Paris School of Economics

Analysis and Policy in Economics program

Master Thesis

August, 2021

Sovereign Default and the Golden Rule of Public Finance

Author:
Venance Riblier

Supervisor:
Gilles Saint-Paul

Referee:
Daniel Cohen

Abstract

This thesis studies the golden rule of public finance in an endogenous default model with public capital. The golden rule requires the sovereign to finance its expenditures with tax revenues but allows for debt-financed investments. It prevents to conduct counter-cyclical fiscal policies that lead to capital liquidation and potentially to default. Moreover, the golden rule reshapes the default incentives. With sovereign debt backed by public capital, bond prices are more sensitive to investment decisions. Thus, the rule prompts the sovereign to sustain a higher and smoother stock of public capital, which dampens economic fluctuations and delivers welfare gains. In addition, it reduces the default frequency and yields enhanced bond prices. The results of the simulations also suggest that the golden rule achieves more sustainable debt and higher output than a simple debt ceiling rule.

JEL Classification: E32, E62, F34, H63

Contents

Acknowledgements

I am particularly grateful to my supervisor Gilles Saint-Paul for his guidance throughout this project. I thank Daniel Cohen for having accepted to be the referee of this thesis. I would also like to thank the participants of the macroeconomics workshop and in particular the professors Broer and Ferriere. I am indebted to Professor Coimbra for his advices on computational methods. Lastly, I thank Thomas Bezy and Theodore Humann for the rich discussions we had.

1 Introduction

Improving the sustainability of sovereign debts in emerging market economies is widely recognized as a fundamental challenge. To address it, different fiscal rules have been considered, such as deficit or debt ceilings. The rationale behind these rules is that there exists a threshold level of debt above which the incentives to default are too important and the public debt is no longer sustainable. A well-known weakness of this approach is the difficulty of identifying this debt threshold, which can eventually lead to rule-of-thumb decisions. An additional concern that arises in the case of emerging market economies is the damaging effect that these rules may exert on economic growth. By limiting the government's operations, they may prevent it from financing productive expenditures that could have enhanced output. To overcome these two pitfalls, an alternative to consider is the golden rule of public finance ("golden rule" hereafter). It stipends that unproductive expenditures must be tax financed while productive investments can be debt-financed. In other words, it constitutes a balanced budget rule that does not take into account public investments. As a result, the golden rule does not imply an ad hoc debt ceiling or a constraint on growth-enhancing expenditures. Although this remedy meets the above caveats, its effects on debt sustainability are less clear at first glance. To date, studies on the golden rule have been limited to its effects on capital accumulation, so it remains unclear how it affects borrowing and repayment decisions. The present dissertation aims to fill this gap. For this purpose, I consider the introduction of the golden rule in an endogenous default model with public capital accumulation.

A strength of this approach is that it can rationalize the debt levels and default frequencies that are actually observed in emerging market economies. In addition, including public capital accumulation generates consistent levels of public expenditures and investment. Therefore, the baseline model without the rule constitutes a pertinent benchmark to compare with the one where the golden rule is enforced. Moreover, it features sophisticated mechanisms through which the rule can affect the economy.

The sovereign can emit debt on the international markets to either finance public investments or insure households against income fluctuations through unproductive expenditures. Debt contracts are not enforceable and yield non-contingent repayments. Hence, in the event of adverse income shock, the sovereign defaults on its debt to generate additional resources that are rebated back to the households through unproductive expenditures. Bond prices internalize the counter-cyclical default risk and thus restrict the possibility for the sovereign to emit debt during bad times, which induces a pro-cyclical borrowing policy. Together with a constant tax rate, this implies a strong correlation of the sovereign's resources with output. Thus, to finance a counter-cyclical fiscal policy[1] during bad times the sovereign has to liquidate public capital. Yet, there are strong incentives for the sovereign to accumulate public capital. First, its output-enhancing effect constitutes a long-term insurance device as it helps the sovereign to deliver a higher and smoother consumption path to households. Moreover, as default incentives are shrinking with public capital, the sovereign can benefit from enhanced borrowing conditions when it launches large public investments. Considering this, the prevalence of unproductive expenditures over investments during bad times suggests that the insurance strategy of defaulting sovereigns is too short-term. By contrast, if the sovereign could fully commit to repaying its debt, it would adopt a more long-term oriented insurance strategy. Without default risk, bond prices are constant and borrowing yields a non-contingent resource. Therefore, the sovereign is no longer required to adjust its capital stock to finance expenditures or to improve borrowing conditions. This results in a more stable capital stock and dampened economic fluctuations. Hence, with enforceable debt contracts, the sovereign would achieve a smooth consumption path without resorting to counter-cyclical fiscal policy. This supports the idea that the default risk undermines the sovereign's insurance strategy.

Nevertheless, the results of the experiments suggest that the golden rule can yield improvements. Specifically, it pushes back the horizon of the sovereign's insurance strategy. As it reshapes the default incentives, the rule renders bond prices more sensitive to public investments. This prompts the sovereign to sustain a higher stock of public capital to avoid the states with deteriorated bond prices. Moreover, with a contingent constraint on public expenditures, there are no longer room for counter-cyclical fiscal policy in bad times. This cuts incentives to adjust the capital stock to finance public expenditures as the rule isolates the latter from the former. This results in dampened investment fluctuations. Consequently, the golden rule has a stabilizing effect on the entire economy. It induces a smoother consumption path for households and thus welfare gains, even though the sovereign can no longer conduct counter-cyclical fiscal policy. In other words, the golden rule alleviates the cost of the limited commitment as it leads to a long-term insurance strategy, analogous to that obtained with enforceable debt contracts. Moreover, with increased capital stock and dampened fluctuations, the rule induces lower default frequency and spreads. This enables the sovereign to sustain a higher level of debt. By contrast, a debt ceiling rule generates a smaller reduction in the default frequency while imposing a significant decline in the bond stock. This highlights that in terms of debt sustainability, what matters most is its use and not its level. Moreover, a debt ceiling rule prevents the sovereign to launch debt-financed

[1] As the tax rate is assumed to be constant, the term "fiscal policy" refers to the unproductive expenditures policy of the sovereign.

investments in low output states, which brakes the development of the country.

The rest of the dissertation is organized as follows. Section 2 provides the literature review. The baseline model is detailed in Section 3. Section 4 presents the main mechanisms that are at play in the baseline model. The golden rule experiments are analysed in Section 5. Section 6 concludes.

2 Literature Review

This dissertation links with two different stances of the economic literature. First, the model considered here is in line with the endogenous default literature initiated by the seminal paper of Eaton and Gersovitz (1981). As in Arellano (2008), bond repayments are assumed to be non-contingent, which preserves the key properties of its paper such as counter-cyclical default incentives and spreads. This dissertation also links to the analysis of Cuadra et al. (2010) who were the first to endogenize the production and analyse fiscal policy in a default model. As in their analysis, bond prices capture counter-cyclical default incentives and render borrowing less appealing in bad times, leaving the resource side of the government budget constraint pro-cyclical. The same mechanism is at play in the more recent work of Kaas et al. (2020). However, augmenting the model with public capital allows for adjustment within the spending side of the government budget constraint and thus prevents to generate a fully pro-cyclical fiscal policy as in the aforementioned papers.

A more recent stance of the endogenous default literature has already made substantial steps in introducing public capital. It has been initiated by Park (2017) who integrates public capital to the production function in a similar way to that adopted in this model. The contribution of this paper is to consider alternative default costs to the one usually assumed since Arellano (2008) to analyse the possibility that a sovereign defaults in good productivity time. This question is left aside here. By assuming a standard asymmetric default cost, the present model predicts that default occurs during bad times. A further step has been made with the analysis of Gordon and Guerron-Quintana (2018) who evaluates the effect of public investments on default incentives. Consistent with their analysis, here, a larger capital accumulation leads to a reduction in the default frequency. However, these two latter papers depart from the model proposed here as they do not consider public expenditures. The distinction between the different types of government spending has only recently been introduced in endogenous default models, by Asonuma and Joo (2020) and Novelli and Barcia (2021). Their models are close but richer than the one considered here as the former features a renegotiation process and the latter a more sophisticated private sector.

Nevertheless, they both highlight the importance of investment cuts during bad times, which is also a key element in the present analysis, as they allow for counter-cyclical expenditures policy to insure the households. In a nutshell, as the present model sits on the shoulders of endogenous default literature, it successfully reproduces the main stylized facts that this literature has progressively captured.

Endogenous default models have also been used to evaluate the effects of fiscal rules. Hatchondo et al. (2012) investigates the effect of debt ceiling. Their findings suggest that such rules induce important spread reduction. This is consistent with Alfaro and Kanczuk (2017) who introduce quasi-hyperbolic preferences to generate a consistent default frequency without imposing an abnormally low discount factor as in standard endogenous default models. They also found that a debt ceiling rule could induce welfare gains. By contrast, the present model suggests that such a rule would reduce welfare compared to a no-rule case. This divergence in conclusions stems from the fact that income is assumed to be exogenous in these two papers, while here the adjustments that occur through the production weaken the fundamentals of the economy. More precisely, a debt ceiling rule turns to be harmful for public capital accumulation. The resulting output cost outweighs the welfare gains induced by enhanced bond prices and reduced default frequency, which are the only source of welfare gains in the two aforementioned papers. This warns against the limits of evaluating debt-to-output ratio rules without considering the denominator.

Secondly, the present dissertation adds to the literature that have studied the golden rule of public finance. To the best of my knowledge, there is no existing investigation within an endogenous default model. However, its effects have been evaluated through other lens. In particular, in the framework of endogenous growth model as in Minea and Villieu (2009) and Groneck (2010) who both contrast the golden rule with balanced budget rules. They draw opposed conclusions regarding its impact on long-term growth. This is because of the different assumptions made concerning the service of the debt. The former considers that the sovereign services its debt by cutting on productive investment. Conversely, the latter assumes that unproductive expenditures are reduced to finance the debt service so that in the long run it does not outweigh the effect of debt-financed investments allowed by the golden rule. By contrast, both cases are considered in this dissertation. There are important differences between a growth and a default model when assessing the potential benefits of the golden rule. Without income shocks, the former cannot account for the insurance motive of borrowing. Conversely, in an endogenous default model with public capital accumulation, debt is a twofold object that can be used both for financing investment and to smooth households' consumption. This dual nature of the debt is a prerequisite to rationalize the borrowing level observed empirically and hence to capture the response of the sovereign bond policy to the introduction of the golden rule. Moreover, without default risk, a growth model might miss the response of public investments that occur through the effect of the rule on default incentives and bond prices. Eventually, in contrast to the endogenous growth literature, the stochastic nature of the present model allows to analyse how the golden rule affects economic fluctuations.

The golden rule of public finance has also been explored through the lens of DSGE models as in Zeyneloglu (2018) and Shvets (2020). This approach integrates private capital and thus allows for crowding out effect of debt-financed public investments. Their findings suggest that the golden rule limits the crowding out effect on private capital. Therefore, a positive shock to the overall public spending eventually results in enhanced output as the increase in public capital outweighs the crowding out effect on the private one. The model considered here is silent on this point. More generally, endogenous default models are not well suited to account for such crowding out effect. Extending the model to allow for private capital accumulation would require a segmented capital market as corporate bonds are not subject to the sovereign default risk. Thus, the crowding out effect could not go through capital returns and would require more sophisticated feedback between private and public capital to be rationalized. Therefore, the results of this stance of the literature should be taken as a complementary to the present investigation.

3 The Baseline Model

I consider a small open economy populated by a representative consumer. The households supply labor and receive profits from the representative firm. The firm uses labor and public capital as input and faces productivity shocks. Hence, households' income is fluctuating. Yet, they are risk-averse, so the objective of the benevolent government is to insure them against these fluctuations. For this purpose, the sovereign can borrow on the international market and distribute the resulting resources to the households through unproductive public expenditures that directly enter the utility function. In addition, the government can invest in productive public capital. To finance its operations, the government also levies a tax on private consumption. There is no way for the sovereign to fully commit to repaying its debt to international lenders. Hence, at each period it can default on its debt. This event triggers a direct productivity cost and the exclusion of the country from financial markets for a random number of periods.

3.1 The model environment

Households and Firm

The representative consumer is infinitely lived and determines a sequence of private consumption c_t and labor supply l_t choices that maximize her expected utility. In addition, she derives utility from the government's public expenditures g_t. With β the discount factor, her preferences are given by:

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t u(c_t, g_t, 1-l_t) \tag{1}$$

The consumer receives labor income $w_t l_t$ and the profits from the firm Π_t. Private consumption is taxed at a constant rate τ. Therefore, her objective is to maximize (1) subject to the budget constraint: $c_t(1+\tau) = w_t l_t + \Pi_t$. The first-order condition of this problem is:

$$\frac{u_l(c_t, g_t, 1-l_t)}{u_c(c_t, g_t, 1-l_t)} = \frac{w_t}{1+\tau} \tag{2}$$

The firm produces the output $z_t F(k_t, l_t)$ using labor and public capital k_t. This technology is affected by a productivity shock z_t, governed by a Markov transition function $p(z_{t+1}, z_t)$. She takes public capital as given and determines the optimal amount of labor demand at each period to maximize her profits $\Pi_t = z_t F(k_t, l_t) - w_t l_t$. The resulting equilibrium wage is given by:

$$w_t = z_t F_l(k_t, l_t) \tag{3}$$

The government

To insure households against income fluctuations, the government can provide unproductive public expenditures g_t. In addition, it can invest in public capital, which evolves according to the law of motion $k_{t+1} = i_t + (1-\delta)k_t$. Investment decisions affect the future level of economic activity and therefore future fiscal revenues. I assume that this is the only instrument through which the sovereign can adjust its fiscal resources, implying that τ is constant. This rationalizes a fiscally constrained sovereign that can adjust its resources only through debt emissions in the short-term. It should be noted that the objective of the sovereign could also be thought in terms of tax smoothing. Assuming instead that public expenditures are exogenous and that the sovereign controls its tax rate, its objective would be to minimize the convex costs of the distorsive tax. Hereafter, I stick to a consumption smoothing problem with constant τ and adjustable g_t, but it should be kept in mind that it can be seen as a tax smoothing problem.

Debt is assumed to be non-contingent and long-term. At each period, a share λ of the outstanding debt b_t matures and has to be repaid. In addition, the government pays a coupon κ on the remaining stock of debt. Overall, to service its debt, the government pays $[\lambda + (1-\lambda)\kappa]\, b_t$. The sovereign can issue new bonds at price $q(z_t, b_{t+1}, k_{t+1})$. The amount of new debt emitted is defined as the difference between the next period stock of debt and the current remaining stock of debt. Hence, a debt contract generates $[b_{t+1} - (1-\lambda)b_t]q(z_t, b_{t+1}, k_{t+1})$ additional resources for the sovereign. An exogenous debt ceiling $\bar{b}$ is assumed to rule out Ponzi schemes, but is not binding at equilibrium. The aggregation of the government's expenses and resources yields the following budget constraint:

$$i_t + g_t = \tau c_t + [b_{t+1} - (1-\lambda)b_t]\, q(z_t, b_{t+1}, k_{t+1}) - [\lambda + (1-\lambda)\kappa]\, b_t \tag{4}$$

International Financial Market

The financial market is assumed to be competitive and foreign lenders to be risk neutral. As a result, the bond pricing must satisfy the no-arbitrage condition. That is, the profit of selling one bond today, $q(z_t, b_{t+1}, k_{t+1})$, must equal the expected profit of holding it one further period. Conditional on being repaid, one bond yields the repayment λ plus the payment of a coupon κ on the share $1-\lambda$ of the bond that continues to mature. Moreover, this share will be valued at price $q(z_{t+1}, b_{t+2}, k_{t+2})$ the next period. The expected profit is discounted at the one-period risk-free interest rate r. Denoting $\mathcal{R}(b_t, k_t)$ the repayment set, that is, the set of productivity shock realizations for which the government does not default on its debt, the no-arbitrage condition gives the following bond pricing:

$$q(z_t, b_{t+1}, k_{t+1}) = \int_{\mathcal{R}(b_{t+1}, k_{t+1})} \frac{\lambda + (1-\lambda)[\kappa + q(z_{t+1}, b_{t+2}, k_{t+2})]}{1+r} p(z_{t+1}, z_t) dz_{t+1} \tag{5}$$

If the sovereign defaults on its debt, the country is excluded from international markets and cannot enter new debt contracts. The duration of this financial autarky is random and lasts on average $\frac{1}{\theta}$ periods, that is at each period there is a probability θ that the sovereign reintegrates the financial market. Moreover, I assume that during the autarky, the country endures direct costs to its productivity, which is reduced to $h(z_t) \leq z_t$. This assumption is motivated by the evidence of a productivity slump during debt crisis documented by Alonso-Ortiz et al. (2017).

3.2 Recursive Problem

At the beginning of each period, the sovereign observes the realization of the productivity shock and if it has access to financial markets, it can decide whether to default or to stay in the debt contract. This choice is made by comparing the values of the two alternatives. The problem solved by the government when it has access to the financial market is given by:

$$v^o(z,b,k) = \max\left\{v^d(z,k), v^c(z,b,k)\right\} \tag{6}$$

Where $v^c(z,b,k)$ and $v^d(z,k)$ denote the value of staying in the debt contract and the value of being in financial autarky respectively. The former is defined by:

$$\begin{aligned}
&v^c(z,b,k) = \max_{b',k',g}\left(u(c,g,1-l) + \beta\int_{z'} v^o(z',b',k')p(z',z)dz'\right)\\
&s.t.\\
&(1+\tau)c = zF(k,l)\\
&\frac{u_l}{u_c} = \frac{zF_l(k,l)}{1+\tau}\\
&k' + g = \tau c + (1-\delta)k + \left[b' - (1-\lambda)b\right]q(z,b',k') - \left[\lambda + (1-\lambda)\kappa\right]b
\end{aligned} \tag{7}$$

The sovereign takes into account the effect of capital choices on private decisions through the budget constraint and the first order condition of the households. The value of default is given by:

$$\begin{aligned}
&v^d(z,k) = \max_{k',g}\left(u(c,g,1-l) + \beta\int_{z'}[\theta v^o(z',0,k') + (1-\theta)v^d(z',k')]p(z',z)dz\right)\\
&s.t.\\
&(1+\tau)c = h(z)F(k,l)\\
&\frac{u_l}{u_c} = \frac{h(z)F_l(k,l)}{1+\tau}\\
&k' + g = \tau c + (1-\delta)k
\end{aligned} \tag{8}$$

If the sovereign defaults, it does so on the entirety of its debt. As a result, $v^d(z,k)$ is independent from the level of debt at the time of default. In the future, there is a probability θ that the country regain access to financial markets with no outstanding debt and a probability $1-\theta$ that it stays in autarky. The government also takes into account the productivity cost $h(z)$ incurred during autarky.

The decision of default is entirely determined by the comparison of (7) and (8). Consequently, one can define the repayment set as:

$$\mathcal{R}(b,k) = \{z \in Z | v^c(z,b,k) \geq v^d(z,k)\} \tag{9}$$

3.3 Equilibrium Definition

The equilibrium of the model is defined by

- The households' policy functions for consumption $c(z,b,k)$ and labor supply $l(z,b,k)$.
- The government's value functions $v^o(z,b,k)$, $v^d(z,k)$, $v^c(z,b,k)$, and repayment set $\mathcal{R}(b,k)$.
- The government's policy functions for public expenditures $g(z,b,k)$, public capital $k'(z,b,k)$ and borrowing $b'(z,b,k)$.
- A bond price schedule $q(z,b',k')$.

Such that

- Given the government policies, the households' policies maximize (1) subject to their budget constraint.
- Given the bond price and the households' policies, the government's value functions and policies solve (6) to (8).
- The bond price schedule satisfies (5).

3.4 Calibration and Numerical Forms

The model is calibrated to Argentina. As the objective of this dissertation is to evaluate the golden rule in a situation where default can occur, the baseline model has to be nested in an economy whose features allow for such episodes. Moreover, the attention devoted to this country in the endogenous default literature provides guiding evidences on the calibration and on the stylized facts of business cycles to be used to evaluate the quantitative properties of the model.

The frequency considered is the quarter. Hence, the risk-free interest rate is set to $r = 0.01$ which is a standard value used in the literature to match the 3-month real interest rate of U.S. Treasury bills. The long-term structure of the debt follows the analysis of Chatterjee and Eyigungor (2012) where bonds mature with a probability of $\lambda = 0.05$ and coupon payment κ is 3%. This is consistent with a median maturity of 20 quarters and an annual coupon rate of 12%. The tax rate is set to $\tau = 0.18$ to generate a public expenditures to output ratio equal to 12.89% as reported by Kaas et al. (2020). The discount factor β is calibrated to 0.86 to match a default frequency of 3 times per century. A major downside of the default models is that they require abnormally low values of the discount factor to generate a default frequency consistent with the data. To unleash the labor supply from wealth effect and achieve finer replication of business cycles facts I consider Greenwood et al. (1988) utility function:

$$u(c_t, g_t, 1-l_t) = \frac{(1-\omega)}{1-\sigma}\left(c_t - \frac{l_t^{(1+\psi)}}{1+\psi}\right)^{1-\sigma} + \omega\frac{g_t^{1-\sigma}}{1-\sigma} \tag{10}$$

Risk aversion, Frisch elasticity and weight on public consumption are set to $\sigma = 2$, $\frac{1}{\psi} = 2.22$ and $\omega = 0.3$ respectively, following Novelli and Barcia (2021) and in line with the real business cycles literature. I consider a Cobb-Douglas production function with constant returns to scale.

$$F(k_t, l_t) = \zeta k_t^{\alpha} l_t^{1-\alpha} \tag{11}$$

Where ζ is a scaling constant calibrated to normalize to unity the steady-state value of output. The elasticity of output to labor is set to 0.64 which is a standard value. Associated with constant returns to scale, this results into high elasticity of output to public capital. This allows to alleviate the absence of private capital in the model by implicitly assuming that an increase in the stock of public capital will creates incentives for private investments and thus an increase in the stock of private capital. The depreciation rate is set to a standard value $\delta = 0.025$. The productivity process is assumed to be log-normal AR(1) with zero mean.

$$\log(z_{t+1}) = \rho \log(z_t) + \varepsilon_t \tag{12}$$

The autocorrelation of the process is set to $\rho = 0.95$, following Chatterjee and Eyigungor (2012). The standard deviation of the innovation σ_ε is calibrated to match the output volatility, which is estimated to 4.98% by Kaas et al. (2020). The process is discretized using Tauchen and Hussey (1991) procedure. I follow Arellano (2008) in the specification of the default cost:

$$h(z_t) = \min\{z_t, \phi \mathrm{E} z_t\} \tag{13}$$

The asymmetry in the cost implies that the country does not endure any penalty during low productivity shock. As a result, the model generates default only in bad times. The productivity cost parameter ϕ is calibrated to 0.86 to match an average debt service of 5.5% as reported by Chatterjee and Eyigungor (2012). As noted by Aguiar and Gopinath (2006), substantial penalties are also important to generate high debt-to-output ratios. Otherwise, the debt threshold above which the government defaults is to low to sustain the debt levels observed in the data. Eventually, the probability of re-entering the financial market is set to $\theta = 0.1$ which is consistent with an average duration of autarky of 2.5 years, in line with the estimate of Gelos et al. (2011). The table 1 summarizes the calibration of the parameters of the model.

TABLE 1: PARAMETERS VALUE

Parameter	Description	Value	Source or Target
Parameters calibrated			
τ	Tax rate	0.18	Public expenditures to output ratio
β	Discount factor	0.86	Default frequency
σ_ε	Productivity standard deviation	0.01	Output standard deviation
ϕ	Productivity cost	0.86	Debt service
Parameters set to standard value			
r	Risk-free interest rate	0.01	Cuadra et al. (2010)
λ	Maturity probability	0.05	Chatterjee and Eyigungor (2012)
κ	Coupon payment	0.03	Chatterjee and Eyigungor (2012)
σ	Risk aversion	2	Chatterjee and Eyigungor (2012)
$1/\psi$	Frisch elasticity	2.22	Cuadra et al. (2010)
ω	Public consumption weight	0.3	Novelli and Barcia (2021)
$1-\alpha$	Elasticity of output to labor	0.64	Novelli and Barcia (2021)
δ	Depreciation rate	0.025	Novelli and Barcia (2021)
ρ	Productivity autocorrelation	0.95	Chatterjee and Eyigungor (2012)
θ	Reentry probability	0.1	Cuadra et al. (2010)

Note: This table summarizes the parameters used to solve the baseline model.

3.5 Simulations

In this subsection, I evaluate the performance of the baseline model to replicate the key business cycle statistics of the country chosen for the calibration, namely, Argentina. Although a perfect match of the data is not the first purpose of this dissertation, it is essential to ensure that the model generates moments that are consistent with the data so that the upcoming analysis of the golden rule nests in a relevant framework.

To that end, I run 1000 simulations of 4500 periods each and discard the first 500 periods.

The resulting time series are then HP filtered with a smoothing parameter set to 1600. The Table 2 shows the average of key business cycle statistics over all simulations. It reveals that the model is successful in replicating the main moments of the Argentina business cycle.

Overall, the level of the main variables of the government's budget constraint is consistent with the data. Considering long-term debt allows to disconnect the bond stock from the debt service, compared to models where bonds mature in one period. As a result, the model generates a high debt-to-output ratio, whereas short-term debt models fail on this point. However, the endogenous debt ceiling above which the default risk renders bond prices worthless is still too low to allow the model to replicate the debt level observed in the data. Moreover, it fails to generate high enough spreads. Yet, the simulations are also consistent with the economic fluctuations observed in Argentina. This ensures that the golden rule experiments developed in Section 5 have a consistent starting point. This will allow to interpret the magnitude in addition to the signs of the changes induced by the golden rule.

Table 2: Simulations baseline model

Variable	Data	Simulations
Mean values (%)		
Default rate	3	3.09
Annualized spreads	8.76	4.24
Debt service	5.5	5.2
Debt-to-output ratio	100	61.5
Public investment-to-output ratio	5	3.1
Public expenditure-to-output ratio	12.89	12.04
Volatility (%)		
Output	4.94	6.41
Public investment	12.8	8.69
Private consumption	5.18	6.65
Public expenditures	2.32	2.18

Note: This table shows the average of the business cycle statistics over the simulations of the baseline model. They are compared to statistics observed empirically. The data on spreads and debt are taken from Chatterjee and Eyigungor (2012), from Gordon and Guerron-Quintana (2018) for investment and from Kaas et al. (2020) for output, private and public consumption.

4 Decisions of the sovereign in the baseline model

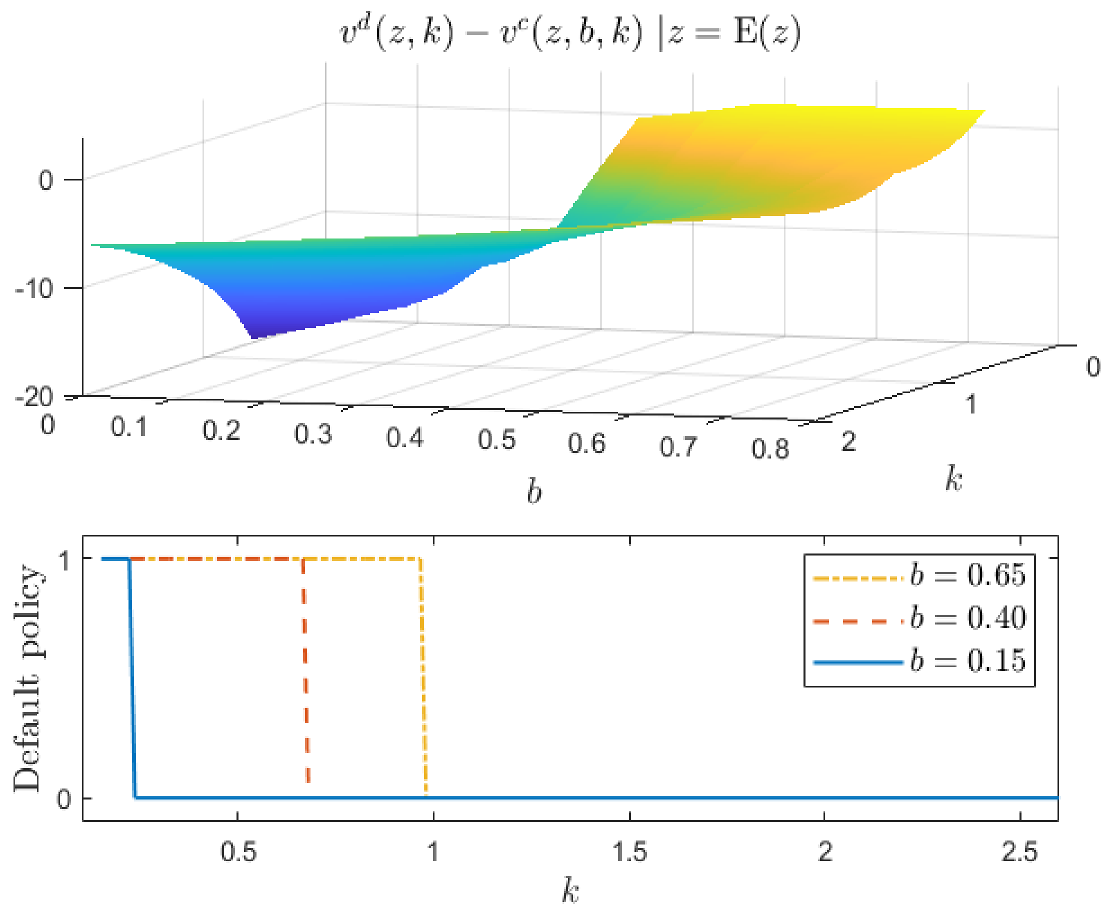
4.1 Default risk, bond price effects and borrowing policy

In this section, I present how introducing public capital in the model affects default risk, bond prices, and borrowing decisions. As noted by Gordon and Guerron-Quintana (2018) the effect of public capital on default incentives is nontrivial. On the one hand, investing today is a way for the government to commit to increase its future tax revenues through a larger fiscal base tomorrow, which increases the chance of repayments. On the other hand, the larger the stock of capital, the more resources the government has to cope with the autarky, increasing the value of default. To elucidate the overall effect of public capital on default risk, one can refer to the upper panel of the Figure 1 which represents the default incentives captured by the difference $v^d(z,k) - v^c(z,b,k)$, with the productivity shock set to its unconditional mean. For low and intermediate values of b, the default incentives are increasing in capital, while they become decreasing for larger values. The reason is that a higher bond stock reduces g through the government budget constraint, which increases the marginal utility of public expenditure. As a result, the marginal effect of public capital on $v^c(z,b,k)$ increases with the bond stock, and after a given point outweighs the marginal effect on $v^d(z,k)$. Overall, accumulating public capital exerts a negative effect on default risk, for a given level of debt. In low debt states, default incentives will increase with k but remain negative. For larger level of debt, where default can be optimal, increasing k reduces default incentives and can even makes them negative. This is consistent with the results of Gordon and Guerron-Quintana (2018), obtained in a model also calibrated to Argentina.

This brings light on the geometry of the repayment set. It was already known to be bounded by a debt ceiling above which it is optimal to default. It is now also limited by a capital floor. Moreover, this capital floor increases with the level of debt (see the lower panel of Figure 1). Consequently, for a given level of z and k the repayment set is shrinking with the level of debt. This property punctuates the literature of endogenous default models since Eaton and Gersovitz (1981) and is preserved when including public capital. It stems from the assumption that if the sovereign defaults, it does so on its entire debt, leaving the value of default $v^d(z,k)$ independent of the debt stock, while the value of staying in the debt $v^c(z,b,k)$ is decreasing in b.

The joint effects of debt and capital on default risk are reflected in the bond price function (see the left panel of Figure 2). Bond prices are decreasing (increasing) in debt (capital) decisions. Therefore, the sovereign has to take into account the combined effect of its decisions on bond prices. When it sustain a high capital stock, it can engage in large debt

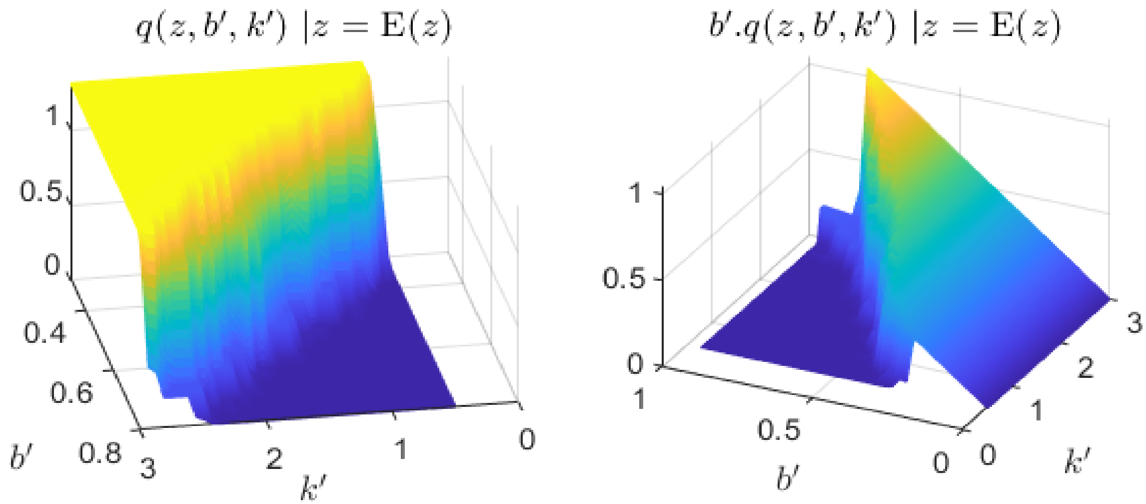
Figure 1: Default incentives and default policy



Note: The upper panel of this figure shows the difference between the value of default and of staying in the debt contract in the baseline model, with the productivity shock set to its unconditional mean. The lower panel represents the optimal default policy of the sovereign in the baseline model, for three different debt levels, with the productivity shock set to its unconditional mean.

contracts while maintaining favourable bond prices. However, if it decides to liquidate its capital stock, it has to reduce its bond stock, otherwise the bond prices will collapse. These price effects shape the sovereign optimal decision as they affect the resources generated by borrowing. This mechanism has been highlighted by Arellano (2008), and can be seen as a Debt Laffer curve. For limited debt emission, the quantity effect dominates, the more bond provides more resources. For too important borrowing, the value of the bond emitted collapse and the resources generated do so. While it only applies to the debt axis in the latter, the Debt Laffer curve is expanded over the capital dimension in this model (see the right panel of Figure 2). What is new here is that if the government emits debt and launches public investments at the same time, its bond emission is more valuable and hence generates more resources for a given level of bonds emitted. On the contrary, if the sovereign liquidates public capital, the same bond issuance would deliver fewer resources.

Figure 2: Bond price function and debt Laffer curve

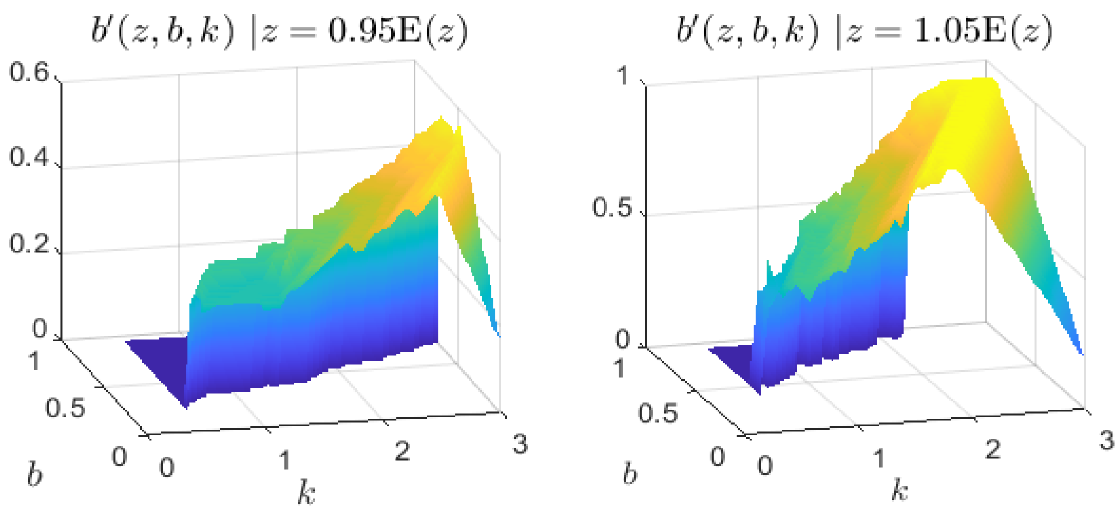


Note: The left panel of this figure shows the bond price function in the baseline model, with the productivity shock set to its unconditional mean. The right panel represents the "Debt Laffer curve", that is the resources generated by borrowing in the baseline model, with the productivity shock set to its unconditional mean.

The price effects that debt and capital decisions exert on the resources generated by borrowing are critical to understand the bond policy of the government. The Figure 3 represents this policy function, for two different realizations of the productivity shock, 5% below and above its unconditional mean. In high capital and low debt states, the bond policy is increasing in b and decreasing in k. That is, the quantity effect dominates. The higher the inherited capital stock, the more resources the government owns, which reduces its need to borrow. Conversely, a higher outstanding debt stock reduces the resources of the government, increasing its need to borrow. However, in smaller capital states, the price effects become dominant. The bond policy is increasing in k. As the capital policy function is increasing in k (see Figure 4), the higher the inherited capital stock, the more the government invests, which enhances the bond prices and prompts the sovereign to borrow more. It can be noted that the capital threshold below which the price effect dominates increases with b. The reason is that when the outstanding bond stock is large, more resources are used for the service of the debt, making additional borrowing necessary even with large capital stock. Thus, the quantity effect locus shrinks with b. Moreover, in the price effect locus, the bond policy is decreasing in b. As the capital policy function is decreasing in b (see Figure 4), the government launches smaller investments in high debt states, which reduces the value of the bond emitted and prompts it to borrow less.

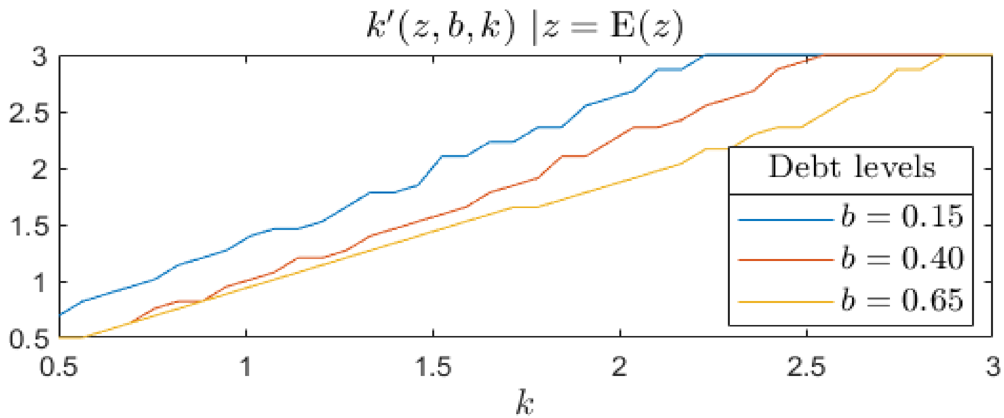
Eventually, it can be observed that the frontier between quantity and price effects locus is pushed back by the productivity shock. With positive productivity shock, bond prices are higher, the resources generated by borrowing are less sensitive to the sovereign's decisions. Thus, the price effects are weaker and the quantity effect locus expands. A positive realization of z also leads the sovereign to issue more bonds, as they deliver further resources. This result rely on the assumption of incomplete financial markets as in Arellano (2008). As debt repayments are non-contingent to the state of the economy, default incentives are lower in good times, which translates into enhanced bond prices.

FIGURE 3: BORROWING POLICY



Note: This figure shows the optimal bond policy in the baseline model, with the productivity shock set 5% below (above) its unconditional mean in the left (right) panel.

Figure 4: Capital policy



Note: This figure shows the optimal capital policy in the baseline model, for three different debt levels, with the productivity shock set to its unconditional mean.

4.2 Public insurance and funding

In this section, I investigate how the sovereign insures the households against income fluctuations, which instruments are used to do so, and how they are financed. At first glance, one could think that the debt is used to instantaneously generate resources in case of bad shock to compensate households through g, and that the sovereign takes advantage of the good periods to reduce its bond stock. However, the insights drawn from the previous subsection suggest that such a counter-cyclical borrowing policy is impossible as the bond prices capture the default risk. The sovereign internalizes that if it issues too much debt during bad periods, bond prices would collapse, and no resources would be generated by external borrowing. Thus, to capture how households' consumption smoothing is achieved, I conduct a perturbation analysis, with three sets of 1000 simulations, each over 40 quarters. Initial values of debt and capital are set to their steady state level and three different scenarios are considered for the productivity shock. In the good scenario, the economy is initially hit by a positive shock whose magnitude is $3\sigma_\varepsilon$. In the second case, a negative shock with the same magnitude hits the economy. Lastly, in the very bad scenario, the productivity declines by $5\sigma_\varepsilon$ from its unconditional mean. For the following periods, productivity shocks are drawn from the discretized distribution of the AR(1) process specified in the equation (12). The Figure 5 shows the average of the 1000 simulations, which can be seen as the impulse response function of the three different scenarios.

In response to the more severe shock, all sources of funding dry up. On the one hand, bond prices are plummeting, borrowing yields no resources so the government cannot emit new debt. On the other hand, output falls and so does tax revenues[2]. Nevertheless, the sovereign has to support the households' utility and hence substantially increases its public expenditures. Two sources of funding are used to afford it. The government simultaneously liquidates public capital and defaults on its debt. After the default, the government is released from its financial obligations. Thus, it can reconstitute its capital stock by running large investments. As in the periods following the default the country is in autarky, the government has to cut on its expenditures to finance these investments. Once it regains access to financial markets after ten quarters on average, it can borrow again and reincrease its expenditures to support households' utility which is still affected by the recession (which has been amplified by the default productivity cost).

However, this perturbation exercise is not the most accurate to understand the conditions that lead to default, as the initial conditions are set to steady state levels, while default generally occurs after a weakening of the economic fundamentals. Thus, to gain further insights on the path leading to default, I isolate all its occurrences in the simulations of the baseline model and store the value of the main economic variables 4 years before and after the default decision. The Figure 6 represents the average dynamics over all isolated episodes. It highlights the insurance motive of the default decision. As output decreases, the government increases its expenditure, sustaining a highly counter-cyclical g. However, because of bond prices deterioration, debt emission is prohibited, so it has to cut investments to finance it. This amplifies the collapse in bond prices and so in resources generated through borrowing. In response, the sovereign must liquidate even more capital to finance the household's insurance. The interaction between investment cuts and the breakdown of bond prices makes debt unsustainable. Overall, the default appears as a last resort insurance instrument which arises when capital liquidation can no longer alleviate the vanishing of bond resources.

In the second scenario with a milder negative shock, the bond prices fall in reaction to the default risk, but not too prohibited levels as in the previous case. Nevertheless, it still leads the government to reduce its borrowing. Coupled with declining tax revenues, this prompts the sovereign to sharply disinvest to finance a counter-cyclical increase in g. As the decline in output is limited, the increase of public expenditures is purely transitory, and they rapidly revert to their steady state levels. This allows the government to reinvest and progressively reconstitute its capital stock. Moreover, the debt reduction reestablishes favourable borrowing conditions. This allows the government to issue new bonds after few quarters, but to support the recovery of their prices, their issuance is very progressive. If the economy is not hit by another adverse shock during this redemption period, it goes back

[2]As tax revenues are proportional to output, the shapes of their IRFs are identical and hence not represented in the figure 5.

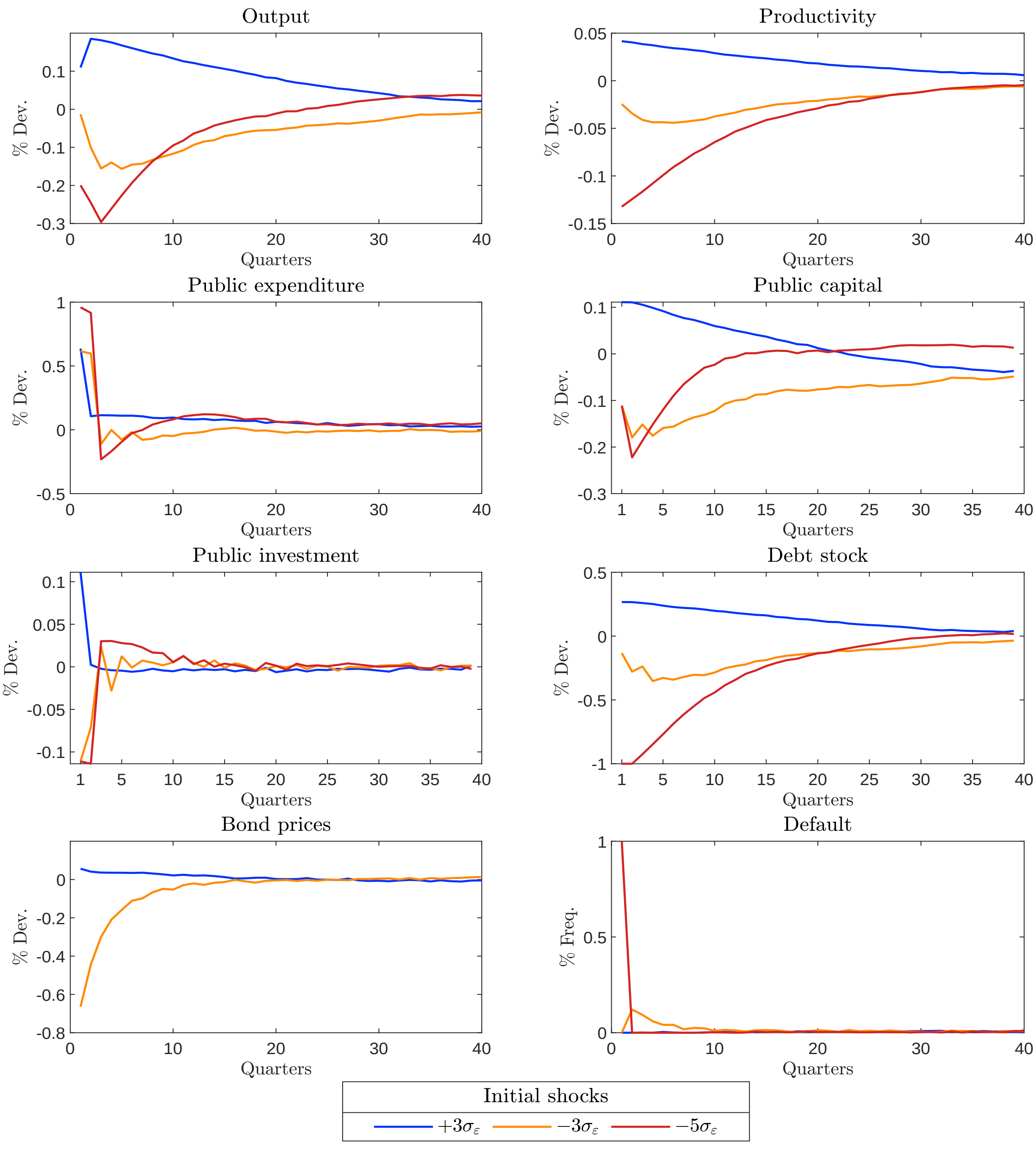
on the path to the steady state. However, if the productivity collapses again while capital stock has not been reconstituted and bond prices have not recovered, the only option left to the government to finance households' insurance is to default on its debt, which occurs in 10% of the simulations. In other words, the insurance strategy of a government facing a mild adverse shock is to make all its possible to avoid default and restore favourable borrowing conditions. This requires to finance the exceptional expenditures by capital liquidation and to adopt a moderate management of the debt during the recovery.

In the last scenario with positive productivity shock, both government resources are increased. Enhanced bond prices leads the sovereign to emit additional debt, and tax revenues soar with output. This translates into a one-time increase in investment, which amplifies the positive effect of the shock on output. The jump in resources also leads to a first significant increase in g, followed by 20 quarters of higher-than-normal expenditures. As the additional capital stock depreciates over time, the effects on output and on bond prices vanish, compelling the government to reduce its borrowing and its expenditures. It is worth noting that in this scenario, capital and bond stocks react in the same way, with a one-time jump at the time of the shock, while public expenditures and tax revenues increases are both long-lasting. In other words, the response of the government to positive shocks is in adequacy with the spirit of the golden rule of public finance, while in bad times, the counter-cyclicity of the government fiscal policy departs from the logic of the rule.

Overall, the insurance strategy of the government does not rely on borrowing in bad times and repaying in good ones. On the contrary, debt turns to be a privilege of the good time. As bond prices internalize the default risk, they constrain debt emissions under adverse shock and encourage borrowing during expansion periods. Consequently, bond issuance is strongly pro-cyclical ($\rho(b, y) = 0.85$ in the simulations of the baseline model). In addition, with a constant tax rate, the entire resource side of the budget constraint of the government is pro-cyclical. This allows to increase both the expenditures and the investments during good times, but in face of bad shock the government must make adjustments in the spending side of its budget constraint. To afford counter-cyclical fiscal policy during recessions, it has to liquidate public capital. However, the effect on bond prices limits the extent to which the government can substitute its different spending. Then, the only instrument left is the default.

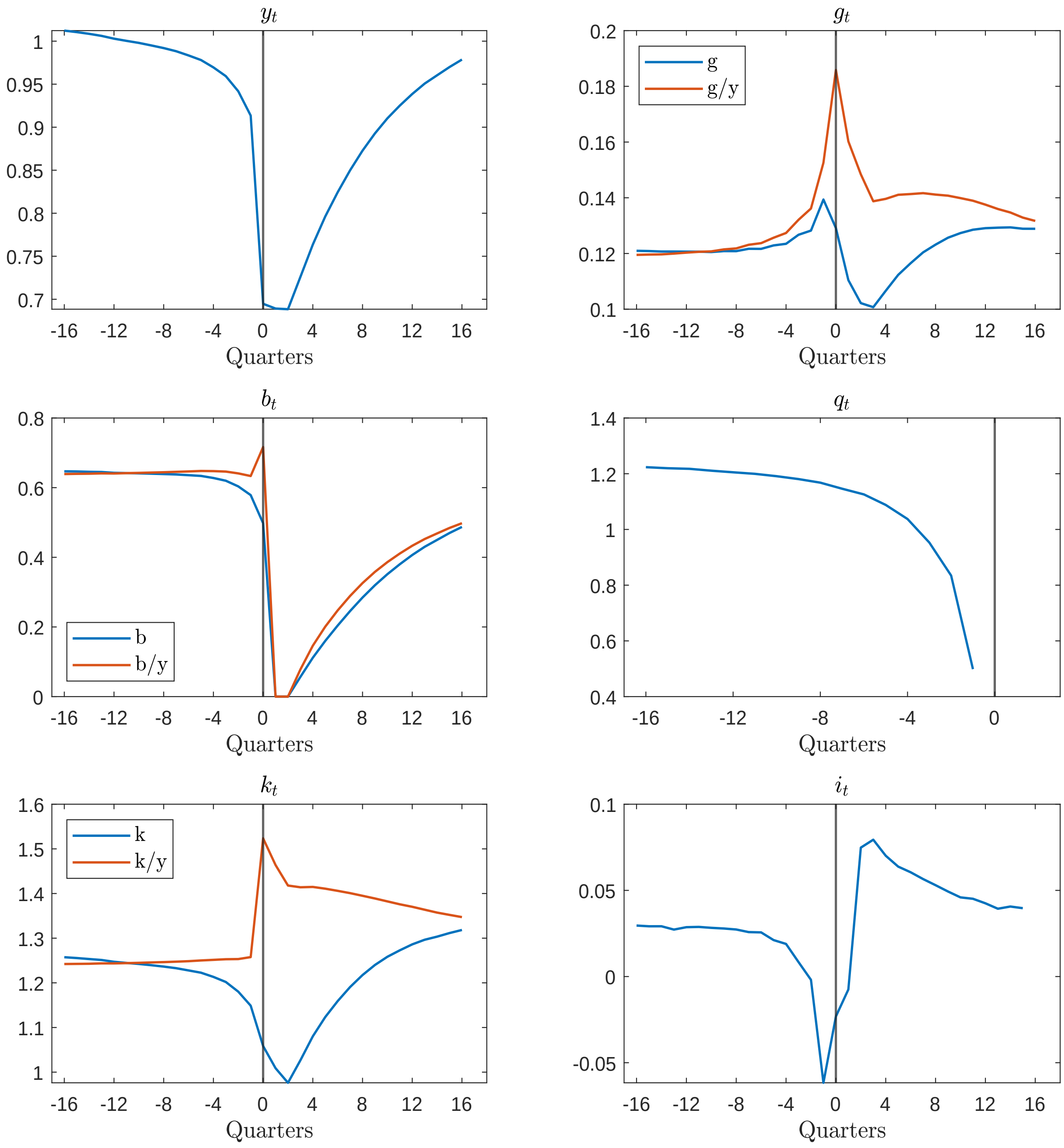
Such an insurance strategy appears to be flawed, as it privileges short-term insurance through unproductive expenditures at the expense of public investment. To better understand the sources of these imperfections, and in particular the role of default risk, the next subsection studies a case where debt contracts are perfectly enforceable.

Figure 5: Response to shocks in the baseline model



Note: This figure shows the average response of the economy to three different productivity shock. The initial shocks considered are 3 standard deviation above and below, and 5 standard deviation below the unconditional mean. For each shock, initial conditions for debt and capital are set to their steady state values, and then 1000 simulations of 40 periods are run. Each panel represents the average reaction of a variable, over all simulations.

FIGURE 6: DEFAULT EPISODES IN THE BASELINE MODEL



Note: This figure represents the average dynamics around default episodes in the simulations of the baseline model. The sovereign defaults in $t = 0$. All the default occurrences in the simulations are isolated to create a sample of periods around default. Each panel represents the average value of a variable, over all default occurrences.

4.3 The cost of limited commitment

I analyse here an alternative version of the model where the government can fully commit to repaying its debt, that is, where there is no default risk. The objective is twofold. First, this exercise delivers further insights on how sovereign decisions are affected by the endogenous bond prices and how costly the lack of commitment is for the country. Furthermore, it provides a second benchmark to evaluate the impact of the golden rule on the economy. Comparing the results of the golden rule experiments with this model in addition to the baseline one allows to evaluate whether the rule can alleviate the cost of default risk by mimicking a situation with enforceable debt contracts.

As the government can fully commit to repaying its debt, the repayment set $\mathcal{R}(b,k)$ is equal to the entire support of the productivity shock Z. As a result, the bond prices are non-contingent on the state of the economy and are given by:

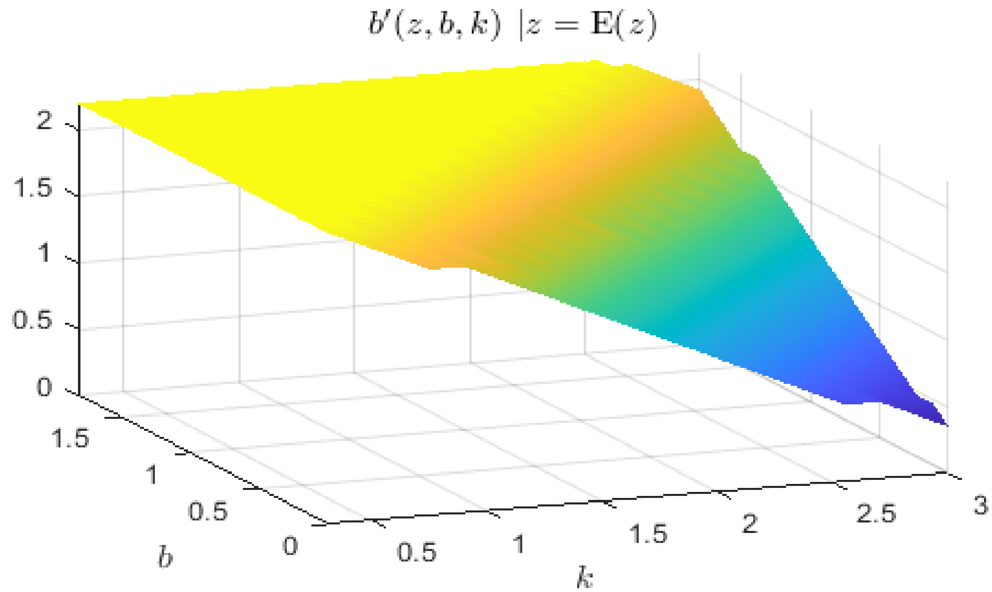
$$q = \frac{\lambda + (1-\lambda)\kappa}{r+\lambda} \tag{14}$$

Moreover, the recursive problem of the government is the one under the debt contract detailed in the equation (7) but with bond prices as defined in the equation (14). Denoting v^{fc} the value function of the sovereign with full commitment, its problem is given by:

$$\begin{aligned}
& v^{fc}(z,b,k) = \max_{b',k',g} \left(u(c,g,1-l) + \beta \int v^{fc}(z',b',k')p(z',z)dz' \right) \\
& s.t. \\
& (1+\tau)c = zF(k,l) \\
& \frac{u_l}{u_c} = \frac{zF_l(k,l)}{1+\tau} \\
& k' + g = \tau c + (1-\delta)k + [b' - (1-\lambda)b]\, q - [\lambda + (1-\lambda)\kappa]\, b
\end{aligned} \tag{15}$$

This problem is solved by using the same numerical approach and calibration as for the baseline model. The analysis of the bond policy under full commitment confirms the intuitions on borrowing decisions developed in Section 4.1. The figure 7 shows this policy function for a productivity shock set to its unconditional mean. As the default risk disappears, the price effects do so. There are no threshold levels of bonds or of capital that could trigger a deterioration of borrowing conditions. Only quantity effects are operant over the state space, meaning that the bond policy is increasing in the outstanding stock of bond and decreasing in the inherited stock of capital. Since there are no longer fluctuations in the borrowing conditions, the rollover risk vanishes. That is, under adverse shock, bond prices are unchanged

Figure 7: Bond policy under full commitment



Note: This figure shows the optimal bond policy in the model with full commitment, with the productivity shock set to its unconditional mean.

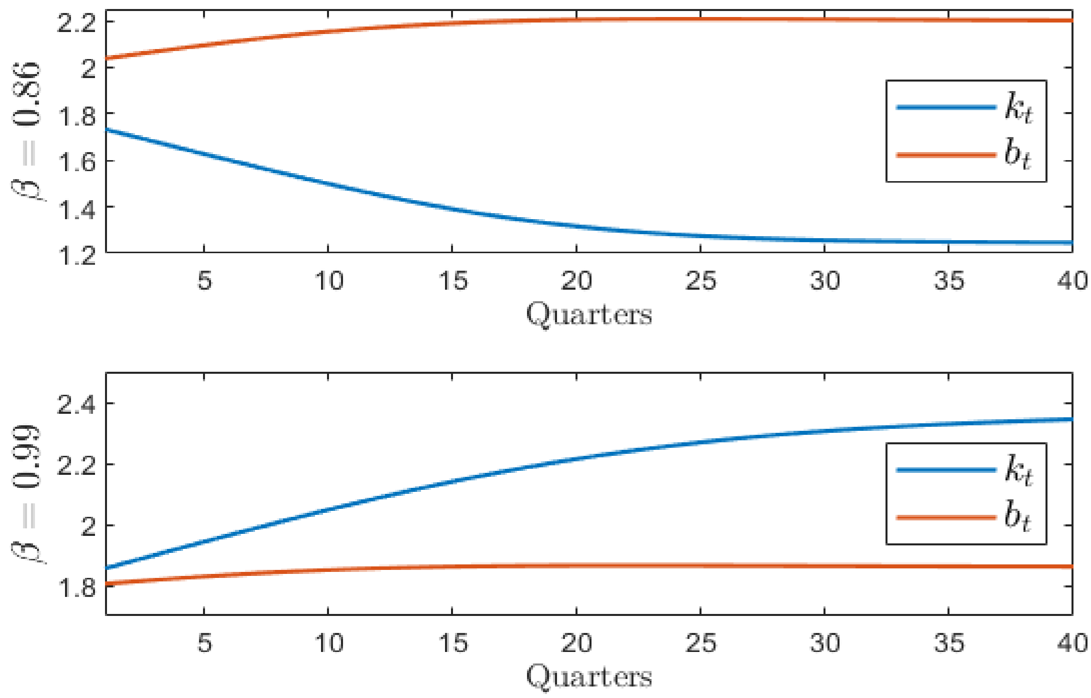
and so there is no difficulty for the government to issue new bonds to service its debt. Hence, there is no mechanism that limits the borrowing capacity of the government. Full commitment destroys the endogenous debt ceiling that was induced by the default risk through the rollover risk. Contrary to the baseline model, the exogenous debt ceiling assumed to rule out Ponzi schemes $b_t \leq \bar{b}$ can be binding at the equilibrium. This possibility should be viewed as the counterpart for the absence of default risk. A limited (and hence potentially binding) exogenous debt ceiling provides a rationale for the country to have access to a commitment device. The results of the simulations for the model with full commitment, shown in the Table 3 reveal that the exogenous debt ceiling is actually binding when the model is solved with the calibration used for the baseline. Because of the low discount factor and constant borrowing conditions, the government continuously increases its bond stock. While the exogenous debt ceiling is not reached, the additional bonds generate enough resources to both service the debt and launch public investments. Once it is reached, after 20 quarters on average (See the upper panel of Figure 8), the government permanently rollover its debt. Because of the high service of the debt induced, the government has less resources, which prompts it to reduce the size of its investments. As the stock of public capital depreciates, it reaches a smaller steady state level, 12% below the one obtained in the baseline model. One might conclude that providing a commitment device to an impatient country could lead it into an over-borrowing trap, with a detrimental effect on public capital accumulation and thus on output. It would suggest that the lack of commitment is actually beneficial for such countries as the induced default risk generates an endogenous debt ceiling that prevents from the over-borrowing trap. However, this conclusion holds only with extremely low values of β such as the one required by the baseline model to generate a consistent default frequency. This raises a question for the class of endogenous default models, whether they actually elicit a significant gap in discount factors between defaulters and non-defaulters countries or whether they fail to capture how default actually emerges? In the latter case, there is no rationale for using the baseline value of β in a model where no default can occur. Thus, I consider a second calibration for the discount factor, with a more standard value $\beta = 0.99$. The last row of the Table 3 shows the results of the simulations when the model with full commitment is solved using this calibration. In this case, the absence of default risk does not trigger an over-borrowing trap. On the contrary, this value of the discount factor implies a higher steady state level of capital than in the baseline model. Favourable and non-contingent borrowing conditions allow the government to launch large debt-financed investments to reach this level of capital. However, maintaining a higher capital stock requires more resources and thus to reduce the debt service, which decreases the steady state bond stock compared to the low discount factor case (see the lower panel of Figure 8). In a nutshell, this more reasonable calibration suggests that the lack of commitment is costly for the country as it weakens its economic fundamentals. Because of the default risk, bond prices fluctuates so borrowing yields unstable resources. Thus, the sovereign lacks source of funding to sustain a higher level of capital. Moreover, default risk creates incentives to use the capital stock as a control variable to adjust the borrowing conditions, which results into volatile public investments, whereas enforceable debt contracts leads to stable capital stock.

Table 3: Simulations full commitment model

Variable	E(y)	E(k)	E(b)	$\sigma(y)$	$\sigma(i)$	$\rho(g,y)$	$\rho(i,y)$	$\rho(b,y)$
Baseline								
$\beta = 0.86$	1.02	1.27	0.62	0.06	0.086	0.42	0.38	0.85
Full commitment								
$\beta = 0.86$	0.92	1.11	2.19	0.02	0.004	0.81	0.02	0.01
$\beta = 0.99$	1.28	1.92	1.89	0.02	0.003	0.97	0.004	0.001

Note: This table shows the average of the business cycle statistics over the simulations of the baseline model, the model with full commitment and $\beta = 0.86$, and with $\beta = 0.99$.

Figure 8: Debt and capital trajectories under full commitment



Note: This figure shows the average of the initial trajectories of debt and capital stocks over the simulations of the model with full commitment, with $\beta = 0.86$ in the upper panel and $\beta = 0.99$ in the lower panel.

It is important to note that even though the two different calibrations of β result in opposed conclusions in terms of capital, output and thus welfare, they generate the same interactions between the components of the government budget constraints. In both cases, the debt is used to invest and reach a stable level of public capital. Once this steady state level of capital is achieved, as there is no rollover risk, the sovereign sustains a constant bond stock which no longer generates resources. That is, the entire borrowing capacities of the country are used for investment, which requires unproductive expenditures to be tax financed. Hence, there is no margin to adjust g, which becomes fully pro-cyclical. The natural behaviour of the sovereign under full commitment is therefore reminiscent of the golden rule of public finance. It tends to silo its budget constraint. On the one hand, it launches initial debt-financed investments and then sustains stable and a-cyclical capital and bond stocks. On the other, it provides pro-cyclical tax-financed expenditures. Therefore, the conclusions drawn from this benchmark full commitment case will be valuable for the investigation of the golden rule. Even though they are not directly applicable to a country that may actually default on its debt, they highlight some of the potential costs and benefits of the rule. Namely, the shift toward a long-term insurance strategy ensures stable and, under reasonable values of β, high economic fundamentals. However, this is to the detriment of short-term insurance, as the government can no longer use debt to compensate households for adverse income shocks. A proper analysis of the golden rule remains necessary as its interaction with default risk may alter these latter conclusions.

5 The Golden Rule of Public Finance

5.1 Motivation and Specification

The golden rule consists in disaggregating the budget constraint of the government. It allows to finance public investments with debt but forbids public deficits for unproductive expenditures, which must be entirely financed by tax revenues. This rule grounds on two main motivations. First, as a fiscal rule, it aims at improving debt sustainability by constraining the ability of the government to run budget deficits. However, since Barro (1990) it is clearer that not all public spending are equal as they exert different effects on output. Thus, a balanced budget rule could turn to be detrimental for economic growth as it can refrain productive investment. The second objective of the golden rule is thus to support economic growth. Therefore, several countries like the United-Kingdom or Germany have allowed to run deficits if they are dedicated to the funding of public investment. This idea also finds support in the economic literature, as in Blanchard and Giavazzi (2004) who advocate for excluding public investment from deficit rules in the Stability and Growth Pact (SGP).

When defining the specification of the golden rule, the elephant in the room is the debt service. On the one hand, one could argue that this is unproductive expenditures and thus must be tax-financed. An alternative view of the rule is that net investment must be financed by net borrowing, which excludes the debt service from the budget balanced constraint. I consider a flexible specification that allows for these two cases and for intermediate ones. A share $1-\gamma$ of the debt service is tax-financed while the remaining share γ can be financed through additional debt. Hence, γ controls for the severity of the constraint. The higher the parameter, the less fiscal space is consumed by the debt service, and thus the less constrained g is. The golden rule constraint writes:

$$g \leq \tau c - (1-\gamma)[\lambda + (1-\lambda)\kappa]b \tag{16}$$

It is important to note that breaking the budget constraint of the government in that way induces an implicit constraint on the public debt:

$$b'.q(z,b',k') \leq i + \gamma[\lambda + (1-\lambda)\kappa]b \tag{17}$$

The golden rule imposes a ceiling on debt emissions, equal to the net investments plus the share of the debt service that can be debt-financed. Again, γ parametrizes the strength of the constraint as it lowers the debt ceiling.

To evaluate the effects of the golden rule I integrate this constraint (16) in the baseline model and solve it with the same calibration. I then repeat the same simulation exercise as the one conducted in Section 3.5. The results are presented and discussed in the following subsection.

5.2 Effects of the golden rule

The table 4 shows the results of the simulations with the golden rule, for three different values of $\gamma = \{0, 0.5, 1\}$. It reveals that the annual default frequency is significantly reduced by the golden rule, falling below 1% in the three scenarios. There are two reasons for this. First, the golden rule reduces default by constraining g in a pro-cyclical way, which prevents the sovereign to insure the households against lower tail risk. When output and tax revenues collapse, the constraint binds and public expenditures remain controlled, contrary to the baseline model. For instance, when $\gamma = 1$ the equation (16) implies a ceiling on the ratio of public expenditures to output equal to $\frac{\tau}{1+\tau} = 15.25\%$. In the baseline model, the average value of $\frac{g}{y}$ is 18.59% at the time of default and 15.26% one quarter before. Under the golden rule, the government can no longer provide such levels of expenditures. Hence, the rule reduces the extent to which the sovereign can smooth households' utility during bad times, the more adverse shocks trigger only limited increases in expenditures. This translates into enhanced pro-cyclicity of g, and a decreased average level of expenditures to output ratio. The corollary is that the government is less likely to be on the path to default, which, as shown in Figure 6, typically involves highly counter-cyclical expenditures prior to repudiation.

Moreover, the golden rule reduces default by prompting the sovereign to sustain a higher stock of public capital. To understand how, one can observe in the Figure 9 that the rule reshapes the repayment set and thus the bond price function. As the public debt is now backed by public investments, bond prices become more sensible to the capital stock. Indeed, debt issuance does not affect the public expenditures so the default incentives become less

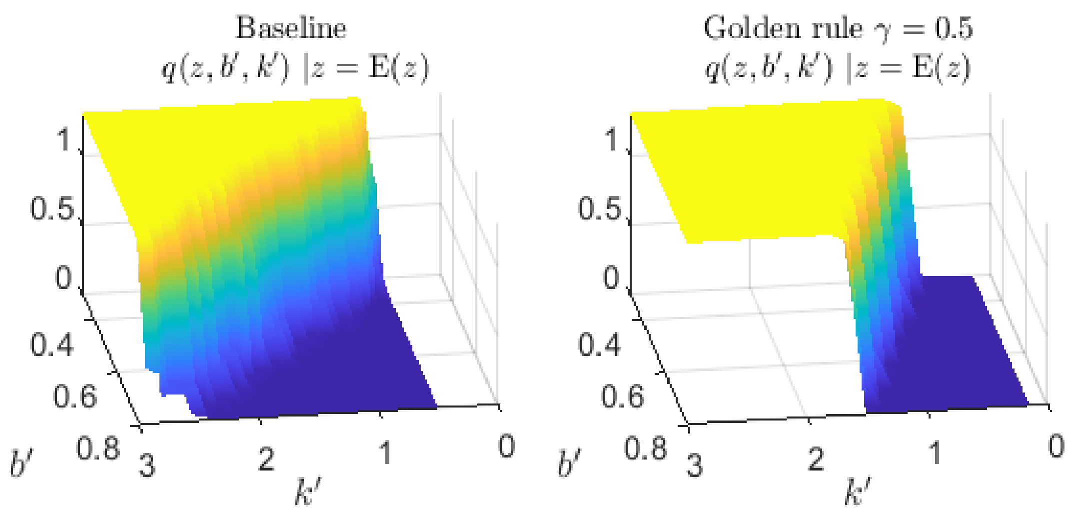
Table 4: Simulations golden rule experiments

Variable	Baseline	$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0$
E(y)	1.02	1.38 (+35%)	1.27 (+24%)	1.20 (+17%)
E(k)	1.27	2.01 (+57%)	1.77 (+38%)	1.65 (+28%)
E(l)	0.66	0.81 (+23%)	0.77 (+16%)	0.74 (+11%)
E(b)	0.62	1.01 (+61%)	0.81 (+30%)	0.62 (+00%)
E(b/y)	0.61	0.73 (+19%)	0.64 (+04%)	0.52 (-15%)
E(g)	0.12	0.15 (+21%)	0.14 (+13%)	0.13 (+09%)
E(g/y)	0.12	0.10 (-10%)	0.10 (-12%)	0.11 (-07%)
E(c)	0.86	1.18 (+35%)	1.08 (+24%)	1.02 (+17%)
E(u)	-4.05	-3.11 (+23%)	-3.36 (+17%)	-3.60 (+10%)
E($default$)	3.09%	0.75% (-2.34pp)	0.26% (-2.83pp)	0.05% (-3.04pp)
E($spread$)	4.24%	0.93% (-3.31pp)	0.28% (-3.96pp)	0.06% (-4.18pp)
$\sigma(y)$	0.064	0.053 (-16%)	0.053 (-17%)	0.033 (-47%)
$\sigma(i)$	0.086	0.077 (-11%)	0.068 (-21%)	0.016 (-81%)
$\sigma(b)$	0.115	0.092 (-20%)	0.067 (-41%)	0.015 (-86%)
$\sigma(g)$	0.021	0.012 (-43%)	0.007 (-64%)	0.004 (-78%)
$\sigma(c)$	0.066	0.051 (-23%)	0.047 (-29%)	0.027 (-58%)
$\sigma(spread)$	0.130	0.046 (-64%)	0.003 (-97%)	0.001 (-98%)
$\rho(g, y)$	0.42	0.76 (+80%)	0.78 (+84%)	0.97 (+130%)
$\rho(b, y)$	0.85	0.72 (-16%)	0.80 (-05%)	0.38 (-55%)
$\rho(i, y)$	0.38	0.33 (-12%)	0.38 (-0.5%)	0.17 (-54%)

Note: This table shows the average of the business cycle statistics over the simulations of the baseline model and the model with the golden rule, for three different values of γ. The percentage changes are computed taking into account all the decimal digits.

sensitive to borrowings. In particular, in low capital states, the sovereign can no longer resort to public debt to enhance g so default becomes a more appealing instrument. Conversely, in high capital states, the default is less attractive as it reduces the resources needed to sustain a large capital stock, given that tax revenues are devoted to public expenditures. As the bond price function is steep and now more sensitive to capital than borrowings, the sovereign has to sustain a higher capital stock than in the baseline model to avoid the states where bond prices are worthless. In return, as the bond price effect of debt is now weaker than the one of capital, the sovereign can sustain a larger stock of bonds. With this new geometry of the repayment set, higher stock of debt and capital yields a reduced default frequency.

FIGURE 9: BOND PRICE FUNCTIONS, BASELINE VS GOLDEN RULE



Note: This figure shows the bond price functions in the baseline model without the rule (left panel) and under the golden rule with $\gamma = 0.5$ (right panel), with a productivity shock set to its unconditional mean.

A fundamental effect of the golden rule is that it dampens the economic fluctuations. A first reason for this is the reduction of the default frequency. Avoiding periods of autarky and the subsequent default costs decreases the drop in output and so reduces the amplitude of the fluctuations. However, there are deeper reasons for the stabilizing effect of the rule. As it limits the room for counter-cyclical fiscal policy, the golden rule cuts incentives to use capital liquidation as a financing instrument for insuring households during recessions. Moreover, as it reduces the default risk, the golden rule improves borrowing conditions and thus reduces incentives to use public investment to adjust bond prices. For these two reasons, the volatility of public investment is reduced compared to the baseline model.

This has two important implications. First, with more stable expenditures, the sovereign has less needs to adjust its resources, which results in smoother borrowing policy. Secondly, smoother investment path yields reduced output fluctuations. This is critically important as this leads to smoother private and public consumption paths. Thus, even though the sovereign can no longer insure households with a counter-cyclical fiscal policy, it achieves a better insurance strategy by smoothing its investments. In other words, the golden rule pushes forward the horizon of the public insurance strategy, which corrects its imperfections mentioned in the previous sections.

Therefore, another key finding that emerges from the different simulations is the welfare gain induced by the rule, which ranges from 10% to 23% as compared to the baseline model. At first glance, one could argue that this is the result of decreased default frequency. To discuss this idea, I exclude all autarky periods from the sample generated by the baseline model and find that welfare is increased only by 2.64%. This suggest that the welfare gains that can be imputed to the avoidance of default periods are small. The main effect is the one of public capital, which by being higher and smoother, increases households' utility.

To understand the difference in the results depending on the value of γ one may recall that this parameter controls for the degree of flexibility left to the government under the golden rule. The lower the parameter, the more stringent the ceiling on g and so the less the sovereign can provide counter-cyclical expenditures during bad times, reducing possibilities to drift on the path to default. As γ decreases, more debt service enters the constraint on expenditures, which reduces the possibility for the sovereign to increase its borrowing. In the most stringent case where $\gamma = 0$ the average level of debt is comparable to the one of the baseline model. On the contrary, a slack constraint allows to sustain a higher level of debt. This affects the steady state level of public capital through bond price effects. Indeed, larger debt can be sustainable only at the condition of a higher stock of public capital. Therefore, as the rule gets slacker, the economy moves toward an equilibrium with more debt and capital.

Although the golden rule globally exerts a stabilizing effect on the economy, it differs depending on the values of γ. When the constraint imposed is stringent and often binding, the margins for adjustment of the sovereign are limited, which results in smoother fluctuations in investment, borrowing, and expenditures decisions. Moreover, as the default is less frequent under a severe rule, spreads volatility vanishes as γ decreases. On the contrary, when the constraint is slack the sovereign can afford a bit of counter-cyclicity in its fiscal policy, which reduces the stabilizing effect of the rule.

In a nutshell, the benefits of the rule differ depending on its severity. On the one hand, its slacker version yields a higher capital stock and output. This results into enhanced welfare as compared to the more severe case. On the other hand, a stringent rule is more efficient to reduce fluctuations and to secure the repayment chances for the foreign lenders.

The comparison of the golden rule with a situation where debt contracts are perfectly enforceable as the one studied in Section 4.3 provides two main insights. First, the golden rule yields an insurance strategy analogous to the one obtained under full commitment. As it prevents against counter-cyclical fiscal policy, it compels the sovereign to adopt a long-term perspective for its insurance strategy. That is, to smooth households' utility through stable investments, which dampen economic fluctuations. Therefore, the golden rule alleviates the cost of limited commitment as it curbs its adverse effects on the sovereign's insurance strategy.

However, contrary to the model with full commitment, the default risk is still present under the golden rule, so the bond price effects are still operating. Thus, the two situations differ in the way they shape investment and borrowing decisions. With enforceable debt contracts and high discount factor, the access to non-contingent resources allows the sovereign to reach a high steady-state level of capital. Under the golden rule, the driving effect differs, the sovereign is prompted to sustain a large stock of public capital to avoid the regions of the state space where borrowing conditions are deteriorated by the default risk. This highlights a strength of the golden rule: it takes advantage of the default risk to recast the sovereign's decisions through bond price effects. In other words, the efficiency of the golden rule might rely on the presence of default risk.

Contrasting the golden rule with the less plausible model with full commitment and low discount factor is also instructive. It reveals that the golden rule safeguards from an over-borrowing trap. As price effects are still applicable, the golden rule still features an endogenous debt ceiling above which borrowing bond prices collapse.

Overall, this comparison yields two lessons: the golden rule takes advantage of the risk of default while correcting the imperfections that it induces on the government's insurance strategy.

5.3 Comparison with debt ceiling rule

To give perspective on the insights drawn on the golden rule, I propose to contrast its effect with the one of a debt ceiling rule. Such rules are often considered to enhance the sustainability of public debts. Nevertheless, they appear are less appealing as they do not account for their spillover on the production side of the economy. I apply the same methodology as for the golden rule, but considering a simple debt ceiling rule:

$$\frac{b}{y} \leq \Gamma \tag{18}$$

Table 5: Simulations debt ceiling experiments

Variable	Baseline	$\Gamma = 0.3$	$\Gamma = 0.4$	$\Gamma = 0.5$
$\mathrm{E}(y)$	1.02	0.65 (-35%)	0.77 (-24%)	0.82 (-18%)
$\mathrm{E}(k)$	1.27	0.64 (-49%)	0.83 (-34%)	0.92 (-27%)
$\mathrm{E}(l)$	0.66	0.48 (-26%)	0.54 (-17%)	0.57 (-13%)
$\mathrm{E}(b)$	0.62	0.16 (-74%)	0.26 (-57%)	0.35 (-43%)
$\mathrm{E}(b/y)$	0.61	0.24 (-59%)	0.34 (-43%)	0.43 (-30%)
$\mathrm{E}(g)$	0.12	0.08 (-33%)	0.09 (-23%)	0.09 (-18%)
$\mathrm{E}(g/y)$	0.12	0.12 (+04%)	0.12 (+01%)	0.12 (+00%)
$\mathrm{E}(c)$	0.86	0.55 (-35%)	0.65 (-24%)	0.70 (-18%)
$\mathrm{E}(u)$	-4.05	-6.27 (-54%)	-5.52 (-36%)	-4.94 (-21%)
$\mathrm{E}(default)$	3.09%	1.33% (-1.76pp)	1.36% (-1.72pp)	1.77% (-1.32pp)
$\mathrm{E}(spread)$	4.24%	1.89% (-2.35pp)	2.40% (-1.84pp)	3.04% (-1.20pp)
$\sigma(y)$	0.064	0.020 (-68%)	0.027 (-57%)	0.037 (-41%)
$\sigma(i)$	0.086	0.011 (-87%)	0.019 (-77%)	0.032 (-62%)
$\sigma(b)$	0.115	0.018 (-83%)	0.033 (-71%)	0.052 (-54%)
$\sigma(g)$	0.021	0.007 (-63%)	0.010 (-50%)	0.013 (-36%)
$\sigma(c)$	0.066	0.020 (-69%)	0.028 (-56%)	0.039 (-40%)
$\sigma(spread)$	0.130	0.08 (-38%)	0.12 (-01%)	0.15 (+16%)
$\rho(g, y)$	0.42	0.45 (+06%)	0.54 (+28%)	0.50 (+19%)
$\rho(b, y)$	0.85	0.52 (-38%)	0.61 (-28%)	0.76 (-10%)
$\rho(i, y)$	0.38	0.22 (-40%)	0.26 (-32%)	0.32 (-15%)

Note: This table shows the average of the business cycle statistics over the simulations of the baseline model and the model with the debt ceiling rule, for three different values of Γ. The percentage changes are computed taking into account all the decimal digits.

The results of the simulations shown in the table 5 suggest that a debt ceiling rule can be harmful for the economic fundamentals of the country. It reduces capital accumulation and output compared to the baseline model (and a fortiori to the golden rule). This highlights a fundamental difference with the golden rule. Whereas the debt ceiling induced by the golden rule is contingent on investment choices, here it relies only on the inherited stock of capital. Thus, even if the government launches large investments, the debt ceiling will remain the same. It does not take into account the increase in bond prices induced by these investments. As a result, in low output states, a debt ceiling rule constrains the sovereign to abandon debt-financed investments, even though they would have enhanced borrowing conditions and debt sustainability. The debt ceiling imposes a constraint based on previous sovereign choices, while the sustainability of the debt is a forward-looking object, which depends on the decisions of the sovereign for the future. The chances of repayment in the future depend on tomorrow's output, not today's. Thus, the golden rule is more efficient in reducing the default frequency and the spreads than a debt ceiling.

The direct consequence is that the debt ceiling rule deteriorates public capital accumulation. It reduces sources of financing when the sovereign needs them to support the country's development. Thus, it cannot reach its full capital potential compared to the baseline model. This results in lower output and consumption. Even though a debt ceiling rule has a dampening effect on economic fluctuations, which leads to a smoother path for households' consumption, it has negative welfare effects as it brakes the country's development.

Another difference between a debt ceiling and the golden rule is the effect they exert on the default frequency. Although they are both effective in reducing it, it is only divided by two with a debt ceiling, while it is at least divided by four with a golden rule. This is because the effects they exert on the default frequency go through different channels. The golden rule reduces default because it yields a higher capital accumulation and prevents the sovereign to insure the households against lower tail risk through counter-cyclical fiscal policy. A debt ceiling rule, however, exerts no constraint on g so the sovereign is still able to insure the households. As a result, the pro-cyclicity of expenditures $\rho(g, y)$ is only slightly increased compared to the baseline model, whereas it was strongly enhanced by the golden rule. With a debt ceiling rule, the decrease in the default frequency stems from the constrained debt-to-output ratio. Constraining public expenditures through the golden rule is more efficient in reducing the default frequency than capping the debt-to-output ratio. This suggests that what matters most for sovereign debt sustainability is how it is used, not is level.

In contrast to a debt ceiling, the golden rule constitutes a growth-oriented and forward-looking rule, which supports debt sustainability and the country's development.

6 Conclusion

This thesis has presented an endogenous default model with public capital accumulation to evaluate the effects of the golden rule of public finance in an emerging market economy. The model features rich mechanisms through which the golden rule can yield improvement. In particular, the comparison with a situation where debt contracts are enforceable reveals that the default risk is detrimental for the insurance strategy of the sovereign. Through bond prices, the default risk creates a motive for the sovereign to use its investment as a control variable on its resources. This induces large output fluctuations. To compensate their detrimental effect on the households' welfare, the sovereign has to conduct counter-cyclical fiscal policies, which may eventually lead to default. In addition, the pro-cyclicity of the value of the bond stock yields an insufficient source of funding which prevents the country to reach a higher level of public capital and thus of output.

The results of this thesis suggest that the golden rule can deliver a significant improvement. Especially, it induces a shift in the horizon of the sovereign's insurance strategy by preventing it to conduct counter-cyclical fiscal policies. Moreover, it takes advantage of the bond price effects to prompt the sovereign to sustain a higher and more stable stock of public capital. This dampens economic fluctuations and achieves a smoother consumption path. This also improves the debt sustainability as it reduces the spreads and the default frequency. Hence, the golden rule yields significant welfare gains.

The golden rule alleviates the cost of limited commitment as it mimics a situation with enforceable debt contracts, where households are insured through high and stable public investments rather than counter-cyclical expenditures. Moreover, it imposes a time-consistent partition of the government's budget constraint. It separates on the one hand the static elements, i.e., tax revenues and public expenditure, and on the other hand the forward-looking elements: debt and public investment. Consequently, the sovereign can benefit from enhanced borrowing conditions when it launches large public investments, irrespectively of its previous decisions. This reduces more the default frequency and the spreads than a simple debt ceiling rule, which can hinder capital accumulation as it prevents to launch large debt-financed investments in low output states.

The most important limitation of this thesis lies in the absence of explicit private capital accumulation. In particular, the simplifying assumption made on the interactions between private and public capital might lead to an overstatement of the effect of public investment on output. Thus, the extent to which the dampening effect that the golden rule exerts on public investment transmits to the production should be qualified. However, this is not likely to change the welfare effects of the rule. Even in an extreme case where public investment does not affect output volatility, the households would still benefit from the increase in the production level. Moreover, such a case would be difficult to rationalize as, on the contrary, public investment is likely to trigger reactions in the private capital stock (e.g., a new highway can induce firms to transfer their buildings). Therefore, extending the model to account for private capital accumulation could help to refine the effect of the golden rule on economic fluctuations.

Another promising avenue for further research is to consider a more sophisticated stochastic process. In this model, the economy faces only transitory shocks. However, as shown by Aguiar and Gopinath (2006), a shift in the growth regime has more effect on the default risk than transitory shocks, so including trend shocks reduces the sensitivity of the bond price function to the sovereign's decision. Therefore, a richer stochastic structure could reduce the gap between the baseline model and the one with the golden rule. Weaker bond price effects create less incentives for the sovereign to use public investment as a way to adjust resources and thus yield a more stable stock of public capital. This could qualify the benefits of the golden rule. Moreover, the insights drawn from the endogenous growth literature advocates for using debt when a transitory shock hits the economy, and using tax for permanent shocks. The golden rule may conflicts with such a policy. Thus, including permanent shock to the model would allow to compare this policy with the golden rule. However, this approach would require to expand the state space of the model over a fourth dimension and thus induces significant computational cost.

References

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of international Economics 69*(1), 64–83.

Alfaro, L. and F. Kanczuk (2017). Fiscal rules and sovereign default. Technical report, National Bureau of Economic Research.

Alonso-Ortiz, J., E. Colla, and J.-M. Da-Rocha (2017). The productivity cost of sovereign default: evidence from the european debt crisis. *Economic Theory 64*(4), 611–633.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review 98*(3), 690–712.

Asonuma, T. and H. Joo (2020). Sovereign debt overhang, expenditure composition and debt restructurings. *Expenditure Composition and Debt Restructurings (August 20, 2020)*.

Barro, R. J. (1990). Government spending in a simple model of endogeneous growth. *Journal of political economy 98*(5, Part 2), S103–S125.

Blanchard, O. J. and F. Giavazzi (2004). Improving the sgp through a proper accounting of public investment. *Available at SSRN 508203*.

Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review 102*(6), 2674–99.

Cuadra, G., J. M. Sanchez, and H. Sapriza (2010). Fiscal policy and default risk in emerging markets. *Review of Economic Dynamics 13*(2), 452–469.

Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies 48*(2), 289–309.

Gelos, R. G., R. Sahay, and G. Sandleris (2011). Sovereign borrowing by developing countries: What determines market access? *Journal of international Economics 83*(2), 243–254.

Gordon, G. and P. A. Guerron-Quintana (2018). Dynamics of investment, debt, and default. *Review of Economic Dynamics 28*, 71–95.

Greenwood, J., Z. Hercowitz, and G. W. Huffman (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, 402–417.

Groneck, M. (2010). A golden rule of public finance or a fixed deficit regime?: Growth and welfare effects of budget rules. *Economic Modelling 27*(2), 523–534.

Hatchondo, J. C., M. L. Martinez, and M. F. Roch (2012). *Fiscal rules and the sovereign default premium*. International Monetary Fund.

Kaas, L., J. Mellert, and A. Scholl (2020). Sovereign and private default risks over the business cycle. *Journal of International Economics 123*, 103293.

Minea, A. and P. Villieu (2009). Borrowing to finance public investment? the 'golden rule of public finance'reconsidered in an endogenous growth setting. *Fiscal Studies 30*(1), 103–133.

Novelli, A. C. and G. Barcia (2021). Sovereign risk, public investment and the fiscal policy stance. *Journal of Macroeconomics 67*, 103263.

Park, J. (2017). Sovereign default and capital accumulation. *Journal of International Economics 106*, 119–133.

Shvets, S. (2020). The golden rule of public finance under active monetary stance: endogenous setting for a developing economy.

Tauchen, G. and R. Hussey (1991). Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica: Journal of the Econometric Society*, 371–396.

Zeyneloglu, I. (2018). Fiscal policy effectiveness and the golden rule of public finance. *Central Bank Review 18*(3), 85–93.

Appendix

The codes to solve and analyse the baseline model and the different experiments are written in MATLAB. They are publicly available in the following repository: https://github.com/venance-riblier/Sovereign-default-and-the-golden-rule-of-public-finance.

The problem is solved by using value function iteration. It appears as the most adapted approach for endogenous default models. As the default decision is made by comparing the value of default and of staying in debt debt contract, it is necessary to compute the value of these two options for each point of the state space, which is done by the value function iteration method. The computational algorithm works as follows. Consider the risk-free bond prices as computed in equation (14) as initial guess. Then at each iteration, compute the value functions for each points of the state space, compare them to compute the repayment set and the bond price function. Iterate until convergence below a tolerance level $10e-6$.