

Krzysztof Ostaszewski: <http://www.krzysio.net>

Author of the “Been There Done That!” manual for Course P/1

<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

Instructor of online P/1 seminar: <http://smartURL.it/onlineactuary>

If you find these exercises valuable, please consider buying the manual or attending the seminar, and if you can't, please consider making a donation to the Actuarial Program at Illinois State University: <https://www.math.ilstu.edu/actuary/giving/>. Donations are tax-deductible to the extent allowed by law.

If you have questions about these exercises, please send them by e-mail to:

krzysio@krzysio.net

P Sample Exam Questions, Problem No. 153 and Dr. Ostaszewski's online exercise posted May 21, 2011

Let X represent the number of customers arriving during the morning hours and let Y represent the number of customers arriving during the afternoon hours at a diner. You are given:

- i) X and Y are Poisson distributed.
 - ii) The first moment of X is less than the first moment of Y by 8.
 - iii) The second moment of X is 60% of the second moment of Y .
- Calculate the variance of Y .

A. 4 B. 12 C. 16 D. 27 E. 35

Solution.

Recall that if a random variable N is Poisson, then $E(N) = \text{Var}(N)$. Let us write

$E(X) = \lambda_X$ and $E(Y) = \lambda_Y$. We are given that

$$\lambda_X = \lambda_Y - 8$$

and

$$\begin{aligned} E(X^2) &= \text{Var}(X) + (E(X))^2 = \lambda_X + \lambda_X^2 = \\ &= 0.6E(Y^2) = 0.6(\text{Var}(Y) + (E(Y))^2) = 0.6(\lambda_Y + \lambda_Y^2). \end{aligned}$$

By substituting $\lambda_X = \lambda_Y - 8$ into the equation given by the relationship of the second moments, we obtain

$$\lambda_Y - 8 + (\lambda_Y - 8)^2 = 0.6(\lambda_Y + \lambda_Y^2).$$

This is a quadratic equation, which simplifies to

$$0.4\lambda_Y^2 - 15.6\lambda_Y + 56 = 0.$$

The two solutions of this quadratic equation are $\lambda_Y = 35$ and $\lambda_Y = 4$. They both appear feasible, but if we recall that $\lambda_X = \lambda_Y - 8$, we realize that $\lambda_Y = 4$ would result in $\lambda_X = -4$, an impossibility. We conclude that

$$\text{Var}(Y) = \lambda_Y = 35.$$

Answer E.

© Copyright 2011 by Krzysztof Ostaszewski.

All rights reserved. Reproduction in whole or in part without express written permission from the author is strictly prohibited.

Exercises from the past actuarial examinations are copyrighted by the Society of Actuaries and/or Casualty Actuarial Society and are used here with permission.