Topics for a master's thesis

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In this document, you will find a selection of topics suitable for preparing your master's thesis under my supervision. Each topic includes a brief explanation and a few references. If you need more information, please feel free to contact me.

FIGURE 1. Here is the list of theses I have supervised.

1. Formanek's theorem

Let K be a field and G a group. The *zero-divisor conjecture* for group rings asserts that the group ring $K[G]$ is a domain if and only if G is torsion-free. The conjecture has been proven affirmative for several classes of groups. In 1973, Formanek proved it for supersolvable groups. References: [**8, 13**].

2. Köthe's conjecture

Köthe's conjecture is an open problem in ring theory, formulated in [**6**] by Gottfried Köthe in 1930. The conjecture can be formulated in various different ways [**7**] and has been shown to be true for various classes of rings, such as polynomial identity rings and right Noetherian rings. References: [**8, 11**].

3. The O'Nan–Scott theorem

The result is one of the most influential theorems in the theory of permutation groups. Briefly, the O'Nan–Scott theorem describes maximal subgroups of symmetric groups. Combined with the classification of finite simple groups, there are so many applications that one cannot even count them! References: [**3, 5, 9**].

4. The Abel–Ruffini theorem

The Abel–Ruffini theorem (also known as Abel's impossibility theorem) states that there is no solution in radicals to general polynomial equations of degree five or higher with arbitrary coefficients. In 1963, Vladimir Arnold discovered a topological proof of the theorem. Arnold's proof does not rely on Galois theory. Reference: [**1**].

5. McKay's conjecture for solvable groups

Counting conjectures play a fundamental role in modern representation theory of finite groups. Among these conjectures, one stands out: McKay's conjecture. John McKay formulated the problem in 1972. The conjecture is known to be true in several cases, including that of solvable groups. Reference: [**12**].

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6. The inverse Galois problem

The Inverse Galois Problem is a fundamental question in mathematics that seeks to determine whether every finite group can be realized as the Galois group of some field extension over the rational numbers. The problem was first explicitly stated by mathematicians in the late 19th and early 20th centuries, notably by David Hilbert. Despite significant progress and partial solutions, the problem remains unsolved for many groups, making it a central topic in algebra and number theory. References: [**10, 14**].

7. Three theorems of Bieberbach

Bieberbach's theorems are fundamental results in the theory of discrete groups of isometries in Euclidean space. These theorems form the basis for the classification of crystallographic groups, which are crucial in mathematics and physics. References: [**2, 4**].

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