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If you find these exercises valuable, please consider buying the manual or attending the seminar, and if you can't, please consider making a donation to the Actuarial Program at Illinois State University: https://www.math.ilstu.edu/actuary/giving/
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Exercise for February 21, 2009

November 2000 Course 1 Examination, Problem No. 32, also Study Note P-09-08, Problem No. 75
The monthly profit of Company I can be modeled by a continuous random variable with density function *f*. Company II has a monthly profit that is twice that of Company I. Determine the probability density function of the monthly profit of Company II.

A. $\frac{1}{2}f\left(\frac{x}{2}\right)$ B. $f\left(\frac{x}{2}\right)$ C. $2f\left(\frac{x}{2}\right)$ D. $2f(x)$ E. $2f(2x)$

Solution.
Let X and Y be the monthly profits of Company I and Company II, respectively. Let us write f_X for the PDF of X, instead of just f. Then $Y = 2X$, so that $X = \frac{1}{2}Y$. Therefore

$$f_Y(y) = f_X(x(y)) \cdot \left|\frac{dx}{dy}\right| = f_X\left(\frac{1}{2}y\right) \cdot \frac{1}{2}.$$

You can, of course, also use the CDF approach. In this case

$$F_Y(y) = \Pr(Y \le y) = \Pr(2X \le y) = \Pr\left(X \le \frac{y}{2}\right) = F_X\left(\frac{y}{2}\right),$$

and

$$f_Y(y) = F_Y'(y) = \frac{d}{dy}F_X\left(\frac{y}{2}\right) = \frac{1}{2}F_X'\left(\frac{y}{2}\right) = \frac{1}{2}f_X\left(\frac{y}{2}\right).$$

Answer A.