Chapter 8 Section 4

TOPICS

• USE TRIGONOMETRIC SUBSTITUTION TO SOLVE AN INTEGRAL

TEXT READING ASSIGNMENT FOR 8.4

PAGE 543,544,545,546,547

TEXT HOMEWORK EXCERCISES FOR 8.4

PAGE 549#7,11,15,21,25,33,35

• USE TRIGONOMETRIC SUBSTITUTION TO SOLVE AN INTEGRAL

In this section we will study how to solve integrals that contain a radicals of the form $\sqrt{a^2-b^2u^2}$, $\sqrt{a^2+b^2u^2}$ and $\sqrt{b^2u^2-a^2}$. The key will be three trigonometric identities $\sin^2\theta+\cos^2\theta=1$ and $\tan^2\theta+1=\sec^2\theta$.

Worked Example 8.4.1

To evaluate an integral using a trigonometric substitution use the steps below.

1) Use the table to identify the appropriate trigonometric substitution.

Radical in integrand	$\sqrt{a^2-b^2u^2}$	$\sqrt{a^2+b^2u^2}$	$\sqrt{b^2u^2-a^2}$
Trigonometric substitution	$x=\frac{a}{b}\sin\theta$	$x=\frac{a}{b}\tan\theta$	$x=\frac{a}{b}\sec\theta$

2) Calculate dx .
3) Substitute x and dx in terms of θ .
4) Integrate (you may consult section 8.3 here).
5) Draw a triangle and label the sides with appropriate functions of x .
6) Rewrite your antiderivative in terms of x (you may need a trigonometric identity here).

Remark Avoid the mistake of making x the independent variable such as $\cancel{u=\sin x}$ or even $\cancel{x=\sin x}$. What makes trigonometric substitution a little different is that x is always the dependent variable in the substitution; the independent variable is always θ .

Student Exercise: Evaluate the integral $\int\frac{\sqrt{4x^2+9}}{x^4}dx$ using trigonometric substitution.

Student Exercise Solution 8.4.2

Remark There is one technicality we have overlooked. When use a substitution like $x = 5\sin\theta$ for $\int\sqrt{25-x^2}\,dx$ as in our first worked example, the integrand is really $\sqrt{25\cos^2\theta} = 5|\cos\theta|$, not $5\cos\theta$. This only becomes a concern when you have a definite integral and want to change the limits of integration to θs. The text avoids this concern by carefully defining the domain of θ for each of the three possible trigonometric substitutions. I avoid this concern by not changing my limits of integration to θs!

In the next worked example, a definite integral requires completing the square and a trigonometric substitution.

Worked Example 8.4.3