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P Sample Exam Questions, Problem No. 153 and Dr. Ostaszewski's online exercise posted May 21, 2011

Let X represent the number of customers arriving during the morning hours and let Y represent the number of customers arriving during the afternoon hours at a diner. You are given:

i) X and Y are Poisson distributed.
ii) The first moment of X is less than the first moment of Y by 8.
iii) The second moment of X is 60% of the second moment of Y.
Calculate the variance of Y.

A. 4 B. 12 C. 16 D. 27 E. 35

Solution.
Recall that if a random variable N is Poisson, then $E(N) = \mathrm{Var}(N)$. Let us write $E(X) = \lambda_X$ and $E(Y) = \lambda_Y$. We are given that

$$\lambda_X = \lambda_Y - 8$$

and

$$E(X^2) = \mathrm{Var}(X) + (E(X))^2 = \lambda_X + \lambda_X^2 =$$
$$= 0.6E(Y^2) = 0.6\left(\mathrm{Var}(Y) + (E(Y))^2\right) = 0.6(\lambda_Y + \lambda_Y^2).$$

By substituting $\lambda_X = \lambda_Y - 8$ into the equation given by the relationship of the second moments, we obtain

$$\lambda_Y - 8 + (\lambda_Y - 8)^2 = 0.6(\lambda_Y + \lambda_Y^2).$$

This is a quadratic equation, which simplifies to

$$0.4\lambda_Y^2 - 15.6\lambda_Y + 56 = 0.$$

The two solutions of this quadratic equation are $\lambda_Y = 35$ and $\lambda_Y = 4$. They both appear feasible, but if we recall that $\lambda_X = \lambda_Y - 8$, we realize that $\lambda_Y = 4$ would result in $\lambda_X = -4$, an impossibility. We conclude that

$$\mathrm{Var}\left(Y\right)=\lambda_Y=35.$$

Answer E.