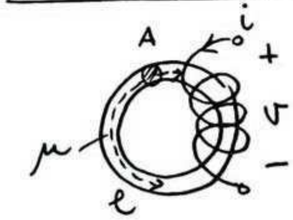
# LECTURE 13

## INDUCTORS & FIRST ORDER CIRCUITS

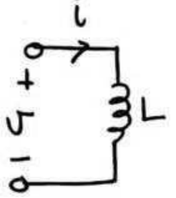



$L = \frac{\mu N^2 A}{l}$

$\lambda = L \cdot i$ magnetic flux

$v = \frac{d\lambda}{dt} = L \cdot \frac{di}{dt}$

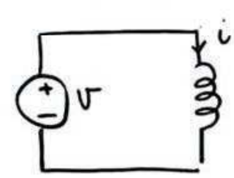
$i = \frac{1}{L} \int_{-\infty}^{t} v(t)\,dt$

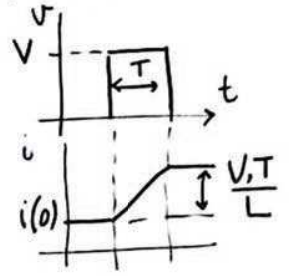



Energy Storage

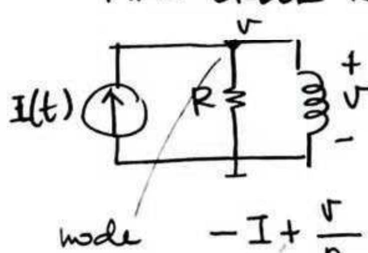
$p = v \cdot i = L \frac{di}{dt} i = \frac{d}{dt}\left(\frac{1}{2} L i^2\right)$

INDUCTOR & VOLTAGE SOURCE

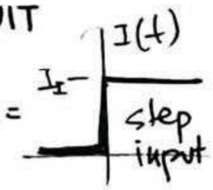



FIRST ORDER RL CIRCUIT



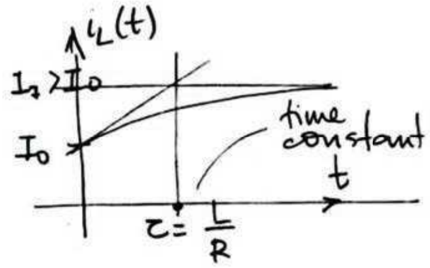

$I(t) =$

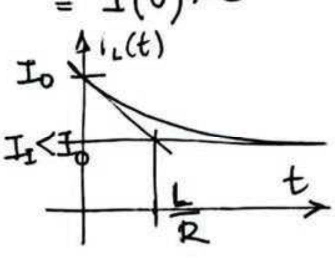



$-I + \frac{v}{R} + i_L = 0$

$$\frac{L}{R} \frac{di_L}{dt} + i_L = I_I \qquad i_L = (I_0 - I_I)\, e^{-\frac{R}{L}t} + I_I =$$

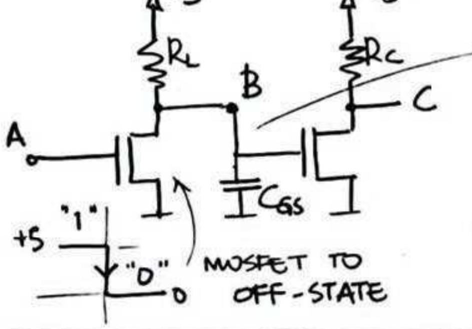
$$= I(0) \cdot e^{-\frac{R}{L}t} + I(+\infty)\left(1 - e^{-\frac{R}{L}t}\right)$$



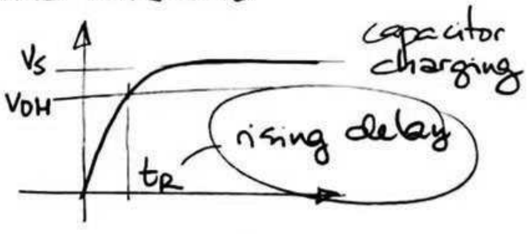



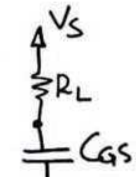

# LECTURE 14

## SPEED OF DIGITAL CIRCUITS



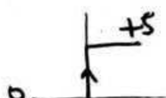

$v_B$



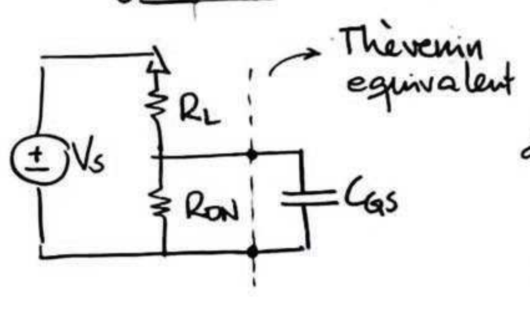

$v_B = V_S\left(1 - e^{-t/R_L C_{GS}}\right)\Big|_{t=t_R} = v_{OH} \qquad t_R = -R_L C_{GS} \ln\left(1 - \frac{V_{OH}}{V_S}\right)$

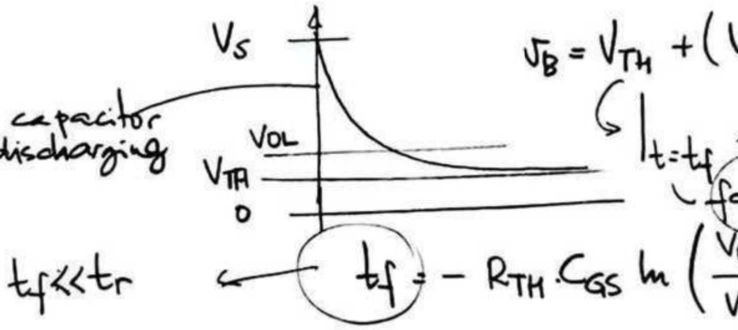
ej. 0,16 nS ) similar
C = 0,1 nS




MOSFET TO ON-STATE

$V_{TH} = V_S \frac{R_{ON}}{R_L + R_{ON}} \qquad R_{TH} = \frac{R_{ON} R_L}{R_L + R_{ON}}$




$v_B = V_{TH} + (V_S - V_{TH}) \cdot e^{-t/R_{TH} C_{GS}}$

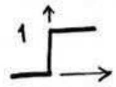
$\Big|_{t=t_f} = V_{OL}$ falling delay

$t_f \ll t_r \qquad t_f = -R_{TH} C_{GS} \ln\left(\frac{V_{OL} - V_{TH}}{V_S - V_{TH}}\right)$

# LECTURE 15 — RAMPS, STEPS & IMPULSES

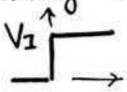
Notation for a step

Unit step



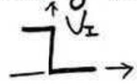

$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

Rising step



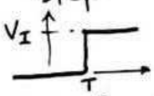

$v(t) = V_I \cdot u(t)$

Falling step




$v(t) = V_I(1 - u(t))$

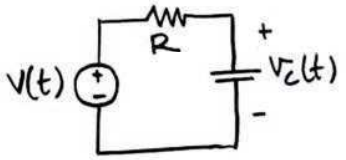
Time translated step

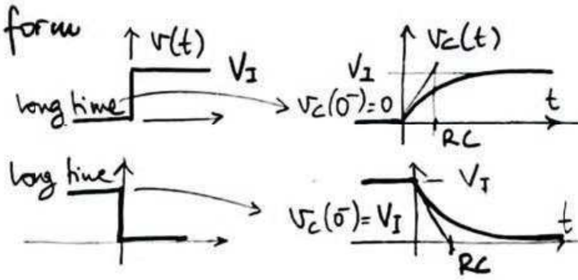



$v(t) = V_T(u - T)$

Response to STEP INPUT

RC circuit. Thèvenin form



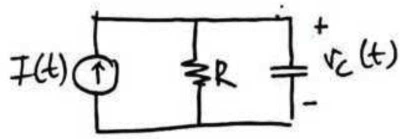



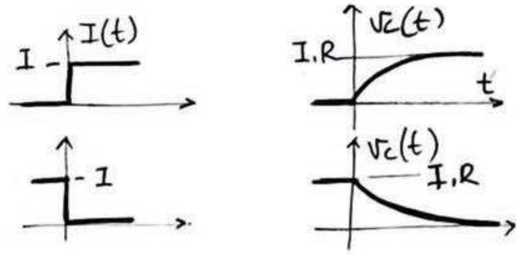

ZERO STATE RESPONSE

$v_c(t) = V_I\left(1 - e^{-t/RC}\right)$

$v_c(t) = V_I \cdot e^{-t/RC}$

RC circuit. Norton form

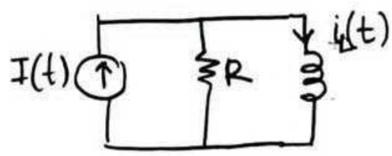


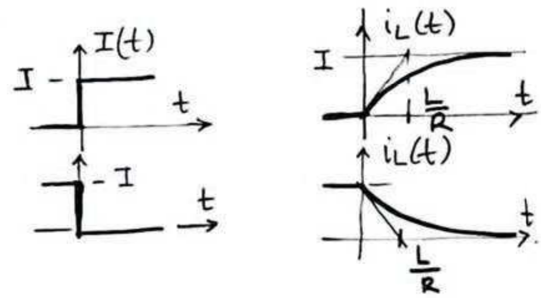




$v_c(t) = I.R\left(1 - e^{-t/RC}\right)$

$v_c(t) = I.R\, e^{-t/RC}$

RL circuit. Norton form

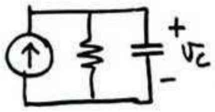


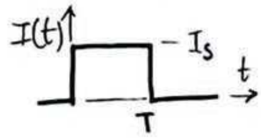

$i_L(t) = I\left(1 - e^{-tR/L}\right)$

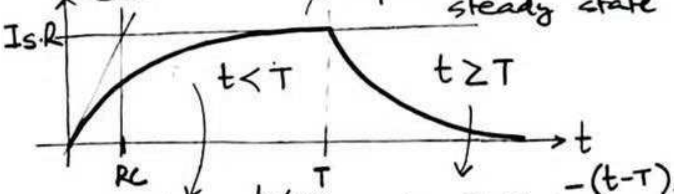
$i_L(t) = I\, e^{-tR/L}$

RC circuit response to PULSE INPUT



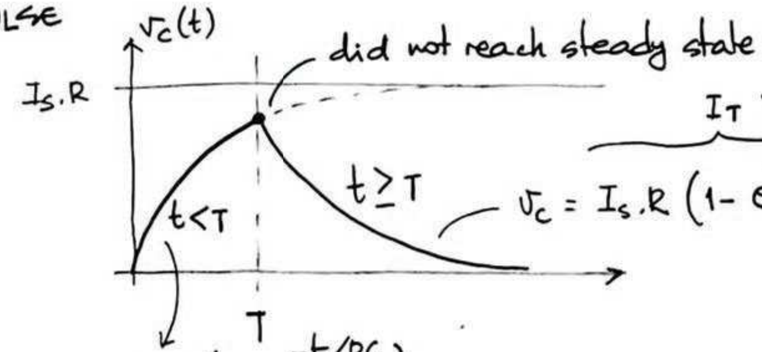

LONG PULSE $T \gg RC$




$v_c = I_s R\left(1 - e^{-t/RC}\right)$ $\qquad v_c = I_s.R\, e^{-(t-T)/RC}$

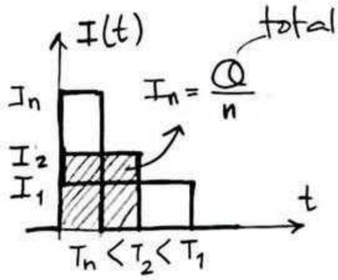
SHORT PULSE




$v_c = \underbrace{I_s.R\left(1 - e^{-T/RC}\right)}_{I_T \text{ value at } t=T} \cdot e^{-(t-T)/RC}$

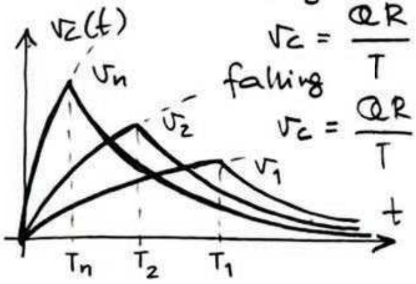
$v_c = I_s.R\left(1 - e^{-t/RC}\right)$

## RC circuit response to <u>IMPULSE</u> input




response →

T→0

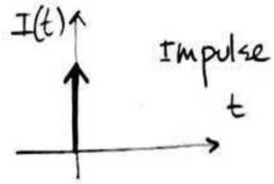



rising

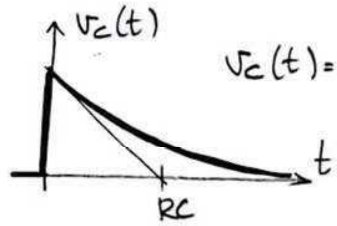
$$v_C = \frac{QR}{T}\left(1 - e^{-t/RC}\right)$$

falling

$$v_C = \frac{QR}{T}\left(1 - e^{-T/RC}\right) \cdot e^{-(t-T)/RC}$$



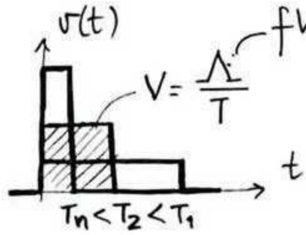

$$I(t) = Q \cdot \delta(t)$$



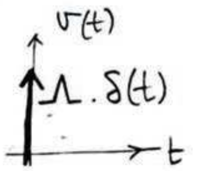

$$v_C(t) = \frac{Q}{C} \cdot e^{-t/RC}$$

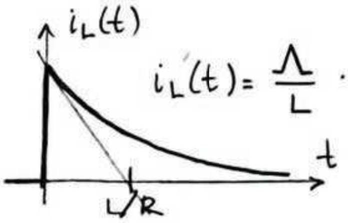
ZERO IMPULSE RESPONSE

## RL circuit response to <u>IMPULSE</u> input




T→0




$$i_L(t) = \frac{\Lambda}{L} \cdot e^{-tR/L}$$

Superposition with initial conditions & source

- Treat initial conditions as sources
- Find response to each source

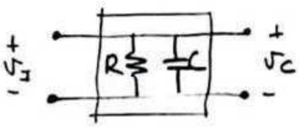
Impulses, steps & Ramps : No source, No initial conditions → LINEAR SYSTEMS

input

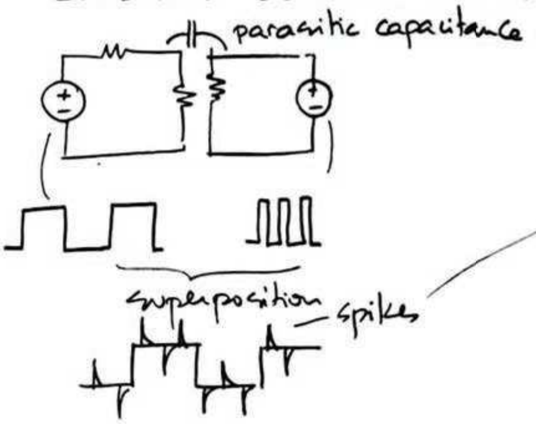

integral & derivative operators are linear

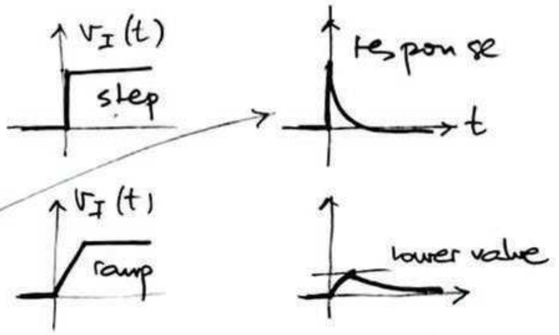
response

$$\begin{array}{ccc} I(t) = \delta(t) & \xrightarrow{\text{RC responce}} & v_C = \frac{1}{C} e^{-t/RC} \\ \int dx \downarrow \ \uparrow d/dx & & \int dx \downarrow \ \uparrow d/dx \\ I(t) = u(t) & \xrightarrow{\text{RC responce}} & v_C = R\left(1 - e^{-t/RC}\right) \end{array}$$



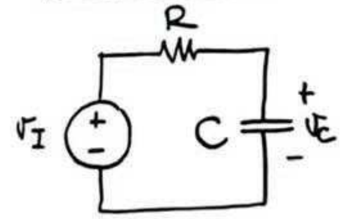

CROSSTALK BETWEEN TWO CIRCUITS



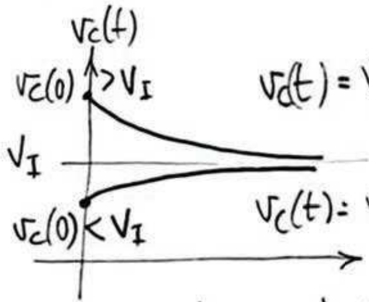
# LECTURE 16

# STATE & MEMORY




$v_I(t) = V_I$ for $t \geq 0$

$v_I(t)$ for $t < 0$




$v_C(t) = V_I + (v_C(0) - V_I) \cdot e^{-t/RC}$

SAME EQUATION

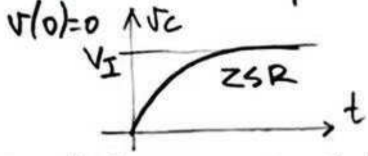
$v_C(t) = v_C(0) + (V_I - v_C(0)) \cdot (1 - e^{-t/RC})$

$v_C = f(v_C(0), V_I)$ $v_C(0)$ – state, captures the past history of $v_I$ in $q = CV$

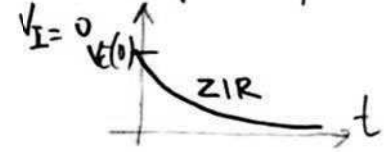
$v_I(t)$ – input

SAME EQUATION

Zero State Responce




Zero Input Responce

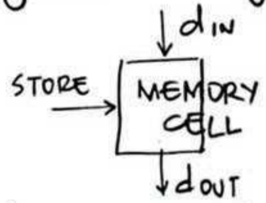


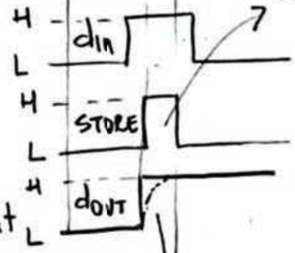

Total Responce

$v_C(t) = V_0 \cdot e^{-t/RC} + V_I(1 - e^{-t/RC})$

$v_C(t) = \text{ZIR} + \text{ZSR}$

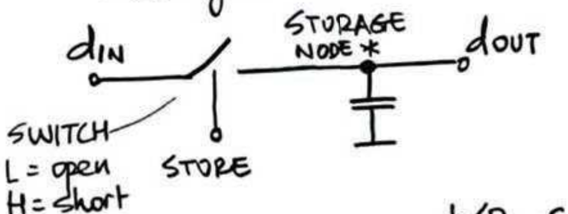
Digital Memory Abstraction



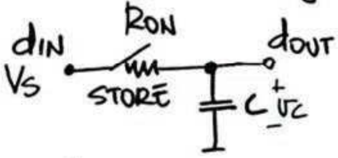




when 'store' goes high, output follows input

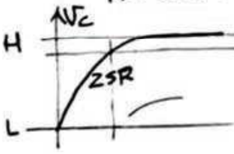
Memory element




Building a Memory Element



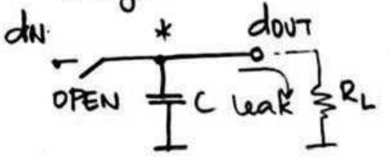

real switch with $R_{ON}$

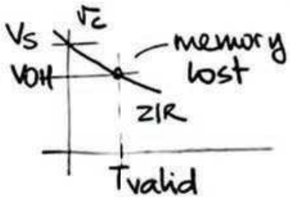



$V_{OH} = V_S\left(1 - e^{-t/R_{ON}C}\right)$

$T_{pulse} \gg T_{min}$ minimum store pulse to reach $V_{OH}$

Storage Time

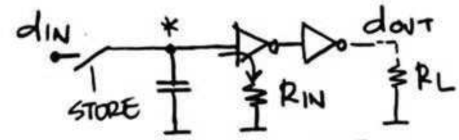



$V_{OH} = V_S \cdot e^{-t/R_LC}$ $T \ll T_{valid}$ to maintain memory

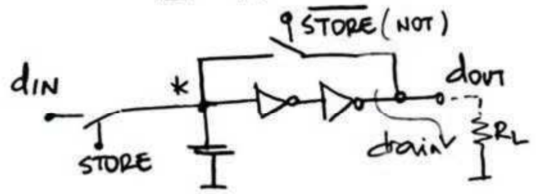
$T_{valid} = -R_LC \ln \frac{V_{OH}}{V_S}$ → THIS DOES NOT WORK!

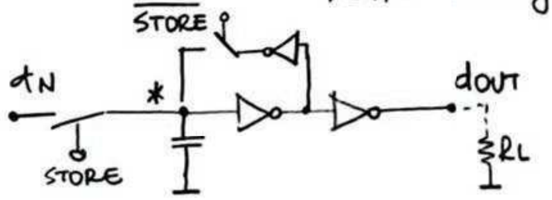
→ THE CIRCUIT IS BETTER




$T_{valid} = -R_{IN}C \ln \frac{V_{OH}}{V_S}$ $R_{IN} \gg R_L$ ~ in the order of 1 ms

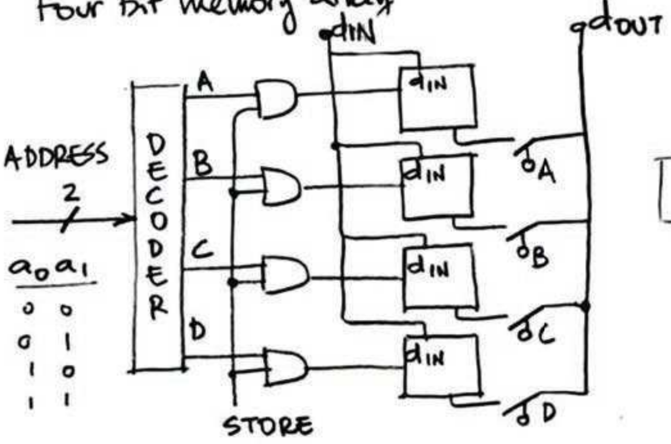
better storage time

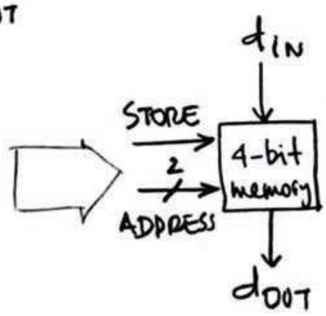


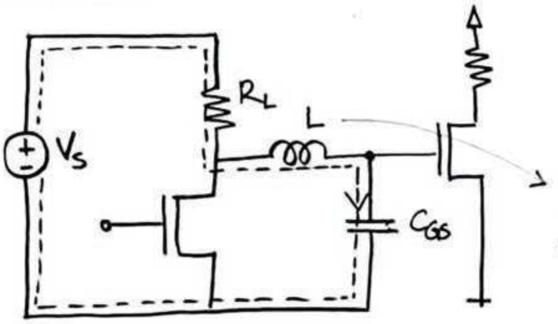

$d_{OUT}$ feedback $v_C$ so the memory lasts forever

Four bit memory array



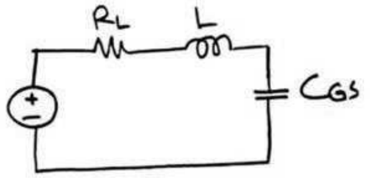
# LECTURE 17 — UNDAMPED SECOND-ORDER SYSTEMS



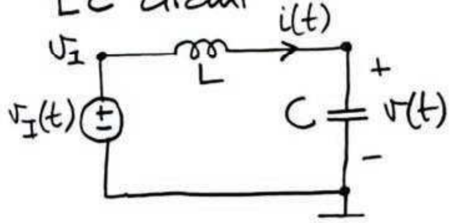

current loop with some inductance

modelled by a lumped inductor L

SECOND-ORDER CIRCUIT



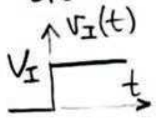

LC Circuit




$i(t) = C \cdot \frac{dv}{dt}$ $\qquad v_I - v = LC \frac{d^2v}{dt^2}$ units time$^2$

$v_I - v = L \cdot \frac{di}{dt}$ $\qquad LC\frac{d^2v}{dt^2} + v = v_I$ LC differential equation

Input step




Initial conditions $v(0)=0$ $i(0)=0$ | ZERO STATE RESPONSE

Differential eq. solution = PARTICULAR + HOMOGENEOUS = TOTAL solution

Particular solution $v_p(t) = V_I$ $\qquad$ Homogeneous solution $LC\frac{d^2v_H}{dt^2} + v_H = 0$ $\qquad v_H = A\cdot e^{st}$

$LCAs^2e^{st} + Ae^{st} = 0$ $\qquad LCs^2 + 1 = 0$ characteristic equation $\qquad s = \pm j\omega_0$ $\qquad \omega_0 = \sqrt{\frac{1}{LC}}$

Total solution with initial conditions (ZSR)

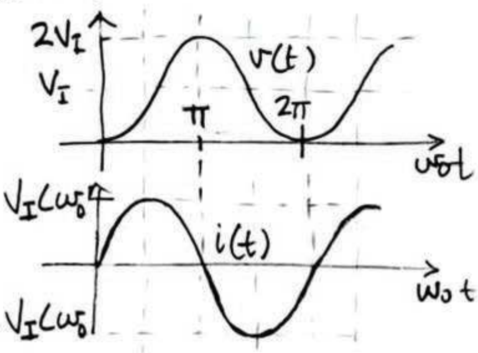
$$v(t) = V_I + A_1e^{j\omega_0 t} + A_2e^{-j\omega_0 t} \qquad v(0) = V_I + A_1 + A_2 = 0$$

$$i(t) = A_1Cj\omega_0e^{j\omega_0 t} - A_2Cj\omega_0e^{j\omega_0 t} \qquad i(0) = A_1Cj\omega_0 - A_2Cj\omega_0 = 0$$

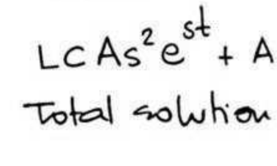
$$\Big\} \; A_1 = A_2 = -\frac{V_I}{2}$$

$$v(t) = V_I - V_I\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) = V_I - V_I\cos\omega_0 t$$

$$i(t) = -V_IC\omega_0\left(\frac{e^{j\omega_0 t} - e^{j\omega_0 t}}{2}\right) = V_I\, C\omega_0 \operatorname{sen}\omega_0 t$$




Zero Input $v_I(t) = 0$ Initial conditions $v(0) = V$; $i(0) = 0$




$$LC\frac{d^2v}{dt^2} + v = 0 \qquad v(t) = A_1e^{+j\omega_0 t} + A_2e^{-j\omega_0 t}$$

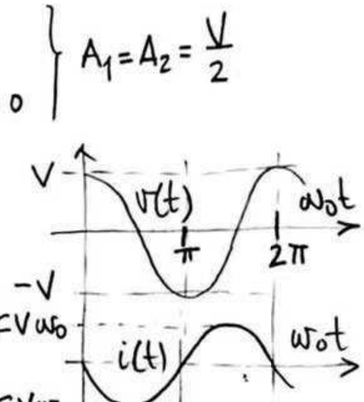
$$v(0) = A_1 + A_2 = V$$

$$i(0) = CA_1j\omega_0 - CA_2j\omega_0 = 0$$

$$\Big\} \; A_1 = A_2 = \frac{V}{2}$$

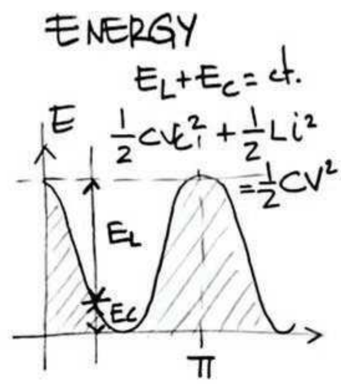
$$v(t) = V\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) = V\cdot\cos\omega_0 t$$

$$i(t) = C\cdot\frac{dv}{dt} = -CV\omega_0 \operatorname{sen}\omega_0 t$$



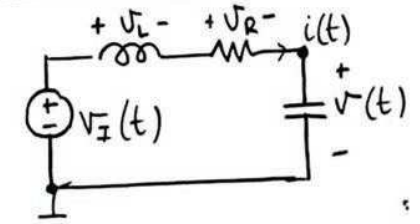

ENERGY

$E_L + E_C = ct.$



# LECTURE 18

## DAMPED SECOND-ORDER SYSTEMS




$i = C \cdot \frac{dv}{dt}$ $\quad v_R = R.i = RC\frac{dv}{dt}$ $\quad v_L = L\frac{di}{dt} = LC\frac{d^2v}{dt^2}$

$v_I(t) = v_L + v_R + v_C = LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v \rightarrow$ constant coefficient ODE

: ZERO STATE RESPONSE $\quad v_I = V_I \cdot u(t)$ $\quad v(0) = 0 \quad i(0) = 0$

ODE Particular solution

$LC\frac{d^2v_P}{dt^2} + RC\frac{dv_P}{dt} + v_p = v_I(t) \Rightarrow v_p = V_I \quad t \geq 0$

ODE Homogeneous solution

$\frac{d^2v_H}{dt^2} + \frac{R}{L}\frac{dv_H}{dt} + \frac{1}{LC}v_H = 0$

Solution trial $\quad$ Characteristic Eq. $\quad$ Canonic form

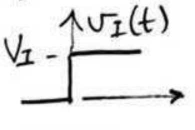
$v_H = A.e^{st} \Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 2\alpha s + \omega_0^2 = 0 \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \alpha = \frac{R}{2L} \Rightarrow$

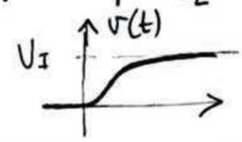
Roots

$\Rightarrow s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \Rightarrow$ ODE Total solution $\quad v(t) = V_I + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

OVERDAMPED CASE $\quad \alpha > \omega_0 \quad s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -\alpha_1 \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -\alpha_2$

$v(t) = V_I + A_1 e^{-\alpha_1 t} + A_2 e^{-\alpha_2 t}$



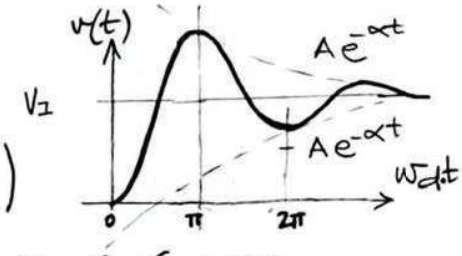




UNDERDAMPED CASE $\quad \alpha < \omega_0 \quad \sqrt{\alpha^2 - \omega_0^2} = j\sqrt{\omega_0^2 - \alpha^2} = j\,\omega_d$ damped natural freq.

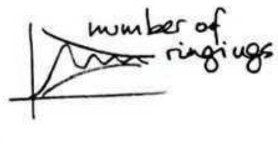
$v(t) = V_I + A_1 e^{-\alpha t} e^{+j\omega_d t} + A_2 e^{-\alpha t} e^{-j\omega_d t} \overset{\dots}{=} V_1 + K_1 e^{-\alpha t}\cos\omega_d t + K_2 e^{-\alpha t}\sin\omega_d t =$

$\overset{\dots}{=} V_I + A e^{-\alpha t}\cos(\omega_d t + \phi)$

$A = -V_I \frac{\omega_0}{\omega_d} \quad \phi = -\arctan\left(\frac{\alpha}{\omega_d}\right)$

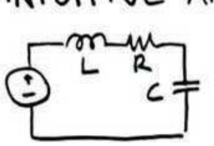



$Q = \frac{\omega_0}{2\alpha}$ = Quality factor




CRITICALLY DAMPED CASE $\quad \alpha = \omega_0 \quad s_1 = s_2 = -\alpha$

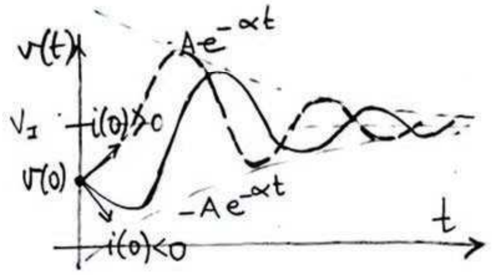

INTUITIVE ANALYSIS



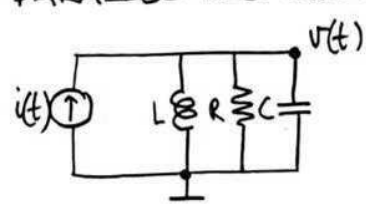

$\omega_0 = \sqrt{1/LC}$

$\alpha = R/2L$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$




PARALLEL RLC CIRCUIT

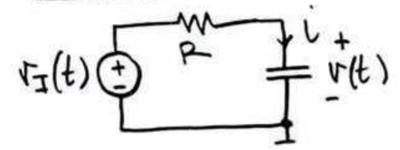



$i(t) = i_L + i_R + i_C = \frac{1}{L}\int_{-\infty}^{t} v\,dt + \frac{v}{R} + C\frac{dv}{dt} \quad \frac{1}{C}\frac{d}{dt}$

Homogeneous solution

$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC} = 0 \Rightarrow \omega_0 = \sqrt{\frac{1}{LC}} \quad \alpha = \frac{1}{2RC} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$

# LECTURE 19 — SINUSOIDAL STEADY STATE




$v_I(t) = \begin{cases} V_i \cos\omega t & t \geq 0 \\ 0 & t < 0 \end{cases}$

$v(0) = 0$ Zero State Response

Fourier analysis:
Any signal can be expressed as a sum of sinusoids.

Sinusoidal response of RC network. ODE: $RC\dfrac{dv}{dt} + v = v_I$

Particular solution: Trigonometric approach $\qquad a\sin\alpha + b\cos\alpha : \sqrt{a^2+b^2}\cos(\alpha+\phi)$

$v_p = A\cos(\omega t+\phi) \qquad -RCA\omega\,\mathrm{sen}(\omega t+\phi) + A\cos(\omega t+\phi) \overset{!}{=} V_i\cos\omega t \qquad \phi = \arctan\frac{b}{a}$

$$A = \frac{V_i}{\sqrt{1+\omega^2R^2C^2}} \qquad \phi = -\arctan(\omega RC)$$

Particular solution: Exponential approach $\quad v_{IS} = V_i e^{st} = V_i e^{j\omega t} \quad v_I = \mathrm{Re}[v_{IS}]$

$v_{ps} = Ae^{st} \qquad RCAse^{st} + Ae^{st} = V_i e^{st} \qquad A = \dfrac{V_i}{sRC+1} = \dfrac{V_i}{1+j\omega RC}$ complex amplitude

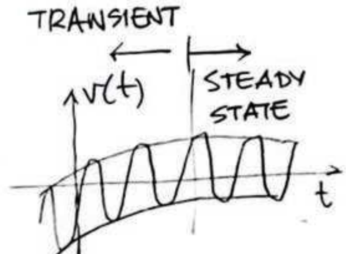
$$v_p = \mathrm{Re}[v_{ps}] = \mathrm{Re}\left[\frac{V_i}{1+j\omega RC}e^{j\omega t}\right] = \frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cos(\omega t+\phi) \qquad \phi = -\arctan(\omega RC)$$

Homogenous solution

$RC\dfrac{dv_H}{dt} + v_H = 0 \qquad v_H = A\cdot e^{-t/RC}$

Total solution & initial conditions

$$v(t) = \frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cos(\omega t+\phi) - \frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cos\phi\, e^{-t/RC}$$

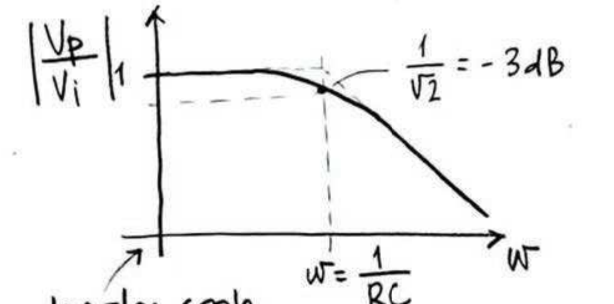



SINUSOIDAL STEADY STATE $\quad t \gg RC \rightarrow v_H = 0 \quad v(t) = v_p$

$$v(t) = \frac{V_i}{\sqrt{1+\omega^2R^2C^2}}\cos(\omega t+\phi) \qquad \phi = -\arctan(\omega RC) \qquad v_p = |V_p|\cos(\omega t + \angle V_p)$$

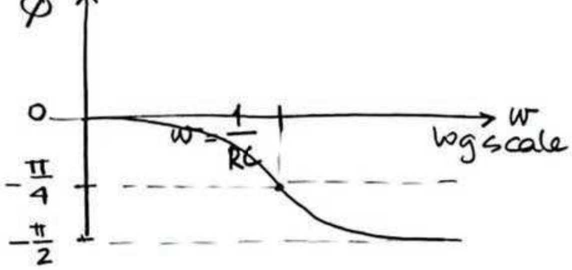
MAGNITUDE & PHASE PLOTS : TRANSFER FUNCTIONS

$$H(j\omega) = \frac{V_p}{V_i} \quad |H(j\omega)| = \left|\frac{V_p}{V_i}\right| = \frac{1}{\sqrt{1+\omega^2R^2C^2}} \qquad \angle H(j\omega) = \phi = -\arctan(\omega RC)$$

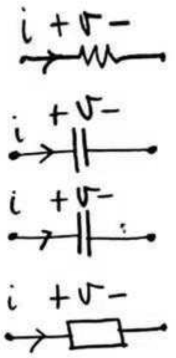



log-log scale

break frequency



# LECTURE 20 THE IMPEDANCE MODEL




$i = I.e^{st}$    $v = i.R$    $V_R.e^{st} = I_R e^{st}.R$    $V_R = R.I_R$ → complex amplitudes of V, I

$v = V.e^{st}$    $i = C.\frac{dv}{dt}$    $I_C.e^{st} = C.V_C s e^{st}$    $V_C = \frac{1}{sC}.I_C$

$s = j\omega$    $v = L.\frac{di}{dt}$    $V_L.e^{st} = L.I_L s e^{st}$    $V_L = sL.I_L$

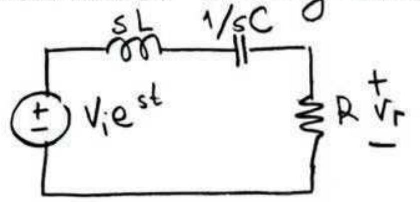
IMPEDANCE    $V = Z.I$ generalized Ohm's Law

$$Z_R = R \quad \Big| \quad Z_C = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{-j}{\omega C} \quad \Big| \quad Z_L = sL = j\omega L$$

IMPEDANCE METHOD SUMMARY

1. Replace sources by their complex amplitudes
2. Replace circuit elements by their impedances (topology REMAINS!)
3. Solve circuit with the 5 circuit analysis methods
4. Obtain the time domain variables (not always) $v_a = |V_a| \cos(\omega t + \angle V_a)$

Sinusoidal steady-state (SSS) response of series RLC network




$$V_r = V_i \frac{R}{R + sL + \frac{1}{sC}} = \frac{V_i \cdot sR/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

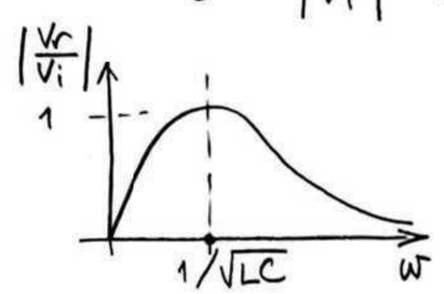
→ RLC series characteristic equation

Frequency response of RLC series circuit $\quad s = j\omega$

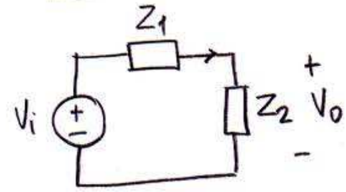
$$\frac{V_r}{V_i} = \frac{j\omega RC}{(1-\omega^2 LC) + j\omega RC} \qquad \left|\frac{V_r}{V_I}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

Amplitude plot

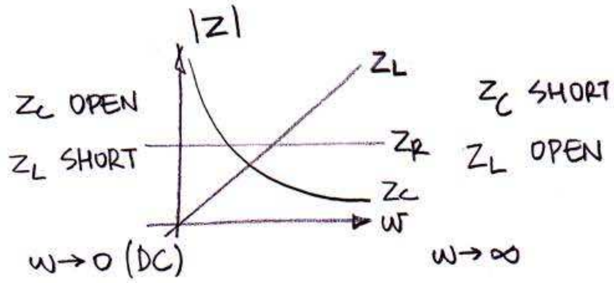
Low $\omega$ $\left|\frac{V_r}{V_i}\right| \approx \omega RC$    High $\omega$ $\left|\frac{V_R}{V_i}\right| \simeq \frac{R}{\omega L}$    $\omega = \frac{1}{\sqrt{LC}}$ $\left|\frac{V_r}{V_i}\right| = 1$

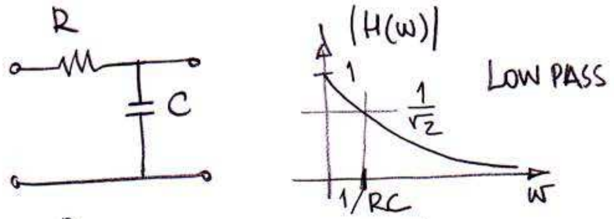


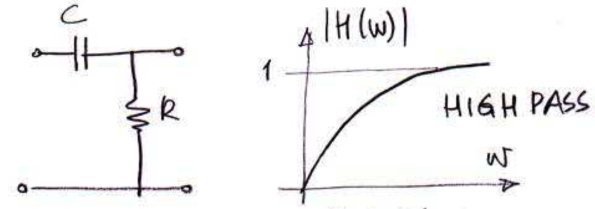
# LECTURE 21 FILTERS

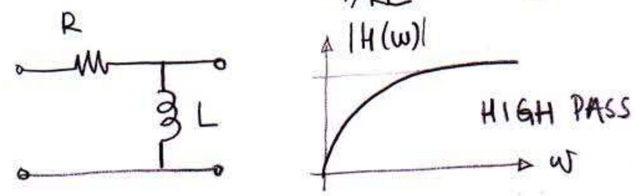


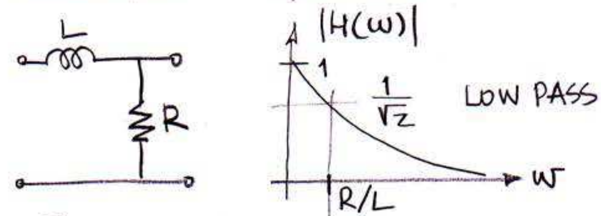

$$H(\omega) = \frac{V_0}{V_1} = \frac{Z_2}{Z_1 + Z_2}$$

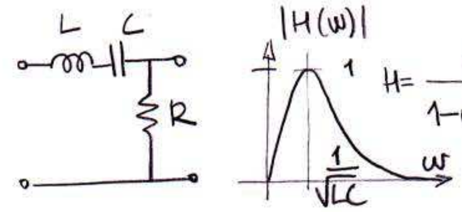


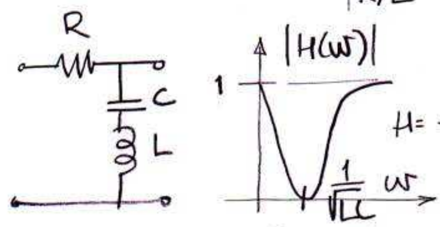



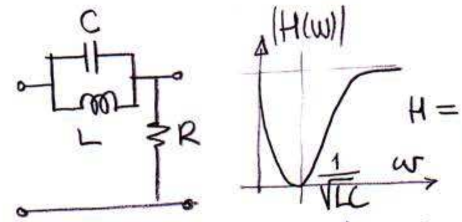

$$H = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

BAND PASS



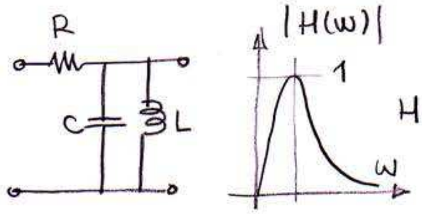

$$H = \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega RC}$$

BAND STOP



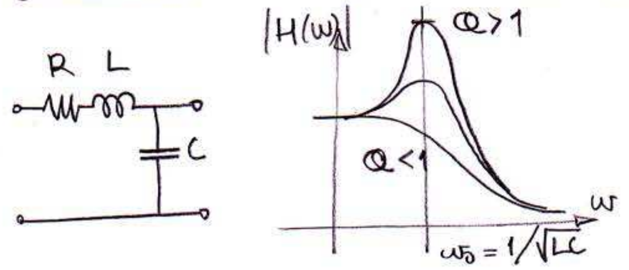

$$H = \frac{1 - \omega^2 LC}{1 - \omega^2 LC + j\omega \frac{L}{R}}$$

BAND STOP




$$H = \frac{j\omega \frac{L}{R}}{1 - \omega^2 LC + j\omega \frac{L}{R}}$$

BAND PASS

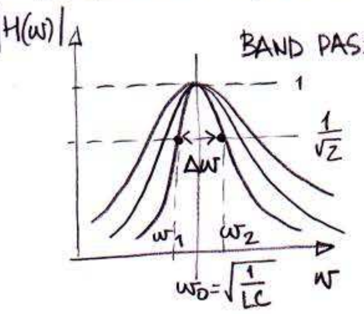



$$H = \frac{1}{1 - \omega^2 LC + j\omega RC} = \frac{\omega_0}{\omega_0^2 - \omega^2 + j2\alpha}$$

$$|H(\omega_0)| = Q = \frac{\omega_0}{2\alpha}$$

Resonance
Quality factor

SELECTIVITY & Q-FACTOR




BAND PASS Series RLC

$$Q = \frac{\omega_0}{\Delta\omega}$$

bandwidth for $|H| = \frac{1}{\sqrt{2}}$ (half power)

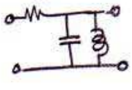
$$\frac{\omega L}{R} - \frac{1}{\omega RC} = \pm 1$$

$$\omega_{1,2} = \pm \frac{R}{2L} + \frac{1}{2}\sqrt{\frac{R^2}{L^2} + \frac{4}{LC}}$$

$$\Delta\omega = \frac{R}{L} = 2\alpha \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

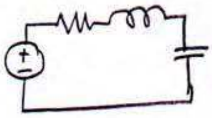
for series RLC

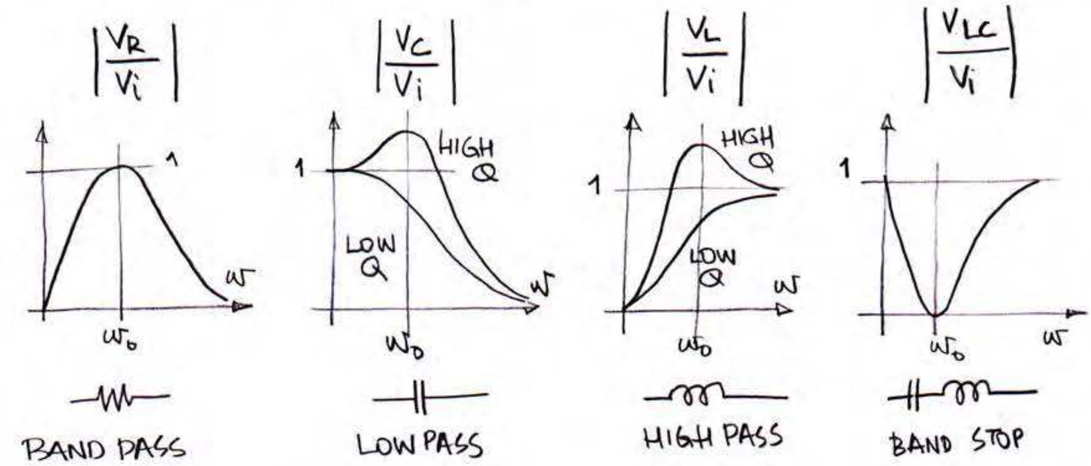
$\frac{1}{RC} = 2\alpha$ for parallel RLC



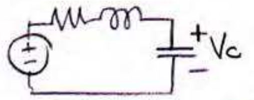
# LECTURE 22 — TIME DOMAIN VS. FREQUENCY DOMAIN ANALYSIS

Different filters in the same circuit

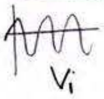




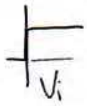

Comparison (Apple & Oranges)



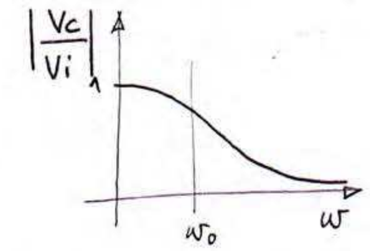

FREQUENCY ANALYSIS

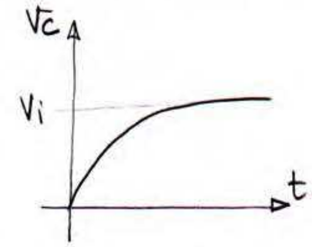



TIME ANALYSIS

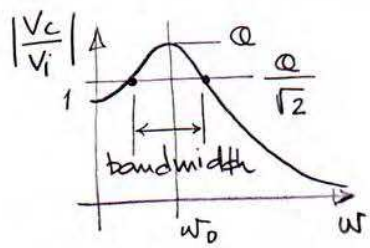


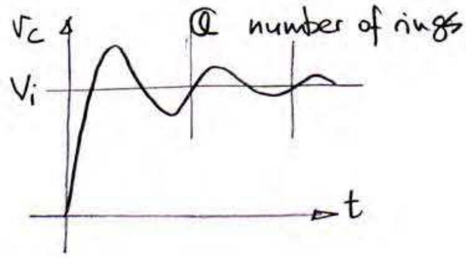

$\alpha > \omega_0$
$Q < \frac{1}{2}$
overdamped



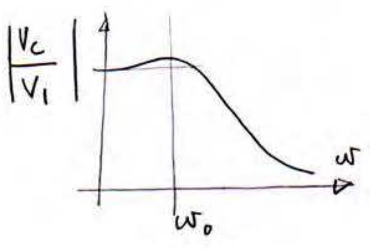



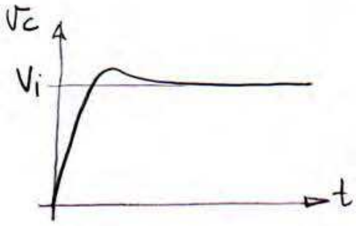

$\alpha < \omega_0$
$Q > \frac{1}{2}$
underdamped



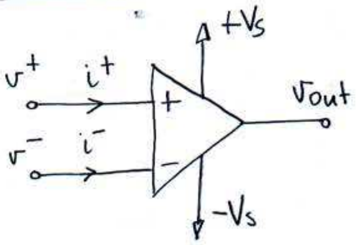




$\alpha = \omega_0$
$Q = \frac{1}{2}$
critically damped



# LECTURE 23 — THE OPERATIONAL AMPLIFIER ABSTRACTION




IDEAL OP-AMP

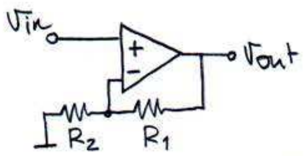
$R_{out} \to 0$

$R_{in} \to +\infty$

$A \to \infty$

no saturation

Noninverting amplifier




$v_{out} = A(v^+ - v^-)$

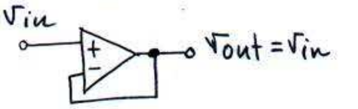
$v^+ = v_{in} \quad v^- = v_{out}\dfrac{R_2}{R_1+R_2}$

$$v_{out} = \frac{A v_{in}}{1 + \frac{AR_2}{R_1+R_2}} \overset{A \gg 1}{\simeq} \frac{R_1+R_2}{R_2} v_{in}$$

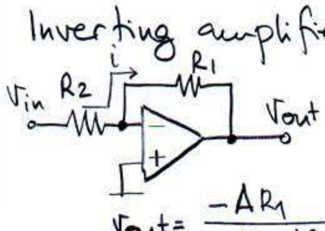
Negative feedback: Virtual short method

$i^+ \simeq 0 \quad i^- \simeq 0 \quad v^+ \simeq v^-$

Voltage follower




Inverting amplifier: virtual ground $v^- = v^+ = 0$




$$\frac{v_{in}}{R_2} = i = -\frac{v_{out}}{R_1} \qquad v_{out} = -\frac{R_1}{R_2} v_{in}$$

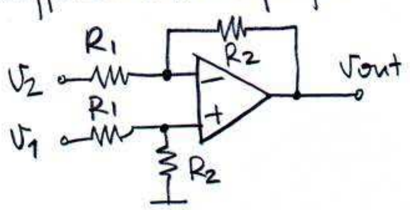
$$v_{out} = \frac{-AR_1}{R_1+R_2+AR_2} v_{in}$$

Input resistance: $R_{IN} = R_2$

$$R_{IN} = \frac{R_1+R_2+AR_2}{1+A}$$

# LECTURE 24 — OPERATIONAL AMPLIFIER CIRCUITS

Differential amplifier



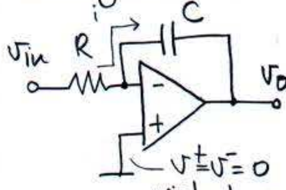

$$v^+ = v_1 \frac{R_2}{R_1+R_2}$$

$$\frac{v_2 - v^-}{R_1} + \frac{v_{out} - v^-}{R_2} = 0$$

$$v^- = \frac{v_2 R_2 + v_{out} R_1}{R_1 + R_2} = v^+ = \frac{R_2}{R_1+R_2} v_1$$

$$v_{out} = \frac{R_2}{R_1}(v_1 - v_2)$$

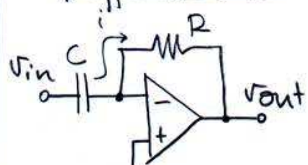
Integrator




$$\frac{v_{in}}{R} = i = -C\frac{dv_{out}}{dt}$$

$$v_{out} = \frac{-1}{RC}\int_{-\infty}^{t} v_{in}\, dt$$

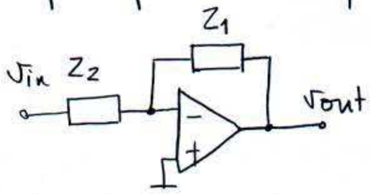
Differenciator




$$C\frac{dv_{in}}{dt} = i = \frac{-v_{out}}{R}$$

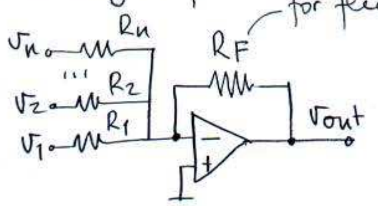
$$v_{out} = -RC\frac{dv_{in}}{dt}$$

Op-Amp Filters: Impedance model




$$v_{out} = -\frac{Z_1}{Z_2} v_{in}$$

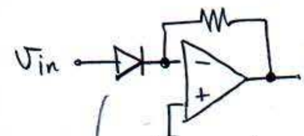
Summing Amp




$$v_{out} = -R_f\left(\frac{v_1}{R_1} + \dots \frac{v_n}{R_n}\right)$$

Non-linear Op-Amp circuits

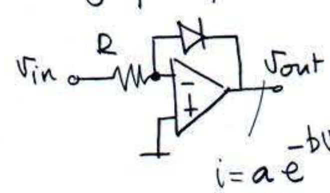
Exponential Op-Amp




$v_{out} = -a \cdot e^{b v_I} \cdot R$

$i = a e^{b v_{in}}$

Log Op-Amp



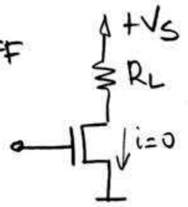

$i = a e^{-b v_{out}}$

$$v_{out} = -\frac{1}{b} \ln \frac{v_{in}}{aR}$$

# LECTURE 26 — ENERGY & POWER

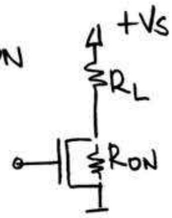
## MOSFET: STATIC POWER

GATE OFF




$P = 0$

GATE ON



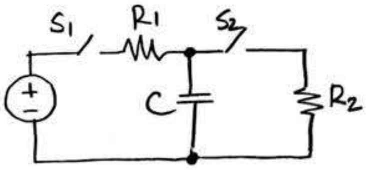

$$P = \frac{V_S^2}{R_L + R_{ON}}$$

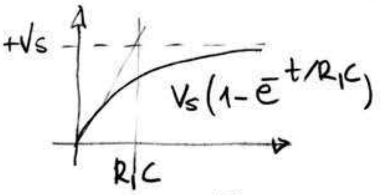
In a circuit with N mosfet, half will be ON and half will be OFF (statistics)

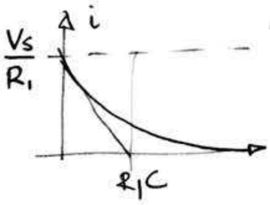
Expected power $P = \frac{V_S^2}{2R_L}$ ($R_L \gg R_{ON}$)

## MOSFET DYNAMIC POWER

$S_1$ short, $S_2$ open $t = [0, T_1]$



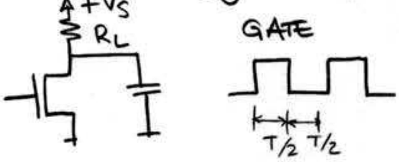

$$E_1 = \int_0^{T_1} P\,dt = \int_0^{T_1} V_S \cdot i\,dt = \frac{V_S^2}{R_1}\int_0^{T_1}(1-e^{-t/R_1C})\,dt = CV_S^2(1-e^{-t/R_1C}) \qquad R_1C = \tau$$

$T_1 \gg R_1C$ then $E_1 = CV_S^2$ <
- $\frac{1}{2}CV_S^2$ stored in C
- $\frac{1}{2}CV_S^2$ dissipated in $R_1$ (independent of $R_1$!)

$S_1$ open, $S_2$ short $t = [T_1, T_1+T_2]$

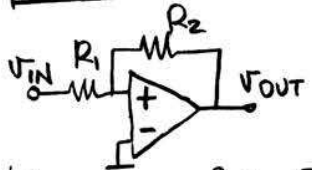
$T_2 \gg R_2C$ then $E_2 = \frac{1}{2}CV_S^2$ dissipated in $R_2$ (independent of $R_2$!)

Total energy dissipated in $T = T_1 + T_2$: $E = CV_S^2$ $\qquad P = \frac{CV_S^2}{T} = CV_S^2 f$



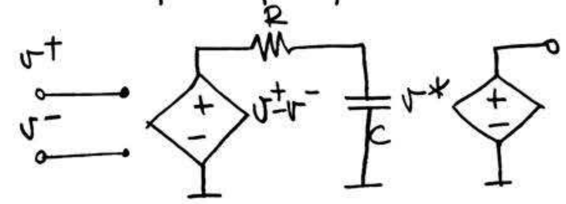

$$\overline{P} = \underbrace{\frac{V_S^2}{2(R_L+R_{ON})}}_{\text{half cycle ON}} + CV_S^2 f \cdot \left(\frac{R_L}{R_L+R_{ON}}\right)^2 \qquad R_L \gg R_{ON} \qquad \overline{P} = \underbrace{\frac{V_S^2}{2R_L}}_{\text{static power}} + \underbrace{CV_S^2 f}_{\text{dynamic power}}$$

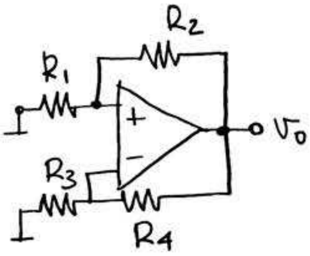
# LECTURE 25 OP-AMPS POSITIVE FEEDBACK




$v_{OUT} = -\frac{R_2}{R_1} v_{IN} \rightarrow$ Static analysis, Not enough nor valid now

Dynamics of an Op-Amp




$$\frac{RC}{A}\frac{dv_0}{dt} + \frac{v_0}{A} = v^+ - v^- =$$

$$= \gamma^+ v_0 - \gamma^- v_0$$

$$\gamma^+ = \frac{R_1}{R_2+R_1} \qquad \gamma^- = \frac{R_3}{R_3+R_4}$$

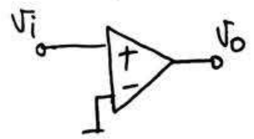
$\gamma^+ < \gamma^-$ STABLE $\quad \gamma^+ > \gamma^-$ UNSTABLE $\quad \gamma^+ = \gamma^-$ NEUTRAL

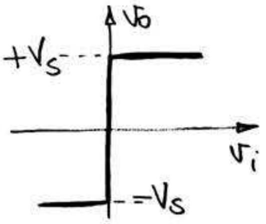
$$v_0 = k e^{-t/T} \qquad T = \frac{RC}{A(\gamma^- - \gamma^+)}$$

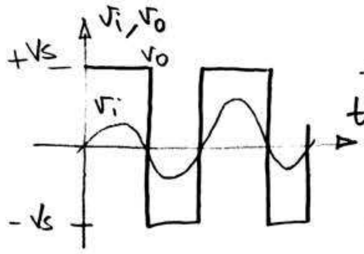
$> 0$ STABLE

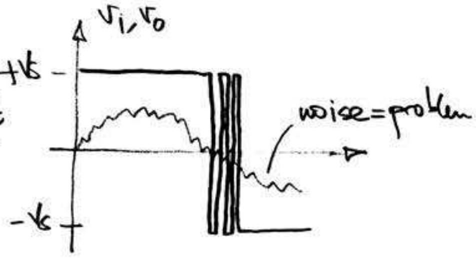
$< 0$ UNSTABLE

Comparator

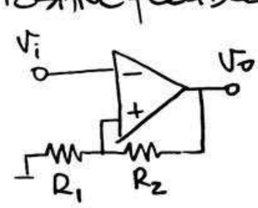


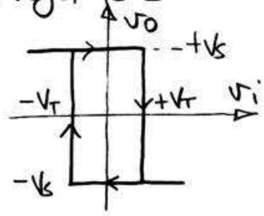



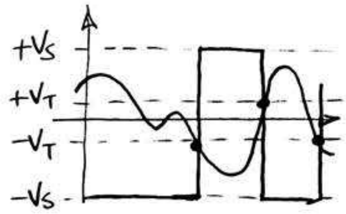

Positive feedback & hysteresis



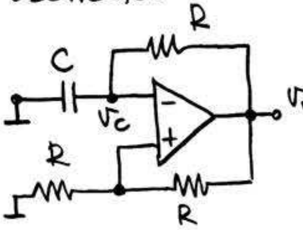



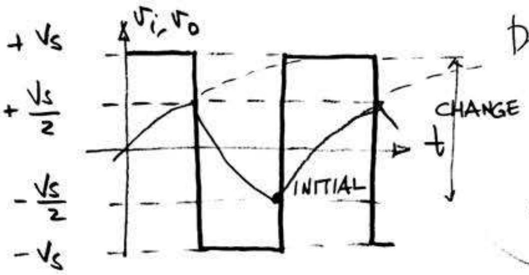

$$V_T = \frac{R_1}{R_1+R_2} V_S$$

threshold




Oscillator




Discretization of time!

When is the signal valid?

$$1 - e^{-t_R/RC} = \frac{2}{3}$$ CLOCK <u>HALF</u> CYCLE

$$\frac{t_R}{RC} = -\ln\frac{1}{3} = \ln 3$$