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Monopoly and Information Acquisition: a Rational Inattention Perspective

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Abstract

In this thesis, I analyse the information acquisition decision of a monopoly firm as a rational information problem. In a two stage game, the firm pays a cost based on the mutual information measure to reduce the uncertainty on the demand she faces. Due to the timing of the model, acquiring information is an investment decision, the firm does not know what information she will receive when she invests. Therefore, the information cannot be used to reinforce her market power from an ex-ante perspective. A better informed monopoly is thus strictly preferable for consumers, as it yields a more relevant quantity setting. I show that the firm underinvests in information, as compared to the optimal level. Nevertheless, the monopoly firm has more incentives than the competitive firm to invest in information. The welfare cost of underinvesting in information can eventually outweigh the cost of imperfect competition.

JEL Classification: D42, D83, L12

1 Introduction

Information, as it reduces uncertainty about the environment is a valuable good. Two of its inherent features distinguish it from a standard good. First, information shapes expectations and decisions. Secondly, it is not straightforward to quantify it. Consequently, the information acquisition cannot be analyzed as a standard transaction. Yet, the need for a clearer understanding of firm's information acquisition behavior is growing with consumers and regulators concerns about data collection. A common source of worry is the possibility for those data to be collected unbeknownst to users. Another one relates more to the use of those data. The suspicion is often directed towards dominant firms, which are presumed to acquire information so as to increase their market power and to extract more consumers surplus.

To evaluate the ground of this last concern, this thesis proposes to investigate the information acquisition decision of a monopoly firm who faces a linear stochastic demand. The objective is to capture what actually motivates a firm with a dominant market position when she acquires information and to evaluate the consequences for the consumers. Such an analysis requires to take into account the two properties of information mentioned above. The way that information affects expectations is closely linked to the nature of the uncertainty. I consider here Gaussian conjugate distributions so that this effect is more tractable. As regards quantifying information, I adopt the mutual information measure, which is extensively used in Rational Inattention literature. Introduced by Sims (2003), the rational inattention analysis accounts for the limited attention paid by the agents to their environment. The problem faced by a rationally inattentive decision maker is the following. She has to take an economic decision, contingent to an uncertain parameter. She receives free and exogenous signals but must determines, before receiving those signals, the quantity of information that they will contain. Signals are free, but information is not. Its cost is commonly represented by a linear function of the mutual information as in Matějka and McKay (2015). An in-depth presentation of rational inattention literature can be found in Maćkowiak et al. (2018). My analysis is conducted through a rational inattentive perspective as it embraces both a two-stage decision procedure and the mutual information based cost function. That settles the question of how measuring the information cost and provides a framework to evaluate its benefits, that is how the firm's profits are impacted by information. Answering this question will allow to define the private value of information to the firm. In the same way, I investigate the effect of information acquisition on consumers, by defining the social value of information. This addresses a first policymaking issue: should information acquisition be encouraged or on the contrary carefully monitored ? I answer it by showing that consumers strictly benefit from information acquisition. That raises a second question for the policymaker, namely the relative efficiency of the monopoly and the competitive firm. If the firms acquire information to strengthen or at least because of their dominant position, what about competitive firms ? I argue that, although the monopoly firm does not acquire information to increase its market power, having a market power provides incentives for information acquisition and so the competitive firm get less informed. Ultimately, I formulate policy recommendation, by investigating the welfare consequences of this result.

Researches on information acquisition in Industrial Organization have already been conducted. A first way of considering it has been developed through principal agent or moral hazard analysis. It induced a specific way of thinking information, where consumers are the only source of information. Indeed, acquiring information in such a framework traduces by trying to impact consumers' decisions to make them reveal information. Far from being inadequate, this conception only captures a facet of the information acquisition problem. Indeed, the consumers are not the only source of information on themselves. Firms conduct market studies, build consumers profile type, or buy data on internet users' preferences directly to websites. Those activities are run independently from consumers, without making them reveal their preferences through their choices. It is this aspect of information acquisition that I propose to examine here.

Another way of thinking firms' information acquisition in industrial organization appeared, especially with the seminal paper of Novshek and Sonnenschein (1982). Closer to approach of this thesis, the firms receive information through exogenous signals and adopt a two-stage decision procedure. They are said to acquire it in the sense they pay to reduce the variance of these signals. I show that using mutual information instead allows for a more flexible way of thinking information acquisition. Under Gaussian conjugate assumption, the mutual information cost can represent either the choice of variance or the choice of number of signals to receive. Their paper has paved the road for research on information sharing in oligopoly. Although it did not give much focus on monopolistic situation these researches have established some regularities regarding the effect of information on consumers. By considering Cournot duopoly, both Vives (1984) and Sakai (1985) showed that the expected consumer surplus increases with information precision when the uncertainty is about demand. The underlying mechanism is that consumers benefit from a better adjustment of the quantities produced. By considering the mutual information instead of the precision, I establish a similar result with the monopoly firm.

Nonetheless, some research investigated the relative efficiency of monopoly firm acquiring information. Hwang (1995) shows that a monopolist chooses a higher precision than a competitive firm. I find an analogous result, but with mutual information: more information is acquired under imperfect competition. However, where he found that a monopoly firm could overinvest in information, I show that information acquisition generates a positive externality for consumers: the monopoly underinvests in information, whatever the nature of competition in my analysis. More recently, Dimitrova and Schlee (2003) analyse the effect of entry on information acquisition. They demonstrate that a higher welfare could be achieved through a well informed monopoly than a duopoly. I extend this result to perfect competition. However, they found that more competition could induce more information acquisition. All those researches consider information acquisition only as a variance reduction. This calls for a new way of thinking, and quantifying information.

This thesis is organized as follow. Section 2 introduces the Shannon entropy and the mutual information, which are used to quantify information. Section 3 defines the information structure of the problem and shows how it links to the choice of attention. In the following section I compute the monopoly equilibrium level of attention as well as the private value of information. In section 5 I evaluate the optimal level of information and show that information acquisition generates a positive externality. The following section discusses the relative efficiency of a competitive firm. The last section concludes.

2 Quantifying information

A common way to quantify information is to use the mutual information which is based on the entropy, introduced by Shannon (1948). For a random variable $\tilde{x}$, the entropy is defined by:

$$H(\tilde{x}) = -\sum_i p_i \log(p_i) \tag{1}$$

This quantity can be understood as the disorder of the distribution of $\tilde{x}$. The higher $H(\tilde{x})$, the more uncertain the distribution of $\tilde{x}$ is. For instance, if $\tilde{x}$ can take only two values, the entropy is maximized when $p = \frac{1}{2}$, that is, when each outcome is equally likely[1]. On the contrary, if the distribution is degenerate ($p = 1$ or $p = 0$), the entropy is $H(\tilde{x}) = 0$, as one knows the outcome with certainty. To reach a greater comprehension of the formula one may consider the following problem. An urn contains four different balls: a red, a blue, a green and a yellow. A first player (the speaker) knows which ball has been drawn and the other (the curious) has to find the colour of the ball, by asking the minimum of questions. The speaker can answer questions only by yes or no (so that answers can be expressed in

[1] In such a case, $H(\tilde{x}) = \ln(2)$ which is equal to one bit if we use the binary logarithm.

bits). At least, the curious must ask two questions to find the colour of the ball. In a first time: is the colour red or blue ? If the answer is yes: is the colour is red ? and if the answer is no: is the colour is green ? This scheme of questions determines the colour of the ball with certainty. The quantity of information transmitted by the speaker is 2 bits since the curious asks only two yes-no questions. In other words, when $\tilde{x}$ follows a uniform distribution $\mathcal{U}(1:4)$ its entropy is $H(\tilde{x}) = \ln(4) = 2$ bits. Considering the same game but with $\frac{1}{p}$ distinct balls in the urn, the curious has to ask κ questions such that $2^\kappa = \frac{1}{p}$. This strategy yields a full partition of the set of possible draws and thus eliminates uncertainty. The number of questions asked is $\kappa = -\ln_2(p)$. Hence, with a uniform distribution, the Shannon entropy is the average number of questions that the curious needs to ask to determine the colour of the ball. As questions are binary, this quantity can be expressed in bits[2].

Measuring uncertainty with Shannon entropy permits to quantify information. More precisely, it allows to measure the information that an observable variable $\tilde{y}$ provides on an unobservable variable $\tilde{x}$. This quantity is the mutual information, defined by:

$$I(\tilde{x}, \tilde{y}) = H(\tilde{x}) - H(\tilde{x}|\tilde{y}) \tag{2}$$

The information can be thought of as the amount of uncertainty about $\tilde{x}$ that is eliminated by the observation of $\tilde{y}$. As for the entropy, computing mutual information requires to define the nature of uncertainty, that is to specify the distribution of observable and unobservable variables, which is done in the following section.

3 Information and attention

In the model proposed in this thesis, the demand parameter α is random. The uncertainty is characterized by Gaussian conjugate distributions. The firm has the following prior belief:

$$\alpha \sim \mathcal{N}(\bar{\alpha}, \sigma_0^2) \tag{3}$$

And she receives n exogenous signals, independent and identically drawn from the distribution:

$$s_i|\alpha \sim \mathcal{N}(\alpha, \sigma^2) \tag{4}$$

As the Gaussian family is self-conjugate, these assumptions regarding priors and likelihood function of signals yield a Gaussian posterior belief. This posterior distribution is denoted

[2] In the following, I consider the natural rather binary logarithm, so the mutual information is expressed in natural units (nats) and not in bits.

$\alpha|\mathcal{I}_n$ with $\mathcal{I}_n = \{s_1, ..., s_n\}$ representing the information set containing all signals received by the firm, and defined by:

$$\alpha|\mathcal{I}_n \sim \mathcal{N}(\mu_{\alpha|\mathcal{I}_n}, \sigma^2_{\alpha|\mathcal{I}_n})$$

$$\mu_{\alpha|\mathcal{I}_n} = a\bar{s} + (1-a)\bar{\alpha} \qquad \sigma^2_{\alpha|\mathcal{I}_n} = (1-a)\sigma_0^2 \tag{5}$$

Where $\bar{s}$ is the average of signals received and a is defined by:

$$a \equiv \frac{n\sigma_0^2}{\sigma^2 + n\sigma_0^2} \in [0,1] \tag{6}$$

This quantity represents the amount of attention paid to signals: it increases with the number of signals or with their precisions. The attention parameter is the weight associated to new informations in the update of posterior belief. By contrast, $1-a$ is the inattention: it determines the share of old beliefs in the posterior. The inattention also determines the value of the posterior variance, as the prior variance, weighted by inattention. Thus, the more attentive the firm is, the more accurate the posterior distribution.

Considering Gaussian conjugate distribution also yields tractable measure of information through mutual information. First, one can express the entropy of the uncertain demand parameter α and its entropy conditional to the information set $\mathcal{I}_n$:

$$H(\alpha) = \ln(\sigma_0\sqrt{2\pi e}) \qquad H(\alpha|\mathcal{I}_n) = \ln(\sigma_{\alpha|\mathcal{I}_n}\sqrt{2\pi e}) \tag{7}$$

Using (2), one can express the mutual information between α and $\mathcal{I}_n$:

$$I(\alpha, \mathcal{I}_n) = \frac{1}{2}\ln(\frac{\sigma_0^2}{\sigma^2_{\alpha|\mathcal{I}_n}}) = \frac{1}{2}\ln(\frac{1}{1-a}) \tag{8}$$

As $a \in [0,1]$ the quantity $I(\alpha, \mathcal{I}_n)$ is defined over the set $[0, \infty[$. One can notice that $a \in [0,1]$ ensure that $I(\alpha, \mathcal{I}_n) \in [0, \infty[$.

In addition, the one-to-one mapping between mutual information and attention allows to consider the problem in either of these quantities. However, attention delivers a more flexible way of thinking information acquisition. From equation (6), one can note that a higher attention can results from either a larger number of signals or a better accuracy of these signals. While it is a commonplace to think information only as the reduction of variance, mutual information allows for two different way of thinking information acquisition: through the quantity (number of signals) or the quality (precision of signals) of information. Furthermore, from the decision maker perspective, those two acquisition procedures are equivalent.

Proposition. *As long as the problem can be reduced to the choice of attention, it is strictly equivalent for the firm to choose the quantity of information for a given quality or conversely to determine her optimal quality of information for a given quantity.*

Indeed, as there is a one-to-one mapping between the quality σ^2 or the quantity n of information and the attention a, when the firm determines her optimal attention, she pins down the optimal value of n given σ^2 or conversely.

Considering attention has an other advantage than delivering a flexible way of thinking information acquisition. Attention makes the link between two keys aspects of information: its quantity (the mutual information) and its purpose: how it is used refine posterior belief. One can illustrate this link with a simple example. Let consider the following: the intercept of the demand is drawn from $\mathcal{N}(\bar{\alpha}, \sigma_0^2)$. The firm has chosen the level of attention $a = \frac{3}{4}$ and has received signals such that $\bar{s} > \bar{\alpha}$. The problem of the firm is now to formulate the best estimate she can. A natural estimator would be $\frac{\bar{\alpha}+\bar{s}}{2}$. Expressed in terms of mutual information, an attention level of $\frac{3}{4}$ is equivalent to one bit of information. That is as if the firm could ask exactly one yes-no question to an external agent who observes the actual draw. The firm naturally asks if the actual draw is closer to $\bar{\alpha}$ or to $\bar{s}$. Admitting than on average signals will be closer than the prior mean, the new estimator of the firm would be $\frac{\bar{s}+\frac{\bar{\alpha}+\bar{s}}{2}}{2} = \frac{3}{4}\bar{s} + \frac{1}{4}\bar{\alpha}$ which exactly corresponds to (5) with $a = \frac{3}{4}$. The bit of information has been used to eliminate one half of the uncertainty (draws between $\bar{\alpha}$ and $\frac{\bar{\alpha}+\bar{s}}{2}$) and so to build a more accurate estimator. This shows how interesting it can be to express the model in terms of attention. By connecting the use of information to its quantity, attention links the profit gain induced by information to its cost, based on its quantity. The trade-off of the firm boils down to the choice of attention.

4 Monopolist's information acquisition

4.1 Attention equilibrium

I consider a monopoly firm, facing linear demand $P(q) = \alpha - \beta q$, where α is uncertain. Uncertainty, firm's belief and information structure are characterized in the previous section. The firm technology is characterized by the cost function $C(q) = \gamma q + \delta q^2$, where δ can be either positive of negative[3] to allow for different returns to scale schemes. Following both the

[3]When considering increasing return to scales: $\delta < 0$, I assume that $|\delta| < \beta$.

rational inattention and the information sharing literature, I consider a two-stage decision procedure. At the ex ante stage, based upon her prior belief, the firm determines the quantity of information she wants to acquire, that is, the attention she will pay to the signals. Then the firm actually receives signals and update her posteriors, consistently with (5) and with the level of attention determined at the first stage. In a second time, at the interim stage, the firm sets the quantity of output to produce. These interim quantities maximize the expected profit, where all information acquired is used to build the expectations. This writes:

$$q(a, \bar{s}) = \arg\max_q \mathrm{E}[(\alpha - \gamma - (\beta + \delta)q)q|\mathcal{I}_n] \tag{9}$$

Which gives:

$$q(a, \bar{s}) = \frac{\mu_{\alpha|\mathcal{I}_n} - \gamma}{2(\beta + \delta)} = \frac{a\bar{s} + (1 - a)\bar{\alpha} - \gamma}{2(\beta + \delta)} \tag{10}$$

The effect of attention on interim quantities reflects its impact on the firm's beliefs. More attention results into a finer adjustment of quantities to new information as it increases the weight put on signals. By contrast, inattention leads to conservative quantity choices, mainly based on the old prior belief.

When choosing how much information to acquire at the first stage, the firm internalizes the effect of attention on interim quantities. At this stage, the objective of the firm is to maximize its expected profit, where expectations are based only on its prior belief. This expected profit is net of information cost $\lambda I(a)$, which is assumed to be linear in the quantity of mutual information acquired. The entire decision problem can be summarized by:

$$\begin{cases} \max\limits_{a \in [0,1]} \mathrm{E}\left[(\alpha - \gamma - (\beta + \delta)q(a, \bar{s}))q(a, \bar{s})\right] - \lambda I(a) \\ s.t. \\ q(a, \bar{s}) = \arg\max\limits_q \mathrm{E}[(\alpha - \gamma - (\beta + \delta)q)q|\mathcal{I}_n] \end{cases} \tag{11}$$

This problem admits a unique global maximum at:

$$a^* = \begin{cases} 1 - \frac{2\lambda(\beta+\delta)}{\sigma_0^2} & if \ \sigma_0^2 > 2\lambda(\beta + \delta) \\ 0 & else \end{cases} \tag{12}$$

Two potential equilibrium can arise. On the first case, the marginal benefit of attention $\frac{\sigma_0^2}{4(\beta+\delta)}$ cannot cover the marginal cost of the first marginal unit of attention[4] $\frac{\lambda}{2}$. In this case full inattention is the best decision for the firm, and no information is processed (it can

[4]One can compute $\frac{\partial \Pi}{\partial a}\Big|_{a=0} = \frac{\sigma_0^2}{4(\beta+\delta)} - \frac{\lambda}{2}$

results from the firm buying no signals, or buying signals with infinite variance). Otherwise, the monopoly has an incentive to acquire information. In this case the level of attention chosen reads as full attention depreciated by the inattention. The equilibrium inattention exactly corresponds to the ratio of marginal costs to benefits when the firm is fully inattentive. This highlights how rational the inattention can be: the level of inattention exactly captures the terms of the trade-off between the positive effect of attention on profits and the information cost. One can note that paying full attention cannot be optimal for the firm (except if information is free: $\lambda = 0$) as it requires an infinite quantity of information. This is an appealing feature of mutual information measure: it ranks full attention (and so full information) as a limit case, attainable only under extreme conditions such as infinite marginal benefit or null marginal cost of information.

4.2 Private value of information

In this framework, the private value of information can be defined as the marginal profit gains induced by a higher attention. To characterize it, one can adapt the formulation of the ex-ante profit and express it as:

$$\Pi(a) = \frac{1}{2}\mathrm{cov}[\alpha, q(a,\bar{s})] + (\beta+\delta)\mathrm{E}[q(a,\bar{s})]^2 - \lambda I(a) \tag{13}$$

As the second term $\mathrm{E}[q(a,\bar{s})] = \frac{\bar{\alpha}-\gamma}{2(\beta+\delta)}$ is independent from a, one can distinguish two different effects of information on ex-ante profit. First, its linear cost. Second, as it enhances the adjustment of the quantities produced to the uncertain demand parameter, it provides a more accurate quantity setting from an ex-ante perspective. Hereafter, I refer to this as the relevance effect. Thus, the private value of information can be viewed as the marginal relevance effect:

$$\mathcal{V}^p = \frac{\sigma_0^2}{4(\beta+\delta)} \tag{14}$$

Several components of the private value of information can be identified. Unsurprisingly, the prior uncertainty ranks among them, as it determines how much the firm needs information. The more uncertain the demand is, the less likely the firm is to choose a relevant quantity setting from an ex ante perspective, and so the more the information is valuable to her.

Another determinant of the value of information is the sensitivity of the demand. The less sensitive the demand is to an increase in prices (the higher β is), the less the information is valuable for the firm. If the demand is highly un-elastic, the profit losses induced by a wrong quantity setting are offset by the weak reactions of consumers demand, so the firm has small incentives to refine its expectations to propose a more relevant quantity setting. In other words, the private value of information is orthogonal to the ability of the firm to extract consumers' surplus.

It is worthwhile noting that the linear marginal cost of production γ doesn't determine the value of information. This is due to the fact that, irrespectively of the amount of information she acquires, the firm will internalizes the linear marginal cost in the same way in the interim quantities. Thus, this term does not affect how the interim quantities are adjusting to the signals, and thereby does not contribute to enhance the relevance of quantity setting.

Nevertheless, the nature of the returns to scale determines the value of information. Decreasing return to scales ($\delta > 0$) exerts an overall negative effect on the interim quantities and thus reduces the weight put on the signals. The intuition is that, with decreasing returns to scale, adjusting quantities can be too costly for the firm, so new information entails a lower change in the quantities produced and thus yields a smaller increase in ex-ante profit. By contrast, increasing return to scales ($\delta < 0$) pushes upward the interim quantities, which increases the leverage effect of attention on profits and thus makes it more valuable to the firm.

The timing of the model implies that when the firm acquires information, she does not receive it instantaneously, but only at the following step of the decision tree, and she does not know what will be the content of the signals she buy. Therefore, in this framework, acquiring information can be viewed as an investment, whose return is uncertain, as the firm does not know how the information she buy will affect her profits. In other words, the private value of information is the expected return of this investment.

5 Underinvestment in information

In this section, I investigate the welfare implications of the information acquisition decisions of the firm. This requires to focus on the expected consumer surplus ($\mathcal{ECS}$). What motivates an ex ante rather than an interim perspective for studying welfare ? The firm is the only agent who acquires information and bears the ensuing cost. Hence, it is natural to think this information as private. We rarely see a firm divulging the results of her market studies to her consumers. Consequently, neither the policymaker, nor consumers have access to the information acquired by the firm. Therefore, studying interim welfare, which assumes to have access to firm's information decisions can be irrelevant. The expected consumer surplus is given by:

$$\mathcal{ECS}(a) = \frac{\beta}{2}\left(\frac{\operatorname{cov}[\alpha, q(a,\bar{s})]}{2(\beta+\delta)} + \mathrm{E}[q(a,\bar{s})]^2\right) = \frac{\beta}{2}\left(\frac{a\sigma_0^2 + (\bar{\alpha}-\gamma)^2}{4(\beta+\delta)^2}\right) \tag{15}$$

A key result is that the consumers always benefit from a higher information acquisition: $\mathcal{ECS}(a)$ is a strictly increasing function. The reason is that consumers also enjoy the relevance effect of information which ensures them, from an ex ante perspective, to obtain more pertinent quantities. This result is important as at first glance, one might have expected that consumers could be hurt by a well informed monopoly, as intuitively if the firm process information on consumers, it is not so much for altruistic motives than for extracting more of their surplus. If information acquisition is actually not detrimental to consumers it is because of the timing of the game. The firm uses her market power to extract consumers' surplus only at the second stage. But as acquiring information is an investment, when the firm invests at the ex ante stage, she does not know the content of the information and so how it will impact her market power: expected prices, and so the ex ante Lerner index are constant with respect to information. So when taking her investment decision, the firm does not take into account the effect of information on her market power. Information is not acquired in order to increase the firm market power and so does not hurt consumers.

Therefore, from the consumers perspective, a full information equilibrium is desirable. But as information is costly the policymaker would be mistaken by forgetting the firm which actually process information. Trying to implement a full information equilibrium is not an achievable goal for an antitrust policy, due to the ensuing infinite cost. That would counterproductively destroy all incentives to acquire information and so results to a full inattention equilibrium. A consistent policy should rather focus on the total ex ante welfare:

$$\mathcal{W}(a) = \frac{3\beta+2\delta}{2}\left(\mathrm{E}[q(a,\bar{s})]^2 + \frac{\operatorname{cov}[\alpha, q(a,\bar{s})]}{2(\beta+\delta)}\right) - \lambda I(a) \tag{16}$$

Welfare decomposes into three part, where the first one represents welfare achieved when no information is processed. The second term captures the additional welfare obtained through information acquisition and the last one account for the information cost paid by the firm. From this point, one can define the social value of information $\mathcal{V}^S$ as the marginal benefit of information on ex ante welfare. But the only margin through which information affects welfare (excluding its cost) is the relevance effect. Hence, like for the private value, the social value of information captures the spillovers of a more accurate output adjustment to the demand. Hence, it is not surprising that the roots of private and social value are identical: prior uncertainty, demand elasticity and returns to scales.

$$\mathcal{V}^S = \frac{\sigma_0^2(3\beta + 2\delta)}{8(\beta+\delta)^2} \tag{17}$$

It is straightforward to see that they exert the same effect on both private and social value. However, while the private value only considers the firm, the social value of information measures how both the firm and consumers enjoy the relevance effect. This results in a positive gap between social and private value of information, leading to the following result

Proposition. *Information acquisition generates a positive externality on consumers and so the monopoly firm underinvests in information. The welfare maximizing level of attention is:*

$$a^o = \begin{cases} 1 - \frac{4\lambda(\beta+\delta)^2}{\sigma_0^2(3\beta+2\delta)} & if\ \sigma_0^2(3\beta+2\delta) > 4\lambda(\beta+\delta)^2 \\ 0 \quad else \end{cases} \tag{18}$$

Where

$$a^o \geq a^*$$

Economic literature has well identified that when information is asymmetric the market is failing. This result suggests that when information is acquired, there is a second market failure, belonging to a distinct class, namely, an externality. It may be worthwhile to note that this model entirely leaves aside the question of asymmetric information. As the signals are exogenously send, all agents have the same information partition. The externality issue only arises from the acquisition of information and not from its distribution among agents. However, many economic situations feature those two market failures. It calls for further research that account for both of them.

Furthermore, this result also has implications on the way that an antitrust policy should be thought. Here, the first best allocation is achieved with standard tools of externality regulation policy, that is subsiding information acquisition so as to align private and social value of information. But such a policy would end up on struggling against firms' market power and their harmful consequences on consumers. Policymaker must choose which market failure she wants to fight: imperfect competition or insufficient information acquisition ? To address this question, one may investigate the behavior of a competitive firm in terms of information acquisition so as to establish the relative efficiency compared to the monopoly firm.

6 Relative efficiency to the competitive firm

The competitive firm faces the same two stage game as the monopoly firm. The only difference is the firm's absence of market power. Consequently, she sets higher interim quantities and generates a smaller ex ante profit as compared to the monopoly firm, for the same level of attention. The competitive firm is thus less incentivized to invest in information. It is instructive to look at the constant return to scale case ($\delta = 0$), where perfect competition induces zero expected profit. Here, a better output adjustment has no effect on profits so information decisions are indifferent to the firm: the private value of information is null and so the competitive firm does not invest in information. Decreasing returns[5] provides an incentive to acquire information, but it is clear that the private value of information for the competitive firm is still smaller than for the monopoly. This leads to the following results:

Proposition. *The competitive firm always invests less in information than the monopolist.*

$$a^* \geq a^c$$

While the previous part has made clear that the firm does not acquire information in order to increase her market power, this result indicates that having a market power provides incentives for investing in information. Indeed both firms differ only in their market positions. In a perfect competition economy, information delivers smaller returns and so the firm acquires less information about her consumers: externality problem gets bigger. This has welfare consequences. One can compute the externality cost (denoted EC) in terms of welfare in both the competitive and the monopoly market and check that the externality is more detrimental to the consumers in a competitive market.

$$EC^c > EC^m \tag{19}$$

With

$$EC^m = \mathcal{W}_o - \mathcal{W}_{eq} \qquad EC^c = \mathcal{W}_o^c - \mathcal{W}_{eq}^c$$

Where $\mathcal{W}_o$ (resp. $\mathcal{W}_o^c$) and $\mathcal{W}_{eq}$ (resp. $\mathcal{W}_{eq}^c$) denote welfare levels at optimum and equilibrium with the monopoly (resp. competitive) firm.

The policymaker faces two market failures: imperfect competition and externality. A successful antitrust policy leading to perfect competition, by solving the first market failure, would increase the damages of the second one. An antitrust policy, by eliminating market power, destroys the incentives for information acquisition and so worsen the externality

[5] I do not consider increasing returns in this part as they are inconsistent with perfect competition.

issue. However, by completing an antitrust policy with a successful externality regulation, it is possible to implement the competitive first best allocation, which exceeds the monopoly one in terms of welfare.

$$\mathcal{W}_o^c > \mathcal{W}_o \tag{20}$$

From a pure theoretical perspective, the competitive first best allocation is achievable and more desirable. But in practice the conclusion may differ. Indeed, such an allocation is the outcome of two succeeding policy, which turns to be not as easy to implement as the theory may suggests. Breaking a monopoly is not costless, neither does fixing the externality. It is clear when one try to implement the optimal subsidy. The value of information depends on the firms' return to scales and so the subsidy does. In actual facts, this technological parameter is private information to the firm, so regulating the externality issue induce at least an information cost for the policymaker, who have to infer the true value of δ. Although fighting against the two market failures delivers a higher welfare, the ensuing costs and the practical difficulties encourage to consider the opposite policy: the *lassez faire* on both market failures. As the externality welfare cost is smaller under imperfect competition, the perfect competition outcome is not necessarily preferable to the monopoly one. One can get convinced by looking at $\Delta = \mathcal{W}_{eq} - \mathcal{W}_{eq}^c$ the difference between welfare levels under monopoly and competitive equilibrium:

$$\Delta = \underbrace{(EC^c - EC^m)}_{\text{Relative externality cost}} - \underbrace{(\mathcal{W}_o^c - \mathcal{W}_o)}_{\text{Dead-weight loss}} \tag{21}$$

The cost of the externality under perfect competition can be sufficiently high, relatively to imperfect competition, to exceeds the dead-weight loss of the monopoly. Indeed the set for which Δ is well defined and where $\Delta > 0$, is non-empty: the monopoly firm, by outperforming the competitive firm in term of information acquisition, can yields a higher expected welfare. Hence the economy can be better off with a *laissez faire* policy than an antitrust policy which fails or does not aim at managing the externality issue.

7 Concluding remarks

In this thesis I proposed a model that analyzes the information acquisition decision of a monopoly firm as a rational inattention problem, by considering a two-stage game, and grounding the information cost on the mutual information measure. Including the Gaussian conjugate assumption allows to capture both the costs and the benefits of information by a unique variable, namely, the attention. I characterize the equilibrium attention level of the monopoly firm and show that it is always higher than that of the competitive firm.

Dealing with a two-stage decision procedure gives to see the information acquisition decision as an investment. A first implication is that the private value of information that I characterize, can be viewed as a return on investment. A further consequence is that, when investing, the firm does not know what information she will receives and so how it will affect her market power. Indeed, at the interim stage, information can push the market power as well downward or upward. Hence, at the ex ante stage, the investment decision is made considering a constant level of market power. Thus, when investing, the firm does not seek to increase her market power (while the market power in itself provides incentive for information acquisition). The firm acquires information only to benefit from its relevance effect. As the consumers also benefit from this effect, the social value of information exceeds the private value, that is to say that information acquisition generates a positive externality for the consumers. I show that the welfare consequences of the underinvesting in information are worst under perfect competition. The relative externality cost can even exceed the welfare loss induced by imperfect competition, so that in the absence of an externality regulation, a better informed monopoly firm can be more desirable in terms of welfare than a competitive one. A higher welfare can also be achieved through a policy focused solely on encouraging firms to invest in information while accepting imperfect competition. Especially since breaking monopoly does not directly lead to perfect competition. This calls for further research on information acquisition in an oligopoly, so as to establish the welfare level that can be achieved in this intermediate case. Other extensions can be considered. Relaxing the Gaussian conjugate assumption would allow for a more general impact of information on expectations and thus on profits. One might also explore a more sophisticated information cost function, still based on mutual information but allowing for non-linearities in the mutual information. This would enhance our understanding of firms' information acquisition behaviour and its consequences on consumers.

Appendix

A Attention equilibrium

Substituting for interim quantities, the problem (11) reduces to

$$\max_{a\in[0,1]} \quad (\beta+\delta)\mathrm{E}[q(a,\bar{s})^2] - \lambda I(a) \tag{22}$$

As $s_i|\alpha \sim \mathcal{N}(\alpha, \sigma^2)$ and $\alpha \sim \mathcal{N}(\bar{\alpha}, \sigma_0^2)$, we have $\mathrm{V}[s_i] = \sigma_0^2 + \sigma^2$ and $\mathrm{cov}[s_i, s_j] = \sigma_0^2$.

$$\mathrm{V}[\bar{s}] = \frac{1}{n^2}\sum_i \mathrm{V}[s_i] + \frac{2}{n^2}\sum_{1\leq i<j\leq n} \mathrm{cov}[s_i, s_j] = \frac{\sigma_0^2+\sigma^2}{n} + \frac{n-1}{n}\sigma_0^2 \tag{23}$$

That leads to:

$$\mathrm{V}[\bar{s}] = \frac{\sigma_0^2}{a} \quad \text{and so} \quad \mathrm{V}[q(a,\bar{s})] = \frac{a\sigma_0^2}{4(\beta+\delta)^2} \tag{24}$$

As $\mathrm{cov}[\alpha, \bar{s}] = \sigma_0^2$ we note that

$$(\beta+\delta)\mathrm{V}[q(a,\bar{s})] = \frac{\mathrm{cov}[\alpha, q(a,\bar{s})]}{2} \tag{25}$$

Which leads to the formulation of the ex ante profit (13). Expressing the ex ante profit in terms of attention yields:

$$\Pi(a) = \frac{a\sigma_0^2 + (\bar{\alpha}-\gamma)^2}{4(\beta+\delta)} + \frac{\lambda}{2}\ln(1-a) \tag{26}$$

Which is a concave function defined on the compact set $a \in [0,1]$. It thus admit a unique global maximum, that is (12).

B Welfare

B.1 Underinvestment in information

Expected consumer surplus (15) is given by $\mathcal{ECS}(a) = \frac{\beta}{2}\mathrm{E}[q(a,\bar{s})^2]$. This gives the expected welfare under monopoly firm

$$\mathcal{W}(a) = \frac{3\beta+2\delta}{8(\beta+\delta)^2}\left(a\sigma_0^2 + (\bar{\alpha}-\gamma)^2\right) + \frac{\lambda}{2}\ln(1-a) \tag{27}$$

From where one can derive (17) and (18). To prove that $a^o > a^*$ one must ensure that we cannot have $a^o = 0$ and $a^* > 0$, that is, that $\sigma_0^2 > 2\lambda(\beta+\delta) \implies \sigma_0^2(3\beta+2\delta) > 4\lambda(\beta+\delta)^2$ which is always true with $\beta > 0$.

B.2 Relative efficiency

The interim problem of the competitive firm is solved by:

$$q^c(a,\bar{s}) = \arg\max_q \mathrm{E}[(P-\gamma-\delta q)q|\mathcal{I}_n] = \frac{a\bar{s}+(1-a)\bar{\alpha}-\gamma}{\beta+2\delta} \tag{28}$$

The ex ante profit of the competitive firm is so:

$$\Pi^c(a) = \frac{a\delta\sigma_0^2+\delta(\bar{\alpha}-\gamma)^2}{(\beta+2\delta)^2} + \frac{\lambda}{2}\ln(1-a) \tag{29}$$

Which is maximized by:

$$a^c = \begin{cases} 1-\frac{\lambda(\beta+2\delta)^2}{2\delta\sigma_0^2} & if\ 2\delta\sigma_0^2 > \lambda(\beta+2\delta)^2 \\ 0 & else \end{cases} \tag{30}$$

As $2\delta\sigma_0^2 > \lambda(\beta+2\delta)^2 \implies \sigma_0^2 > 2\lambda(\beta+\delta)$ we cannot have $a^* = 0$ and $a^c > 0$. Hence $a^* > a^c$ is always true. In a competitive market, the expected consumer surplus is given by:

$$\mathcal{ECS}^c(a) = \frac{\beta}{2}\left(\frac{a\sigma_0^2+(\bar{\alpha}-\gamma)^2}{(\beta+2\delta)^2}\right) \tag{31}$$

One can deduce the expected welfare under perfect competition:

$$\mathcal{W}^c(a) = \frac{a\sigma_0^2+(\bar{\alpha}-\gamma)^2}{2(\beta+2\delta)} + \frac{\lambda}{2}\ln(1-a) \tag{32}$$

From the above quantity, on can derive the optimal level of attention under perfect competition:

$$a^{oc} = \begin{cases} 1-\frac{\lambda(\beta+2\delta)}{\sigma_0^2} & if\ \sigma_0^2 > \lambda(\beta+2\delta) \\ 0 & else \end{cases} \tag{33}$$

With (18), (27), (32) and (33) we compute $\mathcal{W}_o$ and $\mathcal{W}_o^c$. With (12) and (30) we obtain $\mathcal{W}_{eq}$ and $\mathcal{W}_{eq}^c$. Using those values, one can show that (19)

$$EC^m = \frac{\lambda}{2}\left(\frac{\beta}{2(\beta+\delta)} - \ln\left(\frac{3\beta+2\delta}{2(\beta+\delta)}\right)\right) \quad EC^c = \frac{\lambda}{2}\left(\frac{\beta}{2\delta} - \ln\left(1+\frac{\beta}{2\delta}\right)\right) \tag{34}$$

$$EC^c - EC^m > 0 \Leftrightarrow \frac{\beta^2}{2\delta(\beta+\delta)} > \ln\left(\frac{\delta(3\beta+2\delta)}{(\beta+\delta)(\beta+2\delta)}\right) \tag{35}$$

Where the right hand side is always true as $1 + \frac{\beta^2}{2\delta(\beta+\delta)} > \frac{\delta(3\beta+2\delta)}{(\beta+\delta)(\beta+2\delta)}$. Turning now on (20)

$$\mathcal{W}_o^c - \mathcal{W}_o > 0 \Leftrightarrow \frac{\beta^2(\sigma_0^2 + (\bar{\alpha}-\gamma)^2)}{8(\beta+\delta)^2(\beta+2\delta)} > \frac{\lambda}{2}\ln\left(\frac{4(\beta+\delta)^2}{(3\beta+2\delta)(\beta+2\delta)}\right) \tag{36}$$

Which is always true if $1 + \frac{\beta^2(\sigma_0^2+(\bar{\alpha}-\gamma)^2)}{4\lambda(\beta+\delta)^2(\beta+2\delta)} > \frac{4(\beta+\delta)^2}{(3\beta+2\delta)(\beta+2\delta)}$ which reduces to the assumption $\sigma_0^2(3\beta+2\delta) > 4\lambda(\beta+\delta)^2$. Ultimately on can compute $\Delta = \mathcal{W}_{eq} - \mathcal{W}_{eq}^c$

$$\Delta = -\frac{\beta^2(\sigma_0^2 + (\bar{\alpha}-\gamma)^2)}{8(\beta+\delta)^2(\beta+2\delta)} + \frac{\lambda\beta^2}{4\delta(\beta+\delta)} - \frac{\lambda}{2}\ln\left(\frac{(\beta+2\delta)^2}{4\delta(\beta+\delta)}\right) \tag{37}$$

Which can be either positive or negative under the existence condition $2\delta\sigma_0^2 > \lambda(\beta+2\delta)^2$.

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