Chapter 9 Section 3

TOPICS
- USE THE INTEGRAL TEST TO DETERMINE WHETHER AN INFINITE SERIES CONVERGES OR DIVERGES
- USE PROPERTIES OF p-SERIES AND HARMONIC SERIES

TEXT READING ASSIGNMENT FOR 9.3

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TEXT HOMEWORK EXCERCISES FOR 9.3

PAGE 620 # 3,9,13,15
PAGE 621 #21,23,31,33,35
PAGE 622 #59,60,61,65,67,69

- USE THE INTEGRAL TEST TO DETERMINE WHETHER AN INFINITE SERIES CONVERGES OR DIVERGES

Theorem If f is a positive, continuous and decreasing function on $[1,\infty)$ and $a_n = f(n)$ then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} f(x)dx$ converges.

Remark This theorem tells us both <u>necessary and sufficient</u> conditions for $\sum_{n=1}^{\infty} a_n$ to converge.

Remark This theorem does not tell us the sum of the series.
Remark You must be sure to check each of the conditions of this theorem before you apply it.

[Worked Example 9.3.1]

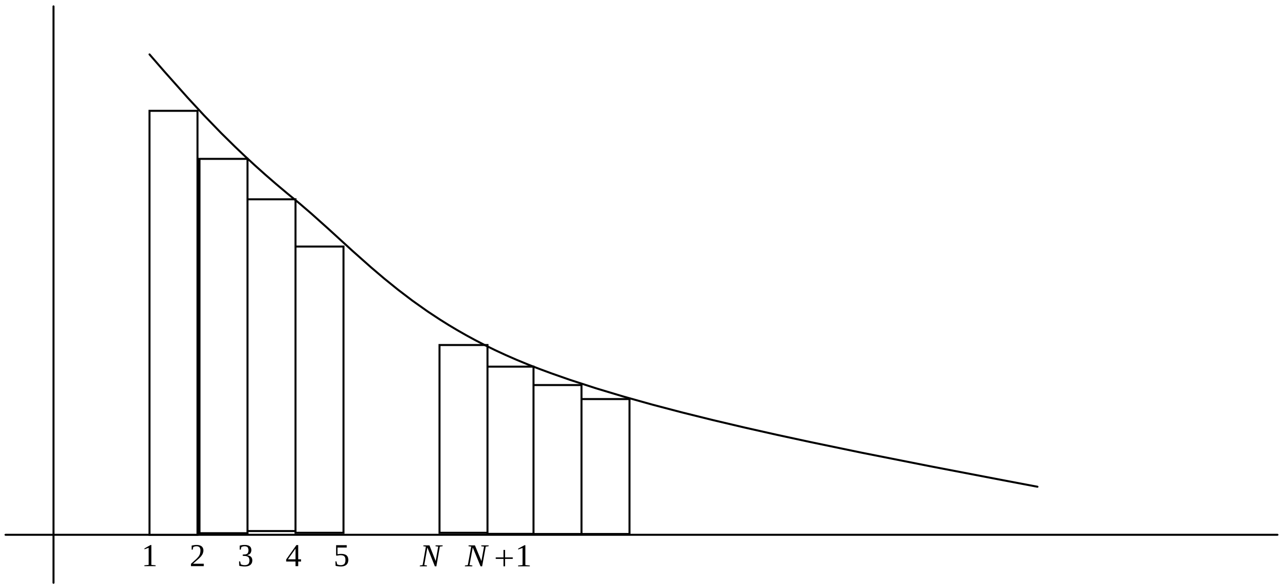
To apply the integral test use the steps below.
1) Verify f is positive and continuous on $[1,\infty)$.
2) Verify f is decreasing on $[1,\infty)$ (show either $a_n \geq a_{n-1}$ is equivalent to a true statement or $f'(x) < 0$ on $[1,\infty)$).
3) Evaluate the improper integral $\int_1^{\infty} f(x)dx$ as in section 8.8.
4) Either $\int_1^{\infty} f(x)dx$ and $\sum_{n=1}^{\infty} a_n$ both converge or both diverge.

Remark Convergence not affected by any finite number of terms. If you have a series that start at a number different than 1, it is still perfectly plausible to apply the integral test.

Student Exercise: Apply the integral test to determine whether the series $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$ converges or diverges.

[Student Exercise Solution 9.3.2]()

The idea behind the integral test can be summarized by a picture. Let's prove if $\int_{1}^{\infty} f(x)dx$ converges then $\sum_{n=1}^{\infty} a_n$ converges. To prove the covers, see text page 617.



The area of the rectangles are $1\cdot f(2), 1\cdot f(3), 1\cdot f(4), 1\cdot f(5), \ldots, 1\cdot f(n), \ldots$. Since $a_n = f(n)$ we have an area inequality $\sum_{n=2}^{\infty} a_n \le \int_{1}^{\infty} f(x)dx$. If $\int_{1}^{\infty} f(x)dx$ converges to L then $\sum_{n=1}^{\infty} a_n \le L - a_1 = L - f(1)$.

In homework problem 59, you are asked to show the remainder $R_N = \sum_{n=1}^{\infty} a_n - S_N$ satisfies $0 \le R_N \le \int_{N}^{\infty} f(x)dx$. Use the picture above as your guide.

In exercise 60, you are asked to prove $\sum_{n=1}^{N} a_n \le \sum_{n=1}^{\infty} a_n \le \sum_{n=1}^{N} a_n + \int_{N}^{\infty} f(x)dx$. Simply replace R_N by $\sum_{n=1}^{\infty} a_n - S_N$ in the inequality in problem 59; $0 \le R_N \le \int_{N}^{\infty} f(x)dx$.

[Worked Example 9.3.3]()

Worked Example 9.3.4

• USE PROPERTIES OF p-SERIES AND HARMONIC SERIES

The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ with $p > 0$ occurs so often, it has its own name it is called the ***p-series***. A special case is when $p = 1$, in which case we get $\sum_{n=1}^{\infty} \frac{1}{n}$ called the ***harmonic series***.

Using the integral test, we get the following results.

Theorem The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$

1) Converges if $p > 1$
2) Diverges if $0 < p \le 1$

Remark The proof is a straight forward application of the integral test. See page 619 for details.

Student Exercise: Which of the following p-series converge?

a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$

b) $\sum_{n=1}^{\infty} \frac{1}{n^{1.01}}$

c) The harmonic series

Student Exercise Solution 9.3.5