SOME NOTES ON THE MATHEMATICAL WORK OF ALBRECHT DURER (1471 – 1528)

R. J. STEEMSON

May 21st this year marked the five hundredth anniversary of the birth in Nuremberg of Albrecht Durer. Although he must be chiefly remembered for his artistic work, including more than seventy paintings and hundreds of woodcuts and engravings, his contributions to science and mathematics must not be forgotten. These lie in his development of the applications of geometry and anatomy to art, and in his introduction of these ideas from Italy into Germany.

He began an apprenticeship to his father who was a goldsmith. However, by the age of 15 he had decided on a career as a painter and became apprenticed to the artist printer Michael Wolgemut. From 1490 onwards he travelled widely in Europe, although his home remained in Nuremberg where he married Agnes Frey in 1494, and died in 1528.

During this period the Renaissance painters in Italy had formulated the empirical laws of perspective and were beginning to apply the study of anatomy to their drawings. Durer travelled to Bologna to learn these laws of perspective and also spent two years studying in Venice. Not only did he introduce these ideas to his native Germany, but also he used his knowledge of geometry to treat perspective from a mathematical point of view. His theory of perspective was published in his book "Unterweissung der Messung mit der Zirkel und Richtscheid" in 1525. This book also contained chapters on linear geometry and on geometry in 2 and 3 dimensions. It contained results on conic sections and regular polygons and described the epicycloid for the first time.

In 1527 he published a book on theory of fortifications, and in 1528 an important work in which he laid the foundations of the science of anthropometry. Although his achievements tend to be eclipsed by those of his Italian contemporary Leonardo da Vinci, they are still of considerable importance, especially in their contribution to the spread of knowledge into central Europe.

QUEENS

Show how 5 Queens can occupy or attack the maximum number of squares possible on an 11 x 11 board.

Note: Queens attack squares in direct horizontal, vertical, or diagonal line to the square they occupy.

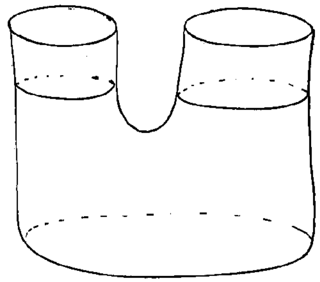
Further note: this further note may be ignored.

THE KNITTING OF SURFACES

BY M. O. REID

Introduction As anyone who has worn a woolly will know, knitting is rather a nice way of representing some 2-dimensional manifolds. The question naturally presents itself: which of the two-manifolds can be knitted without seams? Unfortunately the answer to this is rather dull: they all can be, although not all of them very nicely. Since knitting has got an obvious 'grain', knitting a two manifold provides us with a combing of it. So the question that is interesting is the following: which of the 2-manifolds can be knitted without seams and with only respectable singularities?

For example, let us look at the object below, a cylinder dividing in two. If we were trying to knit this, the branch point would have to have a singularity of the type shown.



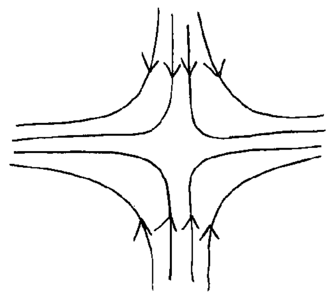


fig 1

This is something that cannot be very elegantly knitted: we will get a hole in the middle.

By contrast a singularity that does come off all right is the one that has been standard knitting practice since the invention of the bobble-cap. Starting with a cylinder we just decrease till there are only half-a-dozen stitches left, slip them all onto the thread, and pull tight.

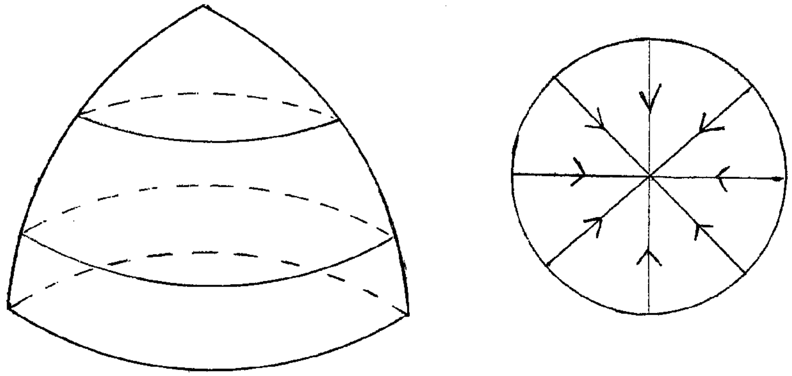


fig 2

This article provides patterns for the sphere, the torus, the klein bottle and one surface-with-boundary, the mobius strip. The torus and the klein bottle can be done with no singularity at all. The other surface than can be nicely combed is the real projective plane. I have a method for doing this but it is long and impossibly messy to describe. All the two-manifolds can now be knitted, by just taking connected sums of toruses and projective planes, using crude techniques, and of course the above untidy singularity.

Remark: the reader may wonder why I am always knitting on the round, rather than back and forth on rows. This is easy to explain: at any stage in the process, the piece of knitting is a 2-manifold with boundary. As is well known, the boundary now has to be a 1-manifold without boundary, i.e. a circle or collection of circles. This explains why I always start "cast on a cylinder of so-many stitches".

Technical Digression I require the use of three rather special techniques. Two of them are deduced from the appearance of a knitted cylinder. It is symmetric for reflections in a horizontal plane, and there is nothing to distinguish one row from any other. So one could have cast on a middle row first, and worked out; or alternatively, put the middle row in last of all.

The first is easy. Using spare (and different colour) wool, cast on a cylinder. Knit a couple of rows. Join in a main colour, and knit, say 10 rows. If the spare wool is now cut away, there is left a further row which can be slipped onto some further needles, and kept for future use.

The second is tricky, although a standard knitting technique (see the P. & B. booklet "Woolcraft" – the section on socks).

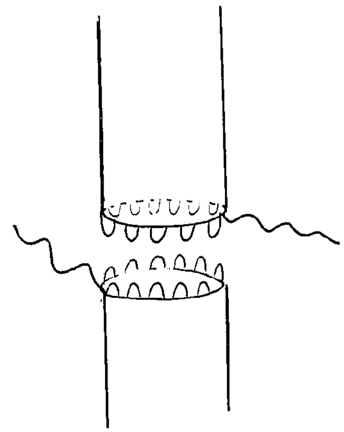
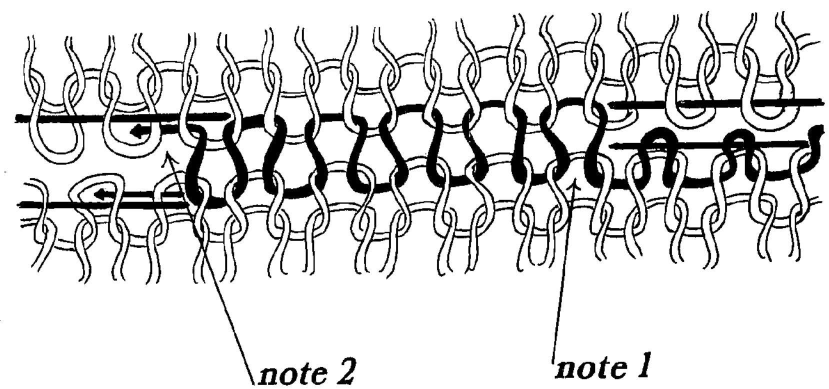


fig 3

As illustrated below, we have two cylinders with right side of work facing. The threads are at opposite sides of the cylinders, one of them being cut to a couple of yards, and threaded onto a bodkin. The process defies explanation, but I hope that the local diagram will make things clear.



In the local diagram note:

(1) The general stitch consists of one threading from front to back to front through the next stitch.

(2) The first stitch is perverse and confusing. The thread is passed through the next stitch from p. side to k. side of work.

I shall refer to this process as "grafting", since this is the standard terminology.

I do feel guilty that this is a cheat since it is "sewing up". I use the following remarks to satisfy my own conscience: a) it is standard b) it is dual to the two-sided casting on, which is irreproachable c) the purist who objects may say that I have cheated. but will not be able to say where, since the "grafting" row is in principle indistinguishable from any other row in which wool was joined, and is in practice indistinguishable, if the grafting was done carefully.

The third special technique is indispensable for making any of the non-orientable surfaces. Since they intersect themselves, we need a process for passing a cylinder through an already existing surface, a "wall". This is not very difficult, but requires a crochet hook. Slip the stitches onto a piece of spare wool, pull first the thread through a chosen hole, then each of the stitches, mounting them onto another piece of spare wool when they are through.

The Patterns requirements: set of four no.8 needles, two ounces of double knitting wool, a crochet hook, a few yards of a different colour scrap wool, kapok for stuffing (from Woolworth's).
The Sphere Cast on (both sides) a cylinder of 30 sts.

*k. 5 rounds, Decrease as follows.

Next round: (k.8 k.2 tog) 3 times
Next round: (k.7 k.2 tog) 3 times
Next round: (k.6 k.2 tog) 3 times
Next round: (k.5 k.2 tog) 3 times
Next round: (k.1 k.2 tog) 6 times
Next round: (k.2 tog) 6 times

Break off thread. Slip last sts onto thread and pull tight, leaving thread on wrong side of work.**

Join in thread at second side of casting on. Knit second hemisphere from *to**, stuffing firmly a few rounds before end if required.

The Torus (a) Cast on (both sides) a cylinder of 30 sts. Knit 80 rows, then half a round. Pick up sts from second side of casting on. Graft to finish, stuffing if required.

(b) Cast on (both sides) a cylinder of 32 sts. Knit one round.

Increase 8 sts in each of the next 8 rows as follows:
Inc in next st, k.3, inc in next st, k.3(5, .. 17) 4 times, knit 8 rounds, then *decrease* 8 sts in each of next 8 rows, as follows:
k.2 tog, k.3, k.2 tog, k.17(15 ... 3) 4 times, knit 7 rounds, knit half a round, graft off.

The Klein Bottle Cast on (both sides) a cylinder of 30 sts, knit 10 rounds. Increase 6 sts in every 4th row, 5 times, as follows:
1st (5th ... 17th) row: k.4(5 ... 8), inc in next sts six times (36, 42 .. 60 sts) others: knit. Knit three rounds.
Form a 'purl' window as follows:

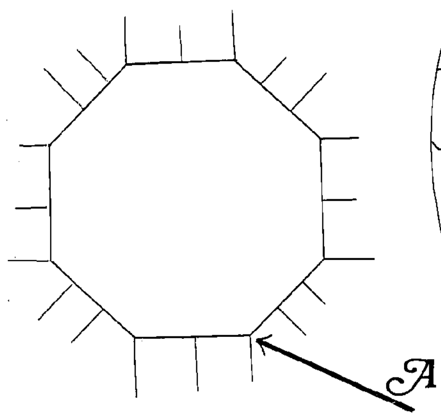
1st round: k.7, p.4, k. to end
2nd round: k.6, p.6, k. to end
3rd round: k.5, p.8, k. to end
4th–7th round: k.4, p.10, k. to end
8th round: as 3rd round
9th round: as 2nd round
10th round: as 1st round

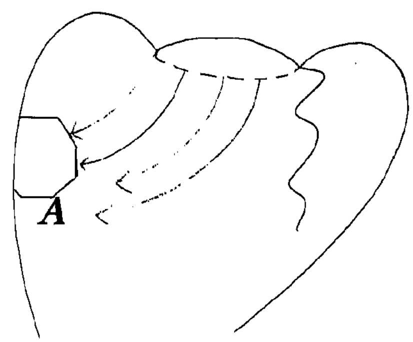
K. 5 rounds, then first five sts of next round onto the end of the last needle, so that the round starts five sts later than previously.

Decrease 5sts every 4th round, 6 times, as follows:

1st (5th .. 21st) row: k.10, (k.8(7, ... 3) k.2 tog) 5 times (55,50 .. 30st) others: knit. K. 5 rounds, then first three stitches of next round. Pass thread through wall at "A" (in diagram) and pass cylinder through the wall at the outside edge of the purl window. K. 50 rounds. Graft off, stuffing.

The Mobius Strip This pattern requires a special weapon – a circular needle 32" long. The method is merely an improvement of "two-sided" casting on such that both sides of the stitches can be used from the beginning.
Using spare wool and a pair of needles, cast on 90 sts.





Using main wool, knit these onto the circular needle. They now cover about 2/3 of the needle, the stitches at the far end from the thread lying on the plastic. The working end of the needle is bent round to the far end to pick these stitches up, and these are *not* slipped off the end as they are knitted. When the row is all picked up, the needle loops the work twice, and the effect is as of ordinary two-sided casting on except that the second row is held on the needles. Knit into this row again. Knit five rows, and cut away the spare row. Cast off. Obviously it is not satisfactory to do a Mobius strip in stocking stitch! However, two-sided casting on cannot possibly work for any rib! (try it and see)

THE ARCHIMEDEANS

The Archimedeans have had a good year, with the evening and tea meetings well attended. The evening meetings included talks by Professor E. C. Zeeman on "Catastrophe Machines" and Professor H. Laster, on "The Propagation of Cosmic Rays". There was also a meeting in the Easter term at which Professor Marshall Hall spoke on "Problems in Arrangements". Tea meetings were very successful, with Dr. J. H. Conway on "Hackenbush, Welter, Prune and other games" and Dr. A. F. W. Edwards' talk: "Probability Theory and Human Genetics".

There was the usual visit to Oxford to play games with the "Invariants" and the Problems Drive in which the "Invariants" visited Cambridge. The visit to the Rutherford High Energy Laboratory at Abingdon was very successful. In the Lent term a dinner was held in the Graduate Centre. Amongst the society's guests were Professor Sir Nevil Mott and Professor J. F. Adams. The Computer Group has had an active year and the Music group and the Bridge group have both met frequently, but the Puzzles and Games Ring died in the Lent Term and its future is uncertain. The Bookshop has continued to thrive.

Speakers for the coming year include Professor C. T. C. Wall on "How to Organise a Tournament". Lady Jeffreys, who will speak at a tea meeting, and an address from Dr. P. Neumann on "The Mathematical Analysis of "1066 and all That" " will be the first meeting of the academic year. There will be a Careers Meeting as usual and also a visit to Oxford.

It is hoped that this year's programme will cater for all tastes. Suggestions for any change in the activities, or for speakers for future years would be most welcome; a book is kept in the Arts School for this purpose.

Simon Anscombe

A CRITICISM OF THE FOOTBALL LEAGUE EIGENVECTOR

D. R. WOODALL

Dr. A. N. Walker has made the following criticism of the eigenvector method of compiling an 'order of merit' after a tournament. He points out that the method only takes account of the team's wins, and not of its losses. Thus a team receives great credit for a win against a good side, but is not penalised for losing to a very bad side. If two teams A and B end up with the same number of points after an 'all-play-all' tournament, and if team A obtained its points against better teams (on average) than team B, it follows that team A must have lost to worse teams (on average). The team that beats the better opponents will benefit by scoring a larger number of points, unless it also loses to some worse opponents.

Because of this, Dr. Walker maintains that in an 'all-play-all' tournament the first order scores (sums of points scored) already take account of the quality of the teams beaten and so provide the fairest possible order. It is for this reason that in tournaments based on, for example, the 'Swiss system', in which not everyone plays everyone else, the usual method of separating ties is to credit a player with the sum of the scores, not of the people he beats, but of all the people he plays against.

The difficulty in the eigenvector method becomes particularly apparent if one considers an 'all-play-all once' tournament, with each team having a technical draw against itself, and with the usual (0-1-2) method of scoring. Then the i'th team's second order score is equal to the sum of *all* the first order scores, *plus* the scores of those teams that team i beats, *minus* the scores of those teams to which team i loses. In other words, team i is penalized more for losing to a good side than for losing to a poor side. To correct this, one should really subtract, not the score $p_j(1)$ of each team j that beats team i, but $C - p_j(1)$ for some constant $C \max p_j(1)$, C presumably being taken equal to the maximum possible number of points that a team can score. Unfortunately this is no longer a simple eigenvector method, but the first convergent could be used to separate ties.

To some extent a person's assessment of methods of scoring must depend on his psychology. A mathematician would tend to regard a draw as a neutral result, a loss as a negative result and a win as a positive result. He would probably expect the method of scoring to reflect this. In particular, he would expect that if all the results in the tournament were reversed, and the 'order of merit was calculated again, then the exact reverse order would be obtained. Of the methods discussed, only the eigenvector method does not have this property. The '0-1-2' method of scoring, however, tends to encourage people to think of a *loss* as a neutral result, a draw as a positive result and a win as a better positive result. This, and the eigenvector method, may accord better with the average person's tendency to forget the bad results and only take account of the good ones.