

# Reading Notes: Comparing ML and SMM

Venetia Wong

April 2020

## Introduction

In this article, I compare the use of the maximum likelihood (ML) estimator against the simulated method of moments (SMM) estimator in the context of the sequential discrete choice model outlined by Eisenhauer et al. [EHM15]. I will first discuss the theoretical performance of the two methods before comparing their empirical performance. Overall, the authors find that the ML estimator has a better performance than the SMM estimator.

## Theoretical Comparison

The SMM is more robust than the ML in obtaining estimators. In this paper, the authors restricted agents to binary choices to avoid the need for simulation and interpolation in obtaining an ML estimator. However, the ML estimator may not be permissible in cases where the computation of the likelihood is too complicated. The problem of computing likelihood is circumvented in the SMM, since the estimator is obtained through simulations.

However, the construction of the SMM can be subjective. After all, researchers have the liberty to choose the moments to match, the number of iterations in the simulation, the moment weighting matrix, and the optimization algorithm. While the moments are often specified, some it is not uncommon for researchers to not declare the number of iterations and the optimization algorithms used, leaving room for doubt on the validity of the estimates.

Nonetheless, in cases where both the ML and SMM methods are able to yield estimators, their respective estimators are both efficient. The efficiency of the ML estimator is easily shown, since its variance achieving the Cramer-Rao bound. Similarly, the SMM estimator is consistent and asymptotically normal. Furthermore, if the score vector for the SMM is correctly specified, the SMM estimator is asymptotically efficient.

## Empirical Performance

Both the ML and SMM estimates achieve a fairly decent cross-section model fit of the average annual earnings for each state and the conditional state frequency. However, the SMM estimates fared worse in fitting the average earnings among late college graduates and late college dropouts, since those were states with few agents. Furthermore, the root mean square error was lower for the ML estimators, indicating that they were more precise. The p-values from the ML estimation suggest that the model was consistent with the data at the 5% level of significance, but this was not the case for the SMM model.

Most glaringly, although the SMM had a good model fit, it failed to correctly estimate the net returns, as a result of its inability to detect the systematic differences in costs faced by agents. Unlike the ML results where all policy predictions were consistent with ground truth, the SMM was overoptimistic about the graduation rate for those induced to enroll in college late.

In addition, the authors investigated sensitivity of the SMM to the choice of the number of replications, weighting matrix, and the optimization algorithm. **Number of iterations:** As expected, the difference between the true parameters drop significantly as the number of iterations increase, but this effect trails off beyond 20 replications. In a finite sample, the difference between moments remain despite a large number of replications. **Weighting matrix and optimization algorithms:** When the identity matrix was used, the criterion function had multiple local minima. This is especially problematic since local minimization algorithms are typically used. In fact, the authors note that the Nelder-Mead algorithm, a local minimization algorithm, is standard practice. Yet, the Nelder-Mead algorithm did not even come close to the minima achieved by the POUNDerS algorithm.

## Conclusion

In conclusion, the ML seems to be a superior estimation strategy in the sequential discrete choice model presented by Eisenhauer et al., as it provides more reasonable estimates, and is more reliable in that, a priori, a researcher need not be concerned with obtaining wrong estimates as a result of a choice of his choice of a weighting matrix, and so forth. However, the ML method requires that the likelihood can be easily computed, which may not necessarily be the case in other types of models. Indeed, the authors acknowledge that the SMM is sometimes necessary, and propose a Monte Carlo exercise for SMM users to build confidence in their implementations.

## References

- [EHM15] Philipp Eisenhauer, James J. Heckman, and Stefano Mosso. Estimation of dynamic discrete choice models by maximum likelihood and the simulated method of moments. *International Economic Review*, 56(2):331–357, 2015.