TOPICS
• USE THE DIRECT COMPARISON TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES
• USE THE LIMIT COMPARISON TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES

TEXT READING ASSIGNMENT FOR 9.4

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TEXT HOMEWORK EXCERCISES FOR 9.4

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• USE THE DIRECT COMPARISON TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES

The most intuitive of the convergence tests is the ***Direct Comparison Test***

Suppose $0 < a_n < b_n$ for all n.

1) If $\sum_{n=1}^{\infty} b_n$ converges so does $\sum_{n=1}^{\infty} a_n$.

2) If $\sum_{n=1}^{\infty} a_n$ diverges so does $\sum_{n=1}^{\infty} b_n$.

Remark The direct comparison test is often used with geometric series, test for divergence and *p*-series.

Worked Example 9.4.1

Student Exercise: Use the Direct Comparison Test to determine the convergence of $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^3+1}}$.

Student Exercise Solution 9.4.2

• USE THE LIMIT COMPARISON TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES

The ***Limit Comparison Test*** is an alternative that can be very useful when the general term a_n is complicated.

It states if $a_n, b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is a finite and positive number then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges.

Example $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$ behaves much like the series with the same leading terms $\sum_{n=1}^{\infty} \frac{5n}{n^2} = \sum_{n=1}^{\infty} \frac{5}{n}$. This last series diverges by direct comparison with the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.

We must check $\frac{5n-3}{n^2-2n+5}$ is positive for all $n \ge 1$. The numerator is clearly positive for all $n \ge 1$. The denominator is always positive because it is positive at $n=1$ and the discriminant of -20 tells us the denominator has no x intercepts.

Now run the limit: $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{5n-3}{n^2-2n+5} \right) \Big/ \left(\frac{5}{n} \right) = \lim_{n\to\infty} \frac{5n^2-3n}{5n^2-10n+25} = 1$. Since this is finite and positive, by the Limit Comparison Test the divergent series $\sum_{n=1}^{\infty} \frac{5}{n}$ implies the series $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$ is also divergent.

Student Exercise: Use the Limit Comparison Test to determine the convergence of $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n}$. Hint: Compare with the geometric series $\sum_{n=1}^{\infty} \frac{1}{2^n}$.

[Student Exercise Solution 9.4.3]