CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a) $y(t) = x(t) + -4\int y(t)dt$ hence,
$\frac{dy}{dt} + 4y = x(t)$

(b) To find y(t) we need to solve this differential equation:

$$\frac{dy}{dt} + 4y = x(t)$$

We can solve this linear first-order differential equation by using **integrating factor** :

$$\mu(t) = ke^{\int p(t)dt}$$

Where $p(t) = 4$, hence the integration factor becomes :

$$\mu(t) = ke^{4t}$$

The solution for the first-order linear differential equation is:

$$y(t) = \frac{\int \mu(t)g(t)dt + c}{\mu(t)}$$

Where $\mu(t) = ke^{4t}$ and $g(t) = x(t) = (e^{-t} + e^{-2t})$. By applying necessary substitutions:

$$y(t) = \frac{\int e^{3t} + e^{2t}dt + c}{e^{4t}}$$
$$y(t) = \frac{e^{3t}/3 + e^{2t}/2 + c}{e^{4t}}$$

To get the solution and find c use the initial conditions :

$$y(0) = \frac{1}{3} + \frac{1}{2} + c = 0 \text{ Hence,}$$
$$c = -\frac{5}{6}$$

The output of the system is:

$$y(t) = \frac{\frac{e^{3t}}{3} + \frac{e^{2t}}{2} - \frac{5}{6}}{e^{4t}}$$
$$y(t) = \frac{1}{3e^{t}} + \frac{1}{2e^{2t}} - \frac{5}{6e^{6t}}$$

2. (a) From the definition of convolution :

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k] = 1$$

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = -1$$

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = -8$$

$$y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k] = 11$$

y[n] is 0 for $n > 4$ and $n < 0$.
The graph is following **Figure1** :

$x[n]$

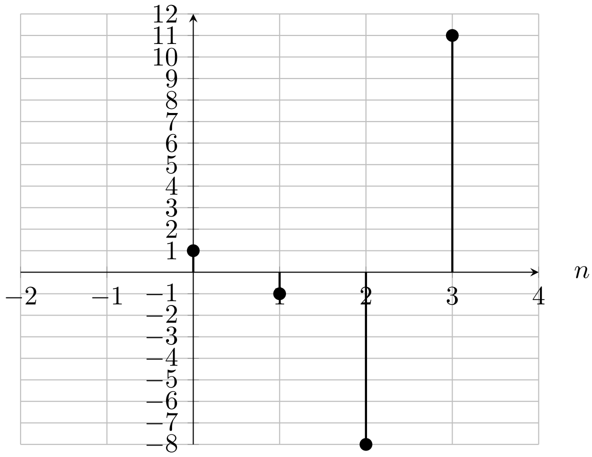

Figure 1: n vs. $y[n]$.

(b) $x'(t) = (u(t) + u(t-1))' = \delta(t) + \delta(t-1)$ put x prime to equation
$y(t) = (\delta(t) + \delta(t-1)) * h(t) = \int_{-\infty}^{+\infty} (\delta(\tau) + \delta(\tau - 1))h(t-\tau)d\tau$
$y(t) = \int_{-\infty}^{+\infty} (\delta(\tau) + \delta(\tau - 1))(e^{-2(t-\tau)} cos(t-\tau)u(t-\tau))d\tau$
$y(t) = \int_{-\infty}^{t} (\delta(\tau) + \delta(\tau - 1))(e^{-2(t-\tau)} cos(t-\tau))d\tau \longleftarrow t > \tau$ take impulses out
$y(t) = e^{-2t} cos(t) + e^{-2t+2} cost(t-1) \longleftarrow t > \tau$

3. (a) By definition of convolution :

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{-3(t-\tau)} u(t-\tau) d\tau$$

Since $u(t)$ is 0 for t ¡ 0, and $u(t-\tau)$ is 0 for $\tau > t$, the integral becomes:

$$y(t) = \int_0^t e^{-\tau} e^{-3(t-\tau)} d\tau$$
$$y(t) = e^{-3t} \int_0^t e^{2\tau} d\tau$$
$$y(t) = e^{-3t} (\tfrac{e^{2\tau}}{2})|_0^t$$
$$y(t) = \tfrac{e^{-t}}{2} - \tfrac{e^{3t}}{2}$$

(b) $y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$
$y(t) = \int_{-\infty}^{+\infty} (u(\tau - 1) - u(\tau - 2))e^{t-\tau} u(t-\tau)d\tau$ take unit steps out
$y(t) = \int_1^t (e^{t-\tau} d\tau \quad - \int_2^t e^{t-\tau} d\tau \longleftarrow t > \tau \; \& \tau > 1 \; for \; left \; integral \; , \tau > 2 \; for \; right \; integral$
$y(t) = e^t (-e^{-t} + e^{-1} + e^{-t} - e^{-2})$
$y(t) = e^t (e^{-1} - e^{-2})$

4. (a) Let's define $u_n = y[n]$, then the equation becomes :

$$u_n - 15u_{n-1} + 26u_{n-2} = 0$$

Since this is homogeneous difference equation, guess:

$$u_n = Aw^n$$

So, the equation becomes:

$$Aw^n - 15Aw^{n-1} + 26Aw^{n-2} \text{ and,}$$
$$w^2 - 15w + 26 = 0$$

There are two roots of this second-order equation, which are :

$$w_1 = \tfrac{15+\sqrt{121}}{2} = 13 \text{ ,}$$
$$w_2 = \tfrac{15-\sqrt{121}}{2} = 2$$

The general solution is that :

$$u_n = A_1 13^n + A_2 2^n$$

By using initial conditions :

$$u_0 = A_1 + A_2 = 10 \text{ ,}$$
$$u_1 = 13A_1 + 2A_2 = 42 \text{ ,hence}$$
$$A_1 = 2 \text{ ,and}$$
$$A_2 = 8$$

By substituting necessary variables, the solution is :

$$y[n] = 2 \times 13^n + 8 \times 2^n = 2 \times 13^n + 2^{n+3}$$

(b) Let's define $u_n = y[n]$, then the equation becomes :

$$u_n - 3u_{n-1} + u_{n-2} = 0$$

Since this is homogeneous difference equation, guess:

$$u_n = Aw^n$$

So, the equation becomes:

$$Aw^n - 3Aw^{n-1} + Aw^{n-2} \text{ and,}$$
$$w^2 - 3w + 1 = 0$$

There are two roots of this second-order equation, which are :

$$w_1 = \tfrac{3+\sqrt{5}}{2} \ ,$$
$$w_2 = \tfrac{3-\sqrt{5}}{2}$$

The general solution is that :

$$u_n = A_1(\tfrac{3+\sqrt{5}}{2})^n + A_2(\tfrac{3-\sqrt{5}}{2})^n$$

By using initial conditions :

$$u_0 = A_1 + A_2 = 1 \ ,$$
$$u_1 = A_1\tfrac{3+\sqrt{5}}{2} + A_2\tfrac{3-\sqrt{5}}{2} = 4 \text{ ,hence}$$
$$A_1 = \tfrac{5+\sqrt{5}}{10} \text{ ,and}$$
$$A_2 = \tfrac{5-\sqrt{5}}{10}$$

By substituting necessary variables, the solution is :

$$y[n] = (\tfrac{5+\sqrt{5}}{10}) \times (\tfrac{3+\sqrt{5}}{2})^n + (\tfrac{5-\sqrt{5}}{10}) \times (\tfrac{3-\sqrt{5}}{2})^n$$

5. (a) $y''(t) + 6y'(t) + 8y(t) = 2x(t)$
in a particular point $Y[s] = x[s]$ then
$S^2Y[s] + 6Y[s] + 8Y[s] = 2x[s]$
$Y[s](s^2 + 6s + 8) = 2x[s]$
$\longrightarrow Y[s] = 2x[s]/(s^2 + 6s + 8)$ since H[s] is he impulse response at x[s], simplifying the Y[s] with X[s] will give us H[s]
$H[s] = 2/(s^2 + 6s + 8) = 1/(s + 2) - 1/(s + 4) \longrightarrow roots\ s_1 = -2\ ,\ s_2 = -4$
$H(t) = (e^{-2t} - e^{-4t})u(t)$

(b) • It is causal. Since if LTI systems are initially at rest they are causal, moreover a system is causal if $h(t) = 0\ for\ t < 0$:
$h(t) = (e^{-2t} - e^{-4t})u(t) = 0\ \forall t < 0$
• Memoryless. Since present impulse response of the function is only dependent on present value of step function and exponentials.
• Stable since the integral of impulse response from minus infinity to plus infinity is not infinity(exponential values decrease as they reach infinity for$t >= 0$, unit step function makes the response 0 for $t < 0$
• Invertible since we can find $h^{-1}(t)$, that makes $h(t) * h^{-1}(t) = \delta(t) \Longrightarrow h^{-1}(t) = \delta(t)/(e^{-2t} - e^{-4t})$