

Problem 6:

Prove that f and g are inverse functions: $f(x) = 5x + 1$, $g(x) = \frac{x-1}{5}$

$$f(g(x)) = 5\left(\frac{x-1}{5}\right) + 1$$

$$f(g(x)) = x - 1 + 1$$

$$f(g(x)) = x$$

$$g(f(x)) = \frac{(5x-1)+1}{5}$$

$$g(f(x)) = \frac{5x-1+1}{5}$$

$$g(f(x)) = \frac{5x}{5} = x$$

$\therefore g(x)$ and $f(x)$ are inverse functions

Problem 44:

Determine whether the function is monotonic: $f(x) = (x + a)^3 + b$

$$f'(x) = 3(x + a)^2 \geq 0 \text{ for all values of } x$$

$\therefore f(x)$ is monotonic, and thus has an inverse function

Problem 48:

Determine whether the function is monotonic: $f(x) = |x + 2|$, $[-2, \infty]$

$$f'(x) = 1 \geq 0 \text{ } [-2, \infty]$$

$\therefore f(x)$ is monotonic, and thus has an inverse function

Problem 74:

Find $(f^{-1})'(a)$, if $f(x) = \frac{1}{27}(x^5 + 2x^3)$ and $a = -11$

$$f'(x) = \frac{1}{27}(5x^4 + 6x^2) > 0, \therefore (f^{-1}) \text{ exists}$$

$$-11 = \frac{1}{27}(x^5 + 2x^3)$$

$$x = 17$$

$$\therefore f(x) = -11 \text{ when } x = -3$$

$$\text{Theorem 5.9 states that } (f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(f^{-1})'(-3) = \frac{1}{f'(-3)}$$

$$(f^{-1})'(-3) = \frac{1}{\frac{1}{27}(-3^4 \times 5 + -3^2 \times 6)}$$

$$(f^{-1})'(-3) = 17$$