(see Chap. 2). From Eq. (6-15), the variation in inductor current is 1.6 A for each output current. Using the 6-A output current, the inductor must be rated for an rms current of

$$I_{L,\,\text{rms}} = \sqrt{6^2 + \left(\frac{0.8}{\sqrt{3}}\right)^2} = 6.02 \text{ A}$$

Note that the average inductor current would be a good approximation to the rms current since the variation is relatively small.

Using $L = 1\ \mu$H in Eq. (6-20), the minimum capacitance is determined as

$$C = \frac{1 - D}{8L(\Delta V_o / V_o)f^2} = \frac{1 - 0.364}{8(1)(10)^{-6}(0.02)(500{,}000)^2} = 0.16\ \mu\text{F}$$

The allowable output voltage ripple of 2 percent is $(0.02)(1.2) = 24$ mV. The maximum ESR is computed from Eq. (6-23).

$$\Delta V_o \approx r_C \Delta i_C = r_C \Delta i_L$$

or

$$r_C = \frac{\Delta V_o}{\Delta i_C} = \frac{24 \text{ mV}}{1.6\ A} = 15 \text{ m}\Omega$$

At this point, the designer would search manufacturer's specifications for a capacitor having 15-mΩ ESR. The capacitor may have to be much larger than the calculated value of 0.16 μF to meet the ESR requirement. Peak capacitor current is $\Delta i_L/2 = 0.8$ A, and rms capacitor current for the triangular waveform is $0.8/\sqrt{3} = 0.46$ A.

6.5 THE BOOST CONVERTER

The boost converter is shown in Fig. 6-8. This is another switching converter that operates by periodically opening and closing an electronic switch. It is called a boost converter because the output voltage is larger than the input.

Voltage and Current Relationships

The analysis assumes the following:

1. Steady-state conditions exist.
2. The switching period is T, and the switch is closed for time DT and open for $(1-D)T$.
3. The inductor current is continuous (always positive).
4. The capacitor is very large, and the output voltage is held constant at voltage V_o.
5. The components are ideal.

The analysis proceeds by examining the inductor voltage and current for the switch closed and again for the switch open.

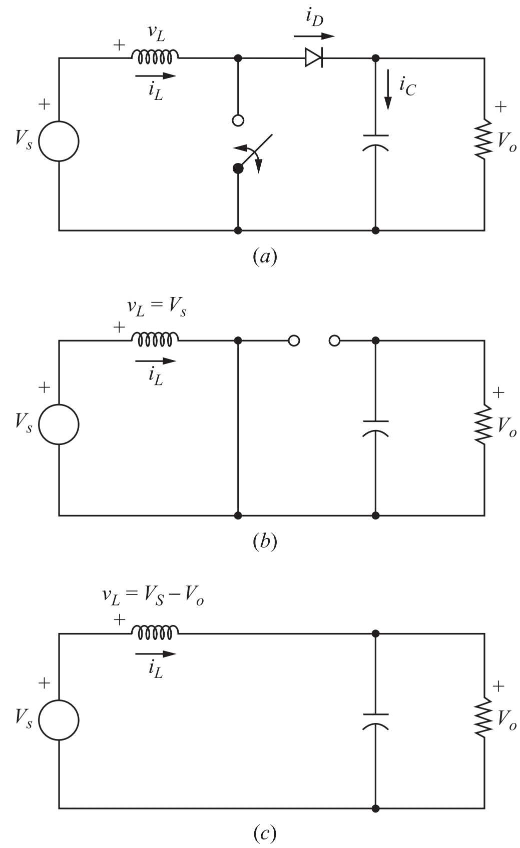

Figure 6-8 The boost converter. (*a*) Circuit; (*b*) Equivalent circuit for the switch closed; (*c*) Equivalent circuit for the switch open.

Analysis for the Switch Closed When the switch is closed, the diode is reverse-biased. Kirchhoff's voltage law around the path containing the source, inductor, and closed switch is

$$v_L = V_s = L\frac{di_L}{dt} \qquad \text{or} \qquad \frac{di_L}{dt} = \frac{V_s}{L} \tag{6-24}$$

The rate of change of current is a constant, so the current increases linearly while the switch is closed, as shown in Fig. 6-9*b*. The change in inductor current is computed from

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L}$$

Solving for Δi_L for the switch closed,

$$(\Delta i_L)_{\text{closed}} = \frac{V_s DT}{L} \tag{6-25}$$

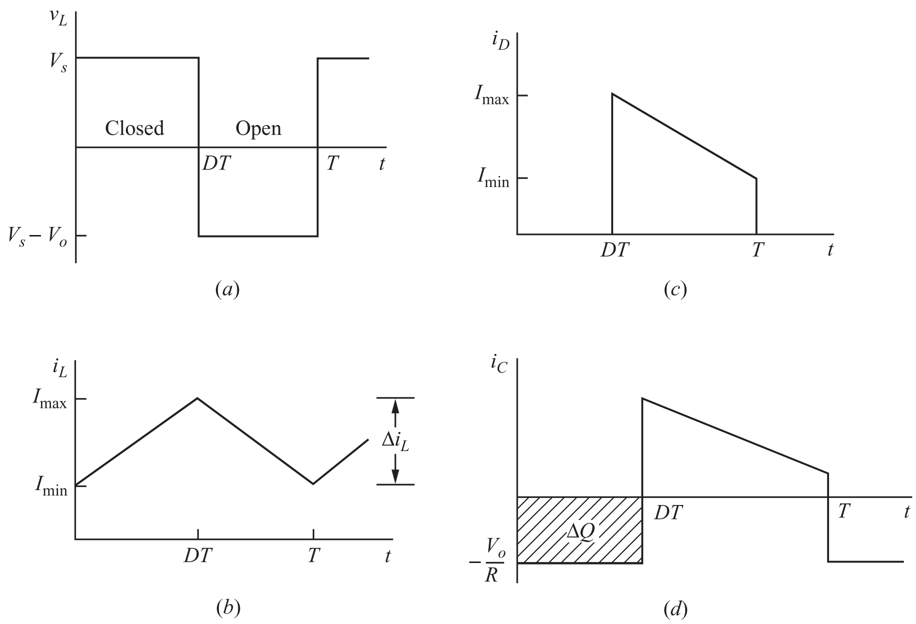

Figure 6-9 Boost converter waveforms. (*a*) Inductor voltage; (*b*) Inductor current; (*c*) Diode current; (*d*) Capacitor current.

Analysis for the Switch Open When the switch is opened, the inductor current cannot change instantaneously, so the diode becomes forward-biased to provide a path for inductor current. Assuming that the output voltage V_o is a constant, the voltage across the inductor is

$$v_L = V_s - V_o = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} = \frac{V_s - V_o}{L}$$

The rate of change of inductor current is a constant, so the current must change linearly while the switch is open. The change in inductor current while the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L}$$

Solving for Δi_L,

$$(\Delta i_L)_{\text{open}} = \frac{(V_s - V_o)(1-D)T}{L} \tag{6-26}$$

For steady-state operation, the net change in inductor current must be zero. Using Eqs. (6-25) and (6-26),

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\frac{V_s DT}{L} + \frac{(V_s - V_o)(1 - D)T}{L} = 0$$

Solving for V_o,

$$V_s(D + 1 - D) - V_o(1 - D) = 0$$

$$\boxed{V_o = \frac{V_s}{1 - D}} \tag{6-27}$$

Also, the average inductor voltage must be zero for periodic operation. Expressing the average inductor voltage over one switching period,

$$V_L = V_s D + (V_s - V_o)(1 - D) = 0$$

Solving for V_o yields the same result as in Eq. (6-27).

Equation (6-27) shows that if the switch is always open and D is zero, the output voltage is the same as the input. As the duty ratio is increased, the denominator of Eq. (6-27) becomes smaller, resulting in a larger output voltage. *The boost converter produces an output voltage that is greater than or equal to the input voltage.* However, the output voltage cannot be less than the input, as was the case with the buck converter.

As the duty ratio of the switch approaches 1, the output voltage goes to infinity according to Eq. (6-27). However, Eq. (6-27) is based on ideal components. Real components that have losses will prevent such an occurrence, as shown later in this section. Figure 6-9 shows the voltage and current waveforms for the boost converter.

The average current in the inductor is determined by recognizing that the average power supplied by the source must be the same as the average power absorbed by the load resistor. Output power is

$$P_o = \frac{V_o^2}{R} = V_o I_o$$

and input power is $V_s I_s = V_s I_L$. Equating input and output powers and using Eq. (6-27),

$$V_s I_L = \frac{V_o^2}{R} = \frac{[V_s/(1 - D)]^2}{R} = \frac{V_s^2}{(1 - D)^2 R}$$

By solving for average inductor current and making various substitutions, I_L can be expressed as

$$\boxed{I_L = \frac{V_s}{(1-D)^2R} = \frac{V_o^2}{V_sR} = \frac{V_oI_o}{V_s}} \tag{6-28}$$

Maximum and minimum inductor currents are determined by using the average value and the change in current from Eq. (6-25).

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2R} + \frac{V_sDT}{2L} \tag{6-29}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2R} - \frac{V_sDT}{2L} \tag{6-30}$$

Equation (6-27) was developed with the assumption that the inductor current is continuous, meaning that it is always positive. A condition necessary for continuous inductor current is for $I_{\min}$ to be positive. Therefore, the boundary between continuous and discontinuous inductor current is determined from

$$I_{\min} = 0 = \frac{V_s}{(1-D)^2R} - \frac{V_sDT}{2L}$$

or

$$\frac{V_s}{(1-D)^2R} = \frac{V_sDT}{2L} = \frac{V_sD}{2Lf}$$

The minimum combination of inductance and switching frequency for continuous current in the boost converter is therefore

$$(Lf)_{\min} = \frac{D(1-D)^2R}{2} \tag{6-31}$$

or

$$\boxed{L_{\min} = \frac{D(1-D)^2R}{2f}} \tag{6-32}$$

A boost converter designed for continuous-current operation will have an inductor value greater than $L_{\min}$.

From a design perspective, it is useful to express L in terms of a desired Δi_L,

$$L = \frac{V_sDT}{\Delta i_L} = \frac{V_sD}{\Delta i_L f} \tag{6-33}$$

Output Voltage Ripple

The preceding equations were developed on the assumption that the output voltage was a constant, implying an infinite capacitance. In practice, a finite capacitance will result in some fluctuation in output voltage, or ripple.

The peak-to-peak output voltage ripple can be calculated from the capacitor current waveform, shown in Fig. 6-9*d*. The change in capacitor charge can be calculated from

$$|\Delta Q| = \left(\frac{V_o}{R}\right)DT = C\Delta V_o$$

An expression for ripple voltage is then

$$\Delta V_o = \frac{V_o DT}{RC} = \frac{V_o D}{RCf}$$

or

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{D}{RCf}} \tag{6-34}$$

where f is the switching frequency. Alternatively, expressing capacitance in terms of output voltage ripple yields

$$C = \frac{D}{R(\Delta V_o/V_o)f} \tag{6-35}$$

As with the buck converter, equivalent series resistance of the capacitor can contribute significantly to the output voltage ripple. The peak-to-peak variation in capacitor current (Fig. 6-9) is the same as the maximum current in the inductor. The voltage ripple due to the ESR is

$$\Delta V_{o,\mathrm{ESR}} = \Delta i_C r_C = I_{L,\max} r_C \tag{6-36}$$

EXAMPLE 6-4

Boost Converter Design 1

Design a boost converter that will have an output of 30 V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50 Ω. Assume ideal components for this design.

■ **Solution**

First, determine the duty ratio from Eq. (6-27),

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

If the switching frequency is selected at 25 kHz to be above the audio range, then the minimum inductance for continuous current is determined from Eq. (6-32).

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\mathrm{H}$$

To provide a margin to ensure continuous current, let $L = 120\ \mu$H. Note that L and f are selected somewhat arbitrarily and that other combinations will also give continuous current.

Using Eqs. (6-28) and (6-25),

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5\ \mathrm{A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2 \text{ A}$$

$$I_{\max} = 1.5 + 1.2 = 2.7 \text{ A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3 \text{ A}$$

The minimum capacitance required to limit the output ripple voltage to 1 percent is determined from Eq. (6-35).

$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48 \ \mu\text{F}$$

EXAMPLE 6-5

Boost Converter Design 2

A boost converter is required to have an output voltage of 8 V and supply a load current of 1 A. The input voltage varies from 2.7 to 4.2 V. A control circuit adjusts the duty ratio to keep the output voltage constant. Select the switching frequency. Determine a value for the inductor such that the variation in inductor current is no more than 40 percent of the average inductor current for all operating conditions. Determine a value of an ideal capacitor such that the output voltage ripple is no more than 2 percent. Determine the maximum capacitor equivalent series resistance for a 2 percent ripple.

■ Solution

Somewhat arbitrarily, choose 200 kHz for the switching frequency. The circuit must be analyzed for both input voltage extremes to determine the worst-case condition. For $V_s = 2.7$ V, the duty ratio is determined from Eq. (6-27).

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{2.7}{8} = 0.663$$

Average inductor current is determined from Eq. (6-28).

$$I_L = \frac{V_o I_o}{V_s} = \frac{8(1)}{2.7} = 2.96 \text{ A}$$

The variation in inductor current to meet the 40 percent specification is then $\Delta i_L = 0.4(2.96) = 1.19$ A. The inductance is then determined from Eq. (6-33).

$$L = \frac{V_s D}{\Delta i_L f} = \frac{2.7(0.663)}{1.19(200{,}000)} = 7.5 \ \mu\text{H}$$

Repeating the calculations for $V_s = 4.2$V,

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{4.2}{8} = 0.475$$

$$I_L = \frac{V_o I_o}{V_s} = \frac{8(1)}{4.2} = 1.90 \text{ A}$$

The variation in inductor current for this case is $\Delta i_L = 0.4(1.90) = 0.762$ A, and

$$L = \frac{V_s D}{\Delta i_L f} = \frac{4.2(0.475)}{0.762(200{,}000)} = 13.1\ \mu\text{H}$$

The inductor must be 13.1 μH to satisfy the specifications for the total range of input voltages.

Equation (6-35), using the maximum value of D, gives the minimum capacitance as

$$C = \frac{D}{R(\Delta V_o/V_o)f} = \frac{D}{(V_o/I_o)(\Delta V_o/V_o)f} = \frac{0.663}{(8/1)(0.02)(200{,}000)} = 20.7\ \mu\text{F}$$

The maximum ESR is determined from Eq. (6-36), using the maximum peak-to-peak variation in capacitor current. The peak-to-peak variation in capacitor current is the same as maximum inductor current. The average inductor current varies from 2.96 A at $V_s = 2.7$ V to 1.90 A at $V_s = 4.2$ V. The variation in inductor current is 0.762 A for $V_s = 4.2$ A, but it must be recalculated for $V_s = 2.7$ V using the 13.1-μH value selected, yielding

$$\Delta i_L = \frac{V_s D}{Lf} = \frac{2.7(0.663)}{13.1(10)^{-6}(200{,}000)} = 0.683\ \text{A}$$

Maximum inductor current for each case is then computed as

$$I_{L,\max,2.7\text{V}} = I_L + \frac{\Delta i_L}{2} = 2.96 + \frac{0.683}{2} = 3.30\ \text{A}$$

$$I_{L,\max,4.2\text{V}} = I_L + \frac{\Delta i_L}{2} = 1.90 + \frac{0.762}{2} = 2.28\ \text{A}$$

This shows that the largest peak-to-peak current variation in the capacitor will be 3.30 A. The output voltage ripple due to the capacitor ESR must be no more than (0.02)(8) = 0.16 V. Using Eq. (6-36),

$$\Delta V_{o,\text{ESR}} = \Delta i_C r_C = I_{L,\max} r_C = 3.3 r_C = 0.16\ \text{V}$$

which gives

$$r_C = \frac{0.16\ \text{V}}{3.3\ \text{A}} = 48\ \text{m}\Omega$$

In practice, a capacitor that has an ESR of 48 mΩ or less could have a capacitance value much larger than the 20.7 μF calculated.

Inductor Resistance

Inductors should be designed to have small resistance to minimize power loss and maximize efficiency. The existence of a small inductor resistance does not substantially change the analysis of the buck converter as presented previously in this chapter. However, inductor resistance affects performance of the boost converter, especially at high duty ratios.

For the boost converter, recall that the output voltage for the ideal case is

$$V_o = \frac{V_s}{1 - D} \tag{6-37}$$

To investigate the effect of inductor resistance on the output voltage, assume that the inductor current is approximately constant. The source current is the same as the inductor current, and average diode current is the same as average load current. The power supplied by the source must be the same as the power absorbed by the load and the inductor resistance, neglecting other losses.

$$\begin{gathered} P_s = P_o + P_{r_L} \\ V_s I_L = V_o I_D + I_L^2 r_L \end{gathered} \tag{6-38}$$

where r_L is the series resistance of the inductor. The diode current is equal to the inductor current when the switch is off and is zero when the switch is on. Therefore, the average diode current is

$$I_D = I_L(1 - D) \tag{6-39}$$

Substituting for I_D into Eq. (6-38),

$$V_s I_L = V_o I_L (1 - D) + I_L^2 r_L$$

which becomes

$$V_s = V_o(1 - D) + I_L r_L \tag{6-40}$$

In terms of V_o from Eq. (6-39), I_L is

$$I_L = \frac{I_D}{1 - D} = \frac{V_o/R}{1 - D} \tag{6-41}$$

Substituting for I_L into Eq. (6-40),

$$V_s = \frac{V_o r_L}{R(1 - D)} + V_o(1 - D)$$

Solving for V_o,

$$\boxed{V_o = \left(\frac{V_s}{1 - D}\right)\left(\frac{1}{1 + r_L/[R(1 - D)^2]}\right)} \tag{6-42}$$

The preceding equation is similar to that for an ideal converter but includes a correction factor to account for the inductor resistance. Figure 6-10*a* shows the output voltage of the boost converter with and without inductor resistance.

The inductor resistance also has an effect on the power efficiency of converters. Efficiency is the ratio of output power to output power plus losses. For the boost converter

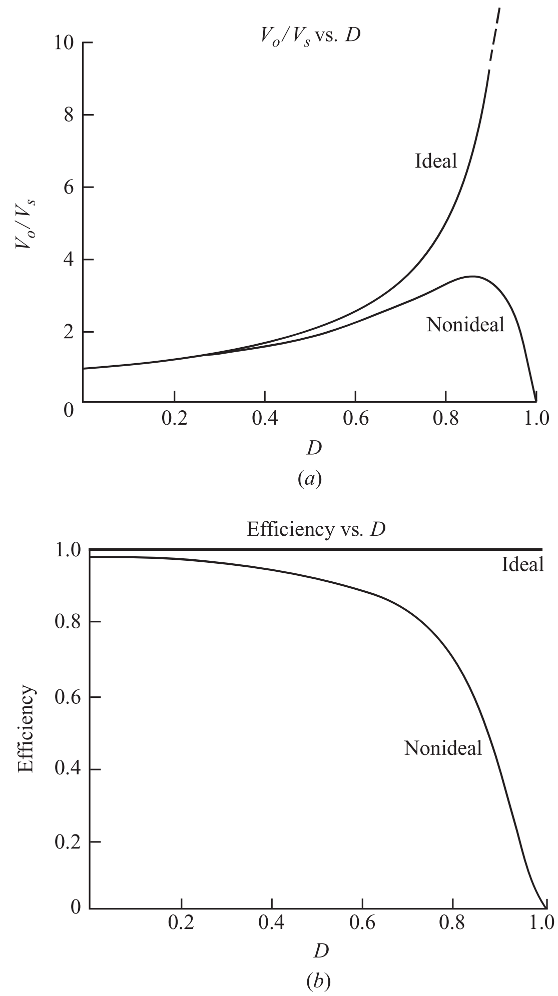

Figure 6-10 Boost converter for a nonideal inductor. (a) Output voltage; (b) Boost converter efficiency.

$$\eta = \frac{P_o}{P_o + P_{\text{loss}}} = \frac{V_o^2/R}{V_o^2/R + I_L^2 r_L} \tag{6-43}$$

Using Eq. (6-41) for I_L,

$$\eta = \frac{V_o^2/R}{V_o^2/R + (V_o/R)^2/(1-D)r_L} = \frac{1}{1 + r_L[R(1-D)^2]} \tag{6-44}$$

As the duty ratio increases, the efficiency of the boost converter decreases, as indicated in Fig. 6-10b.