

- 1) Consider a sequence of coin tosses, and let  $\theta$  be the probability of heads at each toss.
  - a. For some fixed  $k$ , let  $N$  be the number of tosses until the  $k$ th head occurs. Find the ML estimator of  $\theta$  based on  $N$ .
  - b. For some fixed  $n$ , let  $K$  be the number of heads observed in  $n$  tosses. Find the ML estimator of  $\theta$  based on  $K$ .
  
- 2) Let  $D = \{6.5, 8.8, 7.5, 9.2, 9.9, 12.4\}$  be a random sample of size 6 from  $N(\mu, 0.8)$ , i.e. all data points are drawn from Normal distribution with variance 0.8.
  - a. Compute the maximum likelihood estimate for the parameter  $\mu$ .
  - b. Generate %95 confidence interval for the estimated parameter in part (a).
  - c. At least how many samples more are needed in order to estimate our population mean with a distance of 0.5 with %99 confidence?
  
- 3) Let  $p$  denote the true proportion of students who prefers C# programming language to C++. Let  $X$  denote the number of students out of  $n$ , who prefers C#.ol style="list-style-type: none;">- a. Compute the smallest  $n$  for which the probability is 80% that the difference between  $\frac{X}{n}$  and  $p$  is less than 0.02.
- b. Let's say  $\frac{X}{n}$  have a margin of error equal to 0.06, how many more observations are required to have a margin half the size?

- 4) Suppose  $H_0: \mu = 120$  is tested against  $H_1: \mu \neq 120$  where  $\sigma = 10, n = 16$  what P-value is associated with the sample mean  $\bar{x} = 123.3$ . Under what circumstances will  $H_0$  be rejected?

- 5) Let's say we take a sample of size 1 from

$$p_X(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, 2, \dots$$

and want to test:

$$H_0: \lambda = 6 \text{ vs } H_1: \lambda < 6$$

by rejecting  $H_0$  if  $k \leq 2$ .

- a. Calculate the probability of committing a Type I error.
  - b. Calculate the probability of committing a Type II error.
- 6) Check the following measurements for a predictor ( $x$ ) and response variable ( $y$ ) :

$x$	$y$
1	12,6
2	11,6
3	6,8
4	9,2

Find the slope and intercept using least squares estimation.

## REGULATIONS

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your solutions. Otherwise, you will get 0 from that question.
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## SUBMISSION

Submission will be done via ODTUCLASS. Download the given template file, "the3.tex", when you finish your exam upload your "the3.tex" file to ODTUCLASS.