Example 1

Problem OCP:

$$\min_{x(t),u(t),t_f} \int_0^{t_f} 1 d\tau$$

subject to:

$$\dot{x}(t) = u(t), \quad x(0) = 3 \quad x(t_f) = 0 \quad u(t) \le v_{\max} \quad -u(t) \le -v_{\min}.$$

Implementation:

```
clear all
close all
```

Load Parameters

```
CONSTANTS.N = 5; % Order of approximation
CONSTANTS.vmax = 5;
CONSTANTS.vmin = -5;
CONSTANTS.x0 = 3;
CONSTANTS.xf = 0;
```

Initial Guess

```
N = CONSTANTS.N;
%x1 = linspace(CONSTANTS.x0,CONSTANTS.xf,CONSTANTS.N+1)';
x1 = ones(N+1,1);
u1 = ones(N+1,1);
tf = 5;
x0 = [x1;u1;tf];
```

Linear Constraints and UL bounds

```
A=[]; b=[]; Aeq=[]; beq=[]; lb=[]; ub=[];
```

Optimize (fmincon)

```
options = optimoptions(@fmincon,'Algorithm','sqp','MaxFunctionEvaluations',300000);
```

```
tic
[x,f] = fmincon(@(x)costfun(x,CONSTANTS),x0,A,b,Aeq,beq,lb,ub,@(x)nonlcon(x,CONSTANTS),options)
```

```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
```

```
toc
```

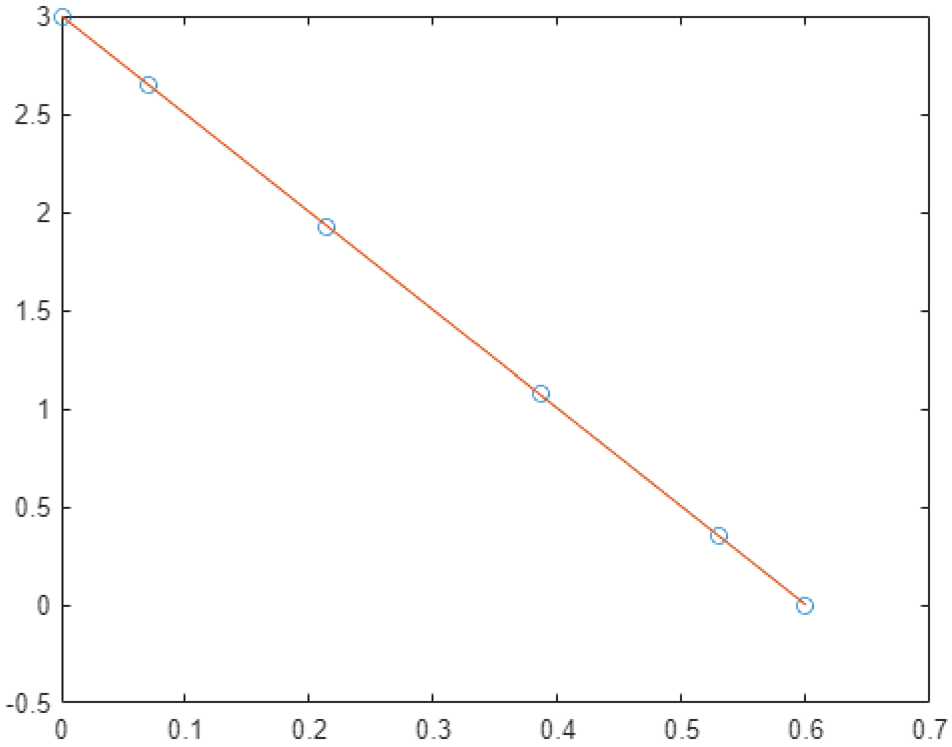
```
Elapsed time is 0.175672 seconds.
```

Plot

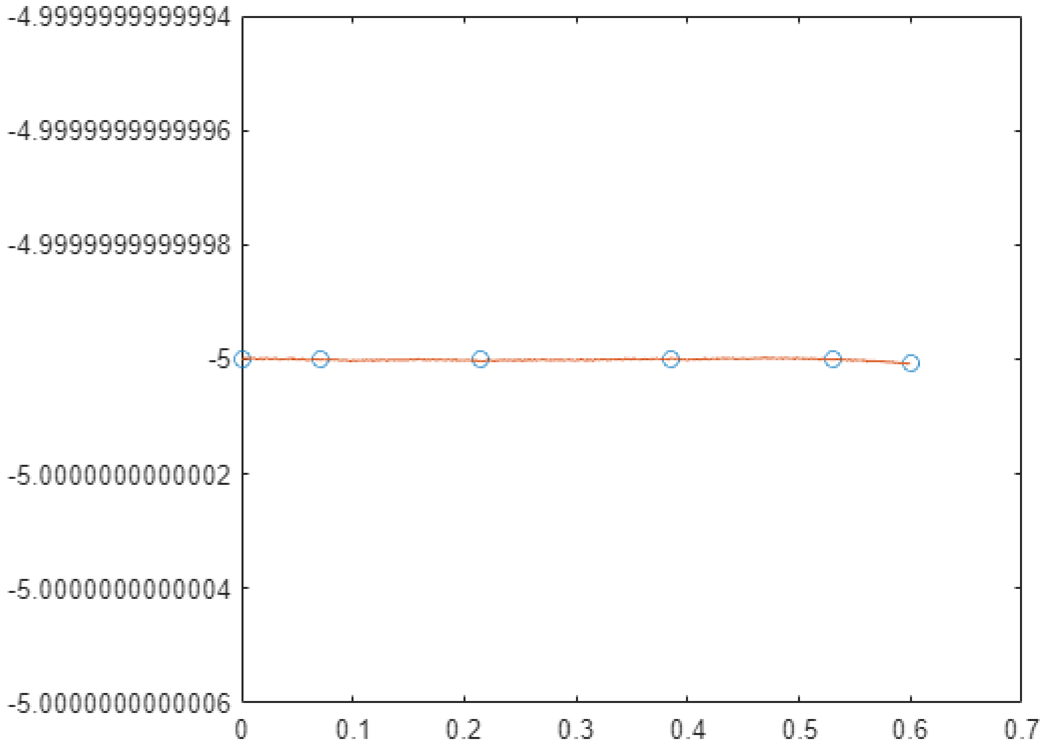
```
N = CONSTANTS.N;
x1 = x(1:N+1);
u1 = x(N+2:2*N+2);
tf = x(end);

[tnodes,w,Diff] = LGL_PS(N,tf);
t = 0:0.001:tf;

figure
plot(tnodes,x1,'o'); hold on
plot(t,LagrangePoly(x1,tnodes,t));
```



```
figure
plot(tnodes,u1,'o'); hold on
plot(t,LagrangePoly(u1,tnodes,t));
```



Cost function

```
function J = costfun(x,CONSTANTS)
N = CONSTANTS.N;

x1 = x(1:N+1);
u1 = x(N+2:2*N+2);
tf = x(end);

J = tf;
end
```

Nonlinear Constraint

```
function [c,ceq] = nonlcon(x,CONSTANTS)
%NONLCON Summary of this function goes here
%   Detailed explanation goes here
N = CONSTANTS.N;

x1 = x(1:N+1);
u1 = x(N+2:2*N+2);
tf = x(end);
```

```
[tnodes,w,Diff] = LGL_PS(CONSTANTS.N,tf);

dyn = x1'*Diff-u1';

nonlcon1 = u1' - CONSTANTS.vmax;
nonlcon2 = -u1' + CONSTANTS.vmin;

c=[nonlcon1'; nonlcon2';];
ceq=[dyn'; x1(1)-CONSTANTS.x0; x1(end)-CONSTANTS.xf;];
end
```