TOPICS

- UNDERSTAND THE DEFINITIONOF A POWER SERIES
- FIND THE RADIUS AND INTERVAL OF CONVERGENCE OF A POWER SERIES
- DETERMINE THE ENDPOINT CONVERGENCE OF A POWER SERIES
- DIFFERENTIATE AND INTEGRATE A POWER SERIES

TEXT READING ASSIGNMENT FOR 9.8

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TEXT HOMEWORK EXCERCISES FOR 9.8

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• UNDERSTAND THE DEFINITIONOF A POWER SERIES

A ***power series centered at c*** is a function of the form $f(x)=\sum_{n=0}^{\infty} a_n(x-c)^n = a_0 + a_1\left(x-c\right)+a_2(x-c)^2 + \cdots$. If we simply say ***power series*** without reference to the center, we will assume the center is $c=0$ and that the function has the form $f(x)=\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$.

Examples

a) The power series $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$ is centered at $c=0$ with $a_n = \frac{(-1)^n}{(2n+1)!}$.

b) The power series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(x-1)^n}{n}$ is centered at $c=1$ with $a_n = \frac{(-1)^{n-1}}{n}$.

Remark When we state the equation $\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \cdots$ we are stating the first term $a_0 x^0 = a_0$ for all x. Of course 0^0 is an indeterminate form. However, it is useful to accept the convention that $0^0 = 1$ when dealing with power series so that we can utilize the very handy summation notation.

• FIND THE RADIUS AND INTERVAL OF CONVERGENCE OF A POWER SERIES

The domain of a power series $\sum_{n=0}^{\infty} a_n(x-c)^n$ centered at $x=c$ is always an interval, called the ***interval of convergence***. There are three cases;

Interval	$x=c$	$c-R<x<c+R$	$-\infty<x<\infty$
Radius	0	R	∞

Remark In case 2, we may include one or both endpoints in the domain.
Remark When finding the interval of convergence we make heavy use of the ratio test.

Worked Example 9.8.1

Student Exercise: Find the radius of convergence for $\sum_{n=1}^{\infty} \frac{n!x^n}{(2n)!}$.

Student Exercise Solution 9.8.2

• DETERMINE THE ENDPOINT CONVERGENCE OF A POWER SERIES

If a power series centered at $x = c$ has a radius of convergence R, with $0 < R < \infty$ then the interval of convergence maybe any of four cases;
1) $c - R < x < c + R$
2) $c - R \le x \le c + R$
3) $c - R \le x < c + R$
4) $c - R < x \le c + R$
To determine the correct interval of convergence, we must inspect the convergence of the power series at both endpoints $x = c - R$ and $x = c + R$ using the techniques from sections 9.1 through 9.7.

Worked Example 9.8.3

• DIFFERENTIATE AND INTEGRATE A POWER SERIES

Theorem Suppose $f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots$ has interval of convergence $c - R < x < c + R$, where $R > 0$. $f(x)$ is differentiable and hence continuous on $c - R < x < c + R$ moreover,

1) $f'(x) = \sum_{n=1}^{\infty} a_n n(x-c)^{n-1} = a_1 + 2a_2(x-c) + 3a_3(x-c)^2 + \cdots$

2) $\int f(x)dx = \sum_{n=0}^{\infty} \frac{a_n(x-c)^{n+1}}{n+1} = C + a_0(x-c) + a_1\frac{(x-c)^2}{2} + a_2\frac{(x-c)^3}{3} + a_3\frac{(x-c)^4}{4} + \cdots$

Remark We may lose or gain endpoints when differentiating or integrating.

Remark This same theorem is valid for a power series with interval of convergence $-\infty < x < \infty$, except the interval does not change when differentiating or integrating.

Worked Example 9.8.4

Student Exercise: Evaluate $f'(x)$ for $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-5)^n}{n5^n}$.

Remark To find $\frac{d}{dx}\left(\sum_{n=0}^{\infty}a_n(x-c)^n\right)$ or $\int\left(\sum_{n=0}^{\infty}a_n(x-c)^n\right)dx$ make sure to treat n as a constant and differentiate or integrate with respect to x.

Remark When evaluating $\frac{d}{dx}\left(\sum_{n=0}^{\infty}a_n(x-c)^n\right)$ or $\int\left(\sum_{n=0}^{\infty}a_n(x-c)^n\right)dx$ be sure to find the interval of convergence including endpoints.

Student Exercise Solution 9.8.5