GROUP HOMEWORK
INTRODUCTORY FINANCIAL MATHEMATICS CLASS - SM141432

1st HOMEWORK

NOVITA PUSPITASARI
NRP 0611 1440000 011

VENANSIUS RYAN TJAHJONO
NRP 0611 1440000 043

YUSRIL IZZA FRIZNAINI
NRP 0611 1540000 048

WINDA FIRDIANA
NRP 0611 1440000 051

Lecturer:
Endah Rokhmati Merdika Putri, S.Si, M.T, Ph.D

DEPARTMENT OF MATHEMATICS
Faculty of Mathematics, Computing, and Data Science
Sepuluh Nopember Institute of Technology
Surabaya 2018

Questions

1. Taken from General Interest Rate, what is happening if $m \to \infty$? Clue: derive the equation $\lim_{m\to\infty} P_0\left(1+\frac{r}{m}\right)^{mn}$.

2. Suppose that you are to receive payments (in thousands of dollars) at the end of each of the next five years. Which of the following three payment sequences is preferable?

   A. 12, 14, 16, 18, 20

   B. 16, 16, 15, 15, 15

   C. 20, 16, 14, 12, 10

   Do your calculation for $r = 5\%$, $r = 10\%$, and $r = 15\%$. Answer it with detail explanation, as a Lender or as a Buyer.

3. A company needs a certain type of machine for the next five years. They presently own such a machine, which is now worth \$6,000 but will lose \$2,000 in value in each of the next three years, after which it will be worthless and unuseable. The (beginning-of-the-year) value of its yearly operating cost is \$9,000, with this amount expected to increase by \$2,000 in each subsequent year that it is used. A new machine can be purchased at the beginning of any year for a fixed cost of \$22,000. The lifetime of a new machine is six years, and its value decreases by \$3,000 in each of its first two years of use and then by \$4,000 in each following year. The operating cost of a new machine is \$6,000 in its first year, with an increase of \$1,000 in each subsequent year. If the interest rate is 10%, when should the company purchase a new machine?

Solutions

1. Will be derived the equation $\lim_{m\to\infty} P_0\left(1+\frac{r}{m}\right)^{mn}$ for knowing the Interest Rate formula for $m\to\infty$.

$$\begin{aligned}\lim_{m\to\infty} P_0\left(1+\frac{r}{m}\right)^{mn} &= P_0\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{mn}\\ &= P_0\lim_{m\to\infty}\left(1+\frac{1}{\frac{m}{r}}\right)^{mn}\\ &= P_0\left(\lim_{m\to\infty}\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{rn}\end{aligned}$$

Then, for any $0<r<1$, we know $p=\frac{m}{r}\to\infty$, then

$$P_0\left(\lim_{m\to\infty}\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{rn} = P_0\left(\lim_{p\to\infty}\left(1+\frac{1}{p}\right)^{p}\right)^{rn}$$

By using the Euler formula $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$, we can conclude that

$$\boxed{\lim_{m\to\infty} P_0\left(1+\frac{r}{m}\right)^{mn}=P_0e^{rn}}$$

2. Firstly, we have to find the present value based on the case. For doing calculation, we used Microsoft Excel. Using formula $PV=FV(1+r)^{-n}$ for $r=5\%$, $r=10\%$, and $r=15\%$

- for $r=5\%$

Present Value	**1**	**2**	**3**	**4**	**5**	**Total PV**
A	11.43	12.70	13.82	14.81	15.67	68.43
B	15.24	14.51	12.96	12.34	11.75	66.80
C	19.05	14.51	12.09	9.87	7.84	63.36

- for $r=10\%$

Present Value	**1**	**2**	**3**	**4**	**5**	**Total PV**
A	10.91	11.57	12.02	12.29	12.42	59.21
B	14.55	13.22	11.27	10.25	9.31	58.60
C	18.18	13.22	10.52	8.20	6.21	56.33

- for $r=15\%$

Present Value	**1**	**2**	**3**	**4**	**5**	**Total PV**
A	10.43	10.59	10.52	10.29	9.94	51.78
B	13.91	12.10	9.86	8.58	7.46	51.91
C	17.39	12.10	9.21	6.86	4.97	50.53

Secondly, based on the result, we can conclude :

- As a Lender, the best option is :
  a. For $r = 5\%$, take option **A**
  b. For $r = 10\%$, take option **A**
  c. For $r = 15\%$, take option **B**
- As a Buyer, the best option is :
  a. For $r = 5\%$, take option **C**
  b. For $r = 10\%$, take option **C**
  c. For $r = 15\%$, take option **C**

After doing some calculation, our group has found few interested things. Let's take a look for $r = 20\%$ and $r = 30\%$

- for $r = 20\%$

Present Value	**1**	**2**	**3**	**4**	**5**	**Total PV**
A	10.00	9.72	9.26	8.68	8.04	45.70
B	13.33	11.11	8.68	7.23	6.03	46.39
C	16.67	11.11	8.10	5.79	4.02	45.69

- for $r = 30\%$

Present Value	**1**	**2**	**3**	**4**	**5**	**Total PV**
A	9.23	8.28	7.28	6.30	5.39	36.49
B	12.31	9.47	6.83	5.25	4.04	37.89
C	15.38	9.47	6.37	4.20	2.69	38.12

Based on the table above, we can conclude :

- As a Lender, the best option is :
  a. For $r = 20\%$, take option **B**
  b. For $r = 30\%$, take option **C**
- As a Buyer, the best option is :
  a. For $r = 20\%$, take option **C**
  b. For $r = 30\%$, take option **A**

From all of the results we had, our group have hypothesis that **for larger** $r > 0$, the best option for a Lender is option C and the best option for a Buyer is option A.

3. Let's take a look for the case. It describes :

   - Old Machine
     a. Worth for \$6,000 and will lose in value \$2000 yearly, then worthless and unusable.
     b. Its operating cost is \$9,000 yearly and increases \$2,000 in each year that it is used.
   - New Machine
     a. Can be bought for \$22,000 in the beginning of any year.
     b. Has 6 years lifetime. Its value decreases by \$3,000 in each of first two years and \$4,000 in the next following year.
     c. Has operating cost \$6,000 in its first year and increases \$1,000 in each subsequent year.

   From the information above, **there are only four types of assumptions** because the old machine's life is three years starts from 0, 1, 2, 3. Afterwards, let's take a look for case 3 when the company bought the machine in the beginning of year 3.

   - First cash flows
     The company doesn't buy a new machine, but use its old machine, so we will have \$9,000 for its operating cost. **[\$9,000]**
   - Second cash flows
     In year 2, the company still operating its old machine, so we will have $\$9,000 + \$2,000$ for its operating cost. **[\$11,000]**
   - Third cash flows
     In year 3, the company decides to buy a new machine. It means that the old machine still worth for $\$6,000 - \$2,000 - \$2,000$ (The company sells the machine and get \$2,000). Knowing from the case, a new machine is purchased for \$22,000 and its operating cost is \$6,000. **[\$26,000]**
   - Fourth cash flows
     In year 4, the company use its new machine which they bought last year, so the company must pay $\$6,000 + \$1,000$ for its operating cost. In the other hand, its new machine will lose in value \$3,000 becomes \$19,000. **[\$7,000]**
   - Fifth cash flows
     In year 5, the company use its new machine which they bought last two year, so the company must pay $\$6,000 + \$2,000$ for its operating cost. In the other hand, its new machine will lose again in value \$3,000 becomes \$16,000. **[\$8,000]**
   - Sixth cash flows
     In year 6, the company decided to sell it's machine and its worth for $\$16,000 - \$4,000 = \$12,000$. So, the company will received \$12,000. **[-\$12,000]**

From the explanation above, we have the sequence *(in thousand dollars)* as written in the textbook $\{9, 11, 26, 7, 8, -12\}$. With the same way, we have the six-year cash flows for case 1, 2, and 4 as follows:

- Case 1: $\{22, 7, 8, 9, 10, -4\}$
- Case 2: $\{9, 24, 7, 8, 9, -8\}$
- Case 3: $\{9, 11, 26, 7, 8, -12\}$
- Case 4: $\{9, 11, 13, 28, 7, -16\}$

To analyze when the company should buy the machine, we use present analysis formula where $PV = FV(1+r)^{-n}$. By applying the formula to all sequences above and we will have the results in table as follows.

Present Value	**1**	**2**	**3**	**4**	**5**	**6**	**Total PV**
A	22.00	6.36	6.61	6.76	6.83	−2.48	46.083
B	9.00	21.82	5.79	6.01	6.15	−4.97	43.794
C	9.00	10.00	21.49	5.26	5.46	−7.45	43.760
D	9.00	10.00	10.74	21.04	4.78	−9.93	45.627

From the result, we have decided the company must buy a new machine at the beginning of year 3 *(two years from now)*.

Note: Excel file for calculation is included in the email.