

Problem 24:

Expand the logarithmic expression:

$$\begin{aligned} \ln(xyz) &= \ln(x \times y \times z) \\ &= \ln(x) + \ln(y) + \ln(z) \end{aligned}$$

Problem 26:

Expand the logarithmic expression:

$$\begin{aligned} \ln\sqrt{a-1} &= \ln(a-1)^{\frac{1}{2}} \\ &= \frac{1}{2} \times \ln(a-1) \end{aligned}$$

Problem 34:

Express as a single logarithm:

$$\begin{aligned} &2[\ln(x) - \ln(x+1) - \ln(x-1)] \\ &= \ln(x^2) - [\ln(x+1)^2 + \ln(x-1)^2] \\ &= \ln \left[\frac{x}{(x+1) \times (x-1)} \right]^2 \\ &= \ln \left(\frac{x^2}{x^2-1} \right) \end{aligned}$$

Problem 42:

Find the limit:

$$\begin{aligned} &\lim_{x \rightarrow 5^+} \ln \left[\frac{x}{\sqrt{x-4}} \right] \\ &= \ln \left[\frac{5}{\sqrt{5-4}} \right] \\ &= \ln(5) \end{aligned}$$

Problem 62:

Find the derivative of the function:

$$y = \ln(\ln(x))$$

$$y' = \frac{\frac{d}{dx}(\ln(x))}{\ln(x)}$$

$$y' = \frac{\frac{1}{x}}{\ln(x)}$$

$$y' = \frac{1}{x \times \ln(x)}$$

Problem 104:

Use logarithmic differentiation:

$$y = \sqrt{\frac{x^2-1}{x^2+1}}, \text{ in the region where } x > 1$$

$$y = \sqrt{\frac{x^2-1}{x^2+1}}$$

$$\ln(y) = \ln \left[\sqrt{\frac{x^2-1}{x^2+1}} \right]$$

$$\ln(y) = \frac{1}{2} \times [\ln(x^2-1) - \ln(x^2+1)]$$

Differentiate each side:

$$\frac{y'}{y} = \frac{1}{2} \times \left[\frac{\frac{d}{dx}(x^2-1)}{x^2-1} - \frac{\frac{d}{dx}(x^2+1)}{x^2+1} \right]$$

$$y' = \frac{y}{2} \times \left[\frac{2x}{x^2-1} - \frac{2x}{x^2+1} \right]$$

$$y' = \frac{1}{2} \times \sqrt{\frac{x^2-1}{x^2+1}} \times \left[\frac{2x}{x^2-1} - \frac{2x}{x^2+1} \right]$$

$$y' = \sqrt{\frac{x^2-1}{x^2+1}} \times \frac{x}{x^4-1}$$