

Krzysztof Ostaszewski: <http://www.krzysio.net>

Author of the BTDT Manual (the “Been There Done That!” manual) for Course P/1
<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

Instructor of online P/1 seminar: <http://smartURL.it/onlineactuary>

If you find these exercises valuable, please consider buying the manual or attending the seminar, and if you can't, please consider making a donation to the Actuarial Program at Illinois State University: <https://www.math.ilstu.edu/actuary/giving/>

Donations will be used for scholarships for actuarial students. Donations are tax-deductible to the extent allowed by law.

If you have questions about these exercises, please send them by e-mail to:

krzysio@krzysio.net

Exercise for February 21, 2009

November 2000 Course 1 Examination, Problem No. 32, also Study Note P-09-08, Problem No. 75

The monthly profit of Company I can be modeled by a continuous random variable with density function f . Company II has a monthly profit that is twice that of Company I. Determine the probability density function of the monthly profit of Company II.

- A. $\frac{1}{2}f\left(\frac{x}{2}\right)$ B. $f\left(\frac{x}{2}\right)$ C. $2f\left(\frac{x}{2}\right)$ D. $2f(x)$ E. $2f(2x)$

Solution.

Let X and Y be the monthly profits of Company I and Company II, respectively. Let us write f_X for the PDF of X , instead of just f . Then $Y = 2X$, so that $X = \frac{1}{2}Y$. Therefore

$$f_Y(y) = f_X(x(y)) \cdot \left| \frac{dx}{dy} \right| = f_X\left(\frac{1}{2}y\right) \cdot \frac{1}{2}.$$

You can, of course, also use the CDF approach. In this case

$$F_Y(y) = \Pr(Y \leq y) = \Pr(2X \leq y) = \Pr\left(X \leq \frac{y}{2}\right) = F_X\left(\frac{y}{2}\right),$$

and

$$f_Y(y) = F'_Y(y) = \frac{d}{dy} F_X\left(\frac{y}{2}\right) = \frac{1}{2} F'_X\left(\frac{y}{2}\right) = \frac{1}{2} f_X\left(\frac{y}{2}\right).$$

Answer A.

© Copyright 2004-2009 by Krzysztof Ostaszewski.

All rights reserved. Reproduction in whole or in part without express written permission from the author is strictly prohibited.

Exercises from the past actuarial examinations are copyrighted by the Society of Actuaries and/or Casualty Actuarial Society and are used here with permission.