Control design for inverted pendulum

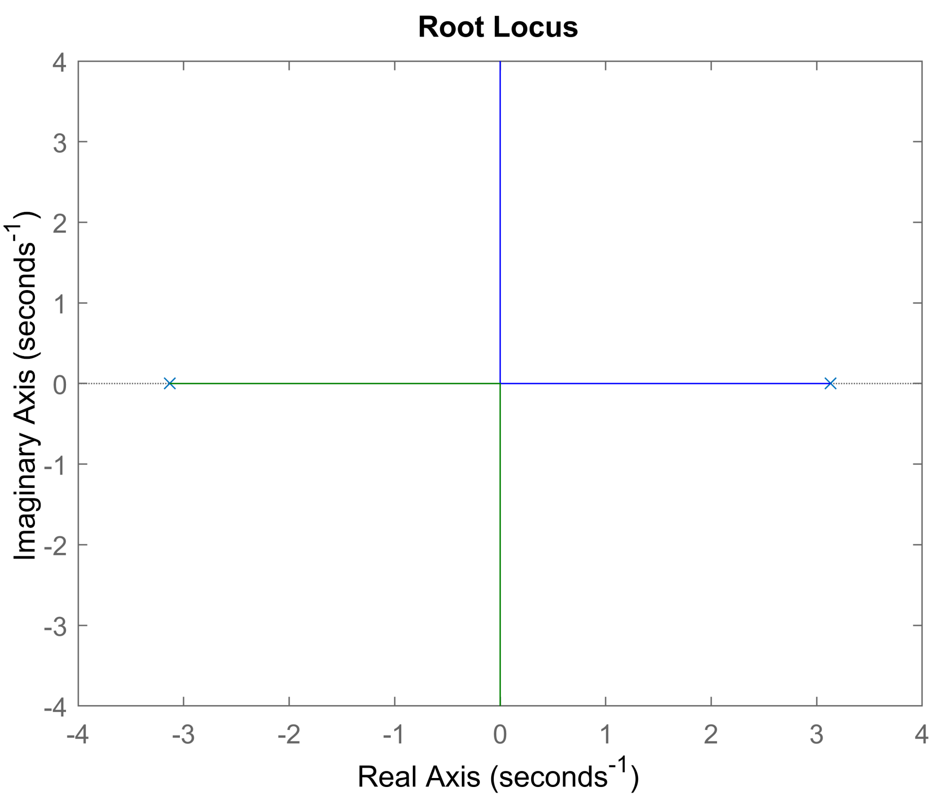
```
clear all
close all

g = 9.8;
L = 1;
s = tf('s');

G = 1/(s^2-g/L);
```

Plot the root locus of G:

```
rlocus(G);
```



Even a large C=K will not bring the poles to the LHP. Therefore, we need to design a dynamic compensator, i.e., a controller with poles/zeros and gain.

PD Controller Design

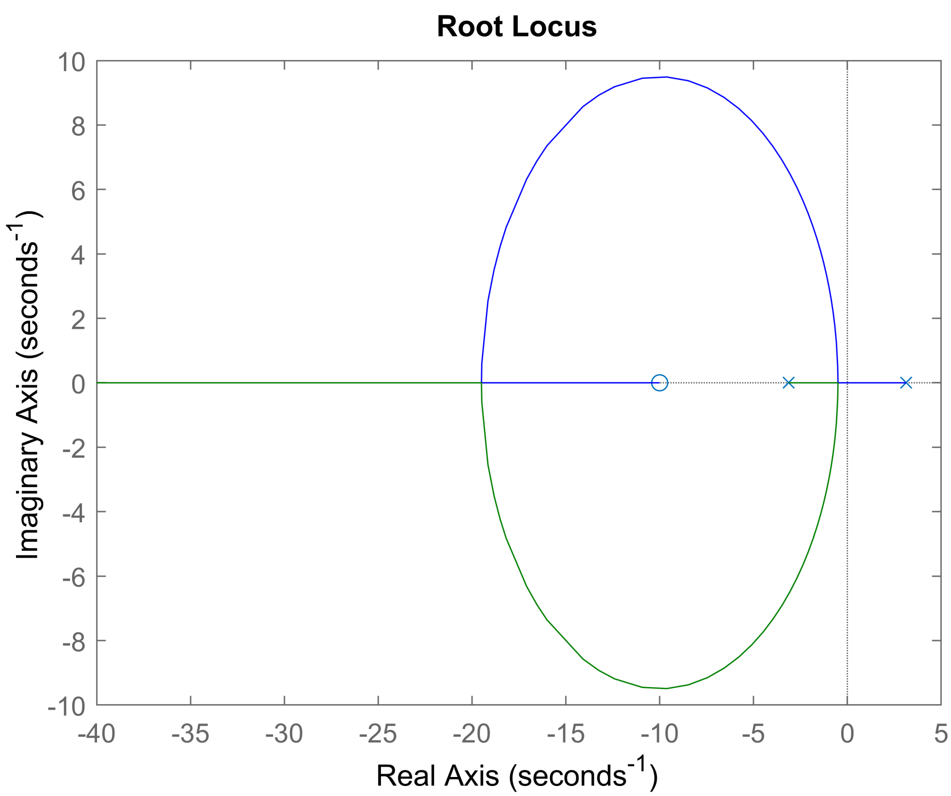
Let's try with a PD controller. The PD controller as introduced in class is given by

$$C(s) = k_p + k_d s = k_p\left(\frac{k_d}{k_p}s + 1\right) = k_p(\mu s + 1)$$

with

$$\mu = \frac{k_d}{k_p}.$$

```
kp = 1; mu = 0.1;
C = kp*(mu*s+1);
rlocus(C*G);
```

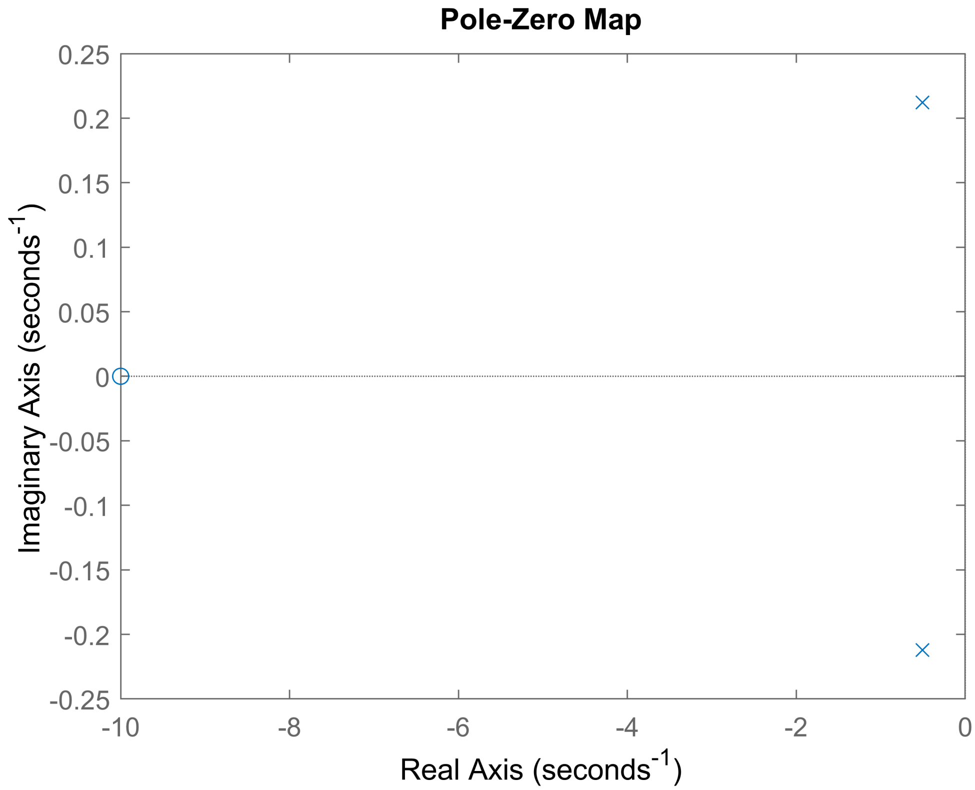


With a gain of 10.1, we will be able to place the closed loop poles at -0.5. Thus, we let

```
C = 10.1*C;
```

Now, we verify that the position of the closed loop poles is as desired

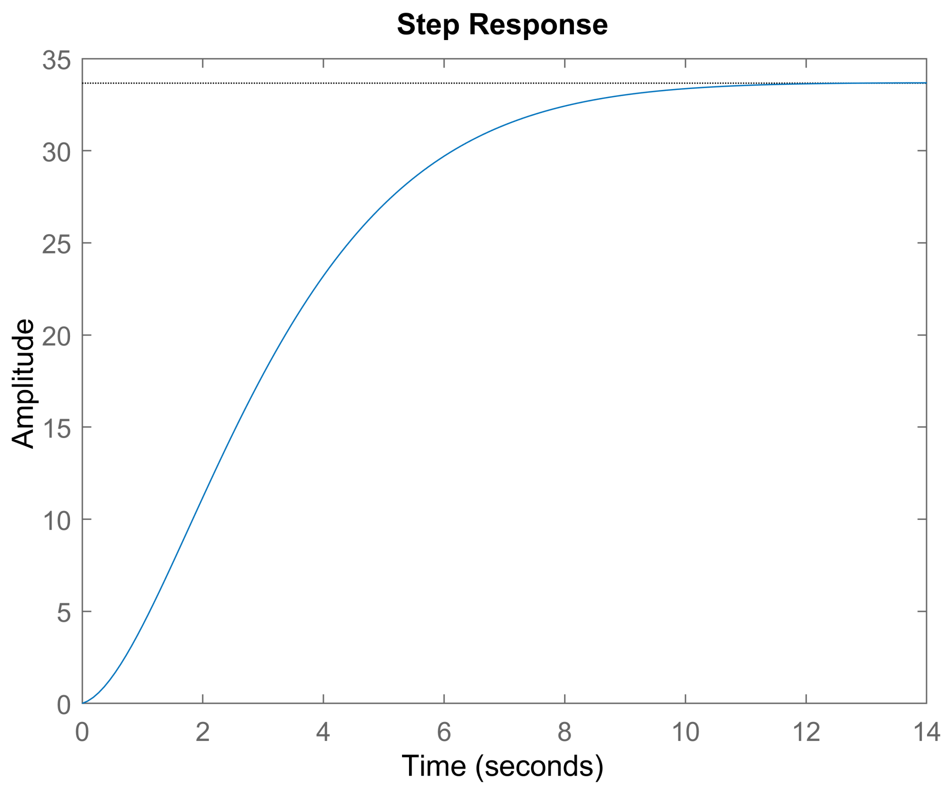
```
pzplot(minreal(C*G/(1+C*G)))
```



PID Controller Design

Assume that the objective is to design a controller that ensures zero steady state error to step inputs. The previous controller will not be able to ensure this, since the system G is type zero, and the controller C will not add any pole at the origin. Let's verify by plotting the step response of the closed loop system:

```
step(minreal(C*G/(1+C*G)))
```



As expected, there is a large steady state error. The final value (or the) can also be computed as follows

$$\lim_{s\to 0} s\frac{1}{1+L(s)}\frac{1}{s} = \frac{1}{1+\frac{1}{-g/L}10.1} = -32.68$$

To ensure zero steady state error to step input we need to increase the type of the system, i.e. we need to add an integrator. This is achieved by implementing a PID controller.

The PID controller is given by:

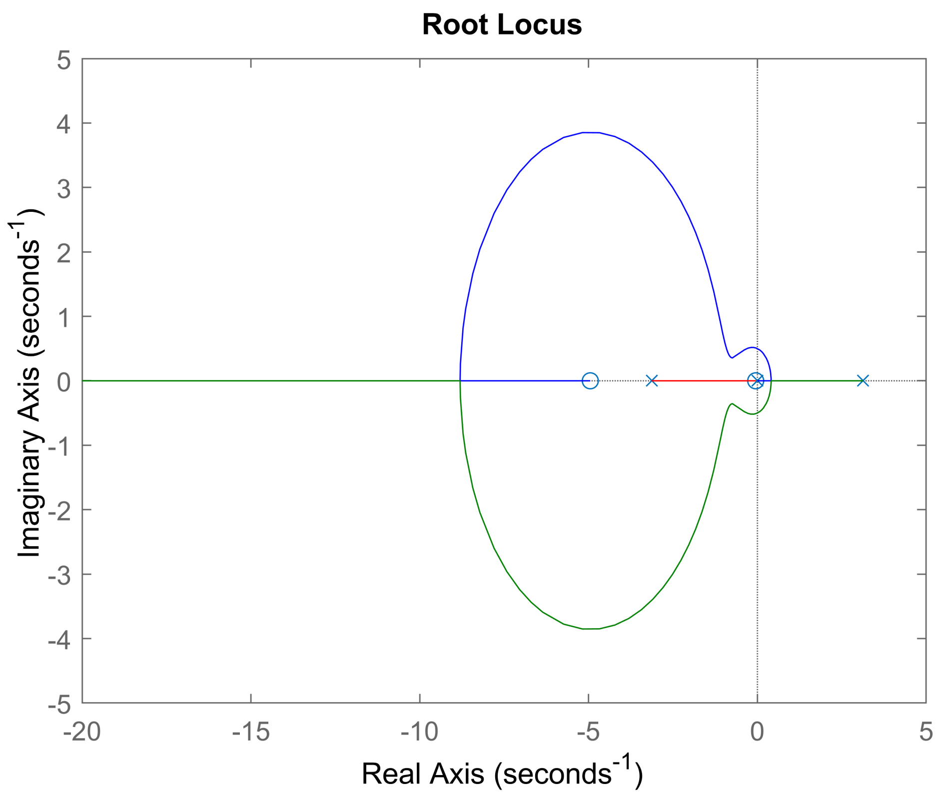
$$C(s) = k_p + k_d s + \frac{k_i}{s} = \frac{k_d s^2 + k_p s + k_i}{s} = k_d\frac{(s^2+as+b)}{s}$$

with

$$a = \frac{k_p}{k_d}, \qquad b = \frac{k_i}{k_d}$$

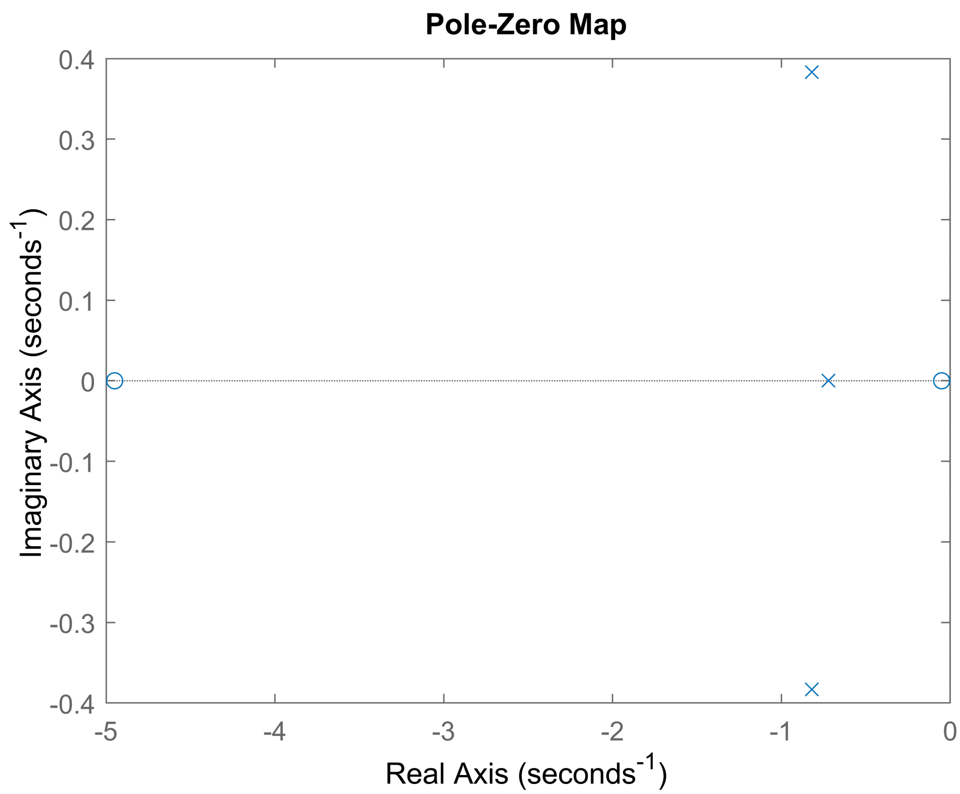
Let's try to design a controller, for example:

```
kp = 1; kd = 0.2; ki = 0.05;
C = kp+kd*s+ki/s;
rlocus(C*G);
```



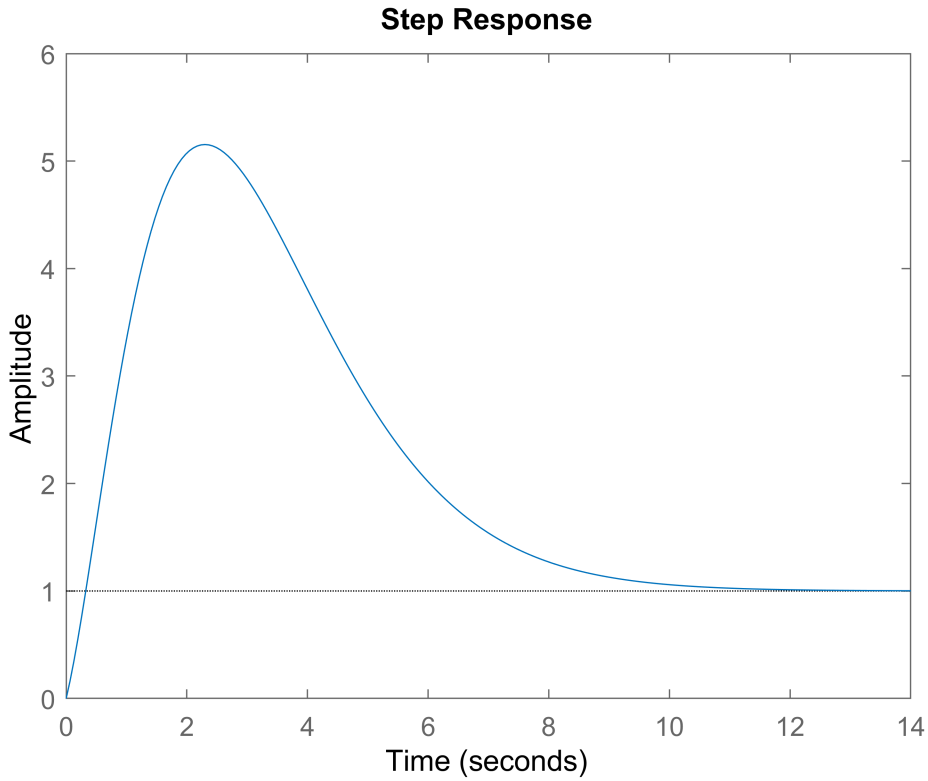
From the root locus, we conclude that by adding a gain of 11.8, the poles of the close loop system will come to the LHP.

```
C = 11.8*C;
pzplot(minreal(C*G/(1+C*G)));
```



The step response of the system will now look like

```
step(minreal(C*G/(1+C*G)));
```



demonstrating zero steady state error.