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Mathematical model for blood flow through a bifurcated artery using couple stress fluid

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ABSTRACT

In this article, the blood flow through a bifurcated artery with mild stenosis is investigated taking blood as couple stress fluid. The artery configuring bifurcation is assumed to be symmetric about the axis of the artery and straight cylinders of finite length. The governing equations are non-dimensionalized and coordinate transformation is used to convert the irregular boundary to a regular boundary. The resulting system of equations is solved numerically using the finite difference method. The variation of shear stress, flow rate and impedance near the apex with pertinent parameters are studied graphically. It has been noticed that shear stress, flow rate and impedance have been changing suddenly with all the parameters on both sides of the apex. This occurs because of the backflow of the streaming blood at the onset of the lateral junction and secondary flow near the apex in the daughter artery.

1. Introduction

The cardiovascular system is an internal flow loop with multiple branches in which a complex liquid circulates. Atherosclerosis, localized narrowing of the blood vessels due to abnormal growth of tissues, is one of the diseases of the cardiovascular system of man. This can cause serious circulatory disorders by reducing or occluding the blood supply. The early atherosclerotic changes of vessel walls and the deposition of platelet thrombi occur preferentially at the entrances of branching arteries [1]. Thus, over the last two decades, extensive research has been carried out to correlate the connection between blood flow, localized genesis and development of atherosclerosis, including experimental, analytical and numerical studies. Studies for both normal and stenotic vessels have been carried out for idealized arteries, idealized arterial bifurcations and branching.

In most of the studies reported in the literature on the blood flow dynamics, blood is assumed to be an incompressible Newtonian fluid. It is well known that most physiological fluids, including blood behave as a non-Newtonian fluid. It has been suggested that blood flow behavior in narrow vessels at low shear rate can be represented by a non-Newtonian fluid. The non-Newtonian behavior of blood is mainly due to the suspension of red blood cells in the plasma. When neutrally buoyant blood cells are contained in a fluid and there exists a velocity gradient due to shearing stress, blood cells have rotatory motion have spin angular momentum in addition to linear angular momentum. Hence, the symmetry of the stress tensor is lost in the fluid motion that is subjected to spin angular momentum. The radius of gyration of the blood cells in such fluids is different from that of the fluid particles. Their difference produces couple stress in the fluid. It is known that such a fluid has spin angular momentum in addition to the couple stress effect. The couple stress fluid model introduced by Stokes [2] has distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglect the size effect of material particles within the continua. These fluids are capable of describing various types of lubricants, blood, suspension fluids, etc.

Mekheimer and Abd Elmaboud [3] discussed the peristaltic flow of a couple stress fluid in an annulus with the application of an endoscope and observed that the trapped bolus decrease in size as one move from couple stress fluid to a Newtonian fluid. Sobh [4] analytically studied the interaction of couple stresses and slip flow on peristaltic transport in a uniform and non-uniform channels and concluded that the peristaltic pumping for the couple stress fluid is more than that of Newtonian fluid. Sahu et al. [5] presented a mathematical analysis to study the effect of a mild stenosis on blood flow characteristics with the representation of blood by couple stress fluid. Nadeem and Akram [6] considered the peristaltic flow of the couple stress fluid in an asymmetric channel under the influence of the induced magnetic field and noticed

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that the pressure rise increases in the peristaltic pumping region to increase in couple stress parameter. Pandey and Choube [7] examined the effect of magnetic field on the peristaltic transport of couple stress fluids through a porous medium and observed that the mean velocity at the channel walls decreases with an increase in the value of couple-stress, permeability parameters and it increases with an increase in the magnetic parameter. Sankad and Radhakrisnamacharya [8] investigated that the time average velocity decreases with viscous damping force and the size of the trapped bolus and decreases with the Hartmann number for the peristaltic transport of couple stress fluid in a channel with different wall properties. Maiti and Misra [9] investigated that the increase in both pumping and pressure by increasing the amplitude ratio, couple stress parameter and also by decreasing the permeability for the peristaltic transport of a couple stress fluid in a porous channel. Srinivasacharya and Srikanth [10] studied the steady streaming effect on the pulsatile nature of couple stress fluid and analyzed that the pressure drop, the shear stress increases with an increase in the size of the catheter and decrease in the couple stress fluid parameter. Tripathi [11] analyzed the peristaltic pumping properties, frictional force, mechanical energy and trapping occurrence for peristaltic hemodynamic flow patterns and concluded that mechanical energy is a decreasing function of couple-stress parameter. Akbar and Nadeem [12] mentioned that the pressure rise increases with an increase in amplitude ratio, couple stress fluid parameter and decrease in thermophoresis parameter for the peristaltic flow of an incompressible couple stress fluid in a two-dimensional uniform tube. Hayat et al. [13] investigated the hall effects on the peristaltic flow of couple stress fluid in an inclined asymmetric channel with heat and mass transfer. They noticed that the size of the trapping bolus decreases with an increase in the value couple stress parameter, Hall parameter and decreases in the value of Hartmann number. Akbar and Nadeem [14] calculated the exact solutions for the peristaltic flow of chyme in intestine using couple stress fluid. Alsaedi et al. [15] considered the flow of couple stress fluid through uniform porous medium and revealed that the volume of the trapped bolus increases and circulates faster with an increase in the value of permeability parameter. Hina et al. [16] reported that the fluid velocity significantly increases with an increase in the couple stress fluid parameter and decreases with an increase in the magnetic field strength for the peristaltic motion of an electrically conducting couple-stress fluid in a channel with complaint walls.

In all the above mentioned papers, the significance of the bifurcation of the arteries was neglected. The study of blood flow in the bifurcated artery is of great scientific attention with respect to the diagnosis of atherosclerosis. All the physiological properties of blood at the bifurcation of the artery severely affected by different parameters. Hence, the aim of the present article is to study the couple stress fluid flow through a bifurcated artery with mild stenosis in the parent lumen. The present study employs a two dimensional model that is tractable computationally although the related physiological situation is three dimensional. The variation of volumetric flow, impedance and shearing stress are analyzed for various values of pertinent parameters involved in the problem.

2. Mathematical formulation

Consider the flow of steady, laminar, incompressible homogeneous blood flow through a bifurcated artery with mild stenosis in its parent lumen. The blood is treated as couple stress fluid of constant density. The stenosis over a length of the artery is assumed to have developed in an axi-symmetric manner and the parent aorta have a single mild stenosis in its lumen as shown in Fig 1. Let (r, θ, z) be the coordinates of the material point in the cylindrical polar coordinate system, of which z is taken as the central axis of

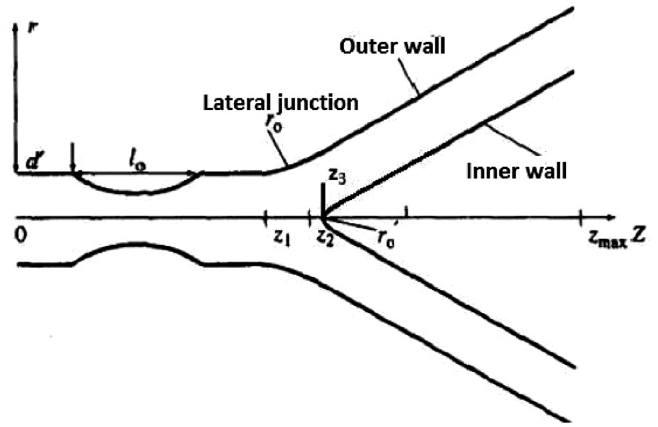

Fig. 1. Schematic diagram of stenosed bifurcated artery.

the parent artery. The arteries, forming bifurcations are symmetrical about the central axis of the parent artery and are straight circular cylinders of restricted length. Curvature is introduced at the lateral junction and the flow divider so that the flow separation zones (if any)can be removed.

The governing equations for the motion of incompressible steady couple stress fluids in the absence of body force and body moments are given by

$$\nabla \cdot \mathbf{q} = 0 \tag{1}$$

$$\rho(\mathbf{q}.\nabla)\mathbf{q} = -\nabla p + \mu \nabla^2 \mathbf{q} - \eta \nabla^4 \mathbf{q} \tag{2}$$

where ρ is the density of the couple stress fluid, η is the couple stress viscosity parameter, $\mathbf{q}$ is the velocity vector and μ is the viscosity.

The force stress tensor τ and the couple stress tensor $\mathbf{M}$ that are present in the theory of couple stress fluid are respectively given by

$$\boldsymbol{\tau} = (-p + \lambda \nabla \cdot \mathbf{q})\mathbf{I} + \mu\left[\nabla \mathbf{q} + (\nabla \mathbf{q})^T\right] - (1/2)\mathbf{I} \times (\nabla \cdot \mathbf{M}), \tag{3}$$

and

$$\mathbf{M} = m\mathbf{I} + 2\eta \nabla(\nabla \times \mathbf{q}) + 2\eta' \left(\nabla(\nabla \times \mathbf{q})\right)^T, \tag{4}$$

where $\mathbf{I}$ is the unit tensor, p is the fluid pressure, m is $1/3rd$ trace of $\mathbf{M}$, The quantity λ is the material constant and η and η' being the constants associated with couple stresses. The dimensions of the material constant λ are that of viscosity, whereas the dimensions of η and η' are those of momentum. These material constants satisfy the following inequalities.

$$\mu \geq 0, \quad 3\lambda + 2\mu \geq 0, \quad \eta \geq 0, \quad \eta' \leq \eta. \tag{5}$$

The mathematical representation of an outer $R_1(z)$ and the inner $R_2(z)$ walls of the bifurcated artery with mild stenosis in parent lumen are ([17,18])

$$R_1(z) = \begin{cases} a & 0 \leq z \leq d' \\ (a - \frac{4\epsilon}{l_0^2}(l_0(z-d') - (z-d')^2) & d' \leq z \leq d' + l_0 \\ a & d' + l_0 \leq z \leq z_1 \\ (a + r_0 - \sqrt{r_0^2 - (z-z_1)^2}) & z_1 \leq z \leq z_2 \\ (2r_1 sec\beta + (z-z_2)tan\beta) & z_2 \leq z \leq z_{max} \end{cases} \tag{6}$$

$$R_2(z) = \begin{cases} 0 & 0 \leq z \leq z_3 \\ (\sqrt{(r_0')^2 - (z - z_3 - r_0')^2}) & z_3 \leq z \leq z_3 + r_0'(1 - sin\beta) \\ (r_0' cos\beta + z_4) & z_3 + r_0'(1 - sin\beta) \leq z \leq z_{max} \end{cases} \tag{7}$$

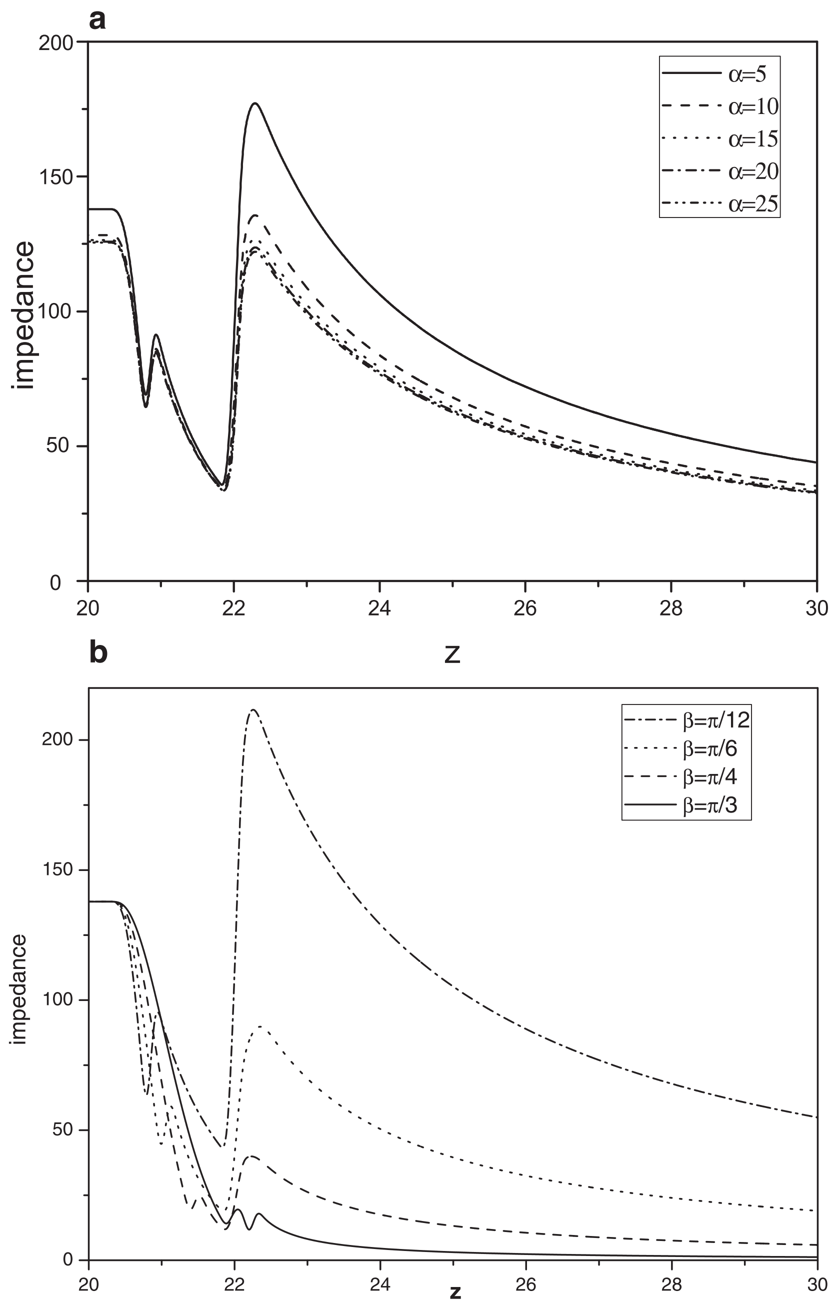

Fig. 2. Variations of impedance with (a) α on both sides of the apex for fixed values of $\sigma = 0.5$, $\beta = \pi/10$ and (b) β on both sides of the apex for fixed values of $\sigma = 0.5$, $\alpha = 5$.

where a is the radius of the parent artery at non-stenosed portion, r_1 is the radius of the daughter artery, r_0 , $r_0^{'}$ are the radii of curvatures for the lateral junction and flow divider respectively, l_0 is the length of the stenosis at a distance $d^{'}$ from the origin, z_1, z_2, z_3 and z_4 are the location of the onset, offset of the lateral junction and the flow divider respectively, β is the half of the bifurcation angle, ϵ is the maximum height of the stenosis at $z = d^{'} + l_0/2$ and z_{max} represents the maximum length of the bifurcated artery under consideration.

The radii of curvature r_0 and $r_0^{'}$ at the lateral junction and flow divider are

$$r_0 = \frac{a - 2r_1 sec\beta}{cos\beta - 1} \qquad \text{and} \quad r_0^{'} = \frac{(z_3 - z_2)sin\beta}{1 - sin\beta} \tag{8}$$

where z_2 , z_3 and z_4 lie on the central axis of the artery and which are functions of half of the bifurcated angle and are defined as

$$z_2 = z_1 + r_0 \sin\beta, z_3 = z_2 + h, z_4 = (z - z_3 - r_0^{'}(1 - sin\beta))tan\beta$$

where h is a small number lying in between 0.1 and 0.5, this is defined for compatibility of the geometry.

Since the flow is taken to be symmetric, all the variables are independent of θ. The non-dimensional variables are defines as follows:

$$\left.\begin{array}{l} r = a\widetilde{r},\ u = \frac{aw_0\widetilde{u}}{L},\ z = L\widetilde{z},\ w = w_0\widetilde{w},\ d = L\widetilde{d}, \\ p = \frac{Lw_0\mu\widetilde{p}}{a^2},\ R_1(z) = a\widetilde{R}_1(\widetilde{z}),\ R_2(z) = a\widetilde{R}_2(\widetilde{z}), \\ \quad r_1 = a\widetilde{r}_1,\ z_1 = a\widetilde{z}_1 etc. \end{array}\right\} \tag{9}$$

where u and w are radial and axial velocity components, L is characteristic length and w_0 is characteristic velocity.

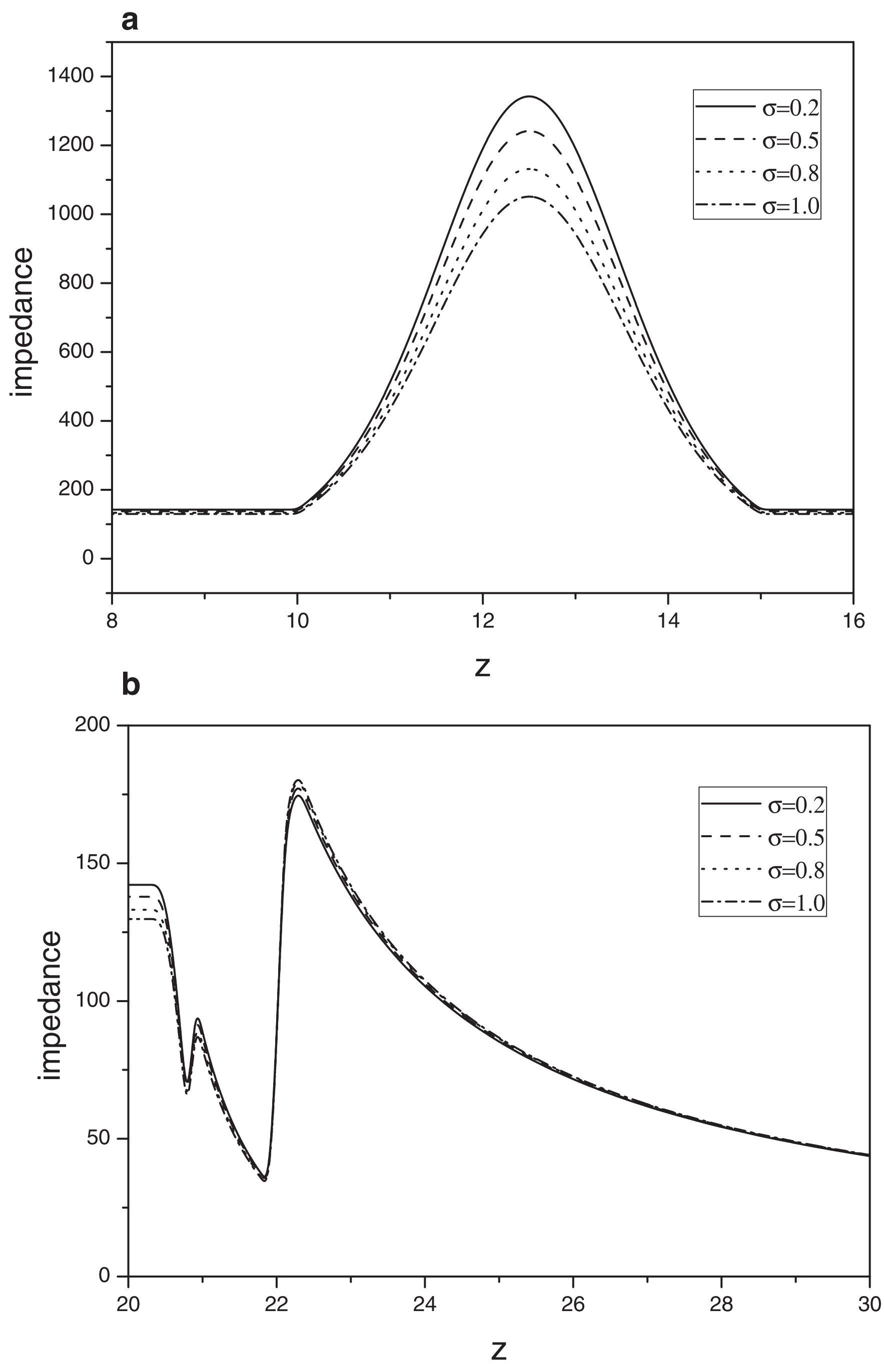

Fig. 3. Influence of σ on impedance in (a) the parent artery and (b) the daughter artery for fixed values of $\alpha = 5,\ \beta = \pi/10$.

It can be shown that the radial velocity is very small when compared with axial velocity and can be omitted for a low Reynolds number flow in an artery with mild stenosis [19]. Therefore, basic Eqs. (1) and (2) in the absence of any radial/rotational flow for the steady, axisymmetric, laminar fully developed flow [20] may be written in non-dimensional form as

$$\frac{\partial p}{\partial r} = 0 \tag{10}$$

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]w - \frac{1}{\alpha^2}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]^2 w = \frac{dp}{dz} \tag{11}$$

where $\alpha^2 = \frac{\mu a^2}{\eta}$ is the couple stress fluid parameter representing the ratio between the radius of the artery and material characteristic length ($\frac{\eta}{\mu}$ has the dimension of length).

The outer and inner walls of the bifurcated artery in non-dimensional form as

$$R_1(z) = \begin{cases} 1 & 0 \le z \le d' \\ (1 - \frac{4\epsilon}{al_0^2}(l_0(z-d') - (z-d')^2) & d' \le z \le d' + l_0 \\ 1 & d' + l_0 \le z \le z_1 \\ (1 + r_0 - \sqrt{r_0^2 - (z-z_1)^2}) & z_1 \le z \le z_2 \\ (2r_1 sec\beta + (z-z_2)tan\beta) & z_2 \le z \le z_{max} \end{cases} \tag{12}$$

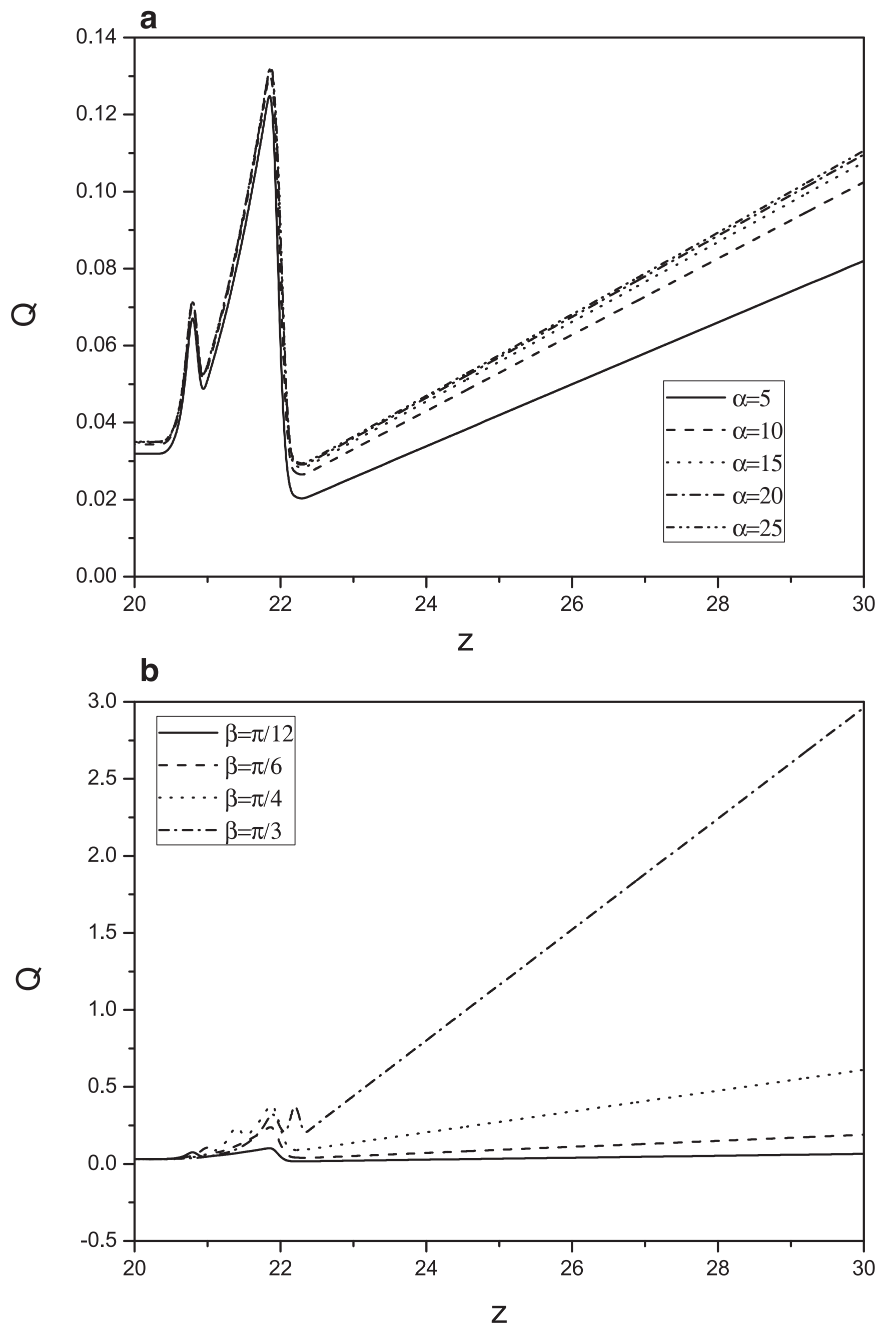

Fig. 4. Variations of flow rate with (a) α on both sides of the apex for fixed values of $\sigma = 0.5$, $\beta = \pi/10$ and (b) β on both sides of the apex for fixed values of $\sigma = 0.5$, $\alpha = 5$.

$$R_2(z) = \begin{cases} 0 & 0 \le z \le z_3 \\ (\sqrt{(r_0')^2 - (z - z_3 - r_0')^2}) & z_3 \le z \le z_3 + r_0'(1 - sin\beta) \\ (r_0' cos\beta + z_4) & z_3 + r_0'(1 - sin\beta) \le z \le z_{max} \end{cases} \quad (13)$$

The boundary conditions associated with physical problem are

$$\left.\begin{array}{l} \frac{\partial w}{\partial r} = 0, \quad \frac{\partial^2 w}{\partial r^2} - \frac{\sigma}{r}\frac{\partial w}{\partial r} = 0, \quad \text{on} \quad r = 0 \quad \text{for} \quad 0 \le z \le z_3 \\ w = 0, \quad \frac{\partial^2 w}{\partial r^2} - \frac{\sigma}{r}\frac{\partial w}{\partial r} = 0, \quad \text{on} \quad r = R_1(z,t) \quad \text{for all } z \\ w = 0, \quad \frac{\partial^2 w}{\partial r^2} - \frac{\sigma}{r}\frac{\partial w}{\partial r} = 0, \quad \text{on } r = R_2(z,t) \\ \quad \text{for} \quad z_3 \le z \le z_{max} \end{array}\right\} \quad (14)$$

where $\sigma = \frac{\eta'}{\eta}$ is the couple stress fluid parameter which is responsible for the effect of local viscosity of particles apart from the bulk viscosity of the fluid μ. If $\eta' = \eta$ then the effects of couple stresses will be absent in a material, which implies that couple stress tensor is symmetric. In this case the Eq. (14) shows that, the couple stresses are disappearing on the inner and outer walls of the bifurcated artery.

In order to solve Eq. (11), let

$$\frac{\partial w}{\partial r} = y_1, \quad \frac{\partial y_1}{\partial r} = y_2, \quad \frac{\partial y_2}{\partial r} = y_3 \quad (15)$$

Therefore Eq. (11) can be written as

$$\frac{1}{\alpha^2}\frac{\partial y_3}{\partial r} + \frac{2}{\alpha^2 r}\frac{\partial y_2}{\partial r} - \left[1 + \frac{1}{\alpha^2 r^2}\right]\frac{\partial y_1}{\partial r} + \left[\frac{1}{\alpha^2 r^3} - \frac{1}{r}\right]\frac{\partial w}{\partial r} = -\frac{dp}{dz} \quad (16)$$

The effect of $R_1(z)$ and $R_2(z)$ from the boundary can be transferred into the governing equations by the following radial coordinate transformation [21]

$$\xi = \frac{r - R_2}{R} \tag{17}$$

where $R(z) = R_1(z) - R_2(z)$. Using this transformation in Eqs. (15) and (16) take the form

$$\left.\begin{aligned}
&\frac{\partial w}{\partial \xi} - R\, y_1 = 0,\\
&\frac{\partial y_1}{\partial \xi} - R\, y_2 = 0,\\
&\frac{\partial y_2}{\partial \xi} - R\, y_3 = 0,\\
&\frac{1}{\alpha^2}\frac{\partial y_3}{\partial \xi} + \frac{2}{\alpha^2(\xi R + R_2)}\frac{\partial y_2}{\partial \xi} - \left[1 + \frac{1}{\alpha^2(\xi R + R_2)^2}\right]\frac{\partial y_1}{\partial \xi}\\
&\qquad + \left[\frac{1}{\alpha^2(\xi R + R_2)^3} - \frac{1}{(\xi R + R_2)}\right]\frac{\partial w}{\partial \xi} = -R\,\frac{dp}{dz}
\end{aligned}\right\} \tag{18}$$

The corresponding boundary conditions, in the transformed coordinates, are

$$\left.\begin{aligned}
&\frac{\partial w}{\partial \xi} = 0,\ \frac{\partial y_1}{\partial \xi} - \frac{\sigma R}{(\xi R + R_2)} y_1 = 0, \text{ on } \xi = 0 \text{ for } 0 \le z \le z_3\\
&w = 0,\ \frac{\partial y_1}{\partial \xi} - \frac{\sigma R}{(\xi R + R_2)} y_1 = 0 \text{ on } \xi = 1 \text{ for all } z\\
&w = 0,\ \frac{\partial y_1}{\partial \xi} - \frac{\sigma R}{(\xi R + R_2)} y_1 = 0 \text{ on } \xi = 0 \text{ for } z_3 \le z \le z_{max}
\end{aligned}\right\} \tag{19}$$

The physical quantities to be analyzed are flow rate, impedance and shear stress for both parent and daughter arteries. The flow rate for both parent artery Q_p and daughter artery Q_d are determined by using

$$Q_p = 2\pi R_i \left[R_i \int_0^1 \xi_i w_{i,j} d\xi_i + R_{2i} \int_0^1 w_{i,j} d\xi_i \right] \tag{20}$$

and

$$Q_d = \pi R_i \left[[R_i \int_0^1 \xi_i w_{i,j} d\xi_i + R_{2i} \int_0^1 w_{i,j} d\xi_i \right] \tag{21}$$

The resistance to the flow (resistive impedance) in parent artery λ_p and daughter artery λ_d are calculated using

$$\lambda_p = \left| \frac{z_3 \frac{dp}{dz}}{Q_p} \right| \ for\ z < z_3 \tag{22}$$

$$\lambda_d = \left| \frac{(z_{max} - z_3)\frac{dp}{dz}}{Q_d} \right| \ for\ z \ge z_3 \tag{23}$$

The mean shear stress is calculated with

$$\tau_{ij} = \frac{1}{R}\frac{\partial w}{\partial \xi} + \frac{1}{4R\alpha^2(\xi R + R_2)^2}\frac{\partial w}{\partial \xi} - \frac{1}{4\alpha^2 R^3}\frac{\partial}{\partial \xi}\left(\frac{\partial^2 w}{\partial \xi^2}\right) - \frac{1}{4R^2\alpha^2(\xi R + R_2)}\frac{\partial^2 w}{\partial \xi^2}. \tag{24}$$

3. Method of solution

The reduced Eq. (18) along with the boundary conditions (19) are solved numerically using finite-difference scheme. A two dimensional computational grid is considered. The stepping process is defined by $z_i = i\Delta z, i = 0, 1, \ldots, n,\ \xi_j = j\Delta\xi, j = 0, 1, \ldots, J$ where Δz, $\Delta\xi$ are step lengths in the axial and radial directions, respectively. If $w_{i,j}$ represents the value of the variable w at (z_i, ξ_j), then the derivatives are replaced by central difference approximations as shown below

$$\begin{aligned}
\frac{\partial w}{\partial \xi} &= \frac{1}{2}\left[\frac{w_{i,j+1} - w_{i,j-1}}{2\triangle\xi}\right], \quad \frac{\partial y_1}{\partial \xi} = \frac{1}{2}\left[\frac{(y_1)_{i,j+1} - (y_1)_{i,j-1}}{2\triangle\xi}\right]\\
\frac{\partial y_2}{\partial \xi} &= \frac{1}{2}\left[\frac{(y_2)_{i,j+1} - (y_2)_{i,j-1}}{2\triangle\xi}\right], \quad \frac{\partial y_3}{\partial \xi}\\
&= \frac{1}{2}\left[\frac{(y_3)_{i,j+1} - (y_3)_{i,j-1}}{2\triangle\xi}\right].
\end{aligned} \tag{25}$$

Substituting (25) into (18), we get the following equations.

$$\left.\begin{aligned}
&w_{i,j-1} - w_{i,j+1} + 2R\triangle\xi\,(y_1)_{i,j} = 0,\\
&(y_1)_{i,j-1} - (y_1)_{i,j+1} + 2R\triangle\xi\,(y_2)_{i,j} = 0,\\
&(y_2)_{i,j-1} - (y_2)_{i,j+1} + 2R\triangle\xi\,(y_3)_{i,j} = 0,\\
&\frac{-1}{2R\triangle\xi}\left(\frac{1}{\alpha^2(\xi R + R_2)^3} - \frac{1}{\xi R + R_2}\right)(w_{i,j-1} - w_{i,j+1})\\
&\quad + \frac{1}{2R\triangle\xi}\left(1 + \frac{1}{\alpha^2(\xi R + R_2)^2}\right)((y_1)_{i,j-1} - (y_1)_{i,j+1})\\
&\quad - \frac{((y_2)_{i,j-1} - (y_2)_{i,j+1})}{R(\triangle\xi)\alpha^2(\xi R + R_2)} - \frac{((y_3)_{i,j-1} - (y_3)_{i,j+1})}{2R(\triangle\xi)\alpha^2} = -\frac{dp}{dz}
\end{aligned}\right\} \tag{26}$$

The Eq. (26) along with the boundary conditions (19) result in a tri-diagonal system of equation and is solved by block elimination method.

4. Results and discussion

In order to explore the effect of couple stress fluid parameters α and σ and the half of the bifurcation angle β on the impedance, volumetric flow rate and wall shear stress, these quantities are calculated numerically using the above computational procedure and analyzed through graphs. For better understanding of the results, we have used the following data ([18,20]): $a = 5$ mm, $d' = 10$ mm, $l_0 = 5$ mm, $\beta = \frac{\pi}{10}$, $r_1 = 0.51a$, $\epsilon = 2$, $\alpha = 5.0$, $\sigma = 0.5$.

The parameter $l = \sqrt{\frac{\eta}{\mu}}$ is the characteristic measure of the polarity of the fluid model and has the dimensions of the length. If l is a function of the molecular dimensions of the liquid, it will vary greatly for different liquids. For example, the length of a polymer chain may be a million times the diameter of water molecule ([2]). One might therefore expect the couple stresses to appear in noticeable magnitude in liquids with very large molecules. α indicates the ratio of the tube radius to the characteristic length, i.e. $(= a/l)$. In the limit $\eta \to 0$ i.e. $\alpha \to \infty$, Eq. (2) reduce to classical Navier–Stokes equation. Hence, for larger values of α, the effect of couple stresses are not significant. The influence of α on impedance in both sides of flow divider is shown in Fig. 2(a). It is noticed from this figure that the impedance is decreasing with an increase in the value of α on both sides of the apex. The impedance in case of couple stress fluid is more than that of a Newtonian fluid. Hence, the effect of couple stresses is to increase the impedance.

The influence of half of the bifurcation angle β on impedance in both sides of flow divider is shown in Fig. 2(b). It is noticed from this figure that the impedance is decreasing with increases in the values of β on both sides of the apex. This may be due to the added resistance to flow caused by the increase in the bifurcation angle and fluid energy lost due to friction, as the fluid changes direction. In general, it is noticed that, the impedance decreases with

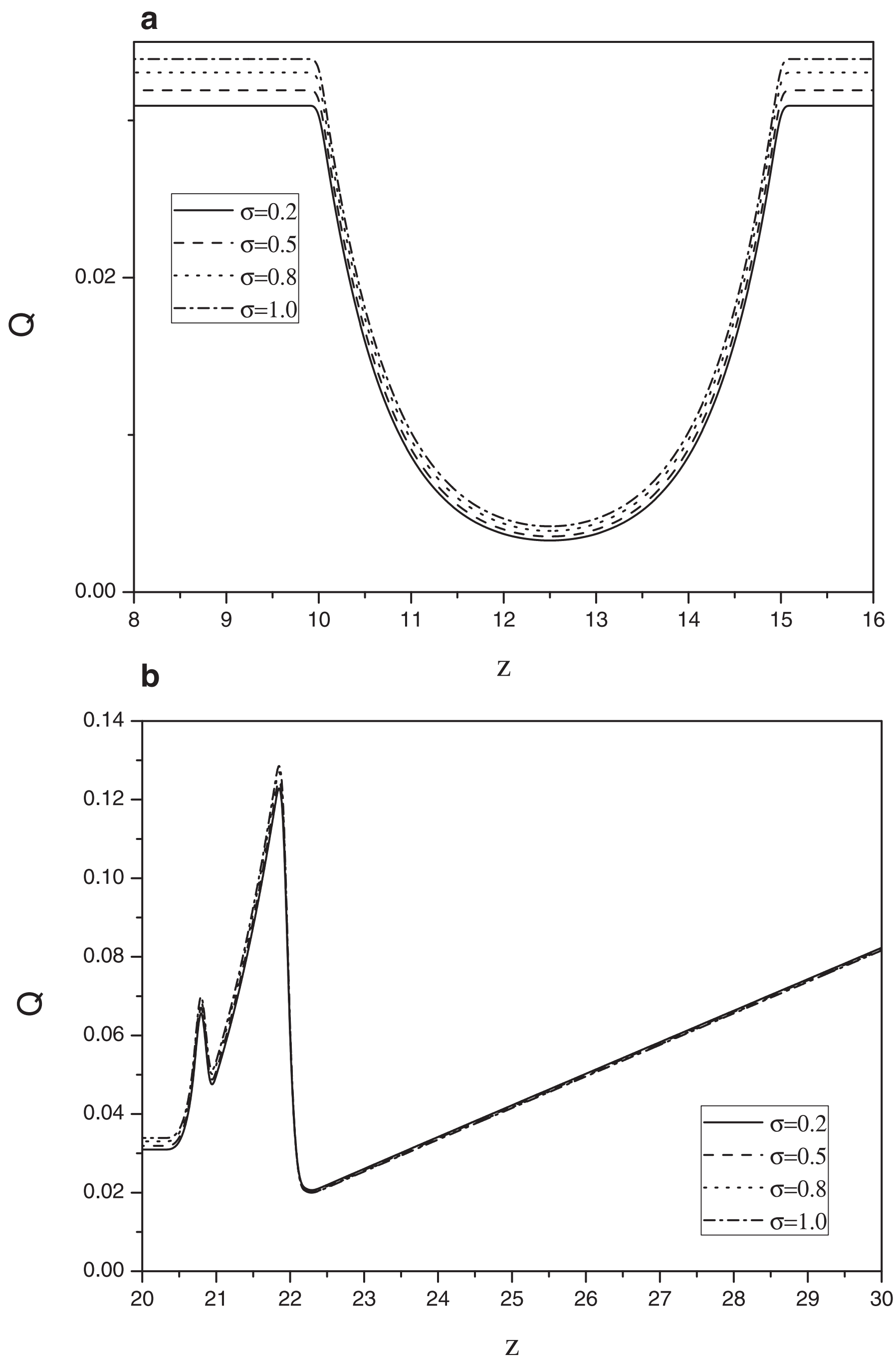

Fig. 5. Influence of σ on flow rate in (a) the parent artery and (b) the daughter artery for fixed values of $\alpha = 5, \beta = \pi/10$.

an increase in the value of z, up to lateral junction, then a slight increase occurs suddenly, and after that gradually decreases till the apex, and then a sudden increase is identified. This is because of diverging of blood flow at the bifurcation of the artery. Thereafter, it is found that the impedance is uniform till z_{max}.

The effect of σ on impedance both in parent and daughter arteries is illustrated in Fig. 3(a) and (b) respectively. It is observed from these figures that the impedance is diminishing in parent artery and increasing in daughter artery with increase in the value of σ. The impedance in case of couple stress fluid is more than that of Newtonian fluid in the parent artery at maximum height of the stenosis. But the effect of σ on the impedance is not significant in the daughter artery. $\sigma = 1$ corresponds to Newtonian fluid case.

The variations of flow rate with α and β on both sides of flow divider are depicted in Fig. 4(a) and (b). From these figures, it is noticed that the flow rate is advancing with advancement in the value of α and β both sides of the flow divider. The flow rate in case of couple stress fluid is less than that of Newtonian fluid. The influence of σ on flow rate both in parent and daughter arteries is explored in Fig. 5(a) and (b) respectively. These figures reveal that the flow rate is increasing in the parent artery and decreasing in the daughter with an increase in the value of σ. It is to be noted that flow rate is increasing with an increase in the value of z, until

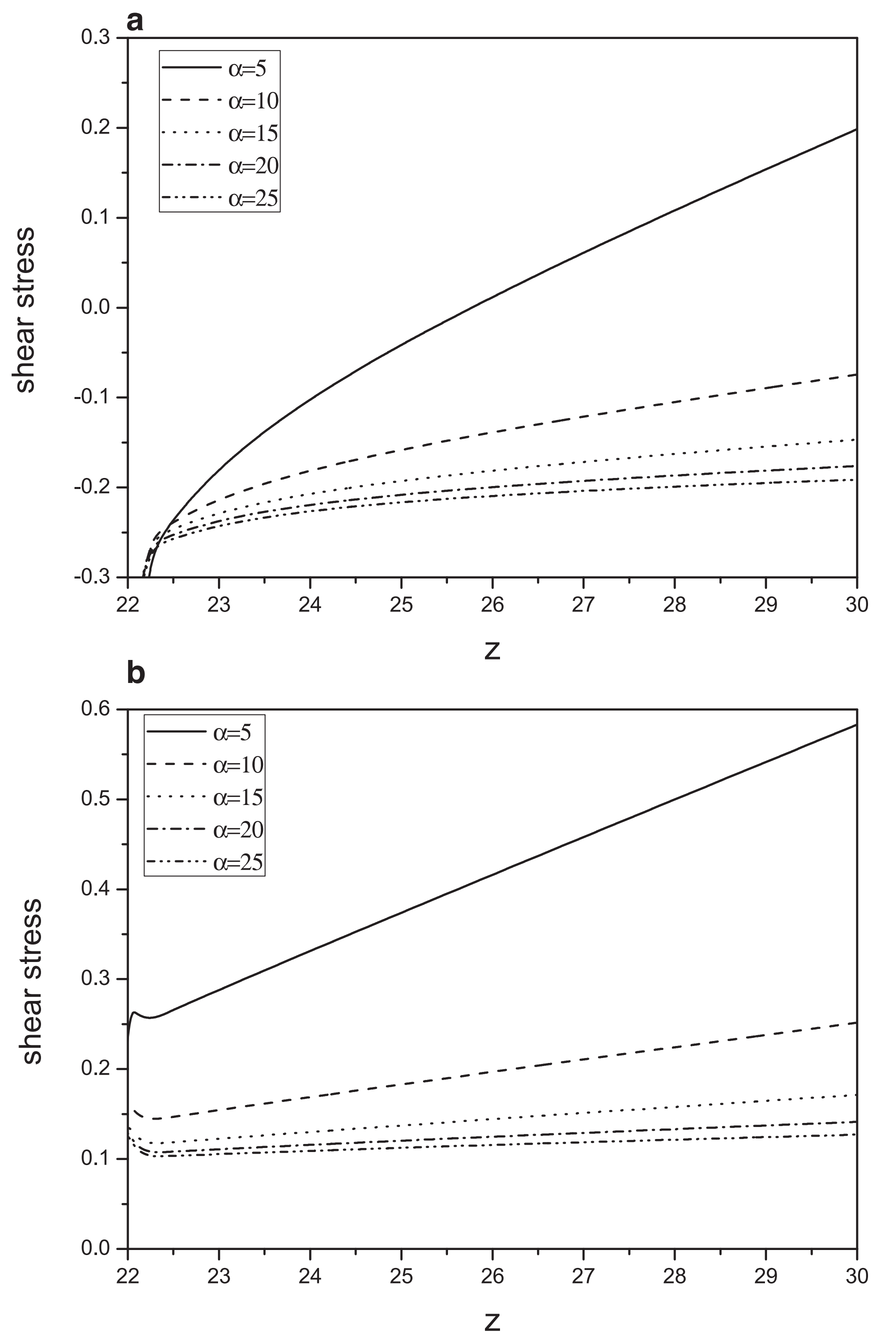

Fig. 6. Variations of shear stress along (a)the inner and (b) the outer walls of the daughter artery with α for fixed values of $\sigma = 0.5$, $\beta = \pi/10$.

inset of lateral junction, then a slight decrease occurred suddenly, and after that gradually increasing till the apex, and then a sudden decrease is identified. This is because of diverging of the blood flow at the bifurcation of the artery. Thereafter, it is found that the flow rate is uniform till z_{max}.

Fig. 6 (a) and (b) illustrate the effect of α on shear stress along the inner and outer walls of the daughter artery. From these figures, it is observed that shear stress is decreasing with an increase in the value of α along the inner and outer walls of the daughter artery. It is interesting to note that the presence of the couple stresses increase the shear stress along the walls of the artery. The effect of β on shear stress along the inner and outer walls of the daughter artery is presented in Fig. 7(a) and (b). It is concluded that shear stress is increasing with the increase in the value of β along the inner and outer walls of the daughter artery. The influence of σ on shear stress along the inner and outer walls of the daughter artery is shown in Fig. 8(a) and (b). From these figures, it is seen that shear stress is increasing along the inner wall and decreasing along the outer wall with an increase in the value of σ.

The present results are compared with the previously published results by Stokes [2] and they are in good agreement as shown in the Table 1.

5. Conclusions

The current facts help us to understand, numerically as well as physically, the influence of β, α and σ on couple stress blood flow through bifurcated artery, which is of great importance in the medical sciences and bio-medical engineering.

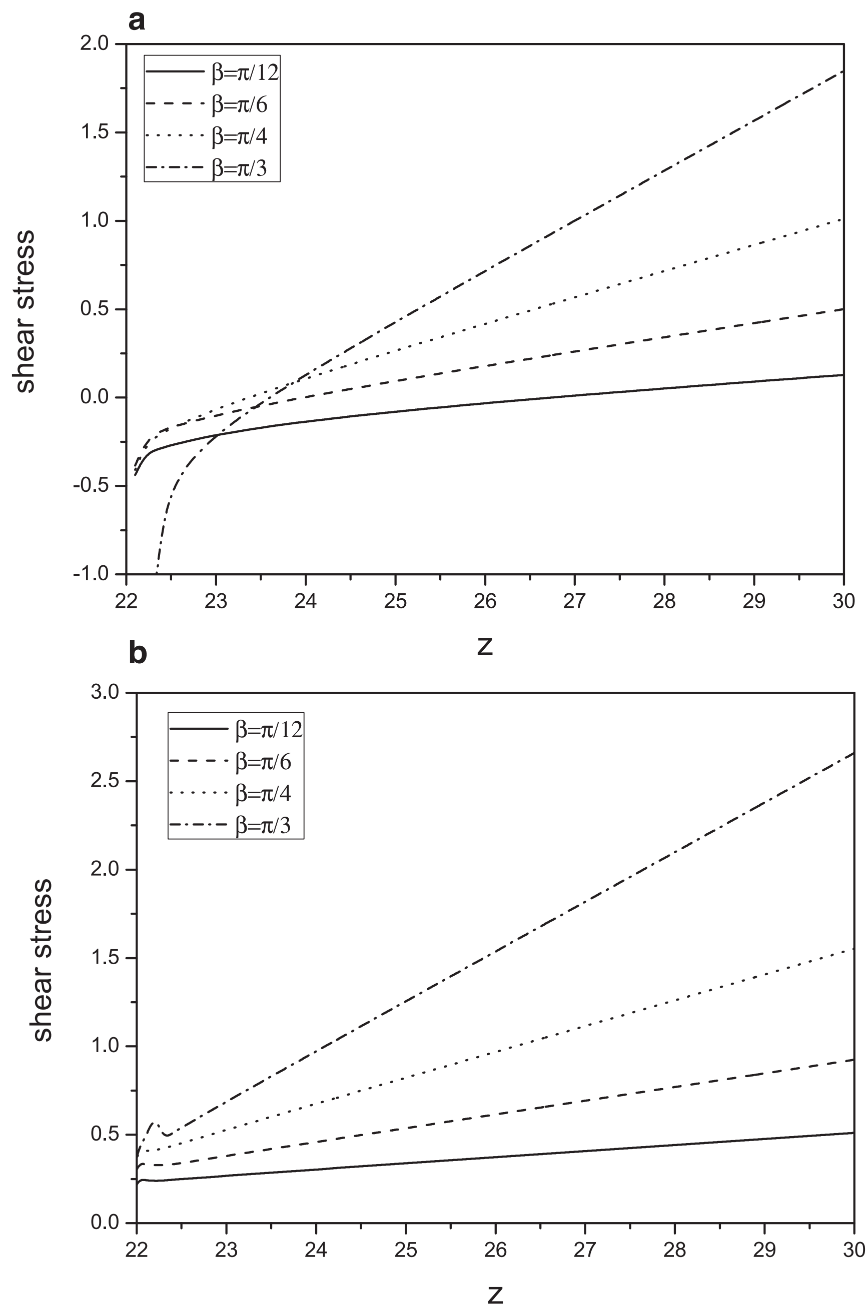

Fig. 7. Influence of β on shear stress along (a)the inner and (b) the outer walls of the daughter artery for fixed values of $\sigma = 0.5$, $\alpha = 5$.

Table 1
Comparison analysis for the velocity calculated by the present method and that of analytical solution [2] for $\alpha = 5.0$, $\frac{dp}{dz} = 2$, $\sigma = 0.5$, $R(z) = 1$ and $R_2(z) = 0$.

Velocity		
ξ	Analytical solution [2]	Present solution
0	0	0
0.1	0.00769	0.00768
0.2	0.01398	0.01398
0.3	0.01856	0.01856
0.4	0.02118	0.02118
0.5	0.02180	0.02179
0.6	0.02049	0.02049
0.7	0.01740	0.01740
0.8	0.01274	0.01273
0.9	0.00679	0.00679
1.0	0	0

- The flow rate increased with an increase in the value of α, β and σ both in parent and daughter arteries.
- The impedance decreased with an increase in the value of α, β and σ both in parent and daughter arteries.
- The shear stress is decreasing with an increase in the values of α, σ and shear stress has increased with advances in the values of β along the inner wall of daughter artery. But along the outer wall of the daughter artery shear stress decreases with increase values of α and shear stress has been increasing with increase in the values of β.

The blood is one which transport glucose, oxygen and other nutrients to the different parts of the body. Especially, muscles required more volume of oxygenated blood to move. The expansion of blood vessel allows higher quantity of blood this implies increase the heart rate, which can reduce development of arterial

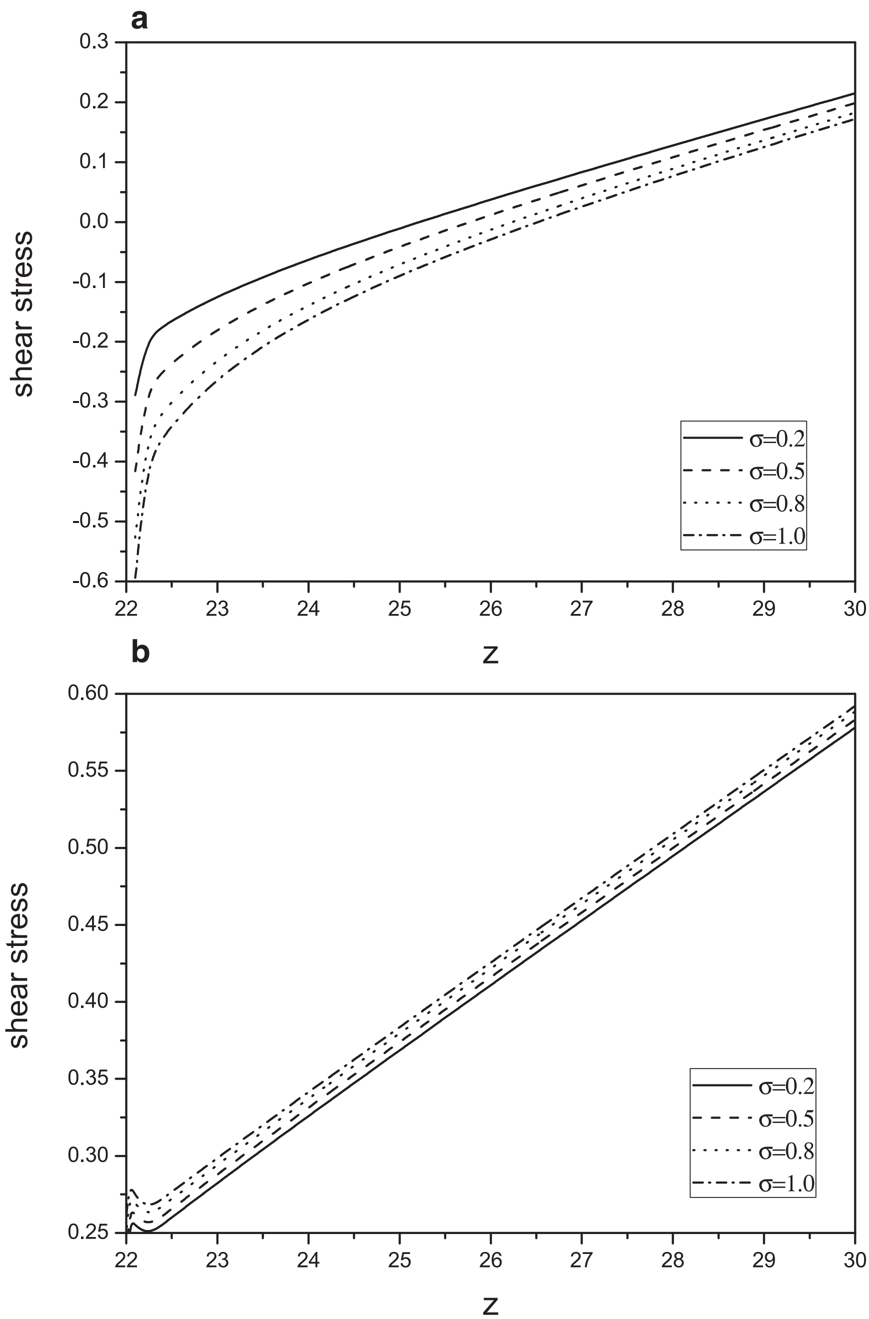

Fig. 8. Variations of shear stress along (a)the inner and (b) the outer walls of the daughter artery with σ for fixed values of $\alpha = 5,\ \beta = \pi/10$.

diseases. The increase of impedance, decreases the flow rate which is helpful to control the blood flow during the surgeries. Therefore, this model is very realistic and is expected to be very useful in predicting and diagnosis of various arterial diseases.

Although the arterial bifurcation geometry plays the important role in determining the flow patterns as well as the wall shear stress distributions, this study has the limitations. First, the geometrical interpretation of axi-symmetric nature of the flow considered. The circular parent artery expands as described by the outer wall, then a conical surface, described by the inner radius, is inserted into this expansion. This is not quite the same thing as the splitting into two daughter arteries. Second, the bifurcation induces a swirl into the flow, which is not considered in this model, but does affect the wall shear stress distributions.

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