CENG 384 - Signals and Systems for Computer Engineers
Spring 2018-2019
Written Assignment 1

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1. (a)

$$
\begin{array}{lrll}
1 & z = & x + yj & \\
2 & \bar{z} = & x - yj & \\
3 & 3z + 4 = & 2j - \bar{z} & \\
4 & 3x + 3yj + 4 = & 2j - x + yj & \\
5 & 4x + 4 = & 0 & (eqn.1) \\
6 & 2y = & 2 & (eqn.2) \\
7 & eqn1 & \implies x = -1 & \\
8 & eqn2 & \implies y = 1 & \\
9 & z = & -1 + j &
\end{array}
$$

(i) $|z|^2 = (-2(j^2)) = 2$
(ii)

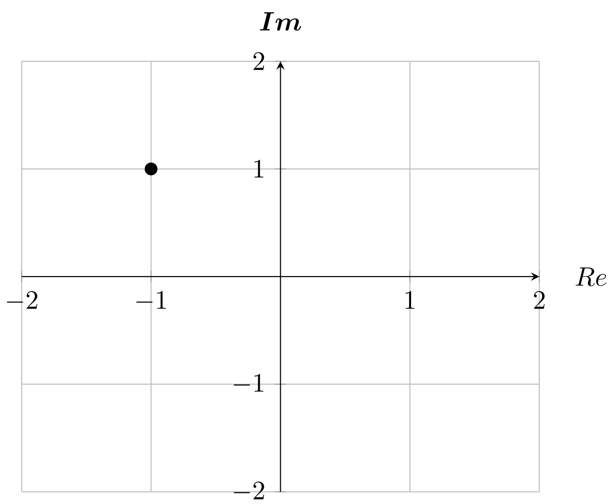

Figure 1: $z = -1 + j$.

(b)

$$
\begin{array}{lrll}
1 & z = & re^{j\theta} & \\
2 & z^3 = & 64j & (eqn1) \\
3 & z = & 4 * (-j) & using\ eqn1\ (eqn2) \\
4 & z = & 4 * e^{-j\pi/2} &
\end{array}
$$

(c)

1	$z = \frac{(1-j)*(1+\sqrt{3}j)}{(1+j)}$	$eqn1$
2	$z = \frac{(1-j)*(1-j)*(1+\sqrt{3}j)}{(1+j)*(1-j)}$	$eqn2$
3	$z = \frac{(-2j)*(1+\sqrt{3}j)}{2}$	$eqn3$
4	$z = \sqrt{3} - j$	$using\ eqn3$
5	$\lvert z\rvert = \sqrt{3+1}$	$by\ definition\ of\ magnitude\ (eqn4)$
6	$\lvert z\rvert = 2$	$using\ eqn4$
7	$\theta = arctan(-1/3)$	$by\ definition\ of\ arctan\ (eqn5)$
8	$\theta = 11\pi/6$	$using\ eqn5$

(d)

1	$z = -je^{j\pi/2}$	$eqn1$
2	$z = -j*(cos(\pi/2)+j*sin(\pi/2))$	$Euler's\ formula\ (eqn1)$
3	$z = -j*(j)$	$from\ the\ eqn2\ (eqn3)$
4	$z = 1$	$using\ eqn3$

2. Figure 2 represents the solution for question 2 .

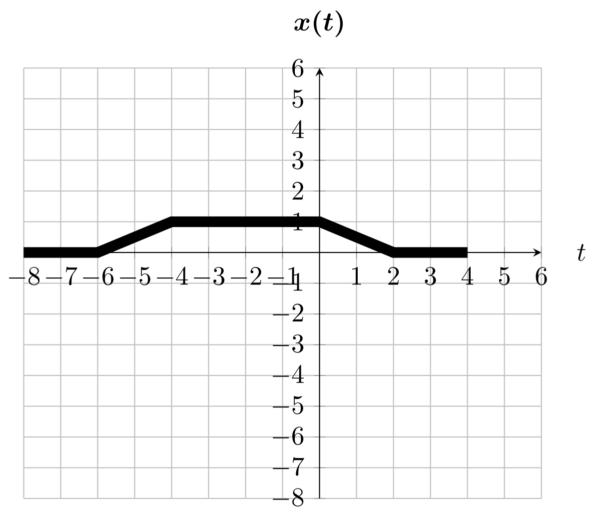

Figure 2: Graph of $y(t) = x(1/2t+1)$

3. (a) Figure 3 represents the solution for question 3.a .

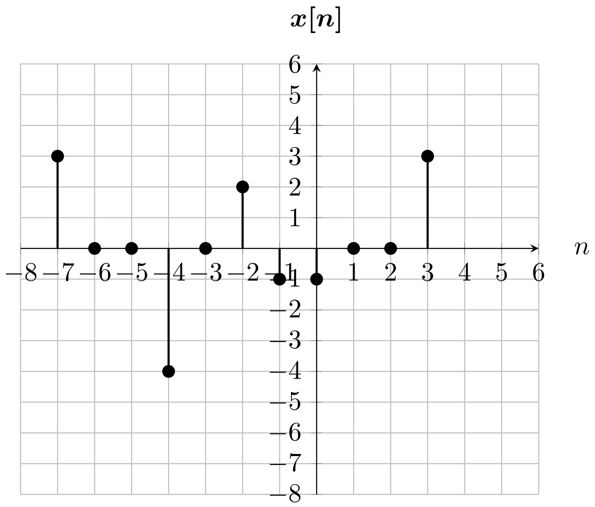

Figure 3: Graph of $y[n] = x[-n] + x[2n+1]$

(b) $x[-n] + x[2n+1] = 3\delta(n+7) - 4\delta(n+4) + 2\delta(n+2) - \delta(n+1) - \delta(n) + 3\delta(n-3)$

4. $x(t) = A * cos(wt - k) \rightarrow T_0 = 2\pi/w_0$
$x(n) = A * cos(\Omega n - k) \rightarrow N_0 = m \times 2\pi/\Omega_0$
Shifting does not effect period

   (a) period of cos $\rightarrow m \times 2\pi/(13\pi/10) = m \times 20/13 \rightarrow {}_{m=13}N_0 = 20$
   period of sin $\rightarrow m \times 2\pi/(7\pi/3) = m \times 6/7 \rightarrow {}_{m=7}N_0 = 6$
   LCM(20,6)=60

   (b) $N = m \times 2\pi/3$ there is no integer m that makes N integer, so this signal is not periodic.

   (c) $T = 2\pi/3 = 2/3$

   (d) $x(t) = -je^{j5t} = -jcos(5t) + sin(5t)$
   period of sin and cos $\rightarrow T = 2\pi/5$
   LCM($2\pi/5, 2\pi/5$)=$2\pi/5$

5. Since $x(n) \neq x(-n)$ and $x(n) \neq -x(-n)$ for all n the signal is not even and not odd.
$odd\{x(n)\} = 0.5\{x(n) - x(-n)\}$
$even\{x(n)\} = 0.5\{x(n) + x(-n)\}$
$x(n) = -\delta(n-1) + 2\delta(n-2) - 4\delta(n-4) + 3\delta(n-7)$
$\delta(-n-k) = \delta(n+k)$
fill the equations and you get
$odd\{x(n)\} = 0.5 * (-\delta(n-1) + 2\delta(n-2) - 4\delta(n-4) + 3\delta(n-7) + \delta(n+1) - 2\delta(n+2) + 4\delta(n+4) - 3\delta(n+7))$
$even\{x(n)\} = 0.5 * (-\delta(n-1) + 2\delta(n-2) - 4\delta(n-4) + 3\delta(n-7) - \delta(n+1) + 2\delta(n+2) - 4\delta(n+4) + 3\delta(n+7))$

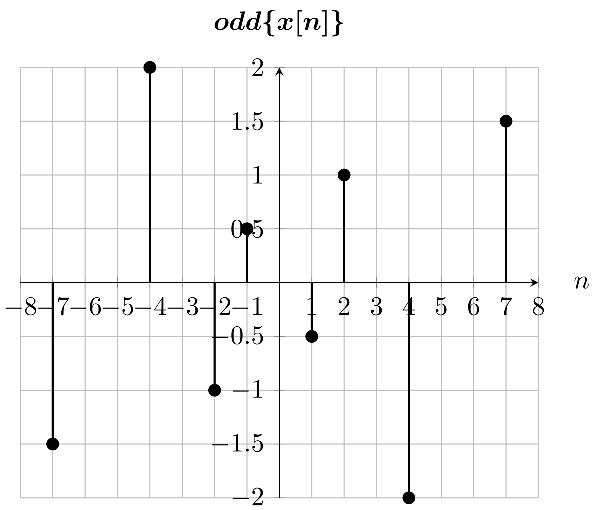

Figure 4: n vs. $odd\{x(n)\}$.

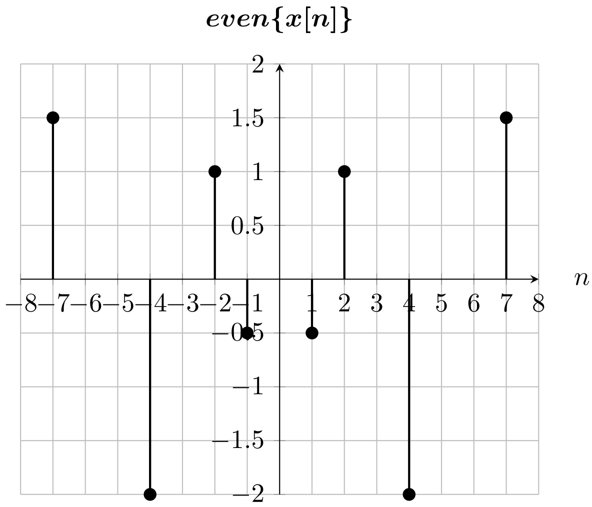

Figure 5: n vs. $even\{x(n)\}$.

6. (a)
   - has memory since for t $\neq$ 3 present value of output is dependent on future or past values of input.
   - stable since all bounded inputs generate bounded outputs.
   - not causal since for t bigger than 3, output is represented by future values of input
   - linear since the superposition principle holds $\rightarrow$ $y_1(t) = a_1x_1(2t-3)$ and $y_2(t) = a_2x_2(2t-3) \rightarrow y_3(t) = a_1x_1(2t-3) + a_2x_2(2t-3) = y_1(t) + y_2(t)$
   - invertible since $y((t+3)/2) = x(2((t+3)/2) - 3) = x(t)$
   - time invariant since, shift input and output by $t_0 \rightarrow$ input becomes $x(2(t-t_0)-3) = x(2t-2t_0-3)$, output becomes $y(t-t_0) = x(2(t-t_0)-3) = x(2t-2t_0-3)$=input

   (b)
   - memoryless since present value of output is represented by only present values of input
   - not stable since output is dependent on input and time, so we can't ensure boundedness of output for bounded inputs.
   - causal since present value of output is represented by only present values of input.
   - linear since the superposition principle holds $\rightarrow$ $y_1(t) = a_1tx_1(t)$ and $y_2(t) = a_2tx_2(t) \rightarrow y_3(t) = t(a_1x_1(t) + a_2x_2(t) = y_1(t) + y_2(t)$
   - invertible since $y(t)/t = x(t)$
   - time invariant since, shift input and output by $t_0 \rightarrow$ input becomes $x(t-t_0)$, output becomes $y(t-t_0) = (t-t_0)x(t-t_0) \neq$input

   (c)
   - has memory since for n $\neq$ 3, present value of output is represented by future or past values of input.
   - stable since all bounded inputs generate bounded outputs.
   - not causal since for n bigger than 3, output is represented by future values of input
   - linear since the superposition principle holds $\rightarrow$ $y_1(n) = a_1x_1(2n-3)$ and $y_2(n) = a_2x_2(2n-3) \rightarrow y_3(n) = a_1x_1(2n-3) + a_2x_2(2n-3) = y_1(n) + y_2(n)$
   - invertible since $y((n+3)/2) = x(2((n+3)/2) - 3) = x(n)$
   - time invariant since, shift input and output by $n_0 \rightarrow$ input becomes $x(2(n-n_0)-3) = x(2n-2n_0-3)$, output becomes $y(n-n_0) = x(2(n-n_0)-3) = x(2n-2n_0-3)$=input

   (d)
   - has memory since present value of output is represented by past values of input(sum of all past inputs).
   - not stable since sum of all past input values can be unbounded, making output unbounded (even if all input values are bounded).
   - causal since present value of output depends on only past values of input.
   - linear since the superposition principle holds $\rightarrow$ $y_1(n) = a_1\sum_{k=1}^{\infty} x_1(n-k)$ and $y_2(n) = a_2\sum_{k=1}^{\infty} x_2(n-k) \rightarrow y_3(n) = \sum_{k=1}^{\infty} a_1x_1(n-k) + a_2x_2(n-k) = y_1(n) + y_2(n)$
   - invertible since $y(n+1) - y(n) = \{x(n) + x(n-1) + x(n-2)...\} - \{x(n-1) + x(n-2), ...\} = x(n)$
   - time invariant since, shift input and output by $n_0 \rightarrow$ input becomes $\sum_{k=1}^{\infty} x(n-n_0-k)$, output becomes $y(n-n_0) = \sum_{k=1}^{\infty} x(n-n_0-k)$=input