Chapter 8 Section 5

TOPICS

• UNDERSTAND THE CONCEPT OF A PARTIAL FRACTION DECOMPOSITION
• USE PARTIAL FRACTION DECOMPOSITION WITH LINEAR FACTORS TO INTEGRATE RATIONAL FUNCTIONS
• USE PARTIAL FRACTION DECOMPOSITION WITH QUADRATIC FACTORS TO INTEGRATE RATIONAL FUNCTIONS

TEXT READING ASSIGNMENT FOR 8.5

PAGE 552,553,554,555,556,557,558

TEXT HOMEWORK EXCERCISES FOR 8.5

PAGE 559#9,13,15,19,21,24,28

• UNDERSTAND THE CONCEPT OF A PARTIAL FRACTION DECOMPOSITION

In precalculus mathematics, much time is spent learning how to add and subtract rational expressions such as $\frac{1}{x-3}+\frac{3}{x^2+1}$. It is an easy exercise to show this answer is $\frac{x^2+3x-8}{(x-3)(x^2+1)}$. However, when it comes to integration, the first expression is easier to integrate. The technique for writing a single fraction into a sum or difference of fractions with linear or irreducible quadratic factors is called ***partial fraction decomposition***.

Worked Example 8.5.1

Worked Example 8.5.2

To find a partial fraction decomposition for $\frac{N(x)}{D(x)}$, use the steps below.

1) Make sure the degree of $N(x)$ is less than the degree of $D(x)$ (if this were not true use long division).
2) Factor the denominator $D(x)$ into all possible linear and irreducible quadratic factors.
3) For a linear factor of $(ax+b)^n$ in $D(x)$, include $\frac{A_1}{ax+b}+\frac{A_2}{\left(ax+b\right)^2}+\cdots+\frac{A_n}{\left(ax+b\right)^n}$ in the decomposition.
4) For a quadratic factor of $(ax^2+bx+c)^n$ in $D(x)$, include $\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{\left(ax^2+bx+c\right)^2}+\cdots+\frac{A_nx+B_n}{\left(ax^2+bx+c\right)^n}$ in the decomposition.
5) Set $\frac{N(x)}{D(x)}$ equal to all terms in the decomposition.
6) Clear the fractions.
7) Form a system of equations by equating all coefficients for each power of x.

8) Solve the system of equations.

Student Exercise: Find the partial fraction decomposition of $\frac{2x^2+x+6}{(x^2+3)^2}$.

Student Exercise Solution 8.5.3

• USE PARTIAL FRACTION DECOMPOSITION WITH LINEAR FACTORS TO INTEGRATE RATIONAL FUNCTIONS

Let's start with an example $\int \frac{14-8x}{6x^2+13x-5}\,dx$.

We check the degree of $N(x)=1$ is less than the degree of $D(x)=2$.
Now factor $D(x)$ into $(3x-1)(2x+5)$.

The partial fraction decomposition is $\frac{14-8x}{(3x-1)(2x+5)} = \frac{A}{(3x-1)} + \frac{B}{(2x+5)}$.

Clear the fractions $14-8x = A(2x+5)+B(3x-1) = (5A-B)+(2A+3B)x$.

Equating all coefficients for each power of x yields the system $\begin{matrix} 5A-B=14 \\ 2A+3B=-8 \end{matrix}$.

Solve the system by multiplying the first equation by 3; $\begin{matrix} 15A-3B=42 \\ 2A+3B=-8 \end{matrix}$,

Eliminate B and we get $17A=34$ so $A=2$ then $5(2)-B=14$ so $B=-4$.

Now the calculus!!! $\int \frac{14-8x}{6x^2+13x-5}\,dx = \int \frac{2}{3x-1} - \frac{4}{2x+5}\,dx = \frac{2}{3}\ln(3x-1) - 2\ln(2x+5) + C = \ln\frac{\sqrt[3]{(3x-1)^2}}{(2x+5)^2} + C$.

Remark The calculus is pretty short in comparison to the decomposition. Now you get a chance to try it!

Student Exercise: Use the fact in student exercise solution 8.5.3 we found that $\frac{x-1}{x^2-6x+9} = \frac{1}{x-3} + \frac{2}{(x-3)^2}$

to evaluate $\int \frac{x-1}{x^2-6x+9}\,dx$.

Student Exercise Solution 8.5.4

• USE PARTIAL FRACTION DECOMPOSITION WITH QUADRATIC FACTORS TO INTEGRATE RATIONAL FUNCTIONS

Worked Example 8.5.5

Student Exercise: Use the fact in worked example 8.5.2 we found that $\frac{2x^2+x+6}{(x^2+3)^2}=\frac{2}{x^2+3}+\frac{x}{(x^2+3)^2}$

to evaluate $\int \frac{2x^2+x+6}{(x^2+3)^2}\,dx$.

Student Exercise Solution 8.5.6