CENG 222
STATISTICAL METHODS FOR CENG
Spring 2019
TAKE HOME EXAM 1

	$Y = 10$		$Y = 20$		$Y = 30$	
	$Z = 0$	$Z = 1$	$Z = 0$	$Z = 1$	$Z = 0$	$Z = 1$
$X = 3$	0.025	0.025	0.03	0.02	0.05	0.15
$X = 5$	0.075	0.050	0.025	0.030	0.020	0.2
$X = 7$	0.04	0.06	0.025	0.050	0.025	0.1

1) Answer the questions **a-e** below according to the **joint probability** table above and you have to show your work by the formulas of **CONDITIONAL PROBABILITY** and **MARGINALIZATION**, otherwise answers will be severely penalized.

- **a.** Compute $P(Z = 0 \mid X = 5, Y = 10)$.
- **b.** Show the marginal probability distribution of $P(X)$.
- **c.** Using the distribution found in **b,** find $\mathbb{E}_X[x]$ and $Var_X[x]$.
- **d.** Show the marginal probability distribution of $P(X \mid Z = 1)$.
- **e.** Using the distribution found in **d,** find $\mathbb{E}_{\{X|Z = 1\}}[x]$ and $Var_{\{X|Z = 1\}}[x]$.

2) Let X and Y be random variables from the following distributions:

$$P_X(k) = c_1 2^{-k}$$
$$P_Y(k) = \frac{c_2 2^{-k}}{k}$$

For $k = 1,2,3 \ldots$

- **a.** Find the constants c_1 and c_2.
- **b.** Compute $P(X \; is \; even)$.
- **c.** Compute $P(X + Y = 6 \mid X \; is \; odd)$ using Bayes Formula.
- **d.** Suppose we have a bag containing balls numbered from 1 to 1000. Suppose somebody draws two balls at random at told that the number total is $b_1 + b_2 = 6$. Compute $P\ (X = b_1, Y = b_2\ )$ using Bayes Formula.

3) Prove the following independence questions:

a. Show whether the random variables X and Y are independent or not in question 1.
b. If event A be independent of itself, i.e. let A and A be independent, then $p(A)$ is either 0 or 1.
c. If event A have $p(A) = 0$ or 1, then A and an arbitrary event B are independent.

4) Let G be a geometrically distributed RV with $P_G(k;p) = (1-p)^{k-1}p, \;\; k = 1,2 \dots$. Answer the following:

a. Show that $P(G = n-1+m \mid G > n-1) = P\,(G = m)$.
b. Show that cumulative distribution function is given by $F_G(n) = P(G \le n) \; = 1 - (1-p)^n$.
c. Now, 3 fair 6-sided dice are thrown repeatedly in an independent fashion. Let G denote the roll on which a sum of 4 is observed for the first time. Use **(b)** to evaluate $P(65 \le G \le 75)$.

5) A chromosome mutation linked with a certain disease is known to occur on the average once in every 10,000 births.

a. Write the binomial formula for computing the probability of exactly 3 of the next 20,000 babies born will always have the mutation. Compute the value.
b. Approximate the probability in **(a)** and state the reasons of your chosen approximating distribution. Is there an agreement between true value and your approximation?

6) Let O be a Poisson distributed RV with $P_O(k;\lambda) = \frac{e^{-\lambda}\lambda^k}{k!}, \;\; k = 0,1,2,\dots$. Answer the following:

a. Show that $\mathbb{E}[O^n] = \lambda\mathbb{E}[(O+1)^{n-1}]$
b. Use **(a)** to find $\mathbb{E}[O^3]$

REGULATIONS

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
3. **Cheating** : We have zero tolerance policy for cheating}. People involved in cheating will be punished according to the university regulations.
4. You must follow odtuclass for discussions and possible updates on a daily basis.
5. **Evaluation**: Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using ``black-box'' technique and manually by assistants, so make sure to obey the specifications.

SUBMISSION

Submission will be done via ODTUCLASS. Download the given template file, "the1.tex", when you finish your exam upload your "the1.tex" file to ODTUCLASS.