# Chapter 9 Section 6

**TOPICS**

- USE THE RATIO TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES
- USE THE ROOT TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES
- REVIEW THE TESTS FOR CONVERGENCE AND DIVERGENCE OF AN INFINITE SERIES

**TEXT READING ASSIGNMENT FOR 9.6**

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**TEXT HOMEWORK EXCERCISES FOR 9.6**

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PAGE 646 #41,43,45,55,59,63,67,77,79

- USE THE RATIO TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES

The ***Ratio Test*** applies to series $\sum_{n=1}^{\infty} a_n$ with non-negative terms $a_n$ .

1) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$ then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

2) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$ then $\sum_{n=1}^{\infty} a_n$ diverges.

3) If $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$ the Ratio Test is inconclusive.

For a proof see page 639.

[Worked Example 9.6.1]

Student Exercise: Use the Ratio Test to determine whether the series $\sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{4n}}{(2n+1)!}$ converges or diverges.

[Student Exercise Solution 9.6.2]

Example If we apply the Ratio Test to $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$ we get $\lim_{n\to\infty} \frac{1}{\sqrt{n+3}} \bigg/ \frac{1}{\sqrt{n+2}} = \lim_{n\to\infty} \frac{\sqrt{n+2}}{\sqrt{n+3}} = 1$. So the Ratio Test is inconclusive.

Let's see if the series converges absolutely as in section 9.5;

$\sum_{n=1}^{\infty}\left|\frac{(-1)^n}{\sqrt{n+2}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n+2}}$. You can either apply the limit comparison test (section 9.4) or the integral test (section 9.2) to show $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n+2}}$ diverges, so $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+2}}$ does not converge absolutely.

Let's see if $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+2}}$ converges conditionally by applying the Alternating Series Test. Since $f(x) = \frac{1}{\sqrt{x+2}}$ has a negative derivative $f'(x) = -\frac{1}{2\sqrt{x+2}}$ on $[1,\infty)$, $a_n = \frac{1}{\sqrt{n+2}}$ is decreasing. The other condition for the Alternating Series Test is easily satisfied $\lim_{n\to\to\infty}\frac{1}{\sqrt{n+2}} = 0$. Hence, $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+2}}$ converges conditionally.

• USE THE ROOT TEST TO DETERMINE WHETHER A SERIES CONVERGES OR DIVERGES

The ***Root Test*** applies to any series $\sum_{n=1}^{\infty} a_n$.

1) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} < 1$ then $\sum_{n=1}^{\infty} a_n$ converges absolutely.

2) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \infty$ then $\sum_{n=1}^{\infty} a_n$ diverges.

3) If $\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$ the Root Test is inconclusive.

The proof is omitted.

Student Exercise: Use the Root Test to determine whether the series $\sum_{n=2}^{\infty}\left(\frac{\ln n}{n}\right)^n$ converges or diverges.

Student Exercise Solution 9.6.3

• REVIEW THE TESTS FOR CONVERGENCE AND DIVERGENCE OF AN INFINITE SERIES

I highly recommend reviewing the summary given on page 644. It covers each test we have covered, their conditions for convergence and divergence and miscellaneous comments.