Chapter 12

Gauss's Law

Electric Flux (Flux of an electric field)

Electric flux is to indicate the flow of electric field through a surface.

$$\Phi_E = \int E\cos\phi \, dA$$
$$= \int E_\perp dA$$
$$= \int \vec{E} \bullet d\vec{A}$$

Φ: Greek letter “phi”

The electric flux through an area = (the magnitude of electric field) x (area)

Surface integral of $E_\perp$ over A or of $\vec{E} \bullet d\vec{A}$

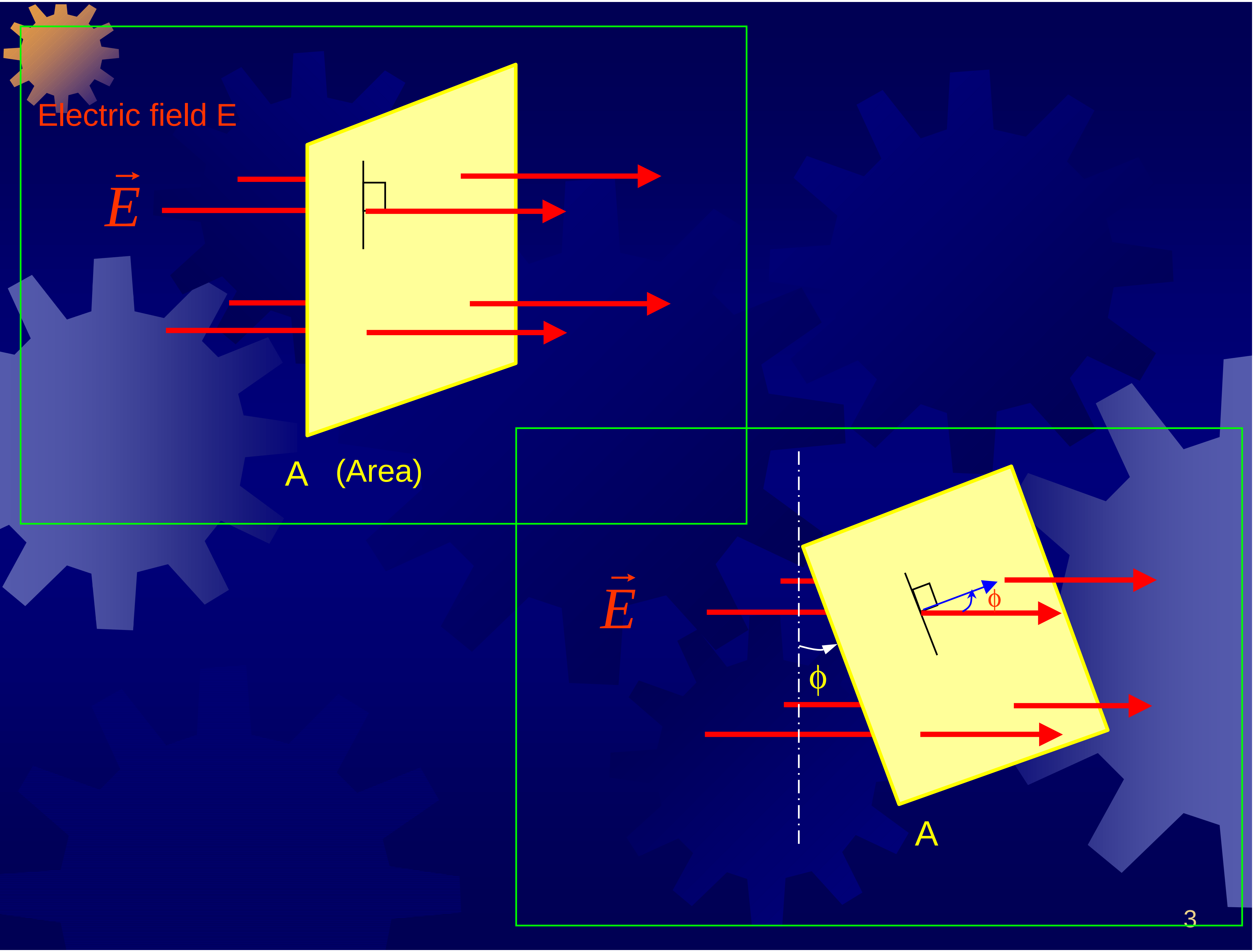
Where,

Φ_E : electric flux (Nm^2/C)

E : electric field (N/C)

$E_\perp$: perpendicular component of the electric field E

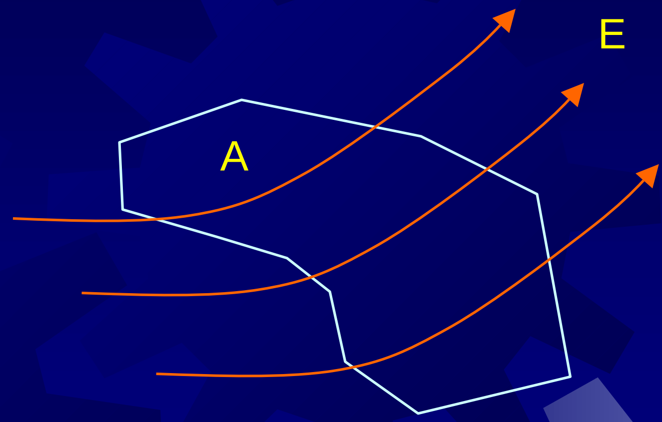
ϕ : the angle between the area vector and E



Electric field E
$\vec{E}$
A (Area)
$\vec{E}$
ϕ
ϕ
A

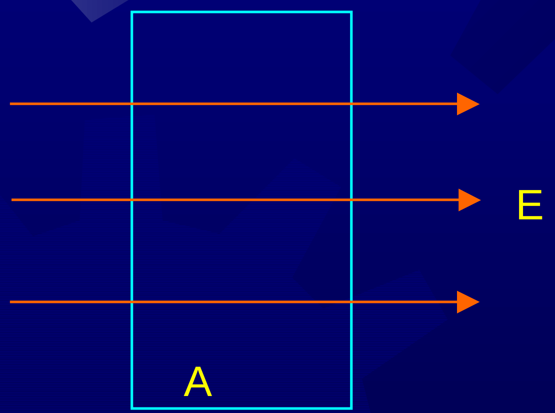
General form of electric flux: Electric flux can be non-uniform

$$\Phi_E = \sum_i \vec{E}_i \cdot d\vec{A}_i = \int \vec{E} \bullet d\vec{A}$$



Uniform electric flux over a flat surface:

$$\Phi_E = \vec{E} \bullet \vec{A}$$



$$\Phi_E = E_{\perp} A$$
$$= EA\cos\phi$$

Gauss's Law

Karl Friedrich Gauss (1777 – 1855)

Gauss's Law is to show the relationship between electric field and electric charge. It is equivalent to Coulomb's Law.

The net (total) electric flux through any closed surface is proportional to the total electric charge inside the surface.

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Surface integral

Total charge enclosed by the surface

Electric flux

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Constant (electrical permittivity)

Component of E normal to surface A

The closed surface is also called "Gaussian surface "

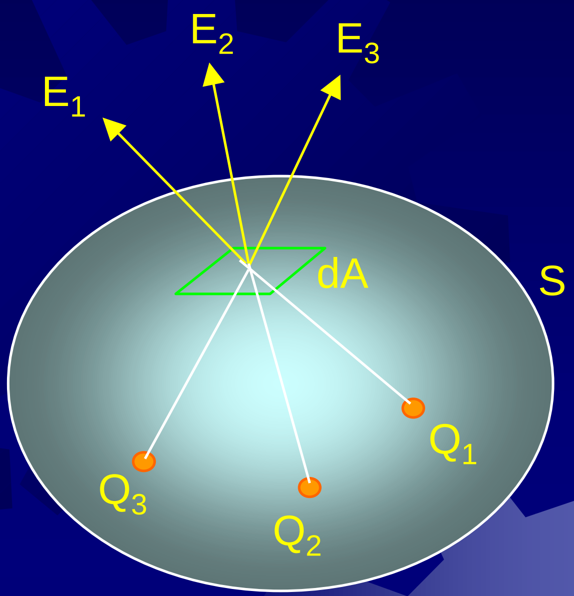
A surface (S) with a number of point charges below it:

$$E = \sum_{i=1}^{3} E_i$$

$$\oint_S E \bullet dA = \oint_S \sum_{i=1}^{3} E_i \bullet dA = \sum_{i=1}^{3} \oint_S E_i \bullet dA$$

$$= \sum_{i=1}^{3} \frac{Q_i}{\varepsilon_0}$$

$$= \frac{Q_{encl}}{\varepsilon_0}$$

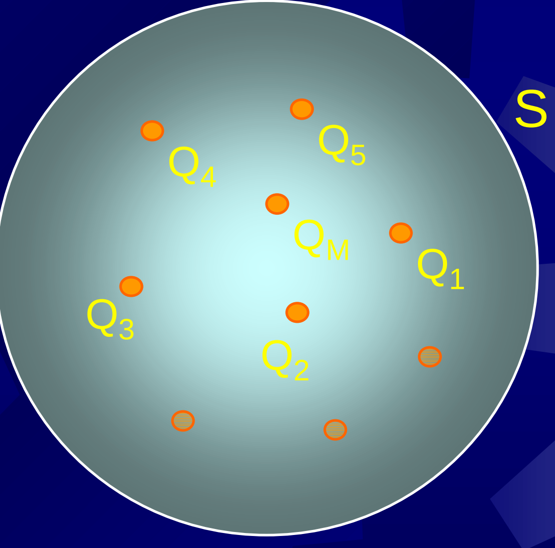


Principle of superposition

Total flux = sum of the flux from each charge

$$\Phi_E = \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0} + \ldots$$

$$\Phi_E = \sum_{i=1}^{M} \frac{Q_i}{\varepsilon_0}$$



Gauss's Law vs Coulomb's Law

Gauss's Law

$$\oint \vec{E} \bullet d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\varepsilon_0 \oint E dA = Q_{enc}$$

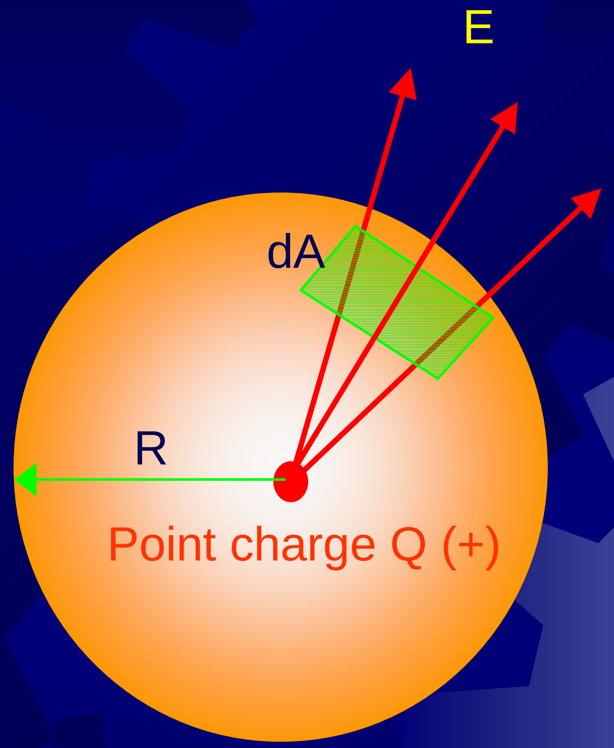
$$\varepsilon_0 E \left(4\pi R^2\right) = Q$$

surface area of a sphere

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}$$

Magnitude of electric field of a point charge

The flux is independent of R



A sphere

Coulomb's Law

$$F_0 = \frac{1}{4\pi\varepsilon_0} \frac{|QQ_0|}{r^2}$$

$$E = \frac{F_0}{Q_0}$$

When there is excess charge, it resides on the surface of conductor (at rest). The net (total) electric field in the material of the conductor is zero.

For a single point charge (Q), the magnitude of electric field (E) is:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

r is the distance from the point charge

For the charge Q on the surface of conductor with a radius R,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Outside the sphere where r > R

$$E = 0$$

Inside the sphere where r < R

Example 12.1: The cube has sides of length L = 10 cm. The electric field is uniform, has a magnitude E = 4 x 10^3 N/C, and is parallel to the xy-plane at an angle of 36.9^o measured from the +x-axis toward the +y-axis. a) What is the electric flux through each of the six cube faces S_1, S_2, S_3, S_4, S_5 and S_6. b) What is the total electric flux through all faces of the cube?

Solution: $(a) \quad \Phi = \vec{E}\cdot\vec{A} = EA\cos\theta \qquad where\ \vec{A} = A\hat{n}$

$\hat{n}_{S_1} = -\hat{j}$ (left) Φ_{S_1} = -(4 x 10^3 N/C)(0.1 m)^2 cos(90 - 36.9^o) = -24 N.m^2/C

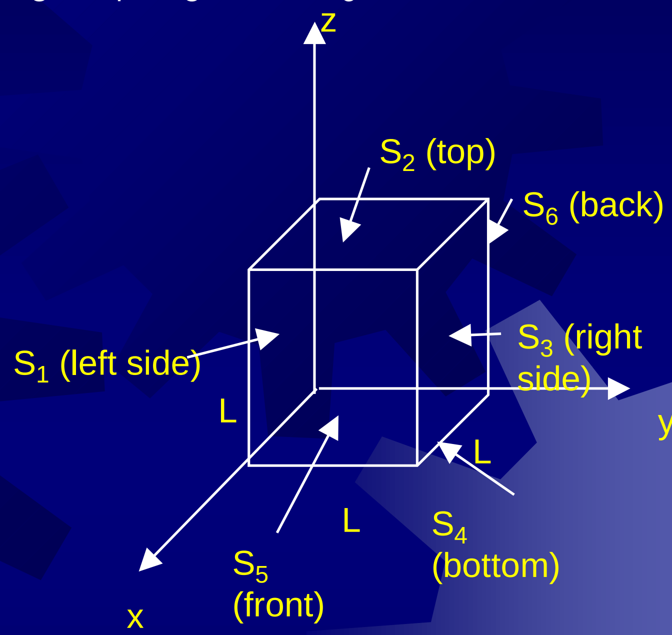
$\hat{n}_{S_2} = -\hat{k}$ (top) Φ_{S_2} = -(4 x 10^3 N/C)(0.1 m)^2 cos 90^o = 0

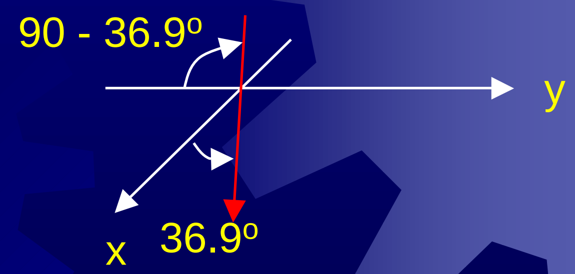
$\hat{n}_{S_3} = +\hat{j}$ (right) Φ_{S_3} = +(4 x 10^3 N/C)(0.1 m)^2 cos (90^o - 36.9^o) = +24 N.m^2/C

$\hat{n}_{S_4} = +\hat{k}$ (bottom) Φ_{S_4} = (4 x 10^3 N/C)(0.1 m)^2 cos 90^o = 0

$\hat{n}_{S_5} = +\hat{i}$ (front) Φ_{S_5} = +(4 x 10^3 N/C)(0.1 m)^2 cos 36.9^o = 32 N.m^2/C

$\hat{n}_{S_6} = -\hat{i}$ (back) Φ_{S_6} = -(4 x 10^3 N/C)(0.1 m)^2 cos 36.9^o = -32 N.m^2 /C





b) The total flux through the cube must be zero; any flux entering the cube must also leave it.

Example 12.2: A metal sphere of radius 0.45 m carries a net charge of 0.25 nC uniformly distributed on its surface. Find the magnitude of the electric field a) at a point 0.1 m outside the surface of the sphere; b) at a point inside the sphere, 0.1 m below the surface.

Solution:

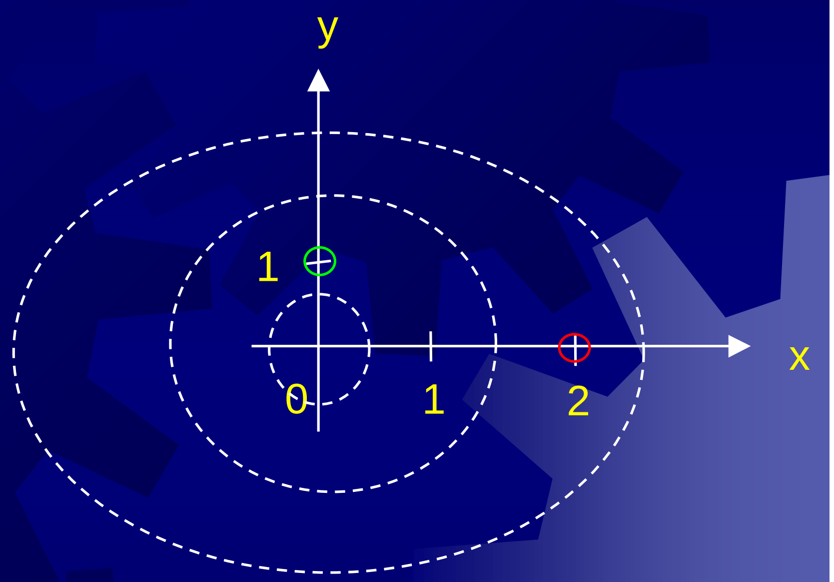
(a) $$E\,(r = 0.450\text{ m} + 0.1\text{ m}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(2.50\times 10^{-10}C)}{(0.550m)^2} = 7.44\,N/C.$$

(b) $\vec{E} = 0$ (inside the sphere)

inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges are not moving).

Example 12.3: A point charge q_1 = 4 nC is located on the x-axis at x = 2m, and a second point charge q_2 = -6 nC is on the y-axis at y = 1 m. What is the total electric flux due to these two point charges through a spherical surface centered at the origin and with radius a) 0.5m? b) 1.5m? c) 2.5m?

Solution:



(a) No charge enclosed so Φ = 0. (r = 0.5m < 1m or 2m where the charges are located.)

$$(b) \quad \Phi = \frac{q_2}{\varepsilon_0} = \frac{-6.00\times10^{-9}\,C}{8.85\times10^{-12}\,C^2/Nm^2} = -678\,Nm^2/C.$$

(Only q_2 is enclosed)

$$(c) \quad \Phi = \frac{q_1 - q_2}{\varepsilon_0} = \frac{(4.00-6.00)\times10^{-9}\,C}{8.85\times10^{-12}\,C^2/Nm^2} = -226\,Nm^2/C.$$

(Both q_1 and q_2 are enclosed)