HOMEWORK 11, ACTS 4306

For each problem, you need to choose a correct answer among 5 given answers.

1. The profit for a new product is given by $Z = 3X - Y - 5$. Let X and Y be independent random variables with $\text{Var}(X) = 1$ and $\text{Var}(Y) = 2$. What is the variance of Z?

[A] 1   [B] 5   [C] 7   [D] 11   [E] 16

2. The time, T, that a manufacturing system is out of operation has cumulative distribution function $F(t) = 1 - (2/t)^2$ for $t > 2$ and it is zero otherwise. The resulting cost to the company is $Y = T^2$. Determine the density function of Y for $y > 4$.

[A] $4/y^2$ [B] $8/y^{3/2}$ [C] $8/y^3$ [D] $16/y$ [E] $1024/y^5$

3. The monthly profit of Company I can be modeled by a continuous random variable with density function f. Company II has a monthly profit that is twice that of Company I. Determine the probability density function of the monthly profit of Company II.

[A] $(1/2)f(x/2)$ [B] $f(x/2)$ [C] $2f(x/2)$ [D] $2f(x)$ [E] $2f(2x)$

4. An actuary models the lifetime of a device using the random variable $Y = 10X^{.8}$ where X is an exponential random variable with mean 1 year. Determine the probability density function $f(y)$, for $y > 0$, of the random variable Y.

[A] $10y^{.8}e^{-y^{-.2}}$ [B] $8y^{-.2}e^{-10y^{.8}}$ [C] $8y^{-.2}e^{-(.1y)^{1.25}}$
[D] $(.1y)^{1.25}e^{-.125(.1y)^{.25}}$ [E] $.125(.1y)^{.25}e^{-(.1y)^{1.25}}$

5. A device containing two key components fails when, and only when, both components fail. The lifetimes, T_1 and T_2, of these components are independent with common density function $f(t) = e^{-t}$, $t > 0$. The cost, X, of operating the device until failure is $2T_1 + T_2$. Which of the following is the density function of X for $x > 0$?

[A] $e^{-x/2} - e^{-x}$ [B] $2(e^{-x/2} - e^{-x})$ [C] $x^2e^{-x}/2$ [D] $e^{-x/2}/2$ [E] $e^{-x/3}/3$

6. A company has two electric generators. The time until failure for each generator follows as exponential distribution with mean 10. The company will begin using the second generator immediately after the first one fails. What is the variance of the total time that the generators produce electricity?

[A] 10   [B] 20   [C] 50   [D] 100   [E] 200

7. A company offers earthquake insurance. Annual premiums are modeled by an exponential random variable with mean 2. Annual claims are modeled by an exponential random variable with a mean of 1. Premiums and claims are independent. Let X denote the ratio of claims to premiums. What is the density function of X?

[A] $(2x + 1)^{-1}$ [B] $2/(2x + 1)^2$ [C] e^{-x} [D] $2e^{-2x}$ [E] xe^{-x}

8. A charity receives 2025 contributions. Contributions are assumed to be independent and identically distributed with mean 3125 and standard deviation 250. Calculate the approximate 90th percentile for the distribution of the total contributions received.

[A] 6,328,000　　[B] 6,338,000　　[C] 6,343,000　　[D] 6,784,000　　[E] 6,977,000

9. An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by distinct policyholders are independent of one another. What is the probability that there is a total of between 2450 and 2600 claims during one-year period.

[A] .68　　[B] .82　　[C] .87　　[D] .95　　[E] 1

10. You are given the following information about N, the annual number of claims for a randomly selected insured: $P(N = 0) = 1/2$, $P(N = 1) = 1/3$ and $P(N > 1) = 1/6$. Let S denote the total annual claim amount for an insured. When $N = 1$, S is exponentially distributed with mean 5. When $N > 1$, S is exponentially distributed with mean 8. Find $P(4 < S < 8)$.

[A] .04　　[B] .08　　[C] .12　　[D] .24　　[E] .25

11. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$E(X) = 50, \ \ E(Y) = 20, \ \ \text{Var}(X) = 50, \ \ \text{Var}(Y) = 30, \ \ \text{Cov}(X, Y) = 10.$$

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these 100 people watch movies or sporting events during this three-month period. Approximate the value of $P(T < 7100)$.

[A] .62　　[B] .84　　[C] .87　　[D] .92　　[E] .97

12. The total claim amount for a health insurance policy follows a distribution with density function $f(x) = (1/1000)e^{-x/1000}$, for $x > 0$. The premium for the policy is set at 100 over the expected total claim amount. If 100 policies are sold, what is the approximate probability that the insurance company will have claims exceeding the premiums collected?

[A] .001　　[B] .159　　[C] .333　　[D] .407　　[E] .460

13. Claim amounts for wind damage to insured homes are independent random variables with common density function $f(x) = (3/x^4)I(x > 1)$, where x is the amount of claim in thousands. Suppose 3 such claims will be made. What is the expected value of the largest of the three claims?

[A] 2025　　[B] 2700　　[C] 3232　　[D] 3375　　[E] 4500

14. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is $f(x, y) = (x+y)/27$ for $0 < x < 3$ and $0 < y < 3$. Calculate the probability that the device fails during its first hour of operation.

[A] .04　　[B] .41　　[C] .44　　[D] .59　　[E] .96

15. X and Y are independent random variables with comnmon moment generating function $M(t) = e^{t^2/2}$. Let $W = X + Y$ and $Z = Y - X$. Determine the joint moment generating function $M(t_1, t_2)$ of W and Z.

[A] $e^{2t_1^2 + 2t_2^2}$ [B] $e^{(t_1 - t_2)^2}$ [C] $e^{(t_1 + t_2)^2}$ [D] $e^{2t_1 t_2}$ [E] $e^{t_1^2 + t_2^2}$