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A macroscopic taxi model for passenger demand, taxi utilization and level of services

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Abstract. In most urban areas taxi services are subject to various types of regulation such as entry restriction and price control. However, effective intervention depends on generating and using suitable information on the demand-supply equilibrium of the taxi market. This paper develops a simultaneous equation system of passenger demand, taxi utilization and level of services based on a taxi service situation found in the urban area of Hong Kong over the last ten years. A set of variables is introduced including number of licensed taxis, taxi fare, disposable income, occupied taxi journey time as exogenous variables and daily taxi passenger demand, passenger waiting time, taxi availability, taxi utilization and average taxi waiting time as endogenous variables. These variables are coupled together through a system of nonlinear simultaneous equations whose parameters are estimated from survey data. The simultaneous equation system can be used to obtain useful regulatory information to assist with the decisions concerning the restriction over the number of taxi licenses and the fixing of the taxi fare structure as well as a range of service quality control.

1. Introduction

In most major cities, taxis are one of the important means of transport and provide flexible and convenient service. In general taxi services are subject to various types of regulation such as entry restriction and fare control. Therefore, one of the important issues in urban taxi services is the determination of licence quantities and fares. However, effective intervention depends on generating and using suitable regulatory information. This should involve an evaluation of the impacts of such intervention on the demand and supply and the level of taxi services. In a realistic taxi market in which demand and supply are interconnected and take place over space and time, such a modeling and evaluation study are challenging and intriguing.

In fact, the economic consequences of taxi regulatory restraints have been examined by many economists in different manners (for most recent studies see, for example, Cairns & Liston-Heyes 1996; Arnott 1996). Those theoretical studies aim to understand the manner in which the demand and supply are equilibrated in the presence of such regulations. However, the analytical models developed so far are highly aggregate and simplified without consideration of the spatial structure of the market. Only until recently Yang and Wong (1998a, 1998b) and Wong and Yang (1998) developed a mathematical model to characterize demand-supply equilibrium of taxi services in a network context. Their model takes explicit account of passenger origin-destination demand pattern and both vacant and occupied taxi movements on the road network.

In the taxi market, the equilibrium quantity (total taxi-hours) of service supplied will be greater than the equilibrium quantity (occupied taxi-hours) demanded by a certain amount of slack (vacant taxi-hours). It is this amount of slack that governs the average passenger waiting time. The expected passenger waiting time is generally considered as an important value or quality of the services received by passengers. This variable affects passengers' decision as to whether or not to take a taxi, and thus plays a crucial role in the determination of the price level and the resulting equilibrium of the market. Furthermore, the demand for and supply of taxi services are interrelated through two intervening variables: taxi availability (as measured by expected passenger waiting time or approximately by vacant taxi headway observed on the roadsides) and taxi utilization (as measured by expected fraction of time that a taxi is occupied or approximately by the fraction of occupied taxis observed on the roadsides). On the demand side, potential passengers will consider taxi availability as well as fare in making their mode-choice decisions. From the supply perspective, taxi firms will operate in response to taxi utilization rate as well as trip revenues and costs. Moreover, taxi availability, through its influence on the level of taxi use, indirectly affects the taxi utilization rate; the utilization rate, through its influence on the level of supply, in turn affects taxi availability. This demand-availability-utilization-supply relation is shown in Figure 1 (Manski & Wright 1976).

This paper is concerned with the modeling of the complex demand-availability-utilization-supply relationship in a taxi market. We develop a system of nonlinear simultaneous equations for passenger demand, taxi utilization and level of services based on a taxi service situation observed in the urban area of Hong Kong over the last ten years. The main objectives of this study are: 1) to establish a system of simultaneous equations based on the data obtained from the Hong Kong Transport Department making use of the concept of queuing theory and the concept of demand and supply equilibrium; and 2) to predict the level of taxi service and conduct sensitivity analysis for the introduction of any new taxi policies by using the established system of equations.

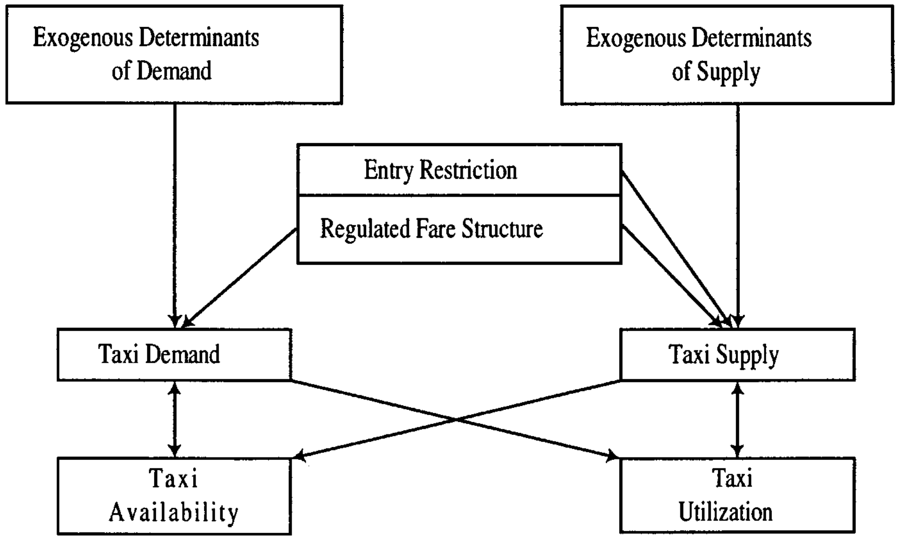

Figure 1. The demand-availability-utilization-supply relation in a taxi market (Source: Manski & Wright 1976).

A set of variables is introduced including number of licensed taxis, taxi fare, disposable income, population, occupied taxi journey time as exogenous variables and daily passenger taxi trips, passenger waiting time, taxi availability, taxi utilization and average taxi waiting time as endogenous variables. These variables are coupled together through a system of nonlinear simultaneous equations whose parameters are calibrated from survey data. The proposed model can predict a number of system performance measures such as utilization rate for taxi and level of service quality, and predict the effects of alternative regulations on system performance. The model can thus be used as a policy tool by regulator to ascertain appropriate taxi regulations such as selection of taxi fleet size and fare structure.

2. Taxi services in Hong Kong

Hong Kong, with a land area of only 1091 square kilometers of which about 15 percent are built-up areas, has a population of more than six million people. Currently 15250 urban taxis, 2838 New Territories taxis and 40 Lantau taxis carry around 1.3 million passengers per day. In the urban area of Hong Kong, taxis make considerable demands on limited road space even when empty and cruising for passengers. Taxis currently form about 25% of the traffic stream overall, in some locations such as Central and Waterloo Road in

Kowloon, taxis form as much as 50% to 60% of the traffic stream during the peak hours (Transport Department 1989, 1993). The urban taxis operate throughout the territory, while the others are fundamentally confined to the rural areas of the New Territories and Lantau Island. In addition to this service area demarcation, the operations of taxis are subject to various regulations as described below (Hong Kong Transport Advisory Committee 1992).

Ownership of a taxi

The ownership of some taxis belongs to several taxi companies while the others belong to the taxi drivers themselves. If a taxi driver cannot afford to own a taxi, he or she may hire one from a taxi company or from a taxi owner. The hire rate is around HK$280–HK$310 per shift. Normally each taxi is operated for two shifts per day and each shift lasts eight to twelve hours. Most of the taxis have a radio-dispatch service, taxi drivers can easily get traffic information from the radio control center.

Service area

The Hong Kong Government sets some regulations on taxi operations, such as limiting New Territories taxis to carry passenger only as far as Tsuen Wan (the western part of Kowloon) or to Shatin (the northern part of Kowloon). They cannot pick up passengers in the urban area. On the other hand, urban taxis can pick up passenger anywhere, either in urban areas or in the New Territories. Furthermore, hiring of taxis and drop-off of passengers are restricted at some prescribed taxi stands in the urban area. This restriction is imposed to avoid interruption to smooth road traffic flow and reduction of limited road capacity due to kerb-side hiring and drop-off. It also avoids unnecessary circulation of empty taxis in the congested urban area.

Taxi licensing system

Taxi licenses have been issued by public tender at a premium since 1964. Taxi licenses are transferable and provide the holders a perpetual operating right. Decision as to whether more taxi licenses should be tendered out (and, if so, how many) is mainly based on annual taxi service survey results with reference to waiting time, queue length and headway between vacant taxis. The impact of additional taxis on the viability of the taxi trade is also taken into account. The current policy is to issue no more than 400 urban taxi licenses every two years subject to further review. Additionally, up to 100 New Territory taxi licenses were issued in 1991–1992, whereas the number of Lantau taxis licenses remain frozen.

The licensing system allows each tender to apply for at most one license in order to prevent monopoly of taxi licenses. The tenders are required to produce documentary proof of financial arrangements to discourage speculative bids. The payment of a license premium can be by installments in order to make ownership more affordable to taxi drivers.

Existing taxi fare policy

It is the government policy to maintain a reasonable fare difference between taxis and other modes of public transport as a measure to regulate the demand for taxis without overburdening limited road space. The existing policy of maintaining a fare difference from other modes of public transport by 5–7 times for urban taxis and 3–4 times for New Territories taxis was endorsed by the Transport Advisory Committee in 1988. In Hong Kong, taxi fares are revised constantly subject to the approval from the government, based on the recommendation of the Transport Advisory Committee. Taxi fares comprise several components: flagfall, incremental charge, waiting time charge and some other surcharges, such as luggage fee, radio call surcharge, fuel supplement and tunnel toll.

3. Taxi service survey in Hong Kong

In Hong Kong, roadside observation surveys and taxi stand surveys are conducted yearly to gather information regarding passenger/taxi waiting time at operative taxi stands and percentage of occupied/vacant taxis and the taxi headway on some major selected road locations (Transport Department 1986–1996). The results of these surveys provide important information for government in policy formulation on taxi services. For instance, the survey results are used for the evaluation of whether taxi service has been improved; whether it is necessary to increase the number of taxis and by how many; and whether to increase or reduce taxi fare.

Coverage and sampling design

Regarding the taxi stand survey, samples of taxi stands are selected randomly. As for the roadside observation surveys, due to resource constraints and other practical considerations, only trunk roads, primary distributors and rural roads are included. The road links are first divided into the same geographical area breakdown as for the taxi stand surveys within which samples of road links are then selected. Longer road links are given higher probability of being selected in the process. The sampling roadside observation points and the taxi stands are selected in such a way as to keep and maintain consistency between consecutive years of survey for the purpose of comparison except for some years in which minor adjustments were made.

Data collection

Roadside observation surveys record the number of taxis together with their occupancy passing roadside checkpoints in every minute between 0700 and 1900 on weekdays. The results of roadside observations provide two useful indices, namely, the average proportion of occupied taxis and the average time headway between vacant taxis. The former index is intended to reflect the utilization level of taxis (a higher proportion of occupied taxis would indicate a higher utilization rate of taxis), while the latter index is used as a measurement of the availability of vacant taxis (a shorter time headway between vacant taxis means a higher availability or a higher flow rate of vacant taxis), thereby reflecting the level of taxi service.

Taxi stand observation surveys involve recording the number of passengers boarding taxis and joining the queue every minute between 0700 and 1900 on weekdays. Observations are carried out simultaneously on taxis arriving and leaving taxi stands. For a particular taxi stand, the total number of boarding passengers together with the average waiting time of passengers and taxis obtained from the results of observation provide an indication of taxi utilization and the level of service.

4. A framework of the simultaneous equation system

The sequence of survey results provides information on the yearly change of passenger demand, taxi service quality and profitability, used by government in taxi service regulation. However, the survey information is purely a *de facto* reflection of the situation of the current taxi services and does not allow a "what if" analysis of the market for introduction of new taxi policies. Fortunately, the availability of the service survey data offers an excellent opportunity to investigate the operation of the taxi market. This paper describes an attempt to utilize the information to model the synchronous inter-relationships among the taxi service variables such as taxi fleet size, fare, demand, passenger and taxi waiting times. Like applications of the simultaneous equation system in other transportation fields (e.g., Ben Akiva & Lerman 1985; Button et al. 1982), once such a model based on the existing data is constructed, sensitivity analysis and prediction of outcomes for the introduction of new taxi policies, such as change in incremental charge of taxi fares and/or the number of taxis, will become possible.

To develop a system of simultaneous equations for taxi services with the Hong Kong survey data, a theoretical framework of the simultaneous equation system is provided here. The general type of the system underlying the simultaneous equations in the taxi market may be represented in the flow chart shown in Figure 2. In general, the system consists of a vector of dependent variables **Y**, and a vector of exogenous variables **X**, and can be described by: $\mathbf{Y} = \mathbf{F}(\mathbf{Y}, \mathbf{X}) + \zeta$ where **F** is a vector function of the structural equations

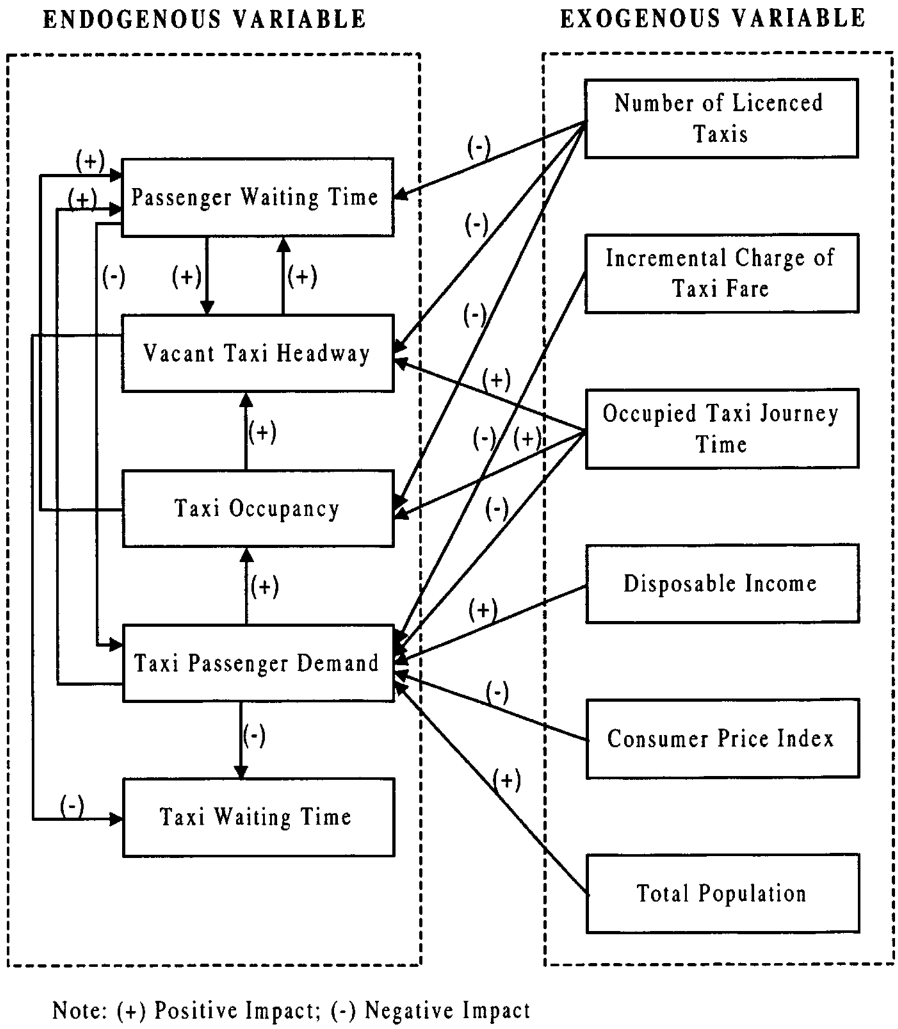

Figure 2. Postulated synchronous relationships among the endogenous and exogenous variables in the simultaneous equation model.

to be specified, ζ is a vector of disturbances. Once the equation system is specified and calibrated from actual data, the expected value $\mathbf{Y}^* = E(\mathbf{Y}/\mathbf{X})$ of the dependent variables **Y** for any given value of the exogenous variable **X** can be obtained by solving the system of equations: $\mathbf{Y}^* = \mathbf{F}(\mathbf{Y}^*, \mathbf{X})$, using the fixed-point algorithm or Newton's method.

In view of the availability of survey data and other relevant information, we consider choosing $\mathbf{Y} = (W_p, H_v, U_t, D_p, W_t)$ where

W_p = average passenger waiting time at taxi stands
H_v = average vacant taxi headway on the roads
U_t = average percentage of occupied taxis on the roads
D_p = average daily number of passenger trips made by taxi
W_t = average taxi waiting time at taxi stands

The predetermined exogenous variables are defined as $\mathbf{X} = (N_t, F, T_t, I_d, P, CPI(A))$ where

N_t = number of licensed taxis
F = incremental charge of taxi fare
T_t = average occupied taxi journey time
I_d = disposable average income per month per worker
P = total population
$CPI(A)$ = 1989/90-based customer price index (A) for transportation.

The observed values of the endogenous variables (1986–1996) are obtained from the survey reports of the level of taxi services (Transport Department 1986–1996), and the values of the exogenous variables (1986–1996) are obtained from annual digest of statistics (Hong Kong Government 1986–1996), annual traffic census and traffic and transport digest (Transport Department 1986–1996). These data are presented in Table 1. From the flow chart in Figure 2, we could have the following equations:

$W_p = W_p(H_v, D_p, N_t, U_t)$ (passenger waiting time)
$U_t = U_t(N_t, D_p, T_t, W_t)$ (percentage of occupied taxis)
$H_v = H_v(N_t, U_t, T_t, W_p)$ (vacant taxi headway)
$D_p = D_p(W_p, F, P, I_d, CPIA, T_t)$ (daily taxi passenger trips)
$W_t = W_t(D_p, H_v)$ (taxi waiting time)

Note that the actual form of the equations is unknown and difficult to establish, the above flow-chart and equations only suggest certain likely linkages among the endogenous and exogenous variables. Some of the linkages may be relatively weak and should be disregarded as insignificant. The final endogenous and exogenous variables requires for each link will be determined based on the statistical significance and sign of the coefficient of the variable providing the links.

Table 1. Basic input data for model calibration.

Year	H_v (min)	W_p (min)	W_t (min)	D_p (trip)	U_t	F (HK\$)	N_t (taxi)	I(HK\$)	I_d (HK\$)	T_t (min)	CPI (A))	P (person)
86	1.2	0.8	4.7	1198000	0.778	0.52	14004	42048	2613	28.8	71.9	5524600
87	1.6	1.2	2.5	1211000	0.796	0.56	14116	47916	3000	28.5	77.7	5580500
88	1.7	1.4	2.7	1216000	0.814	0.56	14334	53628	3383	28.3	81.4	5627600
89	1.8	1.2	2.2	1220000	0.814	0.64	14529	61788	3939	28.0	91.2	5686200
90	1.8	1.2	2.5	1237000	0.826	0.72	14584	72720	4701	27.7	103.4	5704500
91	1.4	0.6	3.5	1249222	0.797	0.72	14719	79716	5199	27.5	116.1	5754800
92	1.4	1.8	3.1	1267000	0.821	0.90	14913	86352	5678	27.2	124.5	5811500
93	2.0	1.0	2.9	1274000	0.831	1.00	14969	95748	6370	26.9	134.8	5919000
94	1.5	0.9	3.6	1275000	0.813	1.00	15247	103452	6947	26.7	147.4	6061400
95	1.2	0.7	4.6	1293000	0.785	1.10	15256	114252	7773	26.4	158.4	6270000
96	1.4	0.7	5.3	1296500	0.810	1.20	15249	124099	8343	26.1	167.8	6421300

Remarks: Only the 1981 and 1992 average taxi journey times are available from Traffic Characteristic Survey (Transport Department, 1982 and 1993), the other years data are approximated by extrapolation.

W_p = average pasenger waiting time at taxi stands
U_t = average percentage of occupied taxis on the roads
W_t = average taxi waiting time at taxi stands
F = incremental charge of taxi fare
I_d = disposable income per month per worker
CPI (A) = 1989/90-based customer price index (A) for transportation

H_{vt} = average vacant taxi headway on the road
D_p = average daily number of trips made by taxi
N_t = number of licensed taxis
T_t = average occupied taxi journey time
P = total population
I = average annual income

5. Model specification & calibration

Many alternative equations have been tried based on the data shown in Table 1. The sample coefficient of multiple determination, R^2, which represents the proportion of the total variation among the values of the variables that can be explained by the fitted model, and the t-value of these variables, which illustrates whether the parameters are significant or not, are used to measure the quality of the equations together with the degree of freedom (Df). Three to five representative equations for each of the endogenous variables are presented below. Note that each equation has been calibrated separately (t-statistics are presented in parentheses below each coefficient).

Average passenger waiting time equation

Passenger waiting time at a particular taxi stand can be approximately derived from queuing theory, by assuming that arrival rate is proportional to the number of passengers (D_p) or average number of passengers per licensed taxi (D_p/N_t), and service rate proportional to the inverse of vacant taxi headway ($1/H_v$) or approximately $(1 - U_t)N_t$. Under standard M/M/1 queuing regime, the average waiting time in the system is $W = 1/(\mu - \lambda)$ or $1/W = \mu - \lambda$ where W is the expected waiting time and μ and λ are service and arrival rates, respectively. With these considerations, the following equations have been attempted and calibrated.

$$\frac{1}{W_p} = \underset{[1.81]}{1.6180}\left(\frac{1}{H_v}\right) - \underset{[-0.07]}{5.1045 \times 10^{-4}}\left(\frac{D_p}{N_t}\right) \tag{1a}$$

$R^2 = 93.46\%$, $Df = 7$

$$\frac{1}{W_p} = \underset{[1.7]}{1.5185}\left(\frac{1}{H_v}\right) + \underset{[0.04]}{1.9308 \times 10^{-8}} D_p \tag{1b}$$

$R^2 = 93.46\%$, $Df = 7$

$$\frac{1}{W_p} = \underset{[3.00]}{1.0181 \times 10^{-3}}(1 - U_t)N_t - \underset{[-1.93]}{0.0218}\left(\frac{D_p}{N_t}\right) \tag{1c}$$

$R^2 = 95.66\%$, $Df = 7$

By comparison in terms of the coefficient of determination, the t-value and the sign of the parameters, equation (1c) is the most suitable one and selected for further consideration.

Average taxi occupancy equation

Average taxi occupancy is defined as the percentage of occupied taxis observed at the roadside and can be considered to be roughly proportional to the ratio of total occupied taxi time to total taxi service time. Total taxi service time is directly proportional to the total number of taxis, and total occupied taxi time is directly proportional to the passenger demand. From the time conservation relation that total occupied taxi time plus total vacant taxi time equals total taxi service time, the following equations have been attempted and calibrated.

$$U_t = \frac{1}{1.9177 - 0.2833\left(\frac{1}{W_t}\right) - 4.7080 \times 10^{-7} D_p} \tag{2a}$$

$$\begin{gathered}[-3.55] \quad [-2.37] \\ R^2 = 66.54\%,\ Df = 6\end{gathered}$$

$$U_t = \frac{1}{2.1111 + 0.2199 W_t - 0.8001 \times 10^{2}\left(\frac{N_t}{D_p}\right)} \tag{2b}$$

$$\begin{gathered}[2.90] \qquad\qquad [-1.09] \\ R^2 = 60.64\%,\ Df = 6\end{gathered}$$

$$U_t = 4.7969 \times 10^{-3} W_t^{-0.0709} N_t^{0.1594} D_p^{0.2621} \tag{2c}$$

$$\begin{gathered}[-3.49]\ [0.22]\ [0.31] \\ R^2 = 71.50\%,\ Df = 5\end{gathered}$$

$$U_t = \frac{1}{1.6282 + 0.0266 W_t - 1.6829 \times 10^{-5} N_t - 1.8345 \times 10^{-7} D_p} \tag{2d}$$

$$\begin{gathered}[3.89] \qquad\qquad [-0.30] \qquad\qquad [-0.24] \\ R^2 = 75.23\%,\ Df = 5\end{gathered}$$

$$U_t = \frac{1}{1.6589 + 0.0271 W_t - 4.0790 \times 10^{-7} D_p} \tag{2e}$$

$$\begin{gathered}[4.37] \qquad\qquad [-2.45] \\ R^2 = 74.85\%,\ Df = 6\end{gathered}$$

By comparison in term of the coefficient of determination, the *t*-value and the sign of the parameters, equation (2e) is the most suitable one and selected for further consideration.

Vacant taxi headway equation

Vacant taxi headway can be considered to be related to the number of licensed taxi and taxi utilization and so on. The following equations have been attepmted and calibrated.

$$H_v = -8.4099 + \underset{[3.99]}{12.3466U_t} \tag{3a}$$

$R^2 = 66.53\%$, $Df = 7$

$$H_v = 8.8250 \times 10^6 \underset{[-4.92]}{(1 - U_t)^{-1.7711}} \underset{[-1.76]}{N_t^{-1.9271}} \tag{3b}$$

$R^2 = 77.64\%$, $Df = 6$

$$H_v = 1.6920 \times 10^{33} \underset{[-5.50]}{(1 - U_t)^{-2.2386}} \underset{[-1.80]}{(T_t + W_p)^{-4.4159}} \underset{[-2.37]}{N_t^{-6.7714}} \tag{3c}$$

$R^2 = 85.48\%$, $Df = 5$

By comparison in term of the coefficient of determination, the t-value and the sign of the parameters, equation (3c) is the most suitable one and selected for further consideration.

Demand equation for average daily taxi trip

The selected passenger demand equation is a traditional direct demand model of the multiplicative kind that has been used extensively in travel demand modeling. The following demand equations with various combinations of the variables have been attempted and calibrated.

$$D_p = 1.3274 \times 10^6 \underset{[-1.09]}{W_p^{-0.0052}} \underset{[14.7]}{F^{0.0893}} \underset{[0.76]}{I^{0.0013}} \underset{[-1.39]}{P^{-0.0033}} \tag{4a}$$

$R^2 = 98.52\%$, $Df = 4$

$$D_p = 1.0634 \times 10^6 \underset{[-0.38]}{W_p^{-0.0023}} \underset{[1.83]}{F^{0.0614}} \underset{[0.83]}{I_D^{0.0232}} \underset{[-0.37]}{P^{-0.0014}} \tag{4b}$$

$R^2 = 98.55\%$, $Df = 4$

$$D_p = 2.6411 \times 10^5 \underset{[0.15]}{W_p^{0.0028}} \underset{[-3.33]}{\left(\frac{F}{I_D}\right)^{-0.1772}} \tag{4c}$$

$R^2 = 63.04\%$, $Df = 6$

$$D_p = 9.8865 \times 10^5 W_p^{0.0009} F^{0.0382} I^{-0.0009} CPIA^{0.0547} \tag{4d}$$

[0.14] [1.03] [–0.33] [1.38]

$R^2 = 98.81\%$, $Df = 4$

$$D_p = 6.0417 \times 10^6 (T_t + W_p)^{-0.5867} \left(\frac{F}{I_d}\right)^{-0.0444} \tag{4e}$$

[2.93] [–0.79]

$R^2 = 83.32\%$, $Df = 6$

Among all equations only equation (4e) has desirable sign of the parameters. This equation is statistically significant at 90% confidence interval, and thus is selected for further consideration.

Average taxi waiting time equation

Similar to the average taxi waiting time, average taxi waiting time at taxi stands can be approximately derived based on the concept of queuing theory. The following equations have been attempted and calibrated.

$$W_t = -1.0807 + 5.2588 \times 10^{-6} \left(\frac{D_p}{H_v}\right) \tag{5a}$$

[5.77]

$R^2 = 80.64\%$, $Df = 7$

$$W_t = -4.1540 \times 10^{-21} H_v^{-1.2093} N_t^{-2.6271} D_p^{5.2585} \tag{5b}$$

[–4.19] [–0.37] [0.64]

$R^2 = 78.71\%$, $Df = 5$

$$W_t = -1.2238 + 5.7372 \times 10^2 \left(\frac{N_t}{H_v D_p}\right) \tag{5c}$$

[5.55]

$R^2 = 79.38\%$, $Df = 7$

$$\frac{1}{W_t} = 8.8434 \times 10^5 \frac{1}{D_p} - 0.5809 \frac{1}{H_v} \tag{5d}$$

[9.76] [–5.32]

$R^2 = 98.94\%$, $Df = 7$

By comparison in term of the coefficient of determination, the *t*-value and the sign of the parameters, equation (5d) is the most suitable one and selected for further consideration.

Final system of simultaneous equations

In summary, the final system of equations selected for further consideration is as follows:

- Passenger waiting time equation

$$\frac{1}{W_p} = 1.0181 \times 10^{-3}(1 - U_t)N_t - 0.0218\left(\frac{D_p}{N_t}\right) \tag{6}$$

- Taxi occupancy equation

$$U_t = \frac{1}{1.6589 + 0.0271W_t - 4.0790 \times 10^{-7}D_p} \tag{7}$$

- Vacant taxi headway equation

$$H_v = 1.6920 \times 10^{33}(1 - U_t)^{-2.2386}(T_t + W_p)^{-4.4159}N_t^{-6.7714} \tag{8}$$

- Taxi trip demand equation

$$D_p = 6.0417 \times 10^6(T_t + W_p)^{-0.5867}\left(\frac{F}{I_d}\right)^{-0.0444} \tag{9}$$

- Taxi waiting time equation

$$\frac{1}{W_t} = 8.8431 \times 10^5 \frac{1}{D_p} - 0.5809 \frac{1}{H_v} \tag{10}$$

Note that the final estimated system of equations (6)–(10) does not necessarily contain all the endogenous and exogenous variables required for all the link shown in the flow chart of Figure 2.

Model application results

Here we present two sets of results from the calibrated system of simultaneous equations. The first set of results includes the observed and predicted values of all endogenous variables and their percentage errors obtained from each individual equation associated with a particular endogenous variable by substituting the observed data of all other endogenous variables into the corresponding equation. The second set of results includes the observed and predicted values of endogenous variables and their percentage errors obtained by solving the system of simultaneous equations using a fixed-point algorithm. The latter case requires use of the values of exogenous variables only, and thus is relevant to the forecasting of future year taxi service situation. These results are shown in Tables 2–6 which shows that the proposed and calibrated system of simultaneous equations replicate the existing data with satisfactory accuracy. Among the five endogenous variables, the accuracy of prediction for passenger demand is the best, while the accuracy of prediction for passenger waiting time is the worst. It can also be observed that the simultaneous equation may not produce unbiased results, the biases for passenger and taxi waiting times are remarkable.

Table 2. Average passenger waiting time.

Year	By individual equation			By simultaneous equations		
	Observed value	Predicted value	Percentage error (%)	Observed value	Predicted value	Percentage error (%)
86	0.80	0.77	–3.9	0.80	1.15	+43.8
87	1.20	0.94	–21.7	1.20	1.18	–1.7
88	1.40	1.16	–17.1	1.40	1.12	–20.0
89	1.20	1.09	–9.2	1.20	1.06	–11.7
90	1.20	1.36	+13.3	1.20	1.14	–5.0
91	0.60	0.84	+40.0	0.60	1.14	+90.0
92	1.80	1.16	–35.6	1.80	1.03	–42.8
93	1.00	1.39	+39.0	1.00	1.11	+11.0
94	0.90	0.93	+3.3	0.90	0.94	+4.4
95	0.70	0.67	–4.3	0.70	1.10	+57.1

Table 3. Average taxi occupancy.

Year	By individual equation			By simultaneous equations		
	Observed value	Predicted value	Percentage error (%)	Observed value	Predicted value	Percentage error
86	0.778	0.771	–0.9	0.778	0.808	+3.9
87	0.697	0.811	+5.5	0.769	0.811	+5.5
88	0.814	0.809	–0.6	0.814	0.811	–0.4
89	0.814	0.819	+0.6	0.814	0.811	–0.4
90	0.826	0.818	–1.0	0.826	0.816	–1.2
91	0.797	0.804	+0.9	0.797	0.818	+2.6
92	0.821	0.816	–0.6	0.821	0.815	–0.7
93	0.831	0.821	–1.2	0.831	0.820	–1.3
94	0.813	0.809	–0.5	0.813	0.814	+0.1
95	0.785	0.796	+1.4	0.785	0.824	+4.9

Table 4. Average vacant taxi headway.

Year	By individual equation			By simultaneous equations		
	Observed value	Predicted value	Percentage error (%)	Observed value	Predicted value	Percentage error (%)
86	1.20	1.29	+7.5	1.20	1.73	+44.2
87	1.60	1.50	–6.3	1.60	1.77	+10.6
88	1.70	1.64	–3.5	1.70	1.66	–2.4
89	1.80	1.62	–10.0	1.80	1.59	–11.7
90	1.80	1.92	+6.7	1.80	1.70	–5.9
91	1.40	1.46	+4.1	1.40	1.68	+16.7
92	1.40	1.48	+5.7	1.40	1.59	+13.6
93	2.00	2.00	0.0	2.00	1.71	–14.5
94	1.50	1.47	–2.0	1.50	1.49	–0.7
95	1.20	1.17	–2.5	1.20	1.71	+42.5

Table 5. Daily taxi trip demand.

Year	By individual equation			By simultaneous equations		
	Observed value	Predicted value	Percentage error (%)	Observed value	Predicted value	Percentage error (%)
86	1198000	1206338	+0.70	1198000	1198735	+0.06
87	1211000	1211537	+0.04	1211000	1209956	–0.09
88	1216000	1215906	–0.01	1216000	1222921	+0.57
89	1220000	1227355	+0.60	1220000	1232715	+1.04
90	1237000	1239477	+0.20	1237000	1241424	+0.36
91	1249000	1265022	+0.13	1249000	1252198	+0.26
92	1267000	1235517	–2.49	1267000	1255271	–0.93
93	1274000	1265781	–0.65	1274000	1261639	–0.97
94	1275000	1277608	+0.21	1275000	1276203	+0.09
95	1293000	1292773	–0.02	1293000	1280973	–0.93

The model is further tested with the 1996 observational data which were not used for calibration. In 1996, the incremental charge of taxi fare was 1.20 (HK$) per 0.2 km and the total number of taxis was 15249. With these, and other relevant data of exogenous variables, the system of simultaneous equations will self-determine a set of values of the endogenous variables. These values are displayed in Table 7 and compared with the actual observational data. The accuracy of prediction seems to be acceptable except for that of taxi waiting time.

Table 6. Average taxi waiting time.

Year	By individual equation			By simultaneous equations		
	Observed value	Predicted value	Percentage error (%)	Observed value	Predicted value	Percentage error (%)
86	1.70	3.94	–16.2	4.70	2.49	–47.0
87	2.50	2.72	+8.8	2.50	2.49	–0.4
88	2.70	2.59	–4.1	2.70	2.69	–0.4
89	2.20	2.49	+13.2	2.20	2.83	+28.6
90	2.50	2.55	+2.0	2.50	2.69	+7.6
91	3.50	3.41	–2.6	3.50	2.77	–20.9
92	3.10	3.53	+13.9	3.10	2.94	–5.2
93	2.90	2.48	–16.9	2.90	2.77	–4.5
94	3.60	3.26	–9.4	3.60	2.31	–35.8
95	4.60	5.00	+8.7	4.60	2.86	–37.8

Table 7. Test result with the 1996 observational data.

Endogenous variable	W_p (min)	U_t (%)	H_v (min)	D_p (trip)	W_t (min)
Observed value	0.70	81.00	1.40	1296500	5.30
Predicted value	1.23	82.90	1.89	1286202	2.63

Model refinement

Attempts were made to calibrate the system of simultaneous equations simultaneously. Because there exists no standard software for calibration of a general nonlinear simultaneous equation system, model refinement was conducted by introducing a tuning factor to each of the calibrated parameter values in the simultaneous nonlinear equations (6)–(10). Then a constrained nonlinear minimization problem is constructed where a weighted least-squares fitting function is minimized with respect to the tuning factors subject to the simultaneous equations. By substituting the equations into the objective function, the problem becomes an unconstrained nonlinear minimization problem which can be solved by Quasi-Newton Method. Unfortunately, the resulting adjusted system does not improve the accuracy of fitting, and thus is not further reported here.

6. Application for forecasting

One of the major applications of the proposed and calibrated simultaneous system of equations is to conduct sensitivity analysis of the level of taxi service when new taxi policies and/or regulations are introduced. In most cities, taxi regulations include entry restriction and price control, and the economic consequences of regulatory restraints have to be known for effective decision-making. This can be achieved by using the calibrated system of simultaneous equations to predict the changes in the values of the five endogenous variables under various combinations of adjustments in incremental charge of taxi fare and number of taxi.

To do the sensitivity analysis, the other exogenous variable values including the average taxi journey time and the disposable income are fixed and taken from the 1996 data, and the incremental charge of taxi fares and the number of urban taxis are increased by 0.05HK$ and 40 taxis at each step, respectively. Figures 3–7 show the trends of changes in average passenger waiting time,

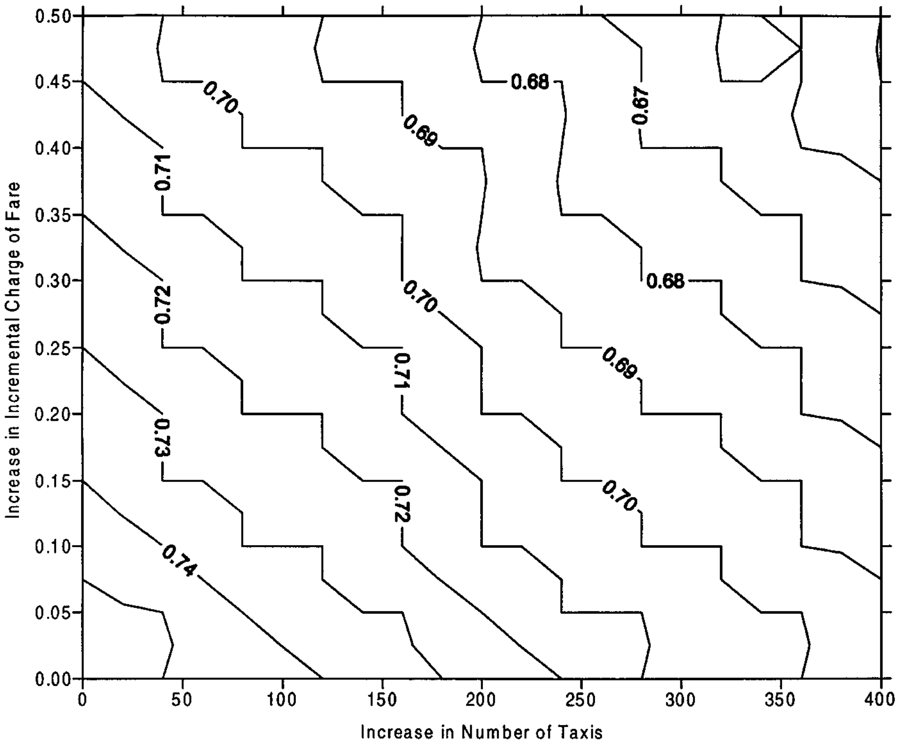

Figure 3. Average passenger waiting time in the space of changes in number of taxi and incremental charge of fare.

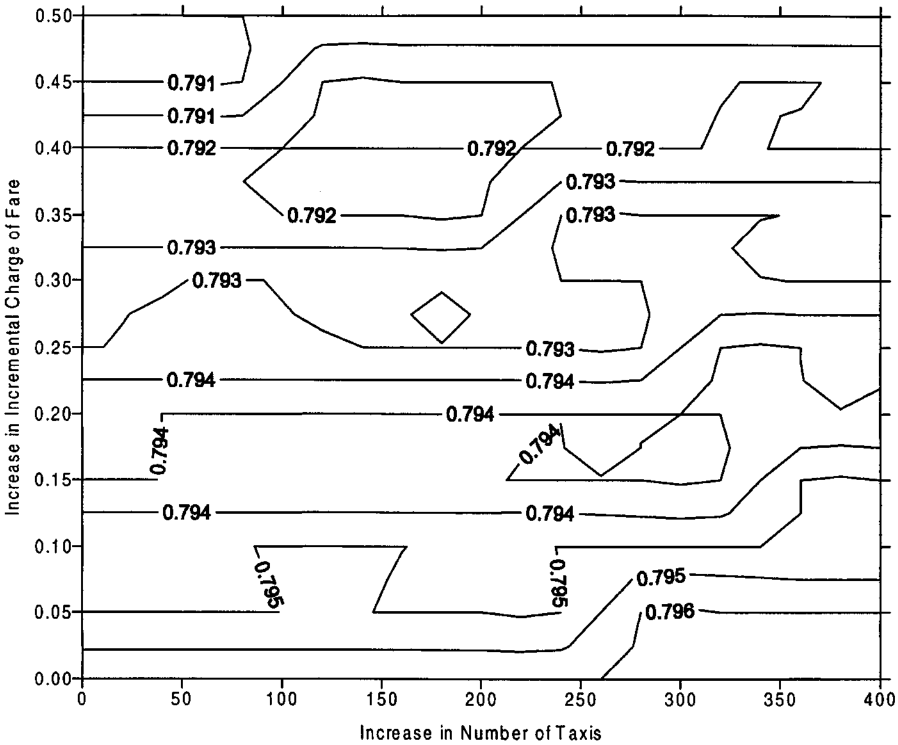

Figure 4. Average taxi occupancy in the space of changes in number of taxi and incremental charge of fire.

average taxi occupancy, average vacant taxi headway, average daily taxi trip demand and average taxi waiting time, respectively. Except for taxi occupancy, all other endogenous variables exhibit particular pattern of changes in the space of changes of number of taxis and incremental charge of fare, and the signs of changes of these variables with taxi fleet size and fare increase coincide with what is expected.

From these figures, for example, the following "what if" policy measures can be inferred from the figures both qualitatively and quantitatively. Average passenger waiting time decreases monotonically with increases in the number of taxis and taxi fare. Average vacant taxi headway decreases monotonically with increase in the number of licensed taxis and marginally decreases with taxi fare. Average daily passenger demand increases with increase in the number of licensed taxis, but decreases with taxi fare. Average taxi waiting time increases with increase in number of licensed taxis, and marginally with increase in incremental charge of taxi fare.

Note that the average taxi occupancy shown in Figure 4 does not exhibit

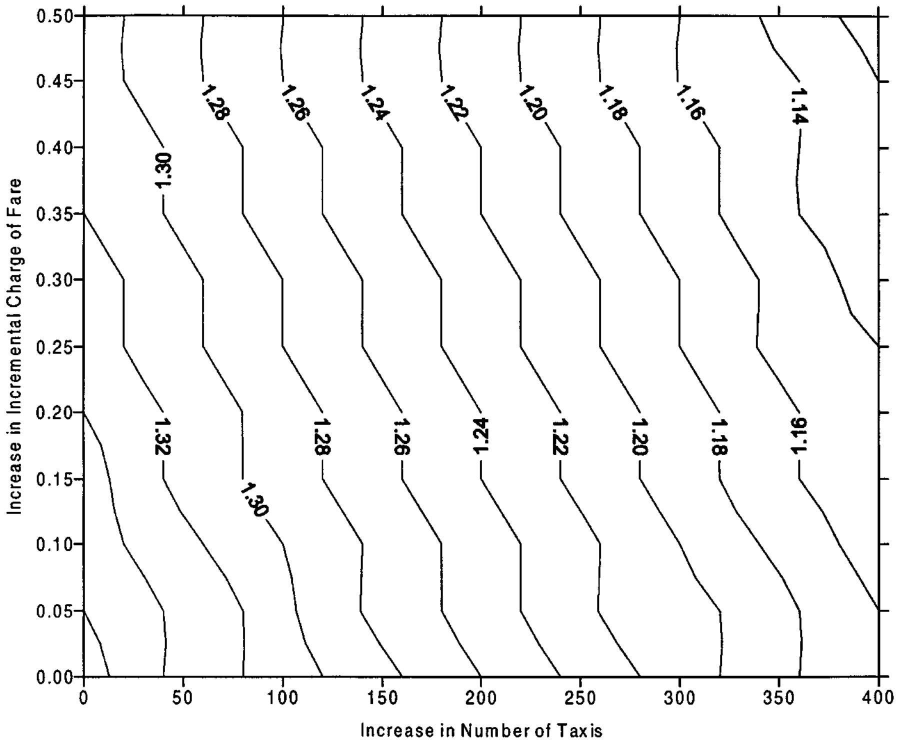

Figure 5. Average vacant taxi headway in the space of changes in number of taxis and incremental charge of fire.

only particular pattern of change. The reason why taxi occupancy does not change monotonically with the number of taxis and fares can be given below. First, taxi occupancy here is defined as the fraction of occupied taxis observed at major roadsides and can be considered only as an approximation of a more exact taxi occupancy, in terms of the fraction of time that a taxi is occupied (the latter cannot be observed directly). Second, increases in the number of taxis will reduce taxi occupancy directly, but on the other hand it will also reduce passenger waiting time and thus increase passenger demand, increase in passenger demand, in turn, increases taxi occupancy. Therefore, with this combined positive and negative effect, taxi occupancy or taxi utilization does not necessarily vary monotonically with number of taxis.

In summary, the prediction results of the simultaneous equation system can be described as fair. Each selected and calibrated structural equation in the system yields intuitively sensible results, which in general offer certain explanations of changes in the endogenous variables. Furthermore, the information presented in these figures is very useful for government

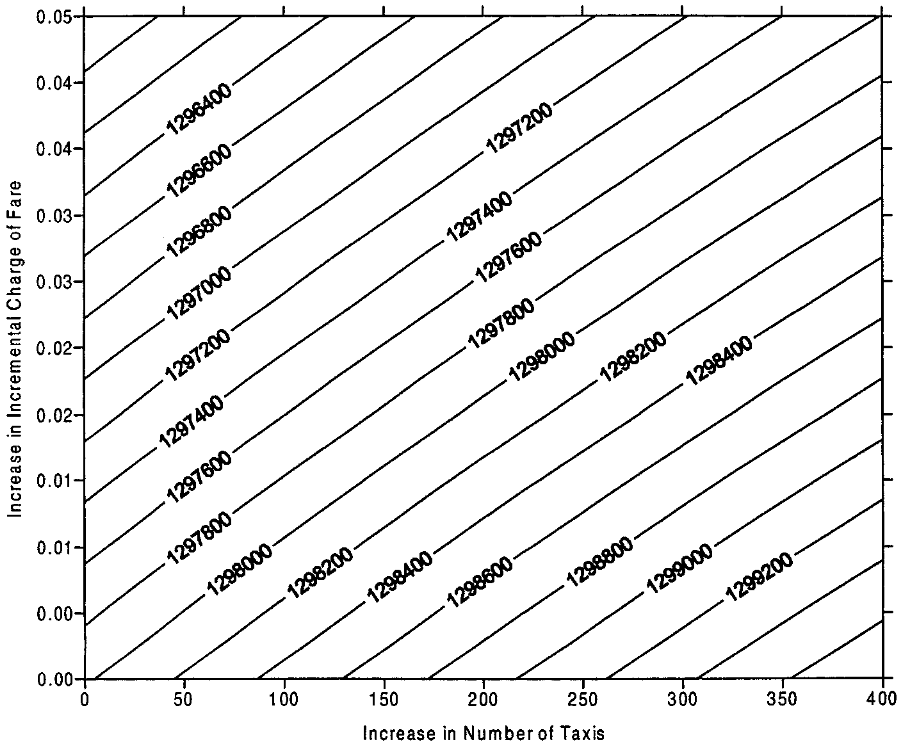

Figure 6. Average daily taxi trip demand in the space of changes in number of taxis and incremental charge of fare.

decision-making concerning taxi regulation. It indicates the sensitivity of taxi services to the regulatory variables of fare and number of taxis. The desirable values of the regulatory variables can be set so as to achieve a certain level of taxi services or taxi utilization in a future year once the values of exogenous variables are known.

7. Conclusions

This paper has developed a system of simultaneous equations for modeling the level of taxi services. The equation system was calibrated with the annual survey data in Hong Kong. It was found that the model parameters are statistically significant and the five equations match the observed data well. The model can be used to predict the future performance measures of taxi services including taxi demand, taxi utilization and service quality once the exogenous variables are given or estimated from other socioeconomic vari-

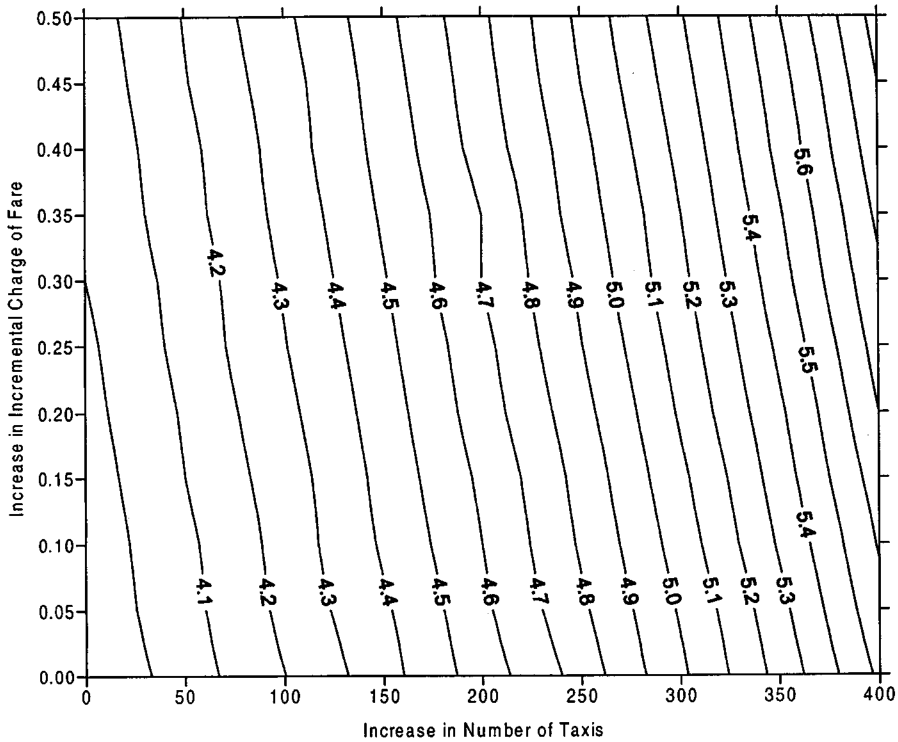

Figure 7. Average taxi waiting time in the space of changes in number of taxis and incremental charge of fare.

ables. Our initial attempt to deal with an important problem makes a good starting point towards understanding of the way in which taxi market operates. Indeed, from our prediction results the endogenous variables in the final specification do appear significant and to exert the sort of influence anticipated from *a priori* reasoning. It is thus encouraging that the model can be used to analyze the impacts of alternative taxi service regulation on system performance and help relevant authority in decision-making.

Nevertheless, we have to reiterate the facts that the complex intervening demand-supply relationships in the taxi markets remain unclear in both theory and practice and there exist no established standard calibration procedures available for a general nonlinear simultaneous equation system. These facts compound the development of an effective taxi service models. Our results do suggest the need to seek better model structure and more efficient estimation method, as more data become available, thereby enhancing the structural equation modeling.

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