**1st HOMEWORK**
**Mathematical Modeling Q Class - KM184701**

# AUTOWASHER SPRING VIBRATIONS MODELING AND ITS SIMULATION

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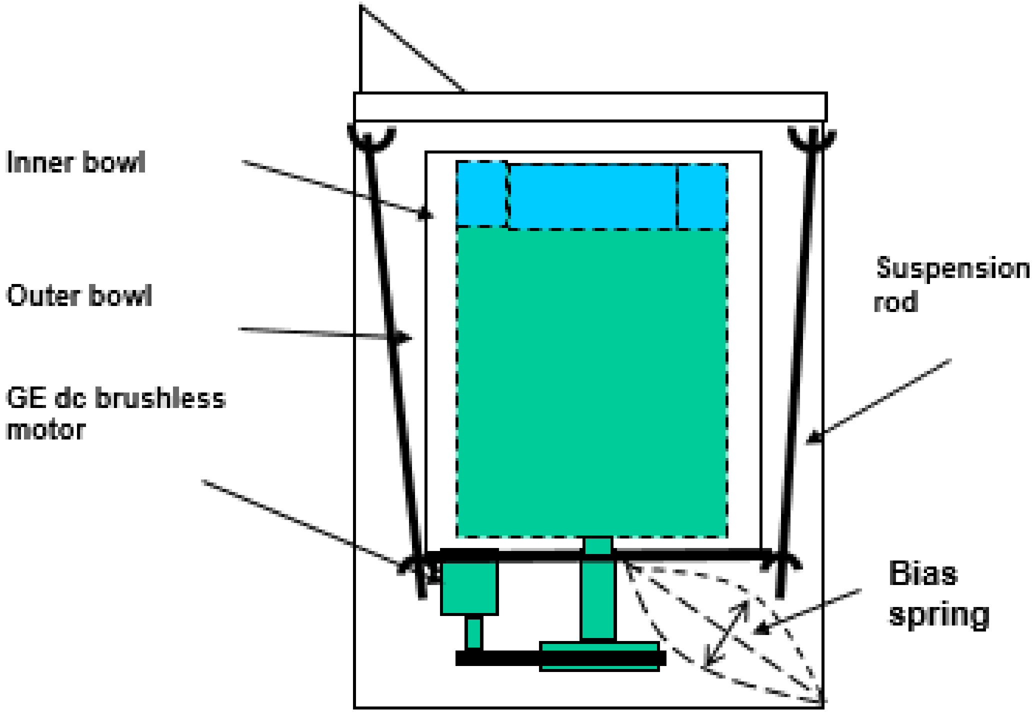
# Autowasher Spring Vibrations Problem

Bias spring resonance problems encountered during the development of the suspension for the F and P electronic autowasher "Gentle Annie" (1985). At 1100 rpm when the washing machine was in spin mode the bias spring would vibrate wildly and make contact with the pulley.




Schematic of autowasher

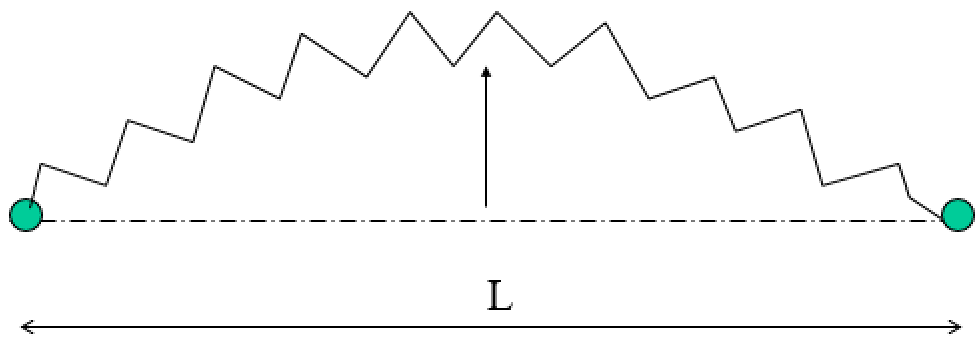
# Part I
# Modeling the Problem

The spring was long and slender like an elastic string. I recalled something I had learned in Engineering Mathematics II (The Predecessor of MM3):

**How to derive the equation governing small transverse vibrations of an elastic string that is stretched to length L and then fixed at its endpoints.**

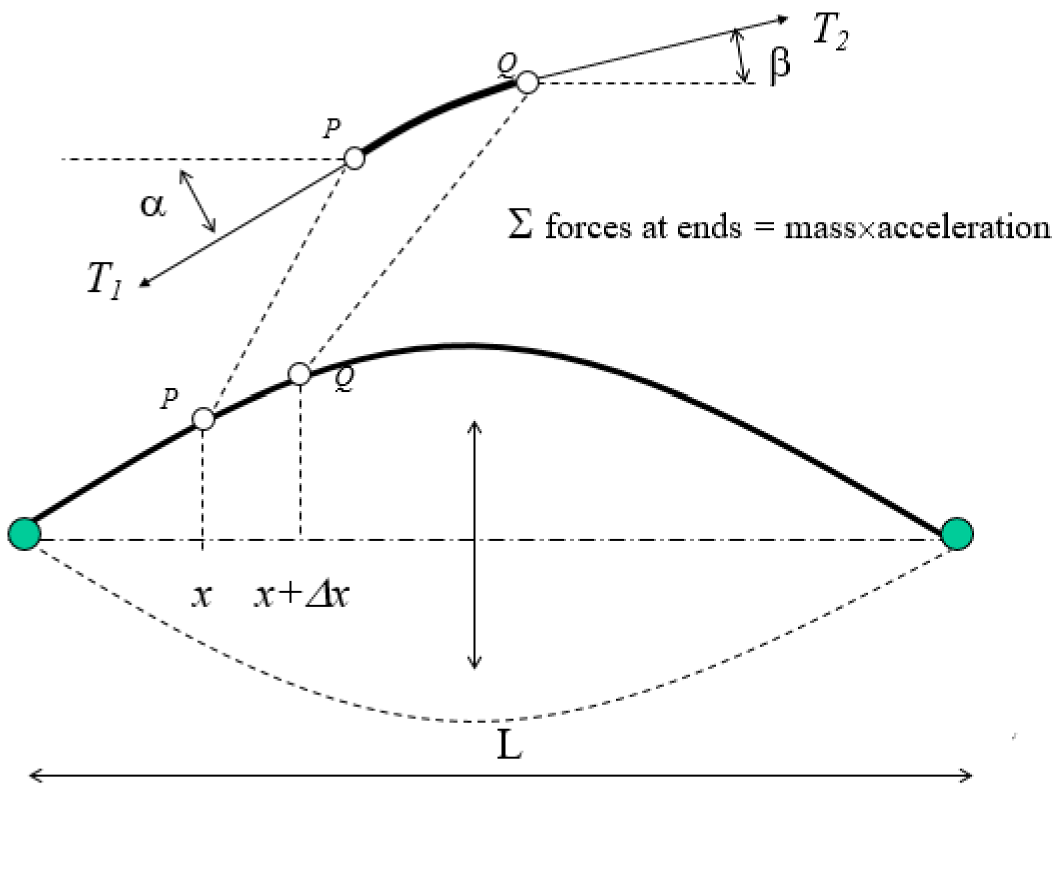
**Assumption**

- Mass per unit length is constant

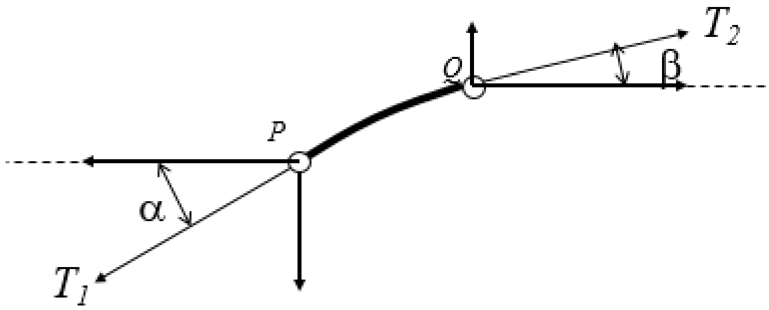



- Gravity can be neglected
- Motion is in one plane

Modelling the spring as an elastic string. Consider a "free body diagram" of at string segment :




Now consider the forces acting on this string segment. we will find the deflection $u(x,t)$ at any point $x$ and $t > 0$.




Note: ρ = linear density of string.

Horizontal Direction :

$$T_1 \cos\alpha \approx T_2 \cos\beta \approx T \qquad (1)$$

Vertical Direction :

$$T_2 \sin\beta - T_1 \sin\alpha = \rho \triangle x \frac{\partial^2 u}{\partial t^2} \qquad (2)$$

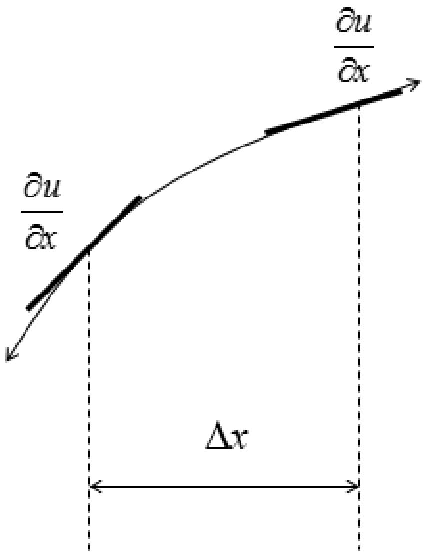
Divided equation (2) by equation (1) to get equation (3) :

$$\frac{T_2 \sin\beta}{T_2 \cos\beta} - \frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \rho \frac{\triangle x}{T} \triangle x \frac{\partial^2 u}{\partial t^2}$$

$$\tan\beta - \tan\alpha = \rho \frac{\triangle x}{T} \triangle x \frac{\partial^2 u}{\partial t^2} \qquad (3)$$

Note:

- $\tan\alpha$ = string slope at $x$ and,
- $\tan\beta$= string slope at$x + \triangle x$




Thus

$$\tan\beta = [\frac{\partial u}{\partial x}]_{x+\triangle x}$$

and get equation (4) :

$$\tan\beta = [\frac{\partial u}{\partial x}]_{x} \tag{4}$$

After dividing equation (3) by $T\triangle x$, and subtituing equation (4) for the tan functions, we have :

$$([\frac{\partial u}{\partial x}]_{x+\triangle x} - [\frac{\partial u}{\partial x}]_{x})(\frac{1}{\triangle x}) = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$$

Now let $\triangle x \to 0$

$$\frac{[\frac{\partial u}{\partial x}]_{x+\triangle x} - [\frac{\partial u}{\partial x}]_{x}}{\triangle x} = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$$

By letting $\triangle x \to 0$, we have obtained the one dimensional wave equation :

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 u}{\partial t^2}$$

Rearrange and set $c^2 = T/\rho$:

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} \tag{5}$$

# Part II

# The Solution to Obtained The Autowasher Spring

Instead of guessing we could start looking for a solution by making the assumption that it will be some function of $x$ multiplied by a function of $t$ :

$$u(x,t) = X(x)T(t)$$

We obtain the solution using the method of

**"Separation of Variables"**

Substitute the solution $u = X(x)T(t)$ into equation (5):

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

You get :

$$X \frac{\partial^2 T}{\partial t^2} = c^2 \frac{\partial^2 X}{\partial x^2} T$$

Now separate variables (get everything that's a function of $x$ on one side and everything that's a function of t on the other):

$$\frac{\frac{\partial^2 T}{\partial t^2}}{c^2 T} = \frac{\frac{\partial^2 X}{\partial x^2}}{X}$$

Both sides are equal to a constant. Call it k:

$$\frac{\frac{\partial^2 T}{\partial t^2}}{c^2 T} = \frac{\frac{\partial^2 X}{\partial x^2}}{X} = k$$

We get the two equations :
Time ($t$) only

$$\frac{d^2 T}{dt^2} - kc^2 T = 0$$

Distance ($x$) only

$$\frac{d^2 X}{dx^2} - kX = 0$$

Choose a sign for the constant that will give you sensible results.
$k$ should be negative
$k = -\lambda^2$
you now get 2 differential equations :

$$\frac{d^2 T}{dt^2} - \lambda^2 c^2 T = 0 \frac{d^2 X}{dx^2} - \lambda^2 X = 0 \tag{6}$$

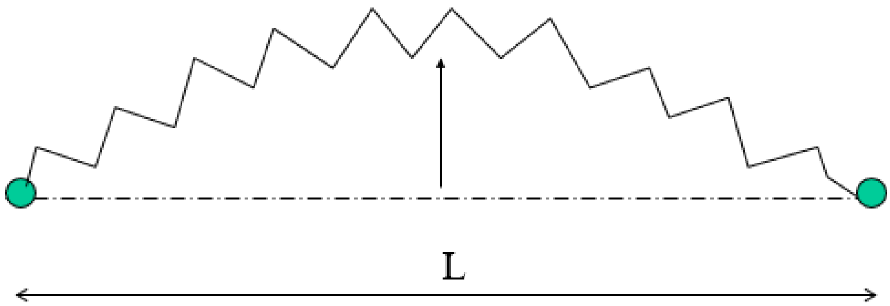
Solutions to $T$ and $X$ are:

$$T = A\sin(\lambda ct) + B\cos(\lambda ct)X = C\sin(\lambda x) + D\cos(\lambda x) \tag{7}$$

The solution for u(x,t) equals X times T:

$$u(x,t) = (Asin(\lambda x) + B\cos(\lambda x)(C\sin(c\lambda t) + D\cos(c\lambda t))) \tag{8}$$

Boundary Conditions :




At t $x = 0, u(0,t) = 0$

$$u(0,t) = X(0)T(t) = B(C\sin(c\lambda t) + D\cos(c\lambda t)) = 0$$

Thus $B = 0$
At t $x = L, u(l,t) = 0$

$$u(L,t) = X(L)T(t) = A\sin(\lambda L)(C\sin(c\lambda t) + D\cos(c\lambda t)) = 0$$

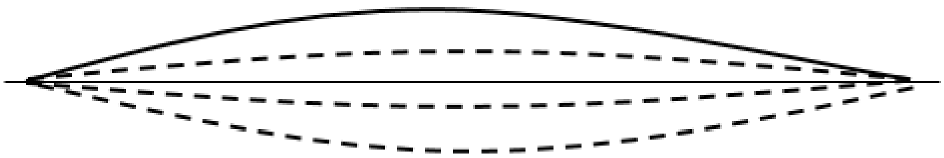
So
$\sin(\lambda L) = 0$
or $\lambda L = n\pi$, where $n = 0, 1, 2, 3, ...$
So

$$u_n(x,t) = A_n \sin\left(\frac{n\pi x}{L}\right)\left[C_n \sin\left(\frac{n\pi ct}{L}\right) + D_n \cos\left(\frac{n\pi ct}{L}\right)\right]$$

or

$$u_n(x,t) = \sin\left(\frac{n\pi x}{L}\right)\left[a_n \sin\left(\frac{n\pi ct}{L}\right) + b_n \cos\left(\frac{n\pi ct}{L}\right)\right] \tag{9}$$

$n = 1, 2, 3, ...$ Note $n = 0$ gives $u_0 = 0$



$u_1(x,t)$

**"$1^{st}$ Spring Mode"**

$$u_1(x,t) = \sin\left(\frac{\pi}{L}\right)\left[a_1 \sin\left(\frac{\pi c t}{L}\right) + b_1 \cos\left(\frac{\pi c t}{L}\right)\right]$$

# Part III
# MATLAB Simulation