Question for THE3

In each part, give a formal description for the Turing machines that computes the given function.

a. $f(x) = \begin{cases} x+1 & \text{if x is odd} \\ \frac{x}{2} & \text{if x is even} \end{cases}$, where $x \geq 1$.

b. $g(x, y) = |x - y|$, where $x, y \geq 1$.

Numbers will be given to TM's in unary. Initially, in part (a), tape is of the form (▷⊔11...11⊔⊔...) where number of 1's equal to x, and in part (b), (▷⊔11...11□11...11⊔ ⊔ ...), where number of 1's before the square symbol represent x and number of 1's after the square symbol represent y. For both parts, initial position of the head is the blank symbol which is between left end symbol and the input. When the machines halt, tape will be of the form (▷⊔11...11⊔ ⊔ ...) where number of 1's equal to the output of the function. Final position of the head does not matter.

In your description, represent the TM's as quintuples (K, Σ, δ, s, H) where K is the finite set of states; Σ is the alphabet containing blank symbol ⊔, input separator symbol □ (for part (b)), left end symbol ▷, and any other symbol you need to compute the functions, but not containing symbols → and ←; δ is the transition function from (K-H)×Σ to K×Σ × {→, ←}; s is the start state and H is the set of halting states.

A solution

a. (K, Σ, δ, s, H) where:

K = $\{s, q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, h\}$

Σ = $\{\sqcup, \triangleright, 1, X, ?, !\}$

H = $\{h\}$

δ is defined as:

$\delta(s, \sqcup) = (q_0, \sqcup, \rightarrow)$
$\delta(q_0, 1) = (q_1, 1, \rightarrow)$
$\delta(q_0, \sqcup) = (q_2, X, \leftarrow)$
$\delta(q_1, 1) = (q_0, 1, \rightarrow)$
$\delta(q_1, \sqcup) = (q_9, \sqcup, \leftarrow)$
$\delta(q_9, 1) = (q_9, 1, \rightarrow)$
$\delta(q_9, \sqcup) = (h, 1, \rightarrow)$
$\delta(q_2, 1) = (q_3, !, \leftarrow)$

$\delta(q_2, \sqcup) = (q_4, \sqcup, \rightarrow)$
$\delta(q_3, 1) = (q_2, 1, \leftarrow)$
$\delta(q_3, \sqcup) = (q_4, \sqcup, \rightarrow)$
$\delta(q_4, 1) = (q_4, 1, \rightarrow)$
$\delta(q_4, !) = (q_4, !, \rightarrow)$
$\delta(q_4, X) = (q_5, X, \leftarrow)$
$\delta(q_4, ?) = (q_4, X, \rightarrow)$
$\delta(q_5, !) = (q_5, X, \leftarrow)$
$\delta(q_5, 1) = (q_6, ?, \leftarrow)$
$\delta(q_5, X) = (q_5, X, \leftarrow)$
$\delta(q_6, !) = (q_6, !, \leftarrow)$
$\delta(q_6, 1) = (q_6, 1, \leftarrow)$
$\delta(q_6, \sqcup) = (q_7, \sqcup, \rightarrow)$
$\delta(q_7, ?) = (q_8, 1, \rightarrow)$
$\delta(q_7, 1) = (q_7, 1, \rightarrow)$
$\delta(q_7, !) = (q_4, 1, \rightarrow)$
$\delta(q_8, 1) = (q_8, 1, \rightarrow)$
$\delta(q_8, X) = (q_8, \sqcup, \rightarrow)$
$\delta(q_8, \sqcup) = (h, \sqcup, \leftarrow)$

b. (K, Σ, δ, s, H) where:

K $= \{s, q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_{10}, h\}$

$\Sigma = \{\sqcup, \rhd, 1, \circ, \Box, \vdash, \dashv, ?, !\}$

H $= \{h\}$

δ is defined as:

$\delta(s, \sqcup) = (q_0, \sqcup, \rightarrow)$
$\delta(q_0, 1) = (q_0, 1, \rightarrow)$
$\delta(q_0, \Box) = (q_0, \Box, \rightarrow)$
$\delta(q_0, \sqcup) = (q_1, \dashv, \leftarrow)$
$\delta(q_1, 1) = (q_2, ?, \leftarrow)$
$\delta(q_2, 1) = (q_2, 1, \leftarrow)$
$\delta(q_2, \Box) = (q_2, \Box, \leftarrow)$
$\delta(q_2, \vdash) = (q_3, \vdash, \rightarrow)$
$\delta(q_2, \sqcup) = (q_3, \sqcup, \rightarrow)$
$\delta(q_3, \Box) = (q_4, \Box, \rightarrow)$
$\delta(q_4, 1) = (q_4, 1, \rightarrow)$
$\delta(q_4, ?) = (q_5, 1, \rightarrow)$

$\delta(q_5, \sqcup) = (q_6, \sqcup, \leftarrow)$
$\delta(q_5, \dashv) = (q_5, \dashv, \rightarrow)$
$\delta(q_3, 1) = (q_7, \vdash, \rightarrow)$
$\delta(q_7, 1) = (q_7, 1, \rightarrow)$
$\delta(q_7, \Box) = (q_7, \Box, \rightarrow)$
$\delta(q_7, ?) = (q_1, \dashv, \leftarrow)$
$\delta(q_1, \Box) = (q_5, \Box, \rightarrow)$
$\delta(q_6, \dashv) = (q_6, \sqcup, \leftarrow)$
$\delta(q_6, \Box) = (q_6, \sqcup, \leftarrow)$
$\delta(q_6, \vdash) = (q_6, \sqcup, \leftarrow)$
$\delta(q_6, \sqcup) = (h, \sqcup, \rightarrow)$
$\delta(q_6, 1) = (q_8, !, \leftarrow)$
$\delta(q_8, 1) = (q_8, 1, \leftarrow)$
$\delta(q_8, \dashv) = (q_8, \dashv, \leftarrow)$
$\delta(q_8, \Box) = (q_8, \Box, \leftarrow)$
$\delta(q_8, \vdash) = (q_8, \vdash, \leftarrow)$
$\delta(q_8, \sqcup) = (q_9, \sqcup, \rightarrow)$
$\delta(q_9, 1) = (q_9, 1, \rightarrow)$
$\delta(q_9, \vdash) = (q_{10}, 1, \rightarrow)$
$\delta(q_9, \dashv) = (q_{10}, 1, \rightarrow)$
$\delta(q_9, \Box) = (q_{10}, 1, \rightarrow)$
$\delta(q_{10}, \vdash) = (q_{10}, \vdash, \rightarrow)$
$\delta(q_{10}, \dashv) = (q_{10}, \dashv, \rightarrow)$
$\delta(q_{10}, \Box) = (q_{10}, \Box, \rightarrow)$
$\delta(q_{10}, 1) = (q_{10}, 1, \rightarrow)$
$\delta(q_{10}, !) = (q_6, \sqcup, \leftarrow)$
$\delta(q_9, !) = (h, 1, \rightarrow)$