Mass Spring Damper - Control Design

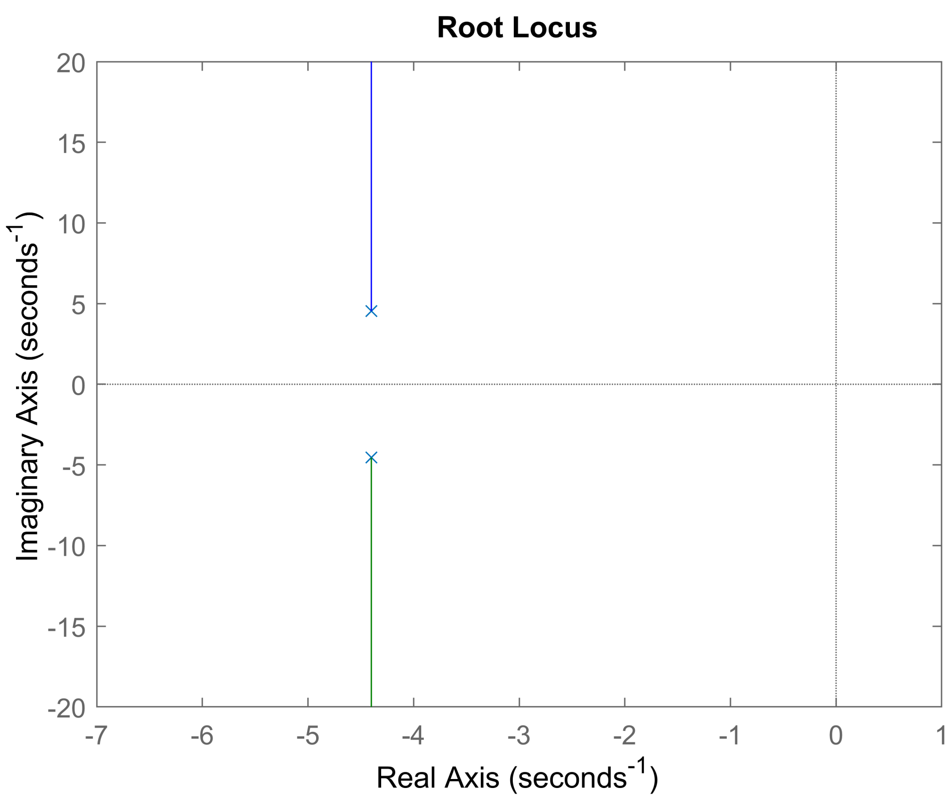
```
clear all
close all

m = 1;
b = 8.8;
k = 40;
s = tf('s');

G = 1/(m*s^2+b*s+k);
```

Plot the root locus of G:

```
rlocus(G);
```

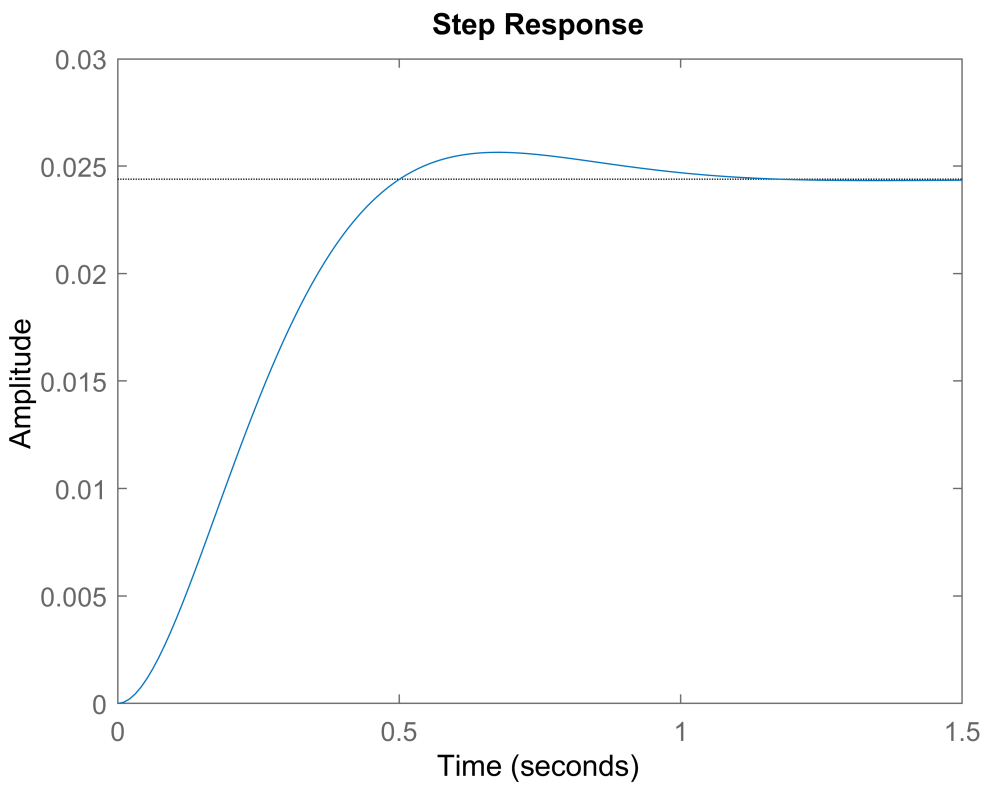


The closed loop system will be stable for any C(s)=K. However, the system will have non-zero steady state error to step inputs, i.e.,

$$\frac{1}{1+L(0)}=\frac{1}{1+K/40}$$

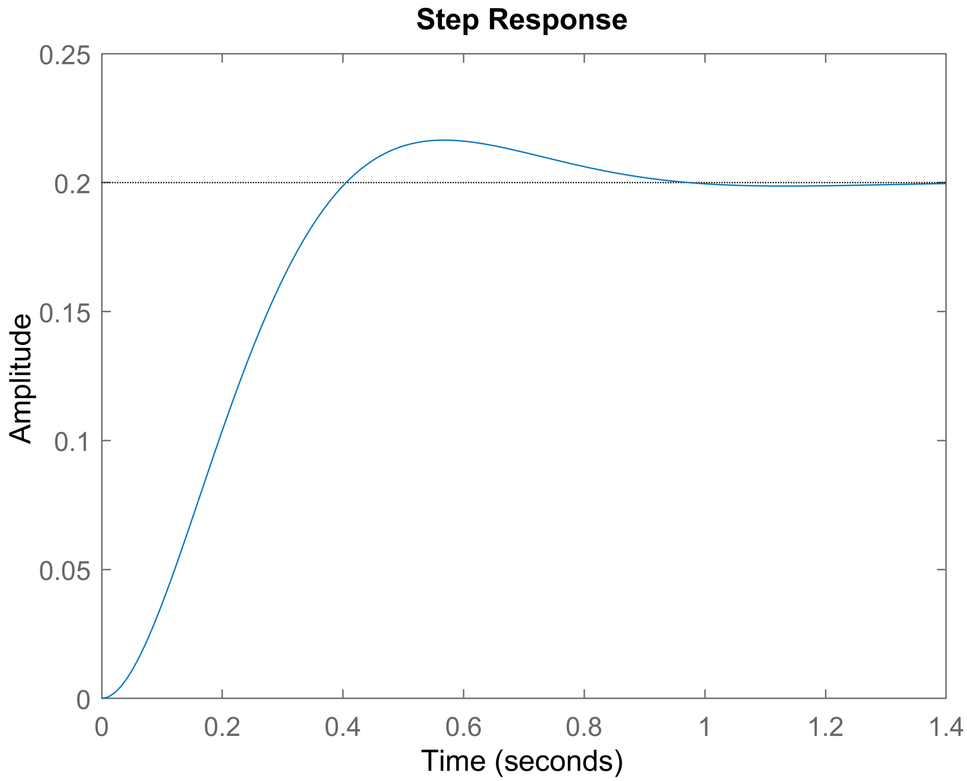
The larger K, the smaller the steady state error. To see this, let's compare

```
K = 1;
step(minreal(K*G/(1+K*G)));
```



and

```
K = 10;
step(minreal(K*G/(1+K*G)));
```



In order to have zero steady state error, we will need to add an integrator term. Let's try with a PI controller.

The PI controller is given by

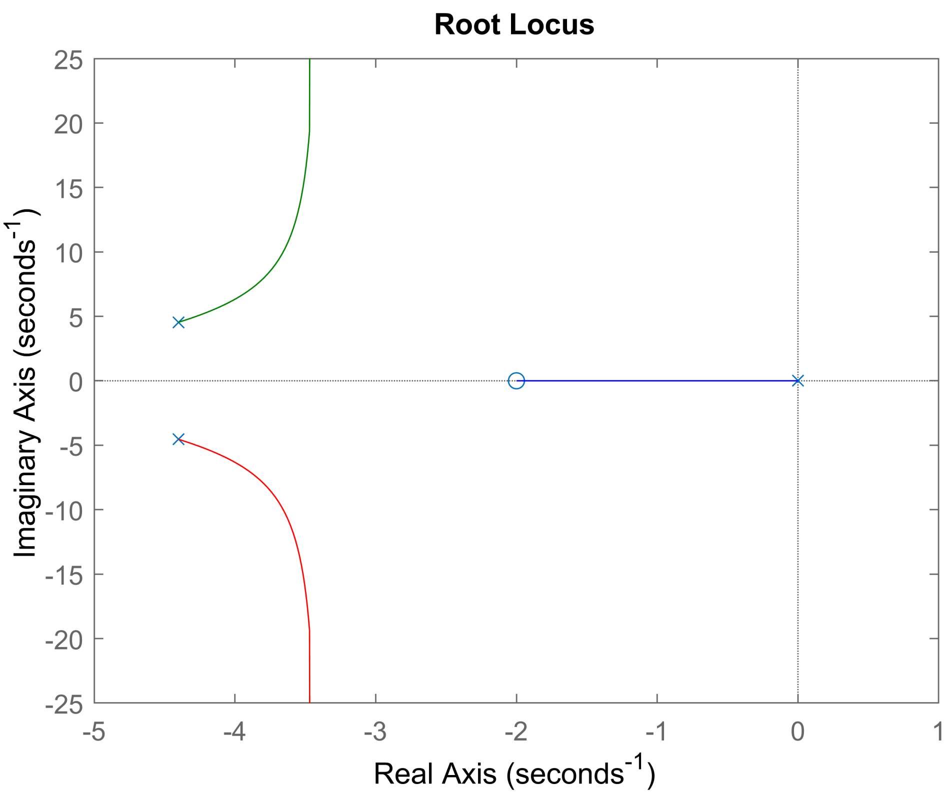
$$C(s) = k_p + \frac{k_i}{s} = k_p \frac{s+a}{s}$$

with

$$a = k_i / k_p$$

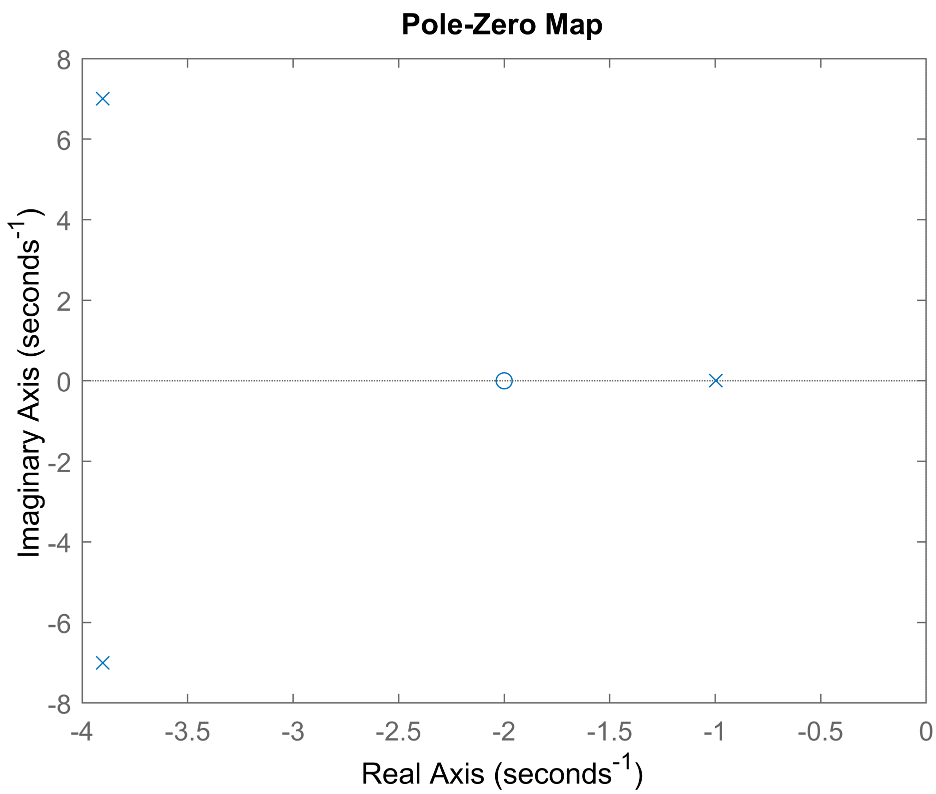
For example, let

```
kp = 1; ki = 2;
C = kp + ki/s;
rlocus(C*G);
```



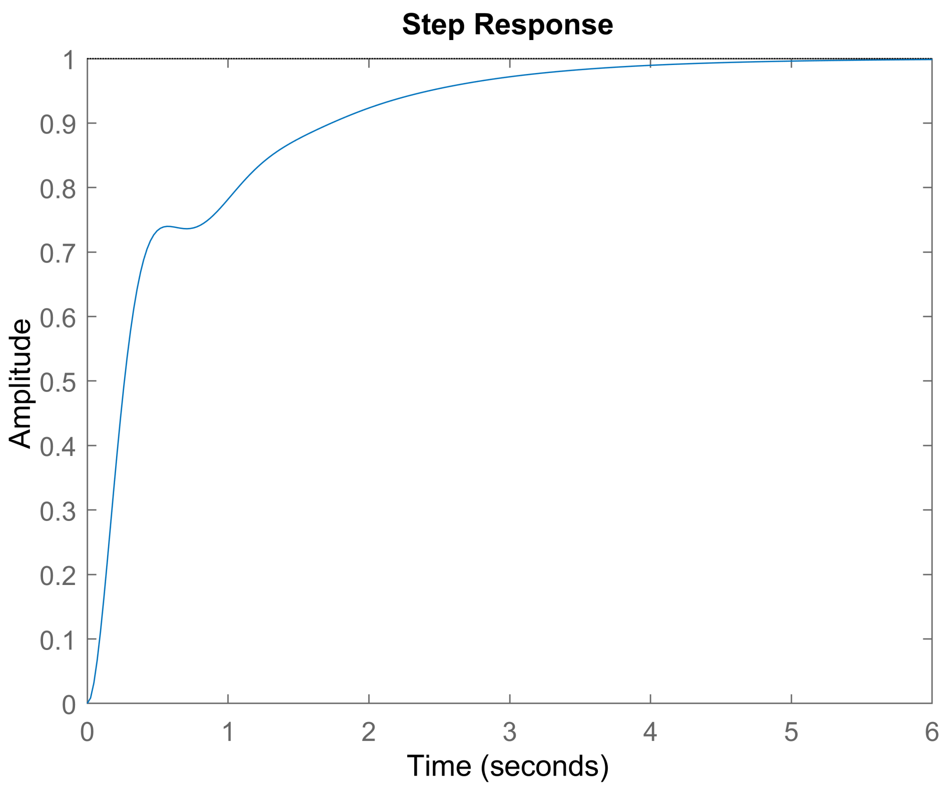
With a gain of 32 we should be able to bring the slowest pole at around -1

```
K = 32;
C = K*C;
pzplot(minreal(C*G/(1+C*G)));
```



Let's verify that the steady state error to step inputs is zero

```
step(minreal(C*G/(1+C*G)));
```


Step Response
Amplitude
Time (seconds)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6