TOPICS
• RECOGNIZE LIMITS THAT PRODUCE INDETERMINATE FORMS
• APPLY L'HOPITAL'S RULE TO EVALUATE A LIMIT

TEXT READING ASSIGNMENT FOR 8.7

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TEXT HOMEWORK EXCERCISES FOR 8.7

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• RECOGNIZE LIMITS THAT PRODUCE INDETERMINATE FORMS

In section 8.8 we will focus our attention on definite integrals where the limits of integration are either at an asymptote or at infinity. In order to do so we need to discus ***indeterminate forms*** like $0/0$, ∞/∞, $\infty-\infty$, $0\cdot\infty$, 1^{∞}, ∞^{0} and 0^{0} in more detail. An indeterminate form is just that; indeterminate, they can end up being any real number, $\pm\infty$ or not exist at all.

L'Hopital's Rule states if $\lim\limits_{x\to c}\frac{f(x)}{g(x)}$ is either $0/0$ or $\pm\infty/\infty$ then, $\lim\limits_{x\to c}\frac{f(x)}{g(x)}=\lim\limits_{x\to c}\frac{f'(x)}{g'(x)}$ provided the limit on the right exists or is $\pm\infty$.

Remark L'Hopital's Rule also applies to limits where $x\to c^{+}$, $x\to c^{-}$, $x\to\infty$ and $x\to-\infty$.
Remark We will see how L'Hopital's Rule can be used to evaluate any of the indeterminate forms above.

In order to apply L'Hopital's Rule c must be contained in an interval (a,b) that where each of the conditions are satisfied, except possibly at c itself;
1) f and g are differentiable.
2) $g'(x)\neq 0$.

Remark In almost every situation the two conditions above are satisfied.

• APPLY L'HOPITAL'S RULE TO EVALUATE A LIMIT

Example Evaluate $\lim\limits_{x\to 1}\frac{\ln x}{x-1}$. If we plug in $x=1$ we get the indeterminate form $0/0$. So by L'Hopital's Rule,

$$\lim_{x\to 1}\frac{\ln x}{x-1}=\lim_{x\to 1}\frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(x-1)}=\lim_{x\to 1}\frac{\frac{1}{x}}{1}=1.$$

Remark Make sure when applying L'Hopital's Rule you <u>do not apply the quotient rule</u>!

Worked Example 8.7.1

In order to apply L'Hopital's Rule to either indeterminate form $\infty - \infty$ or $0 g \infty$ you must manipulate the term in the limit so that you get one of the indeterminate forms $0/0$ or $\pm\infty/\infty$.

Worked Example 8.7.2

Student Exercise: Evaluate $\lim\limits_{x \to 1}\left(x \sin \frac{1}{x}\right)$ by manipulating the term of the limit so that you can apply L'Hopital's Rule. Find this limit.

Student Exercise Solution 8.7.3

Here is an example of how to apply L'Hopital's Rule to the indeterminate form ∞^0.

Example Evaluate $\lim\limits_{x \to \infty}(1+x)^{\frac{1}{x}}$.

By substitution, we get ∞^0. Lets assume the limit exists and call it L, so that $L = \lim\limits_{x \to \infty}(1+x)^{\frac{1}{x}}$. Apply the natural logarithm and you get $\ln L = \ln \lim\limits_{x \to \infty}(1+x)^{\frac{1}{x}} = \lim\limits_{x \to \infty} \ln(1+x)^{\frac{1}{x}} = \lim\limits_{x \to \infty} \frac{\ln(1+x)}{x}$. Since this gives us the indeterminate form ∞/∞ we can now apply L'Hopital's Rule; $\ln L = \lim\limits_{x \to \infty} \frac{\ln(1+x)}{x} \lim\limits_{x \to \infty} \frac{\frac{1}{1+x}}{1} = 0$. Hence, $L = e^0 = 1$.

To apply L'Hopital's Rule to any of the indeterminate forms 1^∞, ∞^0 and 0^0 use the steps below.

1) Set L equal to the limit.
2) Set $\ln L$ equal to the natural logarithm of the limit.
3) Interchange the order of $\ln$ and $\lim$ (this is justified by the fact the natural logarithm function is continuous).
4) Apply the power rule $\ln a^n = n \ln a$.
5) You can now apply L'Hopital's Rule.
6) Your answer is $L = e^{\text{answer in step 5}}$.

Student Exercise: Evaluate $\lim\limits_{x \to \infty}\left(1 + \frac{1}{x}\right)^x$.

Student Exercise Solution 8.7.4