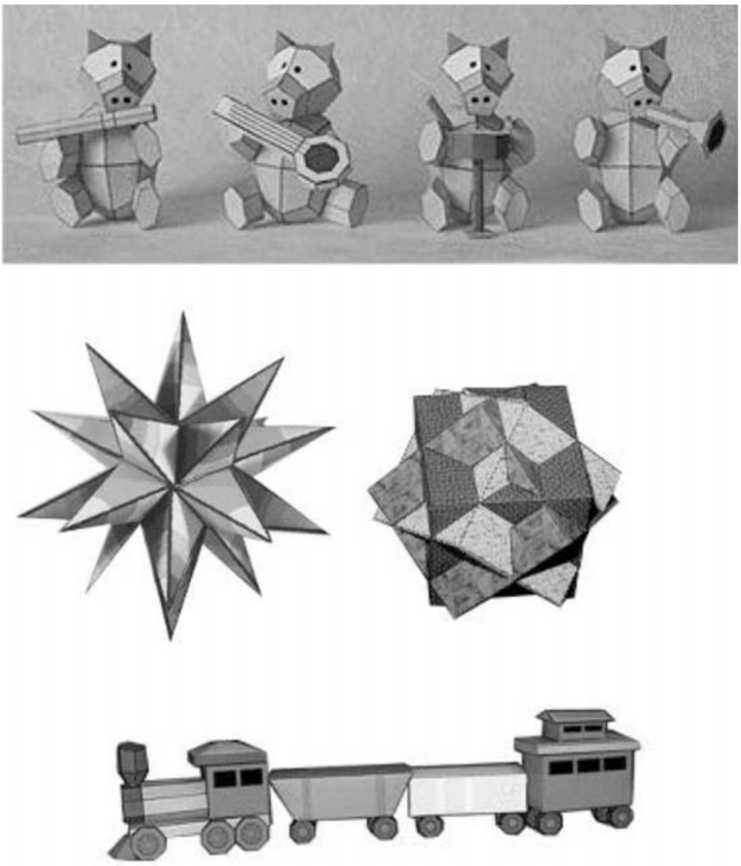

Figure 3. A representative selection of HyperGami models. At top, a quartet of pig musicians. In the middle row, two complex polyhedra: a great stellated dodecahedron (left) and a compound of three cubes (at right). In the bottom row, a four-car paper train, with locomotive and caboose.

berg et al., 1998) and implemented in Java, is based on 'Space Spider,' a mathematical toy of the 1960's (and unfortunately, no longer available). The original Space Spider toy consisted of three orthogonal cardboard squares fitted together to form an octant of space. These three cardboard planes included holes through which multicolored threads could be strung to create approximations of mathematical surfaces. A representative Space Spider construction is shown in Figure 4.

Chen's HyperSpider program is far less elaborate and featureful than HyperGami, but it provides at least the foundation for a powerful design tool. Figure 5 shows a screen shot of the system in the course of a sample project. Again, the user may design string creations on the computer

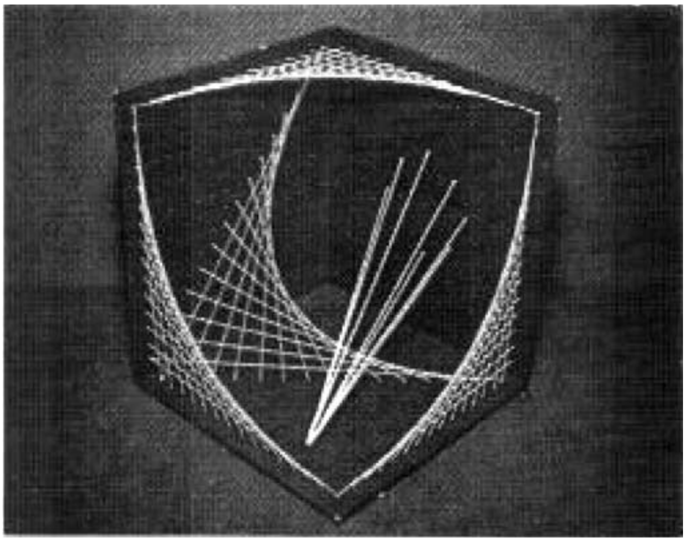

Figure 4. A Space Spider model (from the documentation provided with the original toy) (P2).

screen, employing an 'exploded' view of the three orthogonal planes; the upper right corner of the screen shows a three-dimensional view of the creation under construction. The user may create sculptures with multiple strings, each with its own color; and string patterns may be saved and restored within the program. Once a string sculpture on the screen has been completed to the user's satisfaction, the program will print out frame patterns with color-coded marks corresponding to the strings that pass through them. With the help of these customized frame patterns the user may now more easily employ the 'threading recipe' shown on the screen to produce a physical model. Figure 6 shows a physical string model based on the HyperSpider model.

MachineShop: A Design Application for Mechanical Automata

The third application to be described here, MachineShop, is a system – unlike the previous two – still in a relatively early stage of development. The application is designed by Glenn Blauvelt, and is conceived as a design tool through which students can create mechanical devices, automata, and toys of the sort exemplified by the amazing exhibits of London's Cabaret Mechanical Theatre (Onn and Alexander, 1998).

As envisioned, MachineShop will ultimately consist of a collection, or suite, of modules for creating, browsing, and animating various mechanical elements such as cams, gears, and shafts. At present, only the cam module is sufficiently developed for purposes of illustration; but this module

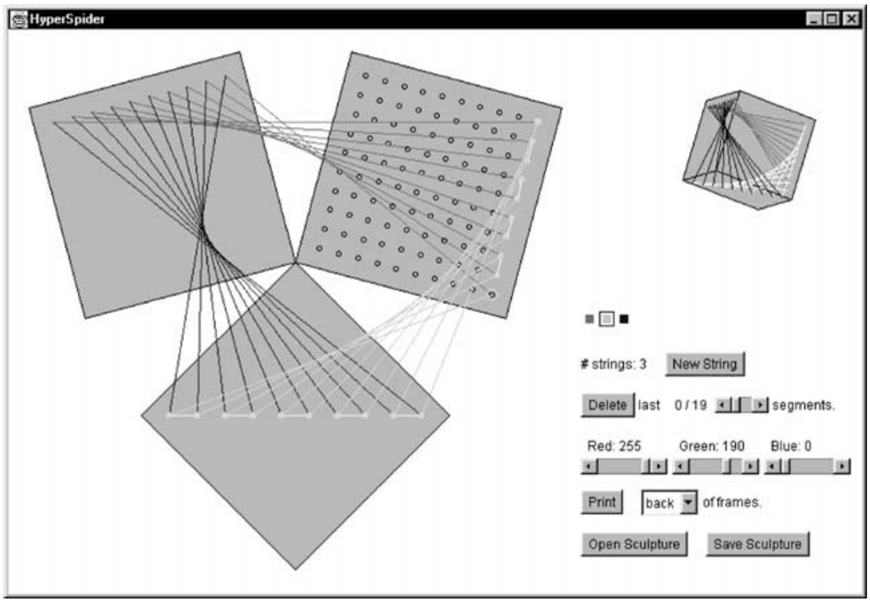

Figure 5. A screenshot of the HyperSpider program in the course of a typical construction scenario. Here, the user is creating a model in which three threads (red, yellow, and blue) are used to compose an overall string sculpture.

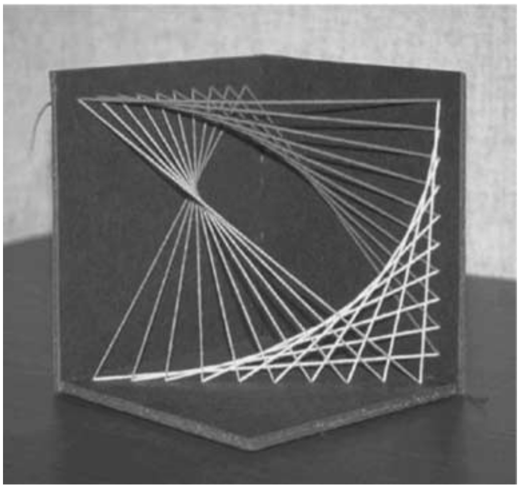

Figure 6. The actual string sculpture specified by the design shown above in Figure 5.

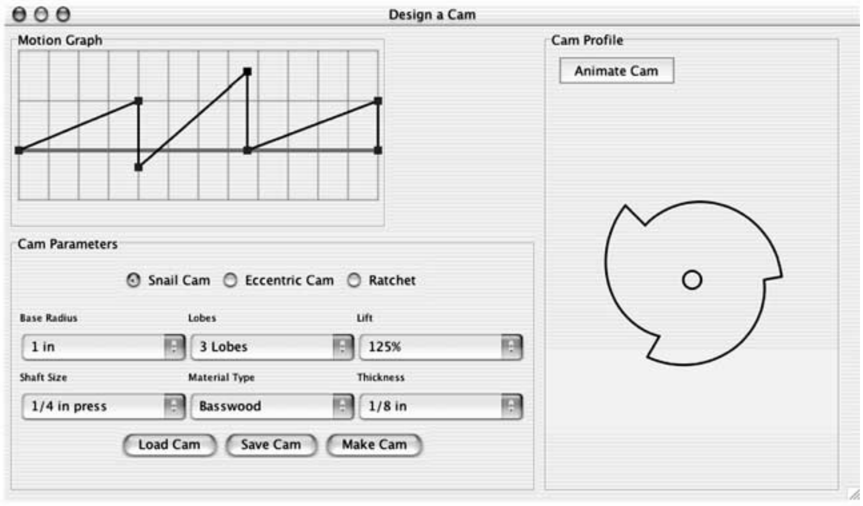

Figure 7. A screenshot of the MachineShop cam-design module in the course of a typical scenario. Here, the user is in the process of creating a customized three-lobed cam. The time graph at upper left shows – as elevation versus time – the path that a cam follower would take if it were sitting atop the steadily-rotating cam under consideration.

should at least provide a springboard for the discussion of output devices in the following sections.

Figure 7 shows a screenshot of MachineShop's cam design module; in this scenario, the user is creating a snail cam with three lobes. The upper left of the window contains a graph; this graph depicts the path that a cam follower would take as the cam turned. In Figure 7, we notice that the follower would execute several vertical drops at the points represented by notches in the cam. The lower left of the window includes various pull-down menus that specify cam parameters (including the number of lobes, base radius, and material – such as basswood – of which the cam is to be constructed). At right, there is a line drawing of the cam and a button that animates the drawing so that the user can view the cam's motion.

A central theme within MachineShop's design is that the user should, wherever feasible, be able to create parts to produce some desired pattern of motion; this is, after all, the reason that a mechanical element is incorporated within some larger automaton. In keeping with this design theme, the rectangles on the graph representation of Figure 7 can be moved interactively on the grid, allowing the user to change the dwell and lift of any lobe while seeing these changes as part of the follower's path. As the user moves these points the cam profile is updated to reflect the cam's new shape.[6] Thus, the cam structure may be designed to fulfill a particular function, as represented by the path of the follower.

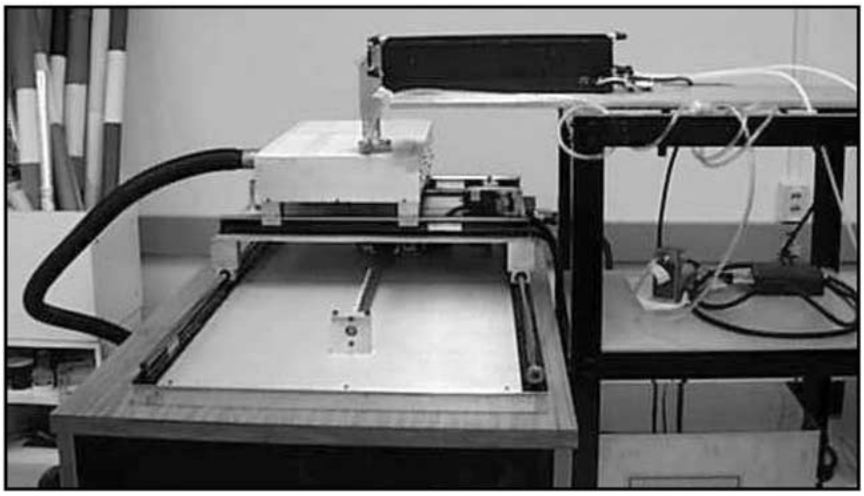

Figure 8. A low-power laser cutter ‘printing out’ the cam of Figure 7.

Figure 9. The result of the printing process in Figure 8 above: the customized cam now rendered in basswood.

Once the user has designed the cam that she wants, she employs the Make Cam button on the screen to create a file that can be employed by a carbon dioxide laser cutter to fabricate the cam in a material such as basswood or foam core. Figure 8 shows a laser cutter producing the cam designed in the earlier figure; and Figure 9 shows the eventual object itself, ‘printed’ in wood.

REFLECTIONS ON THE ROLE OF OUTPUT DEVICES IN MATHEMATICAL CRAFTS

The purpose of the previous section was to introduce three illustrations – three working examples – of software applications focusing on mathematical crafts. In this section, these applications are used as 'objects-to-think-with' for the purpose of reflecting on broader issues of mathematical crafts, educational software design, and output devices. Before proceeding, however, we should admit that such objects-to-think-with are rather a mixed blessing. On the one hand, a provocative illustration can facilitate analogies and generalizations: we can (e.g.) use the applications of the previous section to envision similar applications for other types of mathematical crafts. On the other hand, it is important not to invest too greatly in what is after all only an illustration; an object-to-think-with should not become a straitjacket to our imaginations as designers. Thus, while this section will (in effect) remain conceptually close to the three sample applications, the following section will also attempt to move beyond these applications and explore a larger space of potential output devices and craft applications.

With this caveat in mind, we can begin by noting several important features that all three of the previous section's examples have in common. Most prominently, all three systems share a basic design strategy. In each case, a software application is employed as a means by which students and crafters can *begin* the process of creating a physical object; that is, the software application is used prior to the 'real-world' phase of working with one's hands. A HyperGami polyhedron, or HyperSpider string sculpture, or MachineShop cam, exists first as a data object within the application, and is later translated into physical form.

In this sense, then, all three applications are examples of design tools in which the task of initial conceptualization is largely (though not, in practice, exclusively) accomplished at the computer screen. The user begins by tinkering with the application, moving toward the design of an object that he would like to create in the physical world. This scenario implies in turn several additional points – for example, that the application should strive to inform the user as much as possible about how the physical object will look or behave before the 'hands-on' creation phase. The added value of the application is realized, in large part, by whether it permits the user to create more complex (or expressive, or perhaps mathematically suggestive) craft creations with greater ease than would be possible without the application. To take a specific example: it would be prohibitively difficult and time-consuming to find a folding net for many novel polyhedral forms without the aid of a tool such as HyperGami. By employing HyperGami, a polyhedral modeller can vastly expand the range of creations that he would even bother to attempt.

Another feature shared by all three sample applications is that they are all representative of existing craft traditions in mathematics education. HyperGami builds on a pedagogical tradition of polyhedral modelling that is now centuries old (a set of eighteenth-century cardboard polyhedra can be seen on display at the University of Göttingen in Germany (Mühlhausen, 1993)). HyperSpider and MachineShop build on similarly venerable traditions – and of course the former is directly based on a forty-year-old mathematical toy. In this sense, all three applications at least partially finesse the earlier-mentioned question of whether, and how, such craft activities contribute to mathematical education. That is to say, one may argue that the creation of HyperSpider string sculptures has marginal educational value;[7] but it is hard to argue that HyperSpider has any *less* value than previous or traditional artifacts such as the original Space Spider toy.

Because all three applications are based on existing mathematical craft traditions, they may be said to be (in some sense) conservative. Indeed, none of these systems attempts to break fundamentally new ground in creating novel mathematical crafts; in every case, the original challenge to the software designer was to take some existing craft and design a productive computational enhancement to it. This of course represents a profound challenge in its own right, involving the design of appropriate interfaces, data representations, and formalisms for a particular craft domain; thus, there is no need to apologize for the conservative choices of domains for which these systems were created. And one may argue still further: these applications suggest that there are perhaps still other rich activities to be found within the traditional landscape of crafts, and that many of these activities may be similarly enhanced by computational tools. We will return to these issues in the following section.

Still another feature shared by all three applications is that, for each of them, there is a tight conceptual interweaving between the operation of the application and the strengths and limitations of particular output devices. In other words: existing output devices act as both sparks and constraints to the types of craft activities that educational technologists can currently explore. In the remainder of this section, each of the three applications will be examined in turn, to highlight some specific issue of the relationship between computational craft activities and output devices.

HyperGami and the Burgeoning Power of the Printer

Of the three applications described in the previous section, HyperGami is not only the most mature; it also represents the happiest marriage between mathematical craftwork and computation. In large part, this is due to a factor external to the software itself – namely, the advanced state of the color printer as a personal device. Papercrafts – particularly within the tradition of polyhedral modelling represented by HyperGami – mesh beautifully with the capabilities of commercially available color printers.

It should be pointed out that, from a vantage point of a generation ago, there would be no particular reason to expect that personal color printers would be as plentiful or inexpensive as they have since become.[8] In the mid-1980's, the vast majority of personal printers were single-color (i.e., black) dot-matrix printers; and even as late as the early 1990's, desktop color printers were seen rather as a luxury. Thus, until recently an application such as HyperGami – predicated on the notion that schools, or individual students, would be equipped with a color printer – would have seemed optimistic.

Moreover, it is not only the technology of the printer device itself that has made enormous strides. The selection of materials that may be printed upon has likewise burgeoned. HyperGami polyhedra may thus be printed out upon a huge variety of grades of paper, including glossy photo-quality papers (which in fact are rather difficult to fold); high-fiber letter-quality papers; multicolored papers; or thick cardstock. One can even construct HyperGami polyhedra in materials other than paper. Figure 10 shows a HyperGami sculpture of a glass of iced tea in which the glass and ice cubes have been constructed from plastic transparency; use of this material permits one polyhedron to be placed inside another (transparent) one. Figure 11 represents still another potentiality of current materials: here, the HyperGami nets were printed out onto a fabric-transfer paper (usually employed for making T-shirts), after which they were transferred to fabric and sewn into stuffed 'pillowhedra'.

These HyperGami projects suggest collectively that papercrafting is an educational tradition poised to exploit the relatively mature state of current personal computing technology. Nonetheless, it is worth noting that the advances in printer technology have not at all been conceived with the interests of educators or mathematical papercrafters in mind. The example of plastic transparencies is instructive: this is a standard business technology that finds some utility for HyperGami users as well. Even the more 'recreational' innovations – e.g., fabric-transfer paper, or greeting-card paper – find rather accidental use for educational or mathematical purposes.

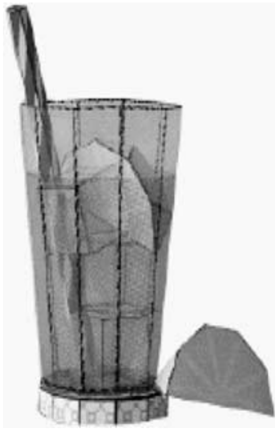

Figure 10. A HyperGami ‘iced-teahedron’ constructed with plastic transparency material (used both in the ‘glass’ and the ‘ice cubes’ inside it).

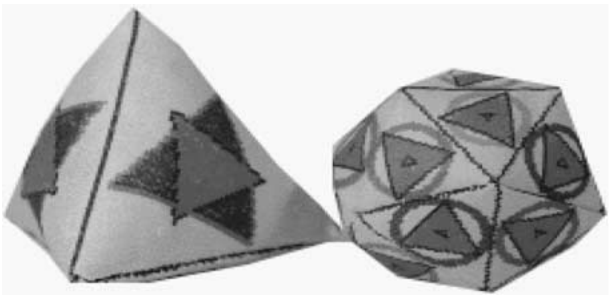

Figure 11. Two HyperGami ‘pillowhedra’. The folding nets were printed out onto fabric transfer paper; and once the nets were transferred to fabric, they were sewn into stuffed polyhedral pillows.

Indeed, there are many plausible innovations that *could* be of use to mathematical papercrafters, but that cannot be found in the current state of personal printing technology. A classic example is the difficulty of printing precisely matched patterns on both sides of a sheet of paper. If one wishes, for instance, to print a decorated folding net for an open surface (such as a bowl) – and thus to print on both sides, interior and exterior, of the folding net – one is faced with the extremely thorny problem of matching exactly the printing pattern on both sides of the sheet. This does not seem, in principle, to be an insurmountable technical challenge for printer hardware (indeed, the publishers of papercraft books often include two-sided patterns), but it is to my knowledge not adequately addressed by current inexpensive personal printers.[9] In the same vein, there are many grades of paper that one might wish, for construction purposes, to run through a personal printer. Standard origami paper cannot be used; nor can specialty handmade papers; nor can extremely delicate papers.

In short, then, while HyperGami represents a highly successful integration between educational software and output device technology, it also hints at the mismatch between the desires of educational crafters and the current state of the art in output device construction. A nagging sense of unmet potential is true even for this most mature of output devices – the printer. We will return to this issue in the following section.

MachineShop and the Infancy of the Personal Laser Cutter

While HyperGami represents an application targeted toward a 'standard' output technology, MachineShop represents an application targeted toward a device still in its infancy – the personal laser cutter. Laser cutters represent an especially interesting case study for those interested in educational technology, both because of their marvelous potential for classroom purposes and their (at least current) drawbacks.

The basic technology of a low-cost laser cutter is similar in principle to that of an x-y pen plotter: a maneuverable laser is directed from above at a thin sheet of substrate material, thereby cutting out customized patterns from the material.[10] This simple design nonetheless has surprising power because of the range of materials that may be employed – materials such as basswood, foam core, cardboard, and certain types of plastic. The uses of these materials for educational purposes are manifold. For a MachineShop user, constructing mechanical automata in wood is generally far preferable to constructing such devices in paper (cardstock automata can be made, but they tend to be fragile and short-lived in comparison to wooden models). Automata, however, are just one instance of the sorts of constructions that students could make with the use of an inexpensive laser cutter. Wooden pieces for jigsaw puzzles (in 2- and 3-D) could be constructed; slotted pieces for wooden animal models (of the sort seen commercially for insects or dinosaurs) would be within reach; customized balancing toys – e.g., the sorts of objects that balance unexpectedly on a shelf or athwart a string – could likewise be printed out with precision. Moreover, under certain circumstances, laser cutters may be used for scoring or engraving materials as well as cutting; thus, not only could (e.g.) mechanical pieces such as gears be cut by a laser cutter, they could potentially be decorated via engraving as well.

One might hope, then, that the near future of personal laser cutters will follow a pattern similar to the recent past of personal color printers. Conceivably, a generation or so from this writing, laser cutters will be as common in classrooms and home workshops as printers are today.

Nonetheless, there are several obstacles to be overcome before that day arrives. Currently, even inexpensive laser cutters are rather expensive items – beyond the reach of most K-12 classrooms or individuals. A reasonable estimate would be that these devices will have to become an order of magnitude cheaper before they can be considered widely affordable.[11] To some degree, there are safety concerns for classroom laser cutters – for example, certain types of plastics are unacceptable for use with these devices because they give off noxious fumes, while other materials may occasionally burn or ignite. Thus, current laser cutters pose requirements for operator training (and ventilation) that may be prohibitive for educational purposes.

In large part, then, the future of educational uses for laser cutters will depend upon whether the designers and manufacturers of these devices begin to view their products as something other than industrial artifacts. A classroom-friendly laser cutter might be designed with particular features geared toward relatively simple projects – e.g., it might be accompanied by its own manufacturer-specific sheets of wood or plastic to avoid the potential problems of using inappropriate materials. Laser cutter technology might thus be seen as one instance of a broader set of output device technologies for which a 'cultural shift' needs to be made in order to accommodate the needs of younger students. We will return to this subject in the next section.

HyperSpider and the Nonexistence of Output Devices for Alternative Materials

In both MachineShop and HyperGami, the basic craft materials of students' constructions (wood and paper, respectively) are the materials operated upon by output devices: HyperGami's paper folding nets are produced by a color printer and MachineShop's mechanical pieces are cut directly from wood by a laser cutter. The HyperSpider user, in contrast, cannot output the string elements of her creation. The utility of HyperSpider, in its current instantiation, thus lies in the ability to ease the user's task by printing out (with appropriate construction 'hints' represented by color coding) the frames through which string will be passed. Additionally, the HyperSpider user may well be assisted by the representation of her construction on the computer screen; that is, a typical user would probably print out (on paper) the graphical screen representation of her construction and use this as a visual 'recipe' to consult while creating a string sculpture (Or, alternatively, she may simply construct the sculpture within viewing distance of the screen and consult the visual 'recipe' on the screen directly).

One could conceivably put different kinds of 'spin,' positive or negative, on the relatively narrow channel of output available to the HyperSpider program. On the one hand, one could argue that there is a positive lesson to be learned here: the example of HyperSpider suggests a certain hardiness, or resourcefulness, to the very idea of using computer applications to work with craft materials. Even when an application cannot 'print out' a craft material such as string, it can nonetheless make use of other means to facilitate craft construction. An application such as HyperSpider may thus print out other sorts of artifacts – such as color-coded frames, in this particular case. There are still other possibilities: a HyperSpider-like design application for working with some troublesome material (e.g., ceramics) might at the very least print out written documentation or diagrammed directions that could aid the crafter in his work.

In other words, the positive lesson of HyperSpider is that conventional output devices (such as desktop printers) may still play an important role in a wide variety of craft activities – activities that use all sorts of materials. But there is a negative – and somewhat less complacent – side to this argument as well. To return to the case of HyperSpider, we could begin by asking the obvious question: why can't we output decorated string from our computer in much the same way that we output (via printers) decorated paper? Suppose, for example, that we wish to have a four-foot-long piece of yarn with evenly spaced 'tick marks' every four inches. Or suppose we would like to have a piece of yarn whose color switches, back and forth, from red to blue every four inches. Or suppose that we wish to have a four-foot-long piece of yarn specifically decorated for a HyperSpider construction, in which the color of the yarn smoothly changes from red to blue over its entire length. Why could we not own a desktop device suggested by the illustration in Figure 12 – that is, a device for which we 'feed in' white yarn and which then produces as output custom-colored yarn?

The device sketched in Figure 12 is a wish, but it is precisely the sort of wish that educational technologists should be actively promoting – particularly those technologists interested in blending computational tools with craft materials. Indeed, one might view the Figure 12 sketch as stemming from a far less ambitious (and less futuristic) impulse than Gershenfeld's 'Personal Fabricator' device mentioned earlier. Engineering a string-output device might not be exactly a trivial task – one might anticipate, for instance, that the range of dyes employed in such a device might be limited, or that the rate at which colored yarn would be output might be quite slow – but the potential benefits for mathematical crafters could be immense.

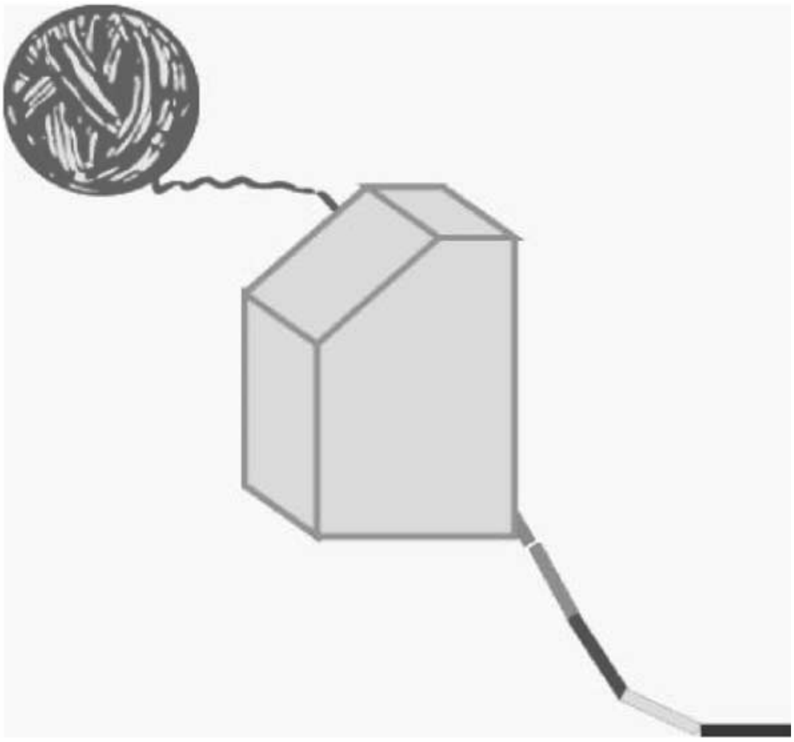

Figure 12. A rough sketch of a hypothetical 'string output' device. The material supplied (upper left) is undecorated yarn or string; the purpose of the device is to dye lengths of the string in computationally-specified patterns. As described in the text, such a device could be exceptionally useful in the practice of mathematical string crafts such as those exemplified by the HyperSpider application (See also the discussion of temari in Section 5 below).

In sum, then, the three sample applications of the previous section highlight distinct issues in the matter of output devices and mathematical crafts. HyperGami illustrates the remarkable utility of color printers for educational crafts – a utility that has been realized almost accidentally over the past decade, with little direct attention to educational issues. At the same time, the types of mathematical papercrafts that one might wish to explore with HyperGami (or similar applications) are often acutely limited by the things that color printers conceivably could (but don't) do. MachineShop suggests the educational potential of an output device – the laser cutter – that is still very much in its infancy. In effect, it seems that the laser cutter is a device that is potentially poised to realize the same sort of explosive growth that the color printer has realized over the past decade; one might even argue that the laser cutter simply needs to be 'personalized,' and brought to the attention of individual hobbyists and amateurs, in much the same way that computer technology was personalized in the mid-1970's. Finally, the example of HyperSpider suggests that there are many potentially new sorts of output devices – devices within the realm of engineering possibility – that could be of intense interest for educational crafts.

THE POTENTIAL LANDSCAPE OF BOTH NEW AND OLD-BUT-REVIVED MATHEMATICAL CRAFTS IN EDUCATION

To this point our discussion of output devices, and their relation to mathematical crafts, has remained close to the three sample applications presented earlier. This section takes a broader view, and explores how novel output devices could vastly expand the power and expressiveness of traditional crafts while making possible the development of never-before-imagined crafts. The purpose of this section, then, is to encourage a certain degree of imaginative free play in the community of educational technologists – both hardware and software designers. In this sense, the ensuing discussion follows in the spirit of Gershenfeld's description of the Personal Fabricator device mentioned earlier; but whereas that hypothetical device represents an extremely ambitious and somewhat all-purpose goal, the guiding image behind this section is that of a relatively near future in which a burgeoning landscape of special-purpose output devices could plausibly be designed.

What Novel Output Devices Could Offer By Way of Traditional Crafts

We can begin our discussion of futuristic output devices by exploring the space of traditional mathematical crafts in education. Essentially, our procedure here is to poke around in the previous century's toy chest of existing mathematical activities, inquiring as we go whether certain of these activities could be enriched by suitably designed computational tools and output devices. What follows is a short list of possibilities; the reader is encouraged to extend this list with ideas drawn from his or her own favorite mathematical crafts.

Paper Braiding – A Paper Tape Output Device
The HyperGami system is used to build polyhedral models from paper; but polyhedral modelling is of course just one of a vast number of mathematical papercraft activities. Among other possibilities, one could mention traditional origami (i.e., forms created simply through paper-folding, without cutting or gluing); pop-up design; design of interesting

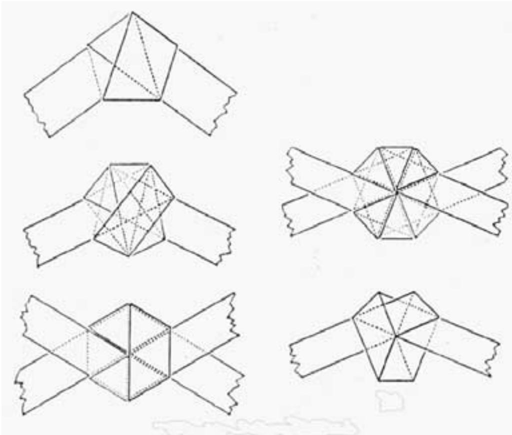

Figure 13. Sample polygonal knots in paper tape (from Cundy and Rollett (1961)).

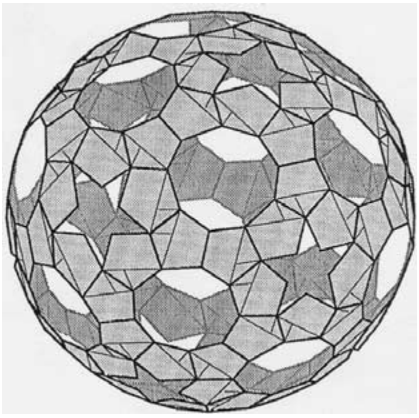

Figure 14. A construction built from twelve pentagonal modules of knotted paper tape (from Strobl (1997)).

topological models (e.g., for Moebius strips); creation of polyhedral dissection puzzles (such as tangrams); and many others.[12]

For the purposes of this discussion, one particularly interesting mathematical papercraft involves the creation of complex forms through weaving and knotting of paper strips. Figure 13, from Cundy and Rollett (1961), shows the basic idea of creating polygonal forms from paper strips. Strobl (1997) describes how these basic knotted forms can be combined into marvelous constructions such as the sphere-like shape (based on the regular dodecahedron) shown in Figure 14.

The marvelous constructions of Strobl are created from long strips of paper tape (such as ticker tape), approximately one inch wide. Such tape can be acquired from specialty papercraft firms, but – to the author's know-

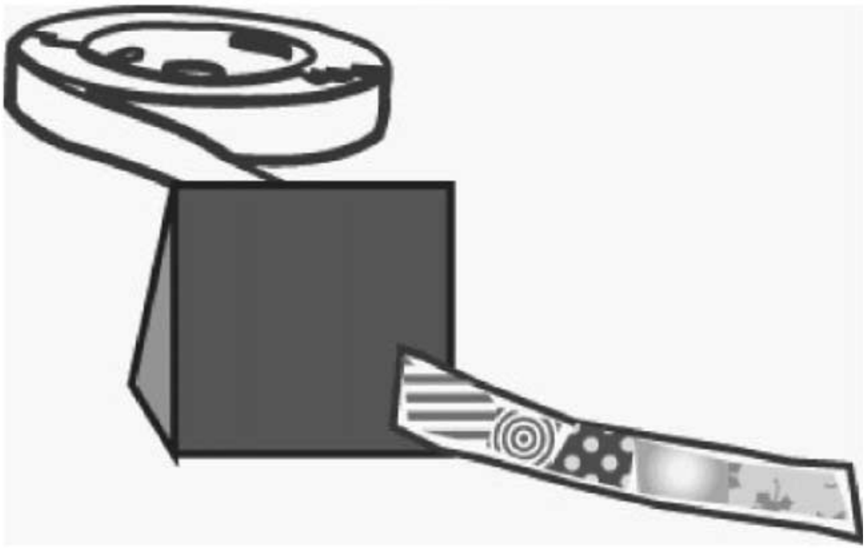

Figure 15. A sketch of an imagined paper-tape printer, designed with an eye toward mathematical crafts such as braiding, flexagon construction, and so forth. A roll of paper tape is supplied as input (toward the top of the figure); decorated tape is output (toward the bottom). The imagined device is simply a printer – but standard color printers (and their associated drivers) are not engineered for paper-tape projects.

ledge – one can only acquire this paper in rather standard colors, with a single color for an entire roll of tape. Moreover – and quite disappointingly – one cannot readily use paper tape as input to a standard color printer.

In short, then, paper tape seems to be a medium of mathematical paper-crafting that would lend itself well to the design of an appropriate output device. Figure 15 shows a sketch of what such a device might look like; the hypothetical tape-printer is actually quite similar to current devices for printing customized tickets. Unlike existing devices, however, the object sketched in Figure 15 is much more geared toward the needs of students and crafters: it would print high-resolution color graphics, and its accompanying driver software would allow users to specify designs such as "a ten-foot-long strip of paper with a smooth color gradient from green to blue". Moreover, the designs printed out to tape could be devised to assist the folder – e.g., by placing small markers on the tape to indicate where and how folding should occur. With an output device of this sort in hand, it would be possible to create computational tools that permit a stunning expansion of paper-tape weaving as a mathematical activity.

Figure 16. Temari ball (from Vandervoort (1992)).

Temari – A String Output Device

In the previous section, we discussed how a string output device could be used to create models of surfaces such as those associated with HyperSpider (and the earlier Space Spider toy). As with paper, however, string is a tremendously versatile medium for mathematical crafting. One remarkable craft – called temari – involves the use of string woven over the surface of a sphere to create gorgeous symmetric patterns. Figure 16, taken from (Vandervoort, 1992), shows a representative temari ball design.

Most temari hobbyists simply follow recipes presented in source books, rather than creating their own original string-weaving designs. But (as with HyperSpider) it should be possible to design computational tools that aid students in the creation of their own personalized and original temari designs. Moreover, with the aid of a string output device such as the one sketched earlier in Figure 12, it would be possible to use custom-colored string to create temari balls of extraordinary complexity and beauty.

Minimal Surfaces through Soap Bubbles – A Wire Output Device

One of the most elegant of mathematical craft activities involves the use of soap films to display minimal surfaces (i.e., surfaces of minimal area given certain geometrical constraints). A cylindrical soap film stretched between two circular wire frames, for example, displays a catenary curve; other beautiful forms are revealed by creating wire frames in polyhedral shapes and dipping these frames into soap solution (see, for example, the lovely photographs in (Hildebrandt and Tromba, 1985)).

It is not inconceivable that an output device might be constructed to facilitate the creation of special-purpose wire frames to be employed for experimentation with soap films. Although the technical problems involved in constructing such a device might be challenging, it seems at least plausible that a device could be designed in which a straight length of wire is input; the purpose of the device would be to bend that length of wire into some desired shape. Possibly, the user could specify a radius of curvature to employ for a particular stretch of wire, and thus create precise two-dimensional wire curves that could then be glued or soldered together to create three-dimensional forms. With the aid of such an output device, the user could specify (on the computer screen) a particular polyhedral wire form that could then be constructed for soap-film experiments. A sketch of the proposed device is shown in Figure 17.

Customized Spirograph Gears – The Personal Laser Cutter

The HyperSpider system, as mentioned earlier, was designed through the inspiration of a traditional toy. Using the same *modus operandi*, one could explore the space of existing mathematical toys, to see how these artifacts could form the basis of novel craft activities. A particularly compelling example is the venerable Spirograph toy (P3). The Spirograph employs plastic gears with holes in them through which colored pens can be placed; by rolling one (pen-equipped) gear around the exterior of another, beautiful kinematic curves may be drawn (cf. Ippolito, 1999).

A natural use of a personal laser cutter – i.e., the affordable student-friendly device envisioned in the previous section – would be to expand the set of existing Spirograph gears by creating one's own wooden gears for use with the 'standard' set. By creating never-before-imagined Spirograph gears, students could be encouraged to think of the toy not as specifying a fixed (though immense) set of curve-drawing possibilities, but rather as the basis of an expandable set of curve-drawing artifacts. In so doing, students might also encounter more challenging mathematical problems than those associated with the standard version of the toy; e.g., it might be possible to pose students with problems in which specialized gears must be created in order to produce a particular curve. Moreover, student-created gears would quite likely exhibit at least some of the social affordances described at the

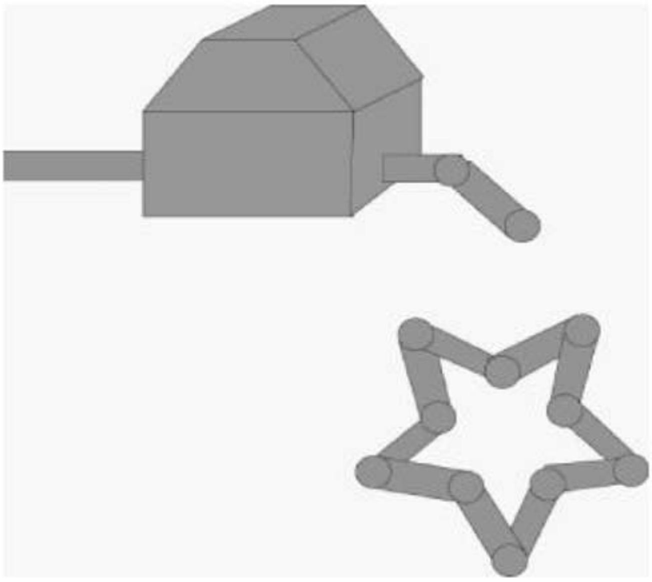

Figure 17. An imagined 'wire output' device for creating computationally-specified patterns realized in bent wire. As portrayed here, the device might be used to bend wire into planar shapes; conceivably, a somewhat more elaborate device could bend wire into nonplanar forms as well. The sketch is intended to suggest a device whose input is a straight length of wire, and whose output is a wire bent by (specified) angles at (specified) intervals. The resulting wire forms – such as the five-pointed star shown toward the bottom – could be used, among other purposes, for the sorts of minimal-surface projects involving soap films mentioned in the accompanying text.

outset of this paper; that is, one could imagine specialized Spirograph gears being collected or traded among peers.

Figure 18 shows, in sketch form, how a laser-cutter-produced gear could be used in conjunction with existing Spirograph gears.

Burr Puzzle Design – The Three-Dimensional Printer

As one final example of a traditional mathematical craft that could be enhanced by novel output devices, we can look to the creation of traditional three-dimensional puzzles such as the 'burr puzzles' described in (Slocum and Botermans, 1992). The basic idea behind such puzzles is that they are composed of interlocking pieces (usually in wood) that fit together to form an interesting geometric shape; the solver's task is to shift the pieces, one at a time, so that the puzzle can be disassembled into separate pieces and reassembled once more. Figure 19, from (Slocum and Botermans, 1992), shows an illustration of a typical burr puzzle.

The example of burr puzzles suggests yet another way of looking at traditional mathematical crafts. Historically, the population of burr puzzle

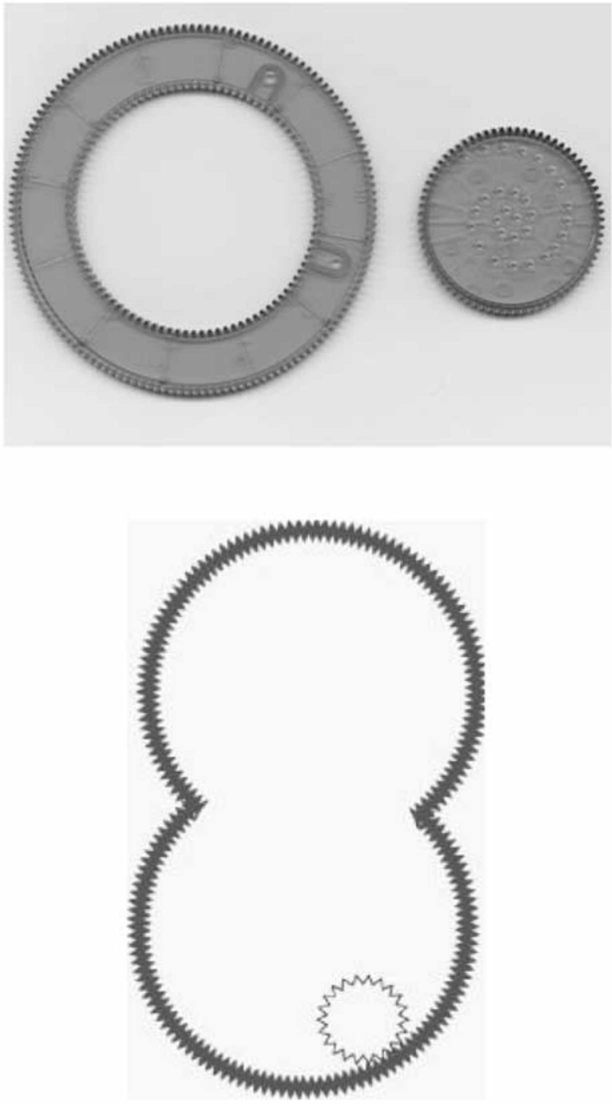

Figure 18. At top, a photograph of two standard Spirograph gears. At bottom, a sketch of a custom-designed large ‘figure-eight’ Spirograph gear within which a smaller standard gear may roll. As conceived, the hypothetical custom-designed gear shown is designed in a desktop software application, and then printed in wood on a laser cutter.

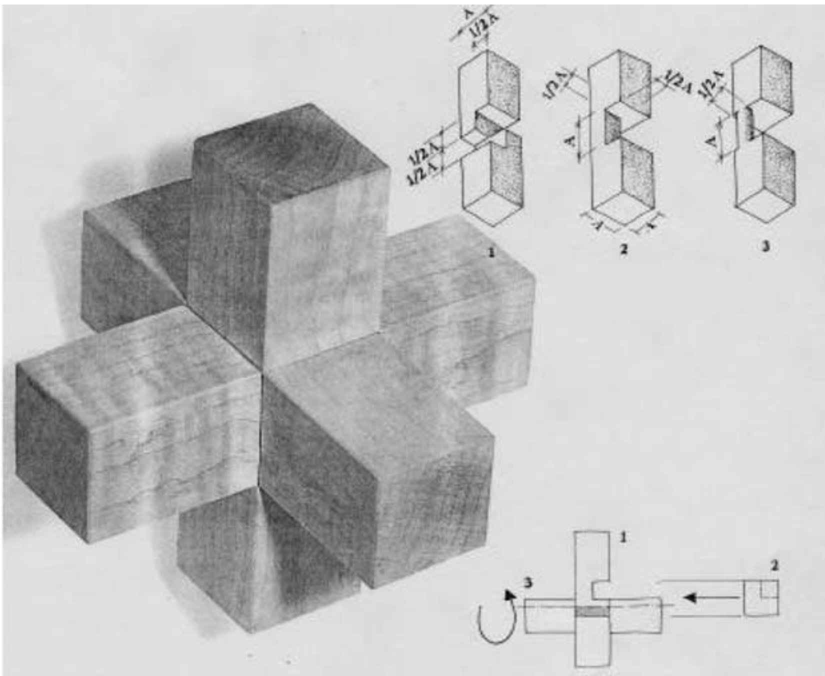

Figure 19. A burr puzzle design (from (Slocum and Botermans, 1992)). The puzzle shown is constructed in wood, from pieces diagrammed in the upper right corner of the figure; but pieces of a similar design could conceivably be printed in plastic on a three-dimensional printer.

designers has been relatively sparse next to the population of puzzle solvers. This is quite probably because the task of burr puzzle design is difficult, both conceptually and physically: a puzzle designer needs, in addition to formidable geometric skills, a certain talent at woodworking.

The advent of computational tools could well cause a cultural shift in the way that students and hobbyists think about burr puzzles (and mathematical puzzles in general). Rather than seeing these artifacts as produced by a small, specialized elite of recreational mathematicians, students could begin to think of puzzles as artifacts within the range of their own creative abilities. Thus, a software design tool might be created to help students create their own three-dimensional burr puzzles; and the pieces of these puzzles could be sent as output to a 'three-dimensional printer' of the sort currently used in industry (and described colloquially in Gershenfeld (1999)).[13]

What Novel Output Devices Could Offer By Way of Futuristic Crafts

In the previous subsection, our focus was on traditional or pre-existing mathematical crafts, and how these activities could be enhanced by appropriately-designed computational media and novel output devices. The following paragraphs, by contrast, look to the possibility of brand-new mathematical crafts – activities that (to the author's knowledge) have never before existed. Although these activities are intended to suggest plausible projects for research and development, the larger point of this discussion is simply to illustrate the ways in which new types of computational output could transform the existing landscape of mathematical crafts.

Height-Varying Mosaics – The Three-Dimensional Printer
One particularly fun use of a three-dimensional printer would be to construct simple mosaic tiles in plastic. Naturally, such tiles could be 'printed out' at a uniform height, to produce a set of (e.g.) square tiles of varying colors and all one centimeter in depth; this would be a set of tiles more or less similar to those found in standard mosaic kits. More interestingly, however, a three-dimensional printer offers a good deal more power and flexibility to the user. Thus, we could imagine a computer application in which the user first designs a function from the x,y-plane to a positive z-value; this could, for example, be a model of a potential function defined over the plane. The user might then print out a set of tiles of varying heights that could then be glued down onto a surface to produce a mosaic model of the desired surface. In addition, these tiles could be color-coded in a variety of ways; one might, for example, color-code the tiles so that 'high potential' corresponds to a red color and 'low potential' corresponds to blue.

Figure 20 shows a sketch of what such a 'three-dimensional mosaic' activity might look like. The essential point here is that a device such as a three-dimensional printer permits the development of new, beautiful, and relatively accessible mathematical crafts. The 'three-dimensional mosaic' would probably be simple enough to provide construction activities for middle-school students (and perhaps even some younger students). The following activity – also based on three-dimensional printing – might be targeted at slightly older students.

L-system Construction – The Three-Dimensional Printer
Another possible use for three-dimensional printing would be the creation of recursive or self-similar three-dimensional forms. A prominent example of such forms arises through the use of Lindenmayer systems (or L-systems) for the design of biologically-inspired shapes (an especially

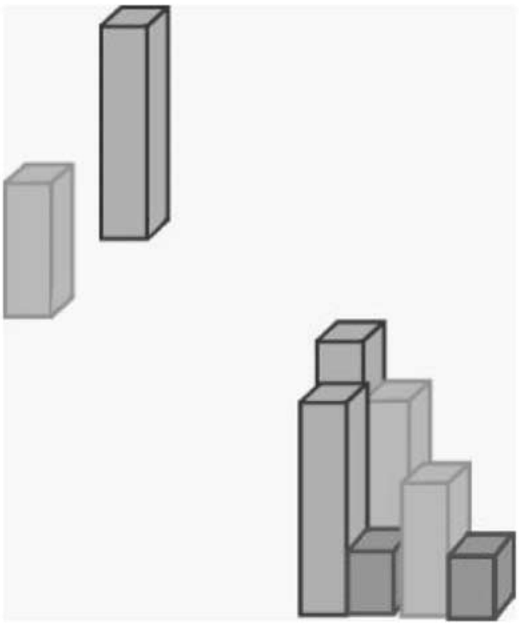

Figure 20. A sketch of a height-varying mosaic kit. Individual pieces in specified colors are printed out in plastic as shown toward the upper left; these pieces could then be assembled into mosaic patterns as suggested by the collection of tiles toward the bottom right. The overall idea is that the user may design height-varying mosaic surfaces on the computer screen, and then print out the appropriate plastic tiles which can then be assembled into the desired mosaic image.

good reference for the use of L-systems to create botanical forms is (Prusinkiewicz and Lindenmayer, 1990)).

A three-dimensional printer could be used as an output device for, say, a ‘tree-design kit’ based on the use of L-systems. The basic idea here is that the user could specify an L-system and create a botanical form on the computer screen; this form could then be decomposed into a set of interlocking ‘branches’ which could be printed out in plastic. The overall system, then, would allow students to custom-design trees and plants on the screen, and then to build physical models of these structures accurately and (relatively) easily. Again, this is one specific example of a somewhat more general theme: namely, that three-dimensional printing allows the design of personalized ‘construction kits’ for the creation of otherwise prohibitively difficult structures such as self-similar or fractal-like models. Figure 21 provides a sketch of the imagined ‘L-system construction kit’.

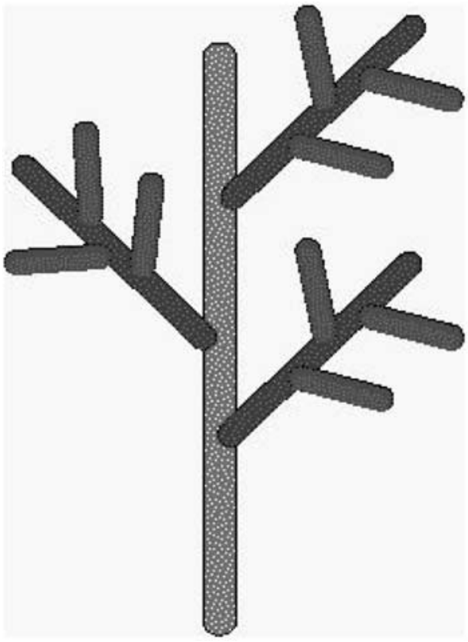

Figure 21. Plastic pieces, printed on a three-dimensional printer, could be assembled into recursive botanical structures as suggested by this sketch. As envisioned, the user employs a software application to create plantlike forms through L-systems; the application then prints out the appropriate pieces which are subsequently assembled together into the desired botanical model.

Topographical Surface Construction – Personalized Laser Cutter

As one final example: a relatively straightforward use of a laser cutter would be to construct customized sets of flat wooden pieces that could be assembled into a variety of three-dimensional forms. Such pieces could be stacked into approximations of (e.g.) cones, spheres, and ellipsoids; clearly, the thinner and more numerous the slices, the finer-grained the approximation to the three-dimensional shape (and the more effortful the construction).[14]

One somewhat more original possibility would be to design surface maps of landscapes on a computer screen, and then to print out a set of flat wooden pieces that could be stacked to construct approximate models of the designed landscape. Figure 22 depicts the general idea. Here, the student has created a topographical map on the screen showing a pair of nearby mountains with a valley between them. The system then prints out a set of wooden pieces (possibly including pegs) that can be used to construct a classroom model of the landscape to any desired level of approximation. The model may then be further elaborated physically – e.g., it might be covered with soil and used as the basis of a realistic geological model.

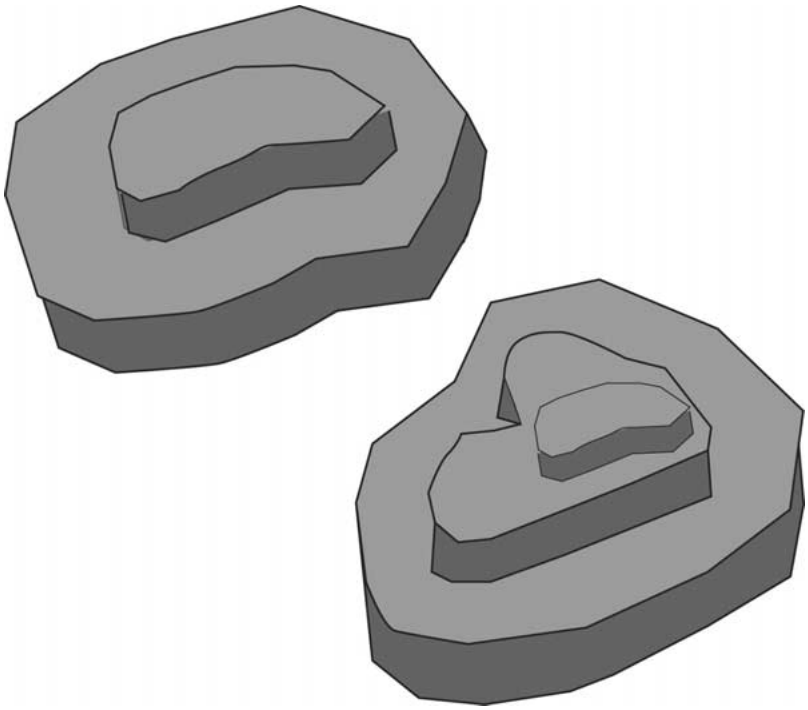

Figure 22. Individual flat pieces are printed out and stacked together to form three-dimensional renderings of contour maps. As envisioned, the user would employ a software application to create customized contour maps; these are translated into sets of flat slices that, once output to a laser cutter, can be assembled into a geological model. The accuracy of the three-dimensional model depends on the width of the slices; fine slices provide a fine-grained approximation of the contour map, while thick slices provide a rougher approximation. Thus, at the cost of some additional construction work, the user may 'tune' the accuracy of the physical model that he/she eventually constructs.

In any event, the essential point here is that materials such as wood (or perhaps foam core) would be natural choices for constructing such a geological model, and could be easily output by a laser cutter; conceivably, a similar activity could be done in the absence of such a device (e.g., by cutting slats of cardboard), but this would be prohibitively tedious for any but the simplest projects.

CURRENT AND FUTURE WORK; OPEN RESEARCH ISSUES

The previous section, as advertised, focused on possibilities rather than actualities. But it should be emphasized that none of the ideas mentioned in those paragraphs requires much of a stretch to the technological imagination. Three-dimensional printers and laser cutters do, after all, exist

– although they are still not quite affordable enough for widespread educational impact. As for devices that output string, wire, or paper tape, these hardly would require profound breakthroughs in engineering; unlike Gershenfeld's Personal Fabricator, these are instead rather modest experimental design efforts waiting to happen.

Indeed, the discussion to this point is perhaps more striking for what it hasn't mentioned than for what it has. There are numerous developments in computational design and materials science that suggest still more exhilarating possibilities for output devices. One central theme involves the inclusion of computational capabilities (such as programmability) within craft items and materials themselves. The development of the programmable Lego brick in Resnick's group at the MIT Media Lab is a foundational effort in this direction (Resnick et al., 1996); these themes have been pursued within the same lab in the development of a number of innovative 'digital manipulatives' (Resnick et al., 1998). In our own group at the University of Colorado, we have begun to pursue similar ideas in the development of 'computationally-enhanced craft items' such as programmable hinges, tiles, and tacks (Wrensch, 2001; Wrensch et al., 2000; Wrensch and Eisenberg, 1998).

To date, efforts such as these have not focused on output devices per se; rather, the emphasis has been on endowing physical artifacts with dynamic behavior. But it is not difficult to envision a near future in which these two areas of development begin to dovetail. One provocative technology-under-development involves the use of conductive inks that permit users to print out digital circuits directly onto a plastic substrate (Mihm, 2000); a similarly exciting technology, 'digital paper' permits the use of paper-thin flexible display surfaces (Mann, 2001); yet another development, recently reported upon in the popular press (Eisenberg, 2001), permits an exceptionally thin chemical battery to be printed onto cardboard or plastic. The ultimate convergence of these technologies portends marvelous new possibilities for computationally-enhanced papercrafts in which the craft material itself can be 'decorated' with circuits and power sources that give constructions interesting behaviors. Conceivably, one could create polyhedral forms that display (e.g.) changing color patterns or programmed Logo-style turtle paths over their surfaces; or one might imagine self-flexing origamic sculptures. One might even take this speculation into other realms beyond paper (e.g., engraving circuit elements into wood, or adding flexible power sources to string); but in any event, the integration of computationally enriched materials and suitably enhanced output devices arguably heralds a new golden age for mathematical crafts.

Nor are computationally-enhanced materials the only types of new materials that might be productively woven into mathematical craftwork. The creative possibilities of such relatively new materials as shape memory alloys, temperature sensitive films, fiber optics, conductive ceramics, among others, have barely been explored. Even more traditional materials, such as plastic, rubber, or glass, are substantially underrepresented in educational crafts. It might be possible, for example, to develop output devices that could 'print out' acrylic prisms and lenses suitable for educational projects in optical crafting (e.g., the design of optical illusions, specialty kaleidoscopes, and the like). Commercial holographic printers currently exist, but – like three-dimensional printers – these too are not yet hobbyist-level devices; ultimately, a personal holographic printer could be used to produce still other stunning optical effects (including kinetic holographic effects), which could be printed onto flexible surfaces to explore still other directions in papercrafting.

Before concluding this futuristic vein of thought, it is also worth noting that the output device's counterpart in the neglected 'peripheral' landscape – namely, the input device – is likewise worthy of some attention for educational purposes. Here, the possibilities revolve around the use of new types of constructive materials as 'physicalized sketching devices,' substrates for decoration, or as novel types of 'keyboards' for communication with desktop applications. Just to take one relatively mundane example: a student might use a tangible geometric construction kit to create a sample shape, which could then be directly 'read in' to a desktop machine. A desktop application could then describe the shape's symmetry and potential relations to other polyhedra; or it could suggest other polyhedra that might be created with the same ensemble of pieces that the student just employed. Such scenarios are just the beginning of a longer reflection – appropriate for another occasion – on the future of input devices and mathematical crafts.

The ongoing efforts of our own small research group at the University of Colorado are intended to explore a variety of the directions discussed in this paper. Of the three systems described earlier, MachineShop and (to a lesser degree) HyperGami are still in active development – though the former is at a much earlier stage (our fervent hope, if only that at present, is that HyperSpider will likewise be developed further). These systems represent, as discussed earlier, a style of application design in which the initial stages of craftwork are done at the computer screen; still other examples of this application-design philosophy are in the works, including design systems for 'sliceform' construction (cf. Sharp, 1999) and for pop-up engineering. As mentioned shortly before, one active research area involves the integration of computational elements within working craft items. This development does not immediately depend on novel output devices; but ultimately, design systems involving computationally-enhanced craft items should be feasible (one might, for example, be able to design and print out wood or plastic artifacts whose structure is explicitly designed to 'house' sets of computational tiles). At present, the computationally-enhanced craft artifacts that we have constructed are too fragile and experimental to suggest a robust integration with our output-device work.

Finally, we believe that the most intriguing research questions that will emerge from the advent of new and powerful output devices – and from the design tools that make use of them – will be in the realm of developmental cognitive science. Computational systems for domains such as paper-crafting, string sculpture, mechanical automata creation, L-system design, and so forth, are not only educational artifacts – i.e., fun and potentially powerful artifacts for students to use. They are also potential platforms with which cognitive scientists can explore fundamental questions bearing on the relationship of symbolic or abstract computational work and tactile experience in children's mathematical and scientific education.

Just to take one somewhat narrowly-focused example: we have taken early steps toward extending HyperGami, hoping to employ the system as a prototype tool with which to explore questions in spatial cognition and its development. One (still highly experimental) planned addition to HyperGami is the inclusion of a variety of software-based spatial 'advisors' that can suggest polyhedral operations to students; the suggested operations are based on plausible but still heuristic notions of what sorts of polyhedral operations are educationally powerful in the development of spatial intuitions about three-dimensional forms.(Eisenberg et al., 1997) Yet another variant of the 'advisor' idea is to include elements within HyperGami that suggest exercises in spatial visualization or tactile construction, focusing either on system-supplied polyhedra or (opportunistically) on the particular shape that the student is in the course of building.

The larger point here, however, is not what particular near-term enhancements are planned for HyperGami. Rather, the crucial point is that a tool like HyperGami offers us an experimental foundation – and a source of lore – for going back to the questions mentioned toward the outset of this paper: What, if any, are the cognitive benefits of hands-on craft activities in the study of mathematics? What are the mechanisms by which visual/spatial imagery (and perhaps 'kinesthetic imagery' as well) influence mathematical learning, and how are these mechanisms influenced by the practice of crafts? How should we map out or classify the space of physical activities in mathematics education; in particular, how do activities such as papercrafting, woodworking, and so forth compare with the use of mathematical manipulatives along a variety of educational dimensions? What are the heuristics that should guide the design of computational tools for mathematical crafts – i.e., how should we think about balancing and interweaving ‘symbolic’ or abstract design activities and hands-on activities?

By investigating questions such as these, we can use our growing understanding of the cognitive side of mathematical craftwork to make more profitable use of new technologies such as those discussed in this paper. To return to a thought mentioned earlier, we might be in a better, more scientifically grounded position from which to design craft activities or manipulatives focusing on topics in advanced mathematics. We might, for instance, be able to approach a question such as “How would one design a manipulative for complex numbers?” with a better sense of what manipulatives *in general* are for, and where the *particular* difficulties lie in connecting complex numbers with visual or tactile cognition. Similarly, we could begin with a question such as “What might a craft activity look like that could support understanding of Laplace or Fourier transforms?”, and approach this question armed with an understanding of what the educational strengths and weaknesses of craft activities are in general, and how these lend themselves to the particular difficulties of understanding and using transforms.

Eventually, then, we may collectively begin to view educational technology less through the lenses of the traditional ‘CPU/peripheral’ or ‘high-tech/low-tech’ distinctions, and more as a complex ecology of tools, materials, and devices for input, output, construction, computation, sensing, communication, and control – in short, as a discipline of mathematical-artifact creation, informed by a richer cognitive science of mathematical objects, crafts, and languages. In the course of this evolution, we should be able to make better use of the output devices that we are likely to see in our own lifetimes. Just as important, we can ourselves take an active role in creating and experimenting with new output devices and materials, designed expressly for their power and affordances in mathematical education research and practice.

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NOTES

[1] Cf. Von Neumann (1945); Von Neumann (1956); and Berkeley (1949), especially chapters 1–3 and chapter 10, p. 275.

[2] A brief but clear statement of precisely this philosophy may be found in English (1999), p. 23. Much of the lore surrounding the use of manipulatives – especially in geometry education for young children – may be traced to Pestalozzi and Froebl (Brosterman, 1997) in the eighteenth and nineteenth centuries and Montessori in the early twentieth (cf. Montessori, 1912, Ch. 19; Hainstock, 1986, pp. 83–84); the theoretical framework behind the transition from 'concrete' to 'abstract' cognitive representations of mathematical concepts is largely attributed to Piaget (for relatively short but informative discussions, see (Piaget, 1972, pp. 81–91) and (Boden, 1994, pp. 38–42)). Crowley (1987) provides an excellent summary of the influential van Hiele model of geometric education and its use of manipulatives as 'transitional objects' in the sense described here. A recent meta-analysis of manipulative use in mathematics education (showing positive results for the long-term use of tangible, concrete materials) is found in Sowell (1989). Finally, for a good overview of current research issues and controversies surrounding the use of manipulatives, see Ball (1992) and the introductory sections of Hall (1998) and Chao et al. (2000).

[3] It should be pointed out that to the extent that this is a sound pedagogical strategy, it suggests in turn that mathematics educators do themselves a disservice by not turning their attention to the active design of mathematical manipulatives for those advanced concepts that seem to defy easy physical realization. It might be a profitable exercise, in other words, to at least attempt the creation of novel physical manipulatives for concepts such as complex numbers, Fourier transforms, fuzzy sets, and so forth.

[4] Admittedly, there are notable and inspiring exceptions: Henderson and Taimina (2001), for instance, presents a fascinating recipe for weaving a hyperbolic surface out of yarn; Grunbaum and Shephard (1988) discuss problems in group theory arising from the structure of woven fabrics; and, in the realm of papercrafts, one rich source of mathematical problems has been the construction of flexagons (see, for instance, Gardner (1988); Oakley and Wisner (1957)).

[5] HyperGami and JavaGami run on all Apple Macintosh computers with at least 18M of memory. Further information on downloading the software may be found at the website www.cs.colorado.edu/∼ctg/projects/hypergami.

[6] For the types of snail cams shown, the drop from the high point of one lobe to the low

point of the next is vertical. This is captured in the graph by constraining points which share a common horizontal position to move horizontally in unison. See also (Blauvelt and Eisenberg, 2001) for a bit more detail.

[7] Though I would disagree, for reasons previously sketched in Section 2.

[8] As of this writing, a reasonably good inkjet color printer costs less than US$100; while US$400 is sufficient to purchase an excellent photo-quality printer.

[9] Those printers described as 'two-sided' – at least those of which this author is aware – are intended for printing text on both sides of the page. They manage this task by running the page twice through the printer – perfectly adequate for text, but liable to slight imprecision for the papercrafter's 'correspondence problem' described here.

[10] The model shown in Figure 8 in fact employs a stationary laser trained upon a moving table.

[11] A survey of commercial sites on the World Wide Web suggests that the most inexpensive current laser cutters cost approximately US$15,000–$20,000.

[12] Cundy and Rollett's classic book *Mathematical Models* (1961) is the single most valuable resource for mathematical crafts in paper and other materials; of the numerous other references on mathematical papercrafts (Holden, 1971; Miura, 1997), and the many relevant articles of Martin Gardner are particular standouts.

[13] Current three-dimensional printers use a variety of means to output model objects; the 'typical' design (if such a term could be ventured) prints out successive layers of a desired shape in a soft plastic material which is then cured through laser light. The plastics employed in current printers are perhaps still a bit fragile for purposes of burr-puzzle construction, but the range of 'printable' materials will almost certainly expand greatly over the next decade.

[14] A similar idea is described in Offerman (1999) using hand-carved florist's foam. Cundy and Rollett (1961) also mention the construction of contour-map models in passing (p. 15).

PRODUCTS

(P1) *MacScheme*. Lightship Software. Available through Academic Distributing, Dewey AZ. Originally published by LightShip Software, Palo Alto, CA.

(P2) Space Spider (Space Materials Center) Space Materials Center. Space Geometrics Toy. New York.

(P3) Spirograph. Hasbro, Inc.

REFERENCES

Angerame, S. (1999). Math-o'-Lanterns. *Teaching Children Mathematics* (October 1999): 72–76.

Ball, D.L. (1992). Magical hopes: manipulatives and the reform of math education. *American Educator* (Summer) 15–18: 46–47.

Berkeley, E.C. (1949). *Giant Brains, or Machines that Think*. New York: Wiley.

Blauvelt, G. and Eisenberg, M. (2001). Machine Shop: Steps Toward Exploring Novel I/O Devices for Computational Crafts. To appear in *Proceedings of IEEE International Conference on Advanced Learning Technologies (ICALT 2001)* (August). Madison, WI.

Blauvelt, G., Wrensch, T. and Eisenberg, M. (2000). Integrating Craft Materials and Computation. *Knowledge-Based Systems* 13(7–8): 471–478.

Boden, M. (1994). *Piaget*. London: Fontana Press.

Brosterman, N. (1997). *Inventing Kindergarten*. New York: Harry Abrams.

Carroll, W. (1988). *Cross Sections of Clay Solids* (pp. 6–11) (March 1988). Arithmetic Teacher.

Chao, S., Stigler, J. and Woodward, J. (2000). The Effects of Physical Materials on Kindergartners' Learning of Number Concepts. *Cognition and Instruction* 18(3): 285–316.

Chen, T. (1999). *HyperSpider: Integrating Computation with the Design of Educational Crafts*. M.S. Thesis, University of Colorado, Boulder.

Croft, H., Falconer, K. and Guy, R. (1991). *Unsolved Problems in Geometry*. New York: Springer-Verlag.

Crowley, M. (1987). The van Hiele Model of the Development of Geometric Thought. In M. Lindquist and A. Shulte (Eds), *Learning and Teaching Geometry, K-12*. Reston, VA: National Council of Teachers of Mathematics.

Cundy, H.M. and Rollett, A.P. (1961). *Mathematical Models*. London: Oxford University Press.

Csikszentmihalyi, M. (1993). Why we need things. In S. Lubar and W.D. Kingery (Eds), *History from Things*. Washington, DC: Smithsonian Institution Press.

Eisenberg, Anne (2001). What's Next: Batteries Push Paper into Electronic Age. *New York Times* (May 24).

Eisenberg, M. (1996). The Thin Glass Line: Designing Interfaces to Algorithms. *Proceedings of CHI 96* (Vancouver, April): 181–188.

Eisenberg, M. and Eisenberg, A. (2000). Designing Real-Time Software Advisors for 3D Spatial Operations. In S. O'Nuallain (Ed.), *Spatial Cognition* (pp. 185–197). Amsterdam: John Benjamins.

Eisenberg, M. and Eisenberg, Ann N. (1998). Shop Class for the Next Millennium: Education through Computer-Enriched Handicrafts. *Journal of Interactive Media in Education* (http://www-jime.open.ac.uk/) (October).

Eisenberg, M. and Eisenberg, Ann N. (1999). Middle Tech: Blurring the Division Between High and Low Tech in Education. In A. Druin (Ed.), *The Design of Children's Technology* (pp. 244–273). San Francisco: Morgan Kaufmann.

Eisenberg, M. and Nishioka, A. (1997a). Creating Polyhedral Models by Computer. *Journal of Computers in Mathematics and Science Teaching* 16(4): 477–511.

Eisenberg, M. and Nishioka, A. (1997b). Orihedra: Mathematical Sculptures in Paper. *International Journal of Computers for Mathematical Learning* 1: 225–261.

Eisenberg, Michael, Ann Nishioka and Schreiner, M.E. (1997). Helping Users Think in Three Dimensions: Steps Toward Incorporating Spatial Cognition in User Modelling. *Proceedings of 1997 International Conference on Intelligent Interfaces (IUI 97)* (January) (pp. 113–120). Orlando, Florida.

Eisenberg, M., Rubin, A. and Chen, T. (1998). Computation and Educational Handicrafts: a Strategy for Integrative Design. In *Proceedings of International Conference on the Learning Sciences, 1998* (December) (pp. 84–90). Atlanta.

English, L. (1999). Reasoning by Analogy: a Fundamental Process in Children's Mathematical Learning. In L. Stiff and F. Curcio (Eds), *Developing Mathematical Reasoning in Grades K-12*. Reston, VA: National Council of Teachers of Mathematics.

Gardner. M. (1988). *Hexaflexagons and Other Mathematical Diversions*. Chicago: University of Chicago Press.

Gershenfeld, N. (1999). *When Things Start to Think*. New York: Henry Holt.

Hainstock, E. (1986). *The Essential Montessori*. New York: New American Library.
Hall, N. (1998). Concrete Representations and the Procedural Analogy Theory. *Journal of Mathematical Behavior* 17(1): 33–51.
Henderson, D. and Taimina, D. (2001). Crocheting the Hyperbolic Plane. *The Mathematical Intelligencer* 23(2): 17–27.
Hildebrandt, S. and Tromba, A. (1985). *Mathematics and Optimal Form*. New York: Scientific American Library.
Hilton, P. and Pedersen, J. (1994). *Build Your Own Polyhedra*. Menlo Park, CA: Addison-Wesley.
Holden, A. (1971). *Shapes, Space, and Symmetry*. New York: Dover.
Ippolito, D. (1999). The Mathematics of the Spirograph. *The Mathematics Teacher* 92(4): 354–358.
Johnson, A. (1999). Now and Then: Fiber Meets Fibonacci. *Mathematics Teaching in the Middle School* (January) 4(4): 256–262.
Laffan, A.J. (1980). Polyhedron Candles: Mathematics and Craft. *Arithmetic Teacher* (November): 18–19.
Lakoff, G. and Núñez, R. (2000). *Where Mathematics Comes From*. New York: Basic Books.
Mann, C. (2001). Electronic Paper Turns the Page. *Technology Review* (March).
Mihm, S. (2000). Print Your Next PC. *Technology Review* (November/December).
Miura, K. (Ed.) (1997). *Origami Science and Art: Proceedings of the Second International Meeting of Origami Science and Scientific Origami*. Otsu, Japan.
Montessori, M. (1912). *The Montessori Method* (1964). New York: Schocken Books.
Mühlhausen, E. (1993). Riemann Surface – Crocheted in Four Colors. *Mathematical Intelligencer* 15(3): 49–53.
Oakley, C. and Wisner, R. (1957). Flexagons. *American Mathematical Monthly* 64(3): 143–154.
Offerman, T. (1999). Foam Images. *The Mathematics Teacher* 92(5): 391–399.
Onn, A. and Alexander, G. (1998). *Cabaret Mechanical Movement*. London: Cabaret Mechanical Theatre.
Papert, S. (1991). Situating Constructionism. In I. Harel and S. Papert (Eds), *Constructionism*. Norwood, NJ: Ablex.
Piaget, J. (1972). *The Child and Reality* (1976). New York: Penguin Books.
Prusinkiewicz, P. and Lindenmayer, A. (1990). *The Algorithmic Beauty of Plants*. New York: Springer Verlag.
Resnick, L.B. and Omanson, S.F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3, pp. 41–96). Hillsdale, NJ: Erlbaum.
Resnick, M., Martin, F., Sargent, R. and Silverman, B. (1996). Programmable Bricks: Toys to Think With. *IBM Systems Journal* 35(3): 443–452.
Resnick, M., Martin, F., Berg, R., Borovoy, R., Colella, V., Kramer, K. and Silverman B. (1998). Digital Manipulatives: New Toys to Think With. *Proceedings of CHI '98*: 281–287.
Shapiro, S. (2000). *Thinking about Mathematics: the Philosophy of Mathematics*. Oxford, UK: Oxford University Press.
Sharp, J. (1999). *Sliceforms*. Suffolk, UK: Tarquin Books.
Slocum, J. and Botermans, J. (1992). *New Book of Puzzles*. New York: W. H. Freeman.
Sowell, E. (1989). Effects of Manipulative Materials in Mathematics Instruction. *Journal for Research in Mathematics Education* 20(5): 498–505.

Strobl (1997). Symmetry in Two- and Three-Dimensional Origami with Knotted Tape. In K. Miura (Ed.), *Origami Science and Art: Proceedings of the Second International Meeting of Origami Science and Scientific Origami*. Otsu, Japan.

Vandervoort, D. (1992). *Temari: How to Make Japanese Thread Balls*. Tokyo: Japan Publications.

Von Neumann, J. (1945). First Draft of a Report on the EDVAC. Reprinted in B. Randell (Ed.), *The Origins of Digital Computers: Selected Papers* (1973). New York: Springer-Verlag.

Von Neumann, J. (1956). *The Computer and the Brain*. Reprinted 2000, New Haven: Yale University Press.

Welchman-Tischler, R. (1992). *The Mathematical Toolbox*. White Plains, NY: Cuisenaire.

Wrensch, T. (2001). *Programming Computationally-Enhanced Craft Items*. Ph.D. thesis, University of Colorado, Boulder.

Wrensch, T., Blauvelt, G. and Eisenberg, M. (2000). The Rototack: Designing a Computationally-Enriched Craft Item. *Proceedings of DARE 2000 (Designing Augmented Reality Environments)* (pp. 93–101). ACM, Elsinore, Denmark.

Wrensch, T. and Eisenberg, M. (1998). The Programmable Hinge: Toward Computationally Enhanced Crafts. *Proceedings of UIST 98* (November) (pp. 89–96). San Francisco.

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