Student Information

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Answer 1

a.

Lets denote every number the set of rational numbers inside the open interval $(-1, 0)$ contains as $n = \frac{a}{b}$. The elements of this set is:

$a \setminus b$	1	2	3	4	...
-1	$-\frac{1}{1}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	...
-2	$-\frac{2}{1}$	$-\frac{2}{2}$	$-\frac{2}{3}$	$-\frac{2}{4}$	...
-3	$-\frac{3}{1}$	$-\frac{3}{2}$	$-\frac{3}{3}$	$-\frac{3}{4}$	...
-4	$-\frac{4}{1}$	$-\frac{4}{2}$	$-\frac{4}{3}$	$-\frac{4}{4}$	...
...	...	...	...	...	...

In the table, first row infinite elements of set $\mathbb{N}$ except 0, starting from 1 and the first column contains the countably infinite elements of set Z^- except 0. The entries in the upper triangular half of this table (above the diagonal, diagonal is exclusive) shows the numbers $n = \frac{a}{b}$ that is in this set.
These can be counted reverse diagonally starting with $-\frac{1}{2}$ (i.e. 1st element: $-\frac{1}{2}$, 2nd element: $-\frac{1}{3}$, 3rd element $-\frac{2}{3}$, 4th element: $-\frac{1}{4}$, 5th element: $-\frac{1}{5}$, 6th element: $-\frac{2}{4}$, 7th element: $-\frac{3}{4}$, 8th element: $-\frac{2}{5}$, 9th element: $-\frac{1}{6}$ and so on). With this way of counting we will eventually get all elements in the set, which we can index them with unique numbers from $\mathbb{N}$ as $0 \implies -\frac{1}{2}$, $1 \implies -\frac{1}{3}$, $2 \implies -\frac{2}{3}$, $3 \implies -\frac{1}{4}$ ...
With this mapping we can get the one-to-one correspondence between the set of rational numbers inside the open interval $(-1, 0)$ and $\mathbb{N}$.
Since we get one to one correspondence with the set of rational numbers this set is countably infinite.

b.

The set$D$ is empty set, therefore it is finite and countable, because any finite language L is regular , L^* is regular and their concatenation is LL^* is regular. Since $L^+ = LL^*$ L^+ is also regular. So L^+ cannot be regular and nonregular. So $D = \{\}$.

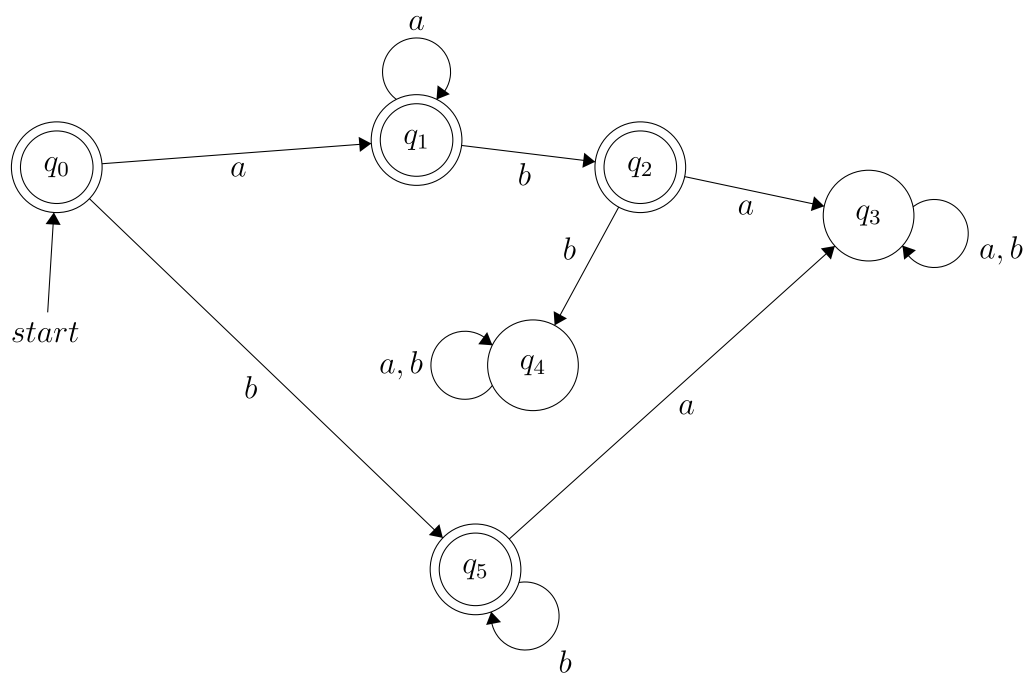
c.

We know that a language is regular iff it is accepted by a FA, and set of nonregular languages over an alphabet of size 2 or more is uncountable. Since the set of all languages on the binary alphabet

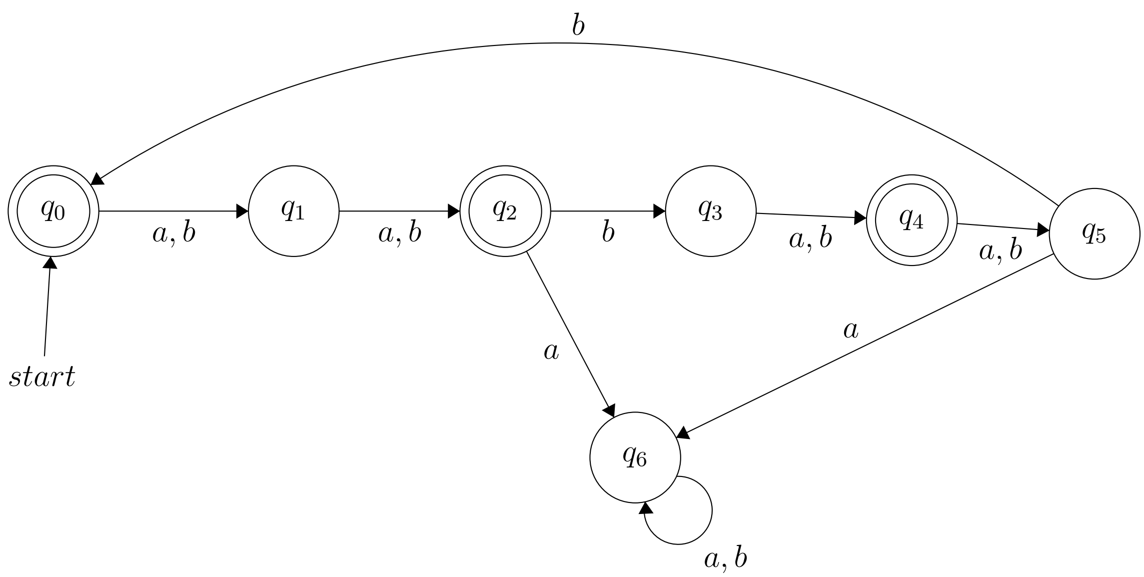
$\sum = \{a, b\}$ which can't be recognized by a FA is the set of all nonregular languages on the binary alphabet of size 2, this set is uncountably infinite.

Answer 2

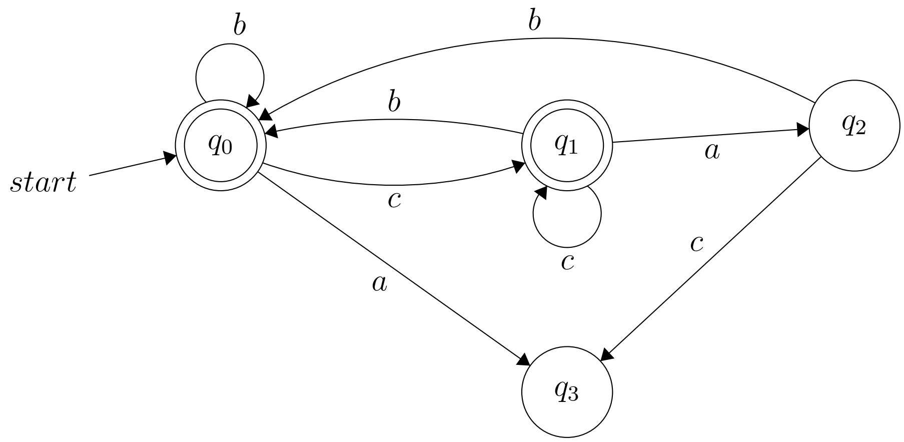
a.



b.

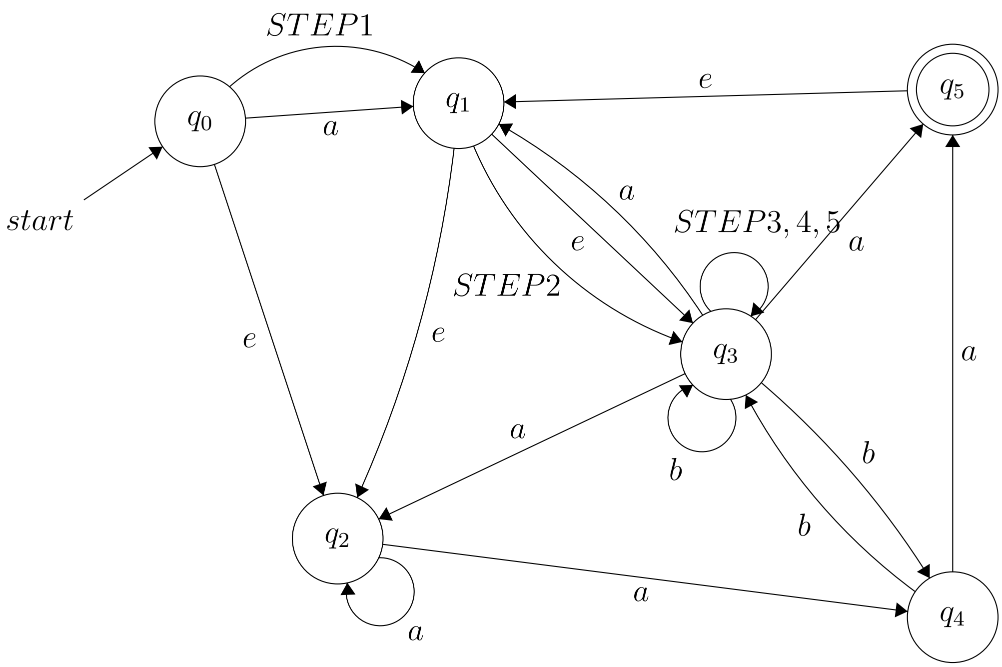


c.



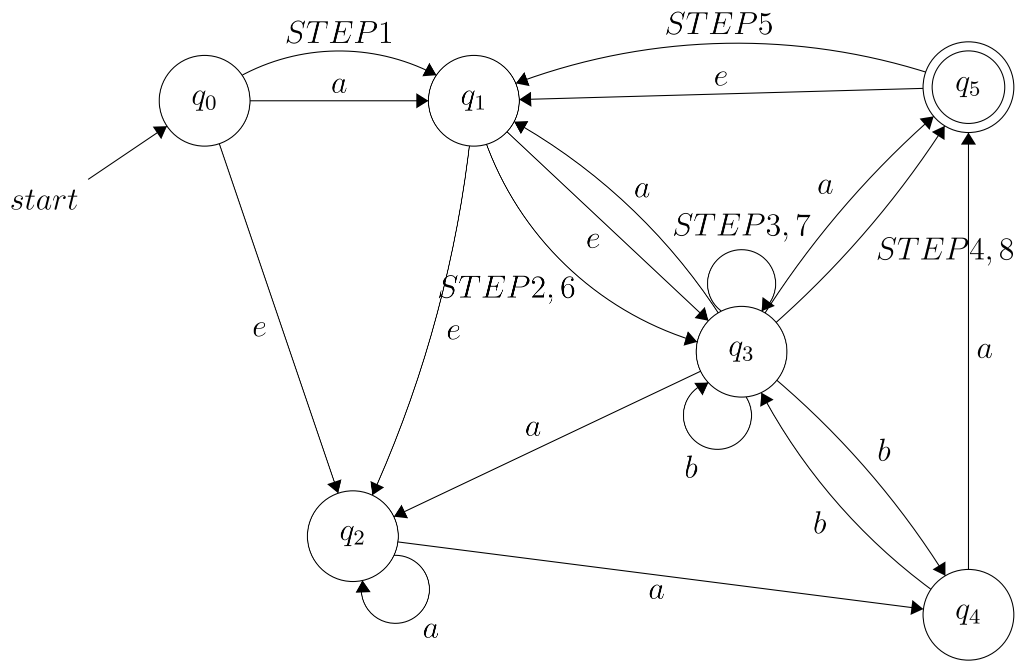
Answer 3

a.



We cant reach to the end-state q_5 with this word so w_1 is not in the language.(Also there is no word that ends with b in this language since there is no empty input or b input coming in to q_5)

b.

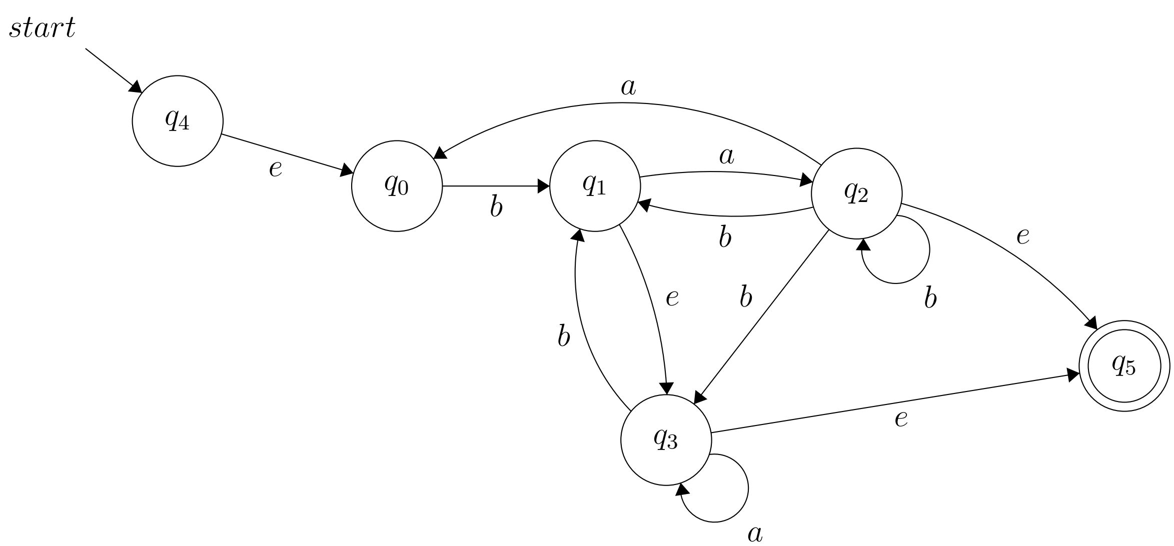


We can reach to the end-state q_5 by following the path $q_0 \implies q_1 \implies q_3 \implies q_3 \implies q_5 \implies q_1 \implies q_3 \implies q_3 \implies q_5$.
So we can read the word *ababa* in this language, therefore this word is in the language.

Answer 4

a.

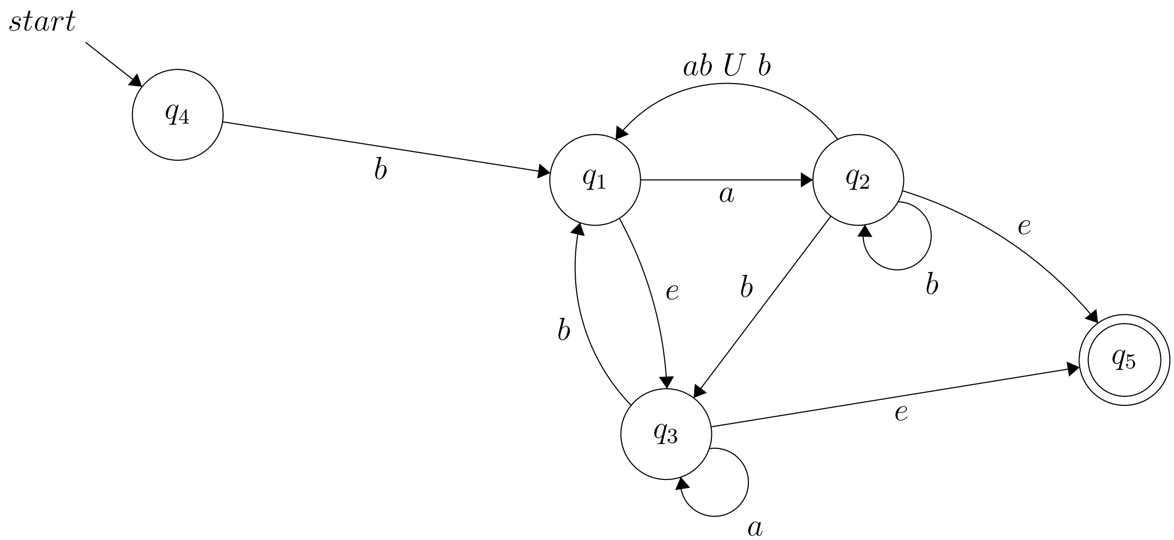
A generalized finite autoamaton (GFA) has one starting state with no incoming transitions; one final state with no outgoing transitions.
Therefore our GFA is of the form:



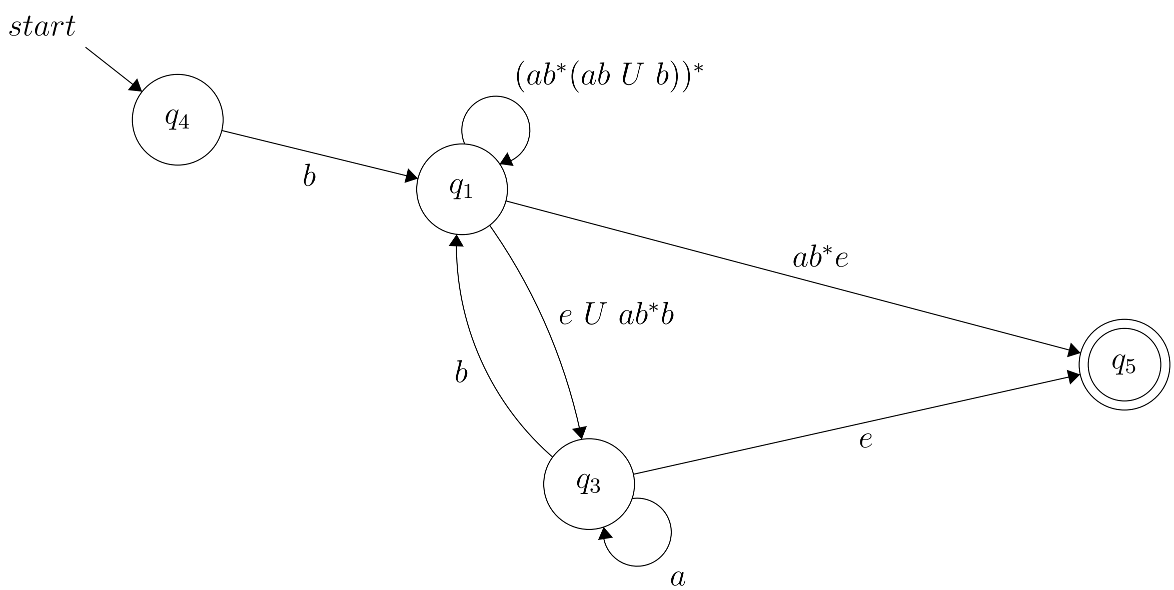
b.

Continuing from the first state given in 4.a:

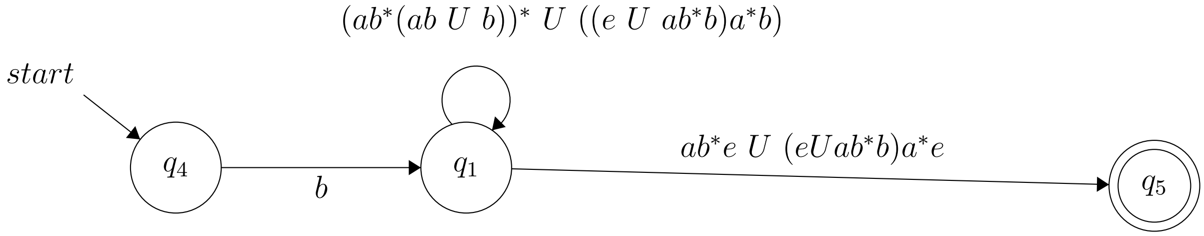
Eleminate q_0 :



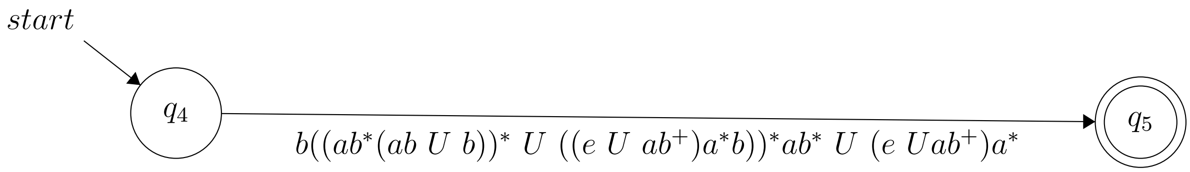
Eleminate q_2:



Eleminate q_3 :



Eleminate q_1 and simplify:



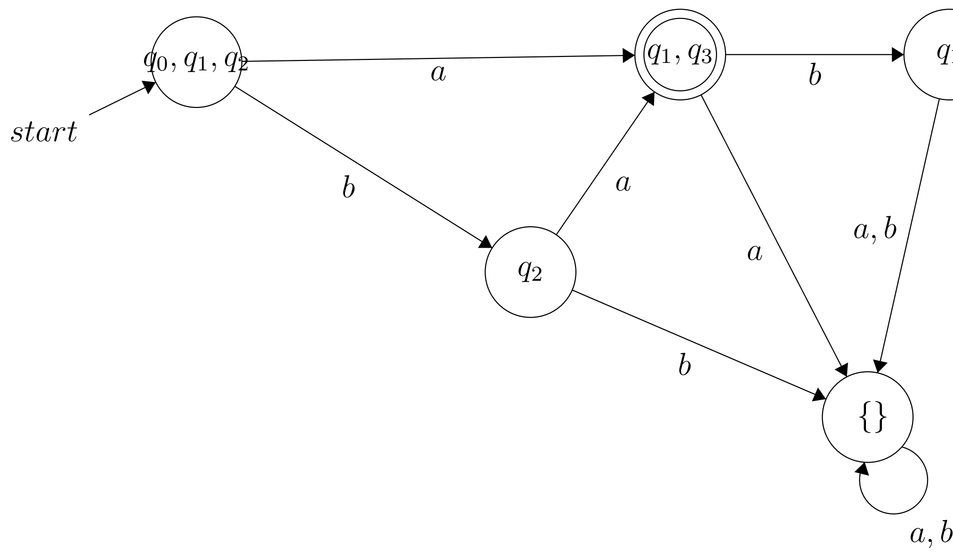
Answer 5

$N = \{k, \sum, \Delta, s, F\}$
$M = \{k', \sum, \Delta, s', F'\}$

a.

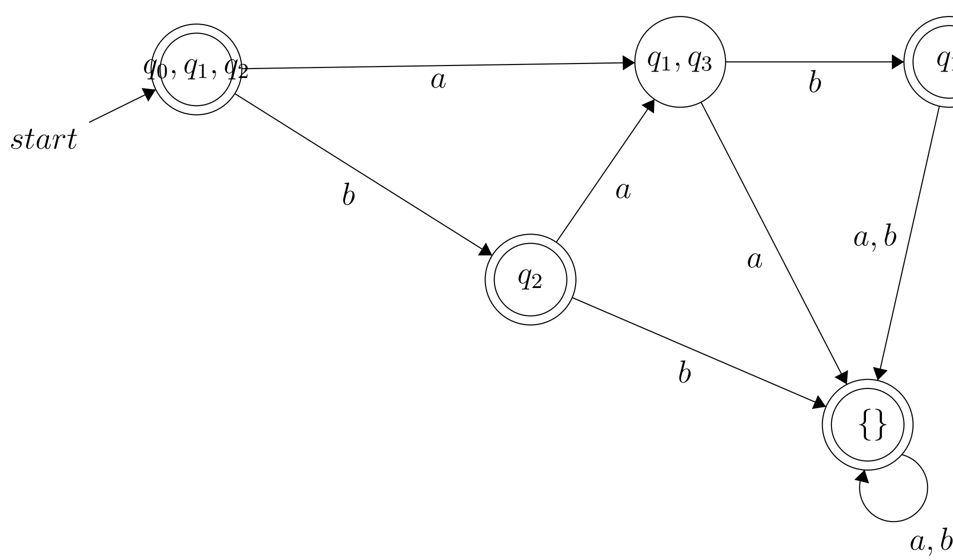
The algorithm is like this: 1- Create a start state of the DFA by taking E(q): (the set of states of N that can be reached from q wihtout reading any symbol from the input tape) with parameter q_0(stand state).
So $s' = E(q_0) = \{q_0, q_1, q_2\}$
2- Each time a new state is generated repeat these:
2.1- Create new states reachable frın tge new states by reading one symbol.
2.2- Apply E() to the new states, possibly resulting in a new state.
3- Finish states of M are those which contain any of the finish states of N(i.e any state that contains q_3)
Applying second step we get:
$E(q_0) = \{q_0, q_1, q_2\} = s'$
$E(q_1) = \{q_1\}$
$E(q_2) = \{q_2\}$
$E(q_3) = \{q_1, q_3\}$
$\delta(E(q_0), a) = E(q_1) U E(q_3) = \{q_1, q_3\}$
$\delta(E(q_0), b) = E(q_2) = \{q_2\}$
$\delta(E(q_1), a) = \{\}$
$\delta(E(q_1), b) = \{\}$
$\delta(E(q_2), a) = E(q_3) = \{q_1, q_3\}$
$\delta(E(q_2), b) = \{\}$
$\delta(E(q_3), a) = \{\}$
$\delta(E(q_3), b) = E(q_1) = \{q_1\}$

So M=



b.

We can get the complement of a DFA just by swapping end-states and non-end-states.
Since N=M, L=L(N)=L(M)
If we take the complement of L we get :



Which is $eU(baUa)((baUbb)(aUb)^*)U(baUa)a(aUb)^*)UbUbb(aUb)^*$ as a RE.
So $\bar{L} = \sum^* - L$ or $\bar{L} = \{w | w \in \sum^* \wedge w \notin L\}$

Answer 6

$L_1 - L_2 = L_1 \cap \bar{L_2}$
Let $L_1 = \{Q_1, \sum, \delta_1, q_1, F_1\}$
$L_2 = \{Q_2, \sum, \delta_2, q_2, F_2\}$
so $\bar{L_2} = \{Q_2, \sum, \delta_2, q_2, Q_2 - F_2\}$(From Question 5.b)

We can assume that alphabets of the L_1 *and* $\bar{L_2}$ are the same i.e $\sum$ is the union of the alphabets of L_1 and L_2.
We can assume that both automata are deterministic since we can convert NFA to DFA(from question 5.a)
Now we should construct an automata L=$L_1 \cap \bar{L_2}$ that simulates both L_1 *and* $\bar{L_2}$.
States of L are pairs of states (p,q) p coming from L_1, q coming from $\bar{L_2}$.
If p goes to s upon reading input s and q goes to t upon reading input s then in our automata L (p,q) will go to (s,t) upon reading input s.
Our start state is pair of start states (q_1, q_2).
Our end-states are pairs that are end-states in their respective automatas i.e (p,q) s.t $p \in F_1, q \in (Q_2 - F_2)$
So our new automata $L = (Q_1 \times Q_2, \sum, \delta, (q_1, q_2), F_1 \times (Q_2 - F_2))$ where $\delta((p, q), a) = (\delta_1(p, q), \delta_2(q, a))$

Answer 7

a.

Assume L is regular
If it is regular there exists some number p which is the pumping length.
$w = a^{p^2} b^* \in L \quad\quad | w | \geq p^2 > p$
Partition w into xyz i.e w=xyz with $| xy | \leq p$ and y shouldn't be empty.
For each different way of partitioning we should show that $xy^i z \notin L$ for some $i \geq 0$
$| xy | \leq p^2$ and y is nonempty
choose $x = a^l \quad y = a^m \quad t = a^{p^2 - l - m}$ where $l + m \leq p \quad and\ m > 0$
choose i=2. We get $xy^2 z = a^{p^2 + m}$
$p^2 + m$ is not a perfect square since $0 > m \leq p$ and difference between p^2 adn next square is always greater than p for all natural numbers.
$xy^2 z \notin L$ so our assumption is wrong and L is not regular.