Problem 4:

$\int_2^3 \frac{2}{x^2} dx$ such that $n = 4$

Trapezoidal Approximation:

$$T = \frac{3-2}{2 \times 4} \times [f(2) + 2 \times f(2.25) + 2 \times f(2.5) + 2 \times f(2.75) + f(3)]$$

$$T = \frac{1}{8} \times (.5 + 2 \times .3951 + 2 \times .32 + 2 \times .2645 + .2222)$$

$$T = .3352$$

Simpson's Approximation:

$$S = \frac{3-2}{3 \times 4} \times [f(2) + 4 \times f(2.25) + 2 \times f(2.5) + 4 \times f(2.75) + f(3)]$$

$$S = \frac{1}{12} \times (.5 + 4 \times .3951 + 2 \times .32 + 4 \times .2645 + .2222)$$

$$S = .3333$$

Exact value:

$$E = \int_2^3 \frac{2}{x^2} dx$$

$$E = -\frac{2}{x}|_2^3$$

$$E = \frac{1}{3}$$

Problem 30:

Solve for n, such that the error is $\leq$.00001 for the approximation of$\int_0^1 \frac{1}{x} dx$

Note that $f' = \frac{-1}{(x+1)^2}$, $f'' = \frac{2}{(x+1)^3}$, $f''' = \frac{-6}{(x+1)^4}$, and $f^{(4)} = \frac{24}{(x+1)^5}$

Trapezoidal Approximation Error:

$$|E| \leq \frac{(b-a)^3}{12 \times n_t^2} \times [max|f''(x)|_a^b]$$

$$.00001 \leq \frac{(1-0)^3}{12 \times n_t^2} \times [max|\frac{2}{(1+x)^3}|]$$

There are no critical points in f''in x = [0,1], so $max|\frac{2}{(1+x)^3}| = f''(0) = 2$

$$n_t^2 \geq \frac{1 \times 2}{12 \times .00001}$$

$$n_t \geq 129.099$$

$$n_t = 130$$

Simpson Approximation Error:

$$|E| \leq \frac{(b-a)^5}{180 \times n_S^4} \times [max|f^{(4)}(x)|_a^b]$$

$$.00001 \leq \frac{(1-0)^5}{180 \times n_S^4} \times [max|\frac{24}{(1+x)^5}|]$$

There are no critical points in $f^{(4)}$in x = [0,1], so $[max|\frac{24}{(1+x)^5}|] = f^{(4)}(0) =$ 24

$$n_S^4 \geq \frac{1 \times 24}{180 \times .00001}$$

$$n_S \geq 115.47$$

$$n_S = 116$$