Empirical Methods in Finance

Project #1: Cointegration and Pair Trading

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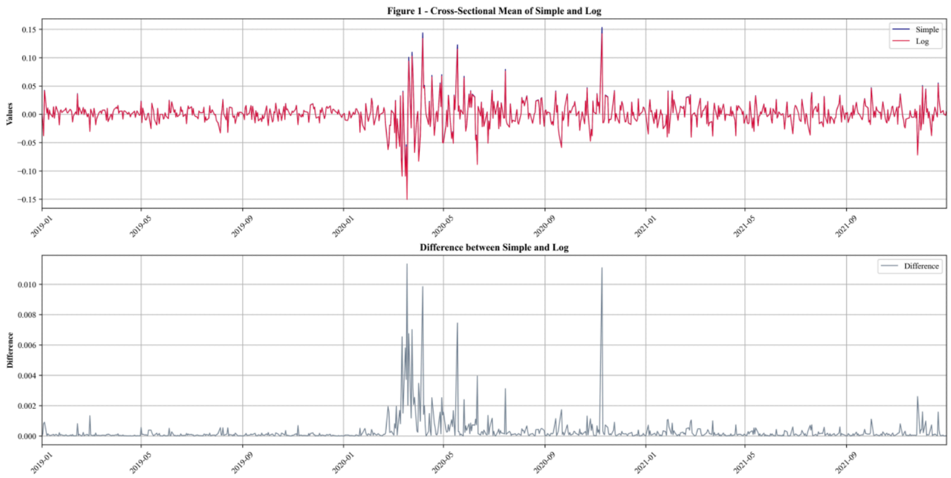
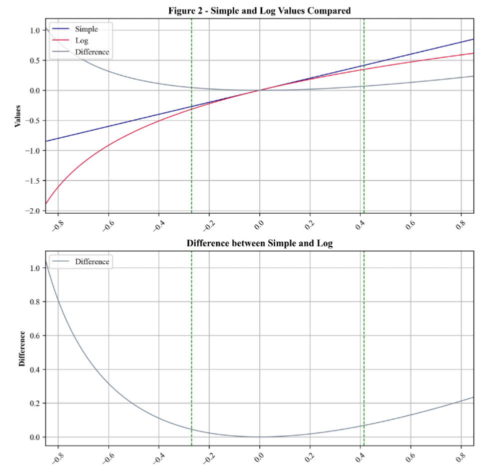
Part 1 - Descriptive Statistics

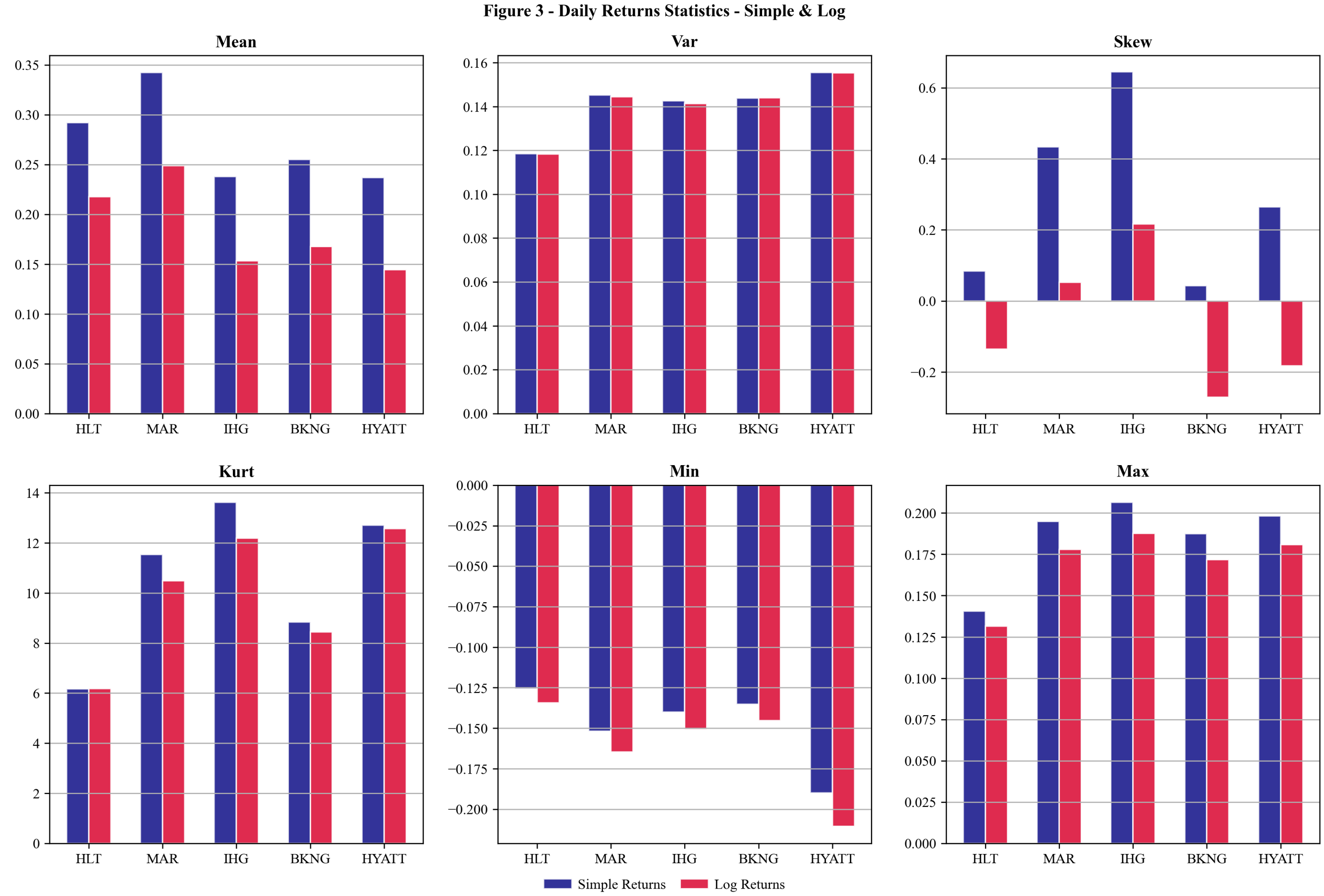
Q1.1 - We start by noting that the use of log returns is appropriate. Figure 1 shows that simple and log returns are essentially identical most of the time. They diverge slightly when returns are extreme, but that difference is often negligible and always very transitory, never surpassing 12% for our daily returns sample. Importantly, log returns systematically attenuate the magnitude of high returns and overstate the magnitude of low returns. We provide Figure 2 demonstrating the magnitude of the difference between simple and log returns as a function of returns. It must be noted, however, that log returns may become less accurate when looking at weekly returns. This is the case because the extreme values of weekly returns can be significantly higher than those of daily returns. The most significant deviation in our sample originates from an extreme weekly return of just over 41%, resulting in a 7% understatement by log returns.





Observing individual moments of distributions reveals that some moments differ significantly while others remain essentially unchanged, as shown in Figure 3:

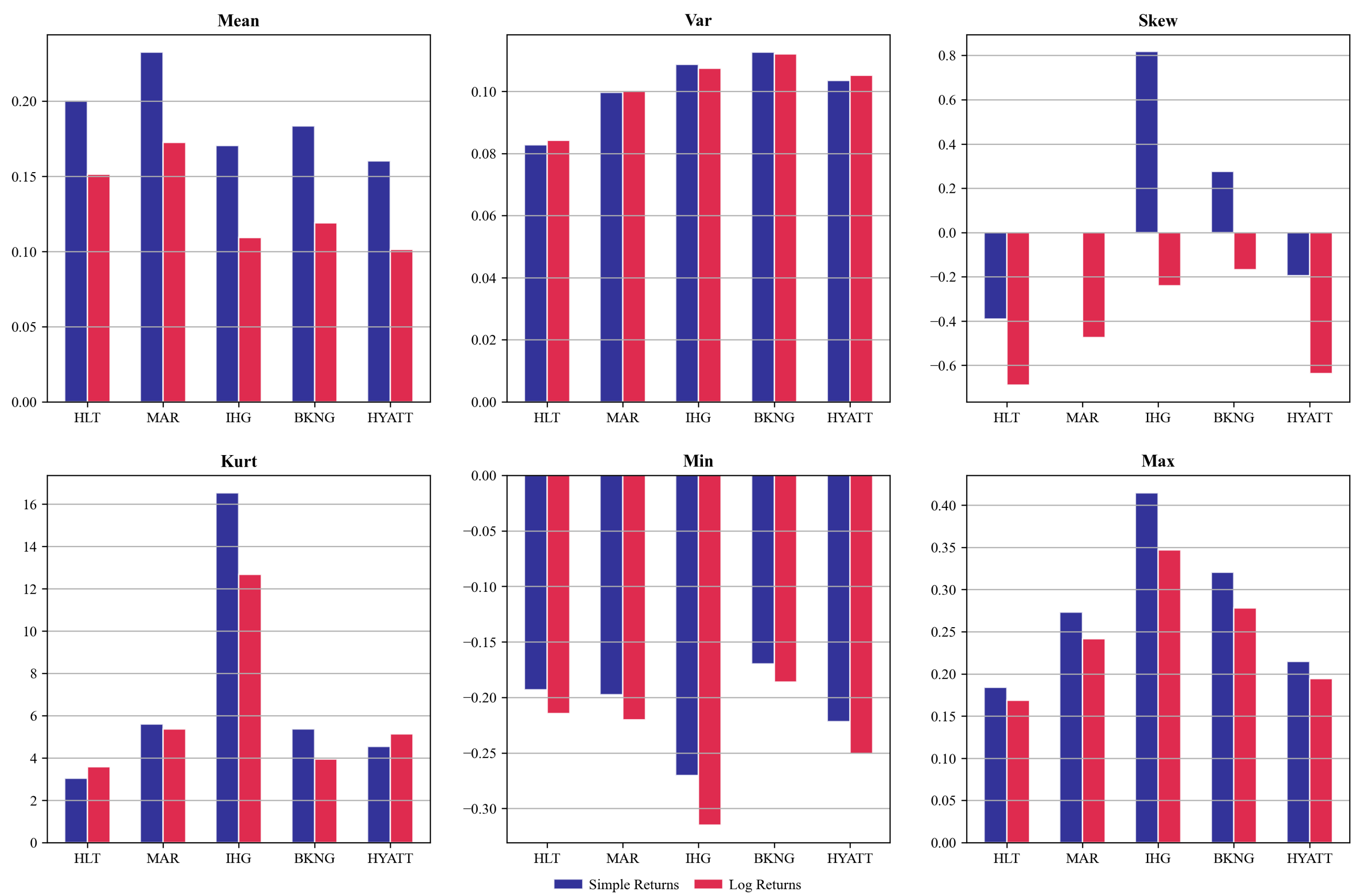
Figure 3 - Daily Returns Statistics - Simple & Log



The sample mean, minimum and maximum observed returns are all systematically lower for log returns, which is to be expected given the aforementioned behaviors of log values. The statistic least affected by the log transformation is the sample variance, with almost identical values for simple and log returns. Excess kurtosis is also lower for log returns. Skewness is the statistic that is most affected by the log transformation, with 3 out of 5 log returns showing a skewness with a negative sign, whereas all simple returns had a positive skew. This highlights that log returns not only understate returns but underestimate returns asymptotically as they become extremely negative.

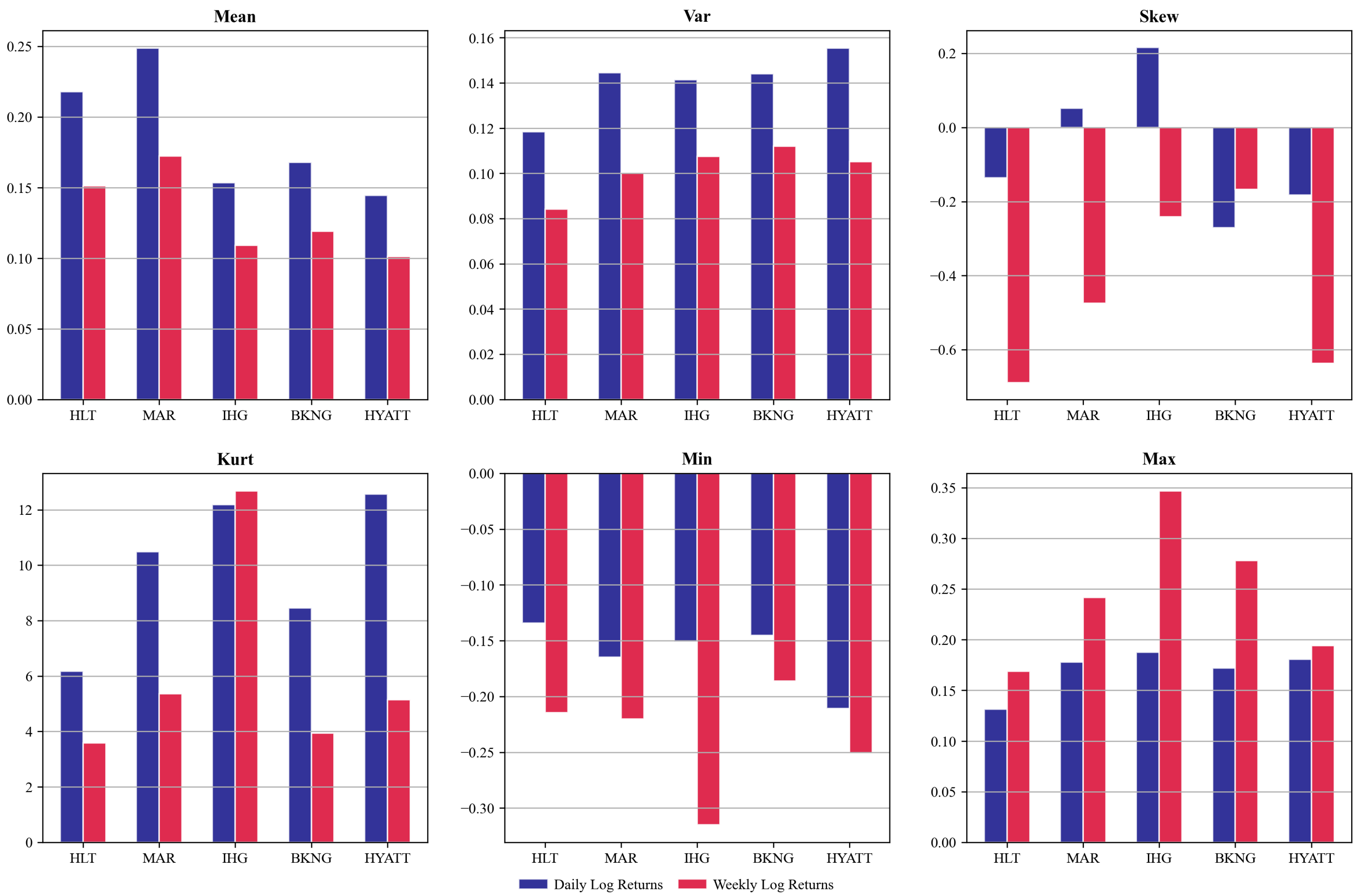
Q1.2 – Comparing the simple return and log return in weekly frequency, unsurprisingly, the differences between the two return methods in most moments are similar to the daily frequency, except for the skewness. To be more precise, weekly returns exhibit similar sample statistic properties regarding sample mean, minimum, and maximum. Moreover, the variance between simple and log returns differs slightly more than in the case of daily returns, but the difference is marginal. However, although both types of return are leptokurtic, the pattern between the differences in kurtosis is ambiguous across different firms. Furthermore, interestingly enough, log returns for all companies are negatively skewed, suggesting that highly negative outcomes occur more frequently for all companies compared to a standard normal distribution. These differences in skewness values between log and simple returns seemingly contradict each other, but again, they are due to the mathematical properties of simple return and log return explained previously. In particular, significant losses can cause the value to approach negative infinity as the log return becomes highly negative when the price ratio gets closer to zero.

Figure 4 - Weekly Returns Statistics - Simple & Log



Q1.3 – When comparing daily and weekly log returns shown in Figure 5, it is evident that the general characteristics persist across the two timeframes. As anticipated from the earlier comparisons, mean, minimum, and maximum returns are consistently lower for weekly log returns. Variance exhibits a slight but noticeable increase in disparity when assessed on a weekly basis. The most significant difference is the negative skewness in weekly log returns across all firms, suggesting an intensified frequency of negative outcomes at a weekly interval, further exaggerated by the log transformation.

Figure 5 - Daily vs Weekly Returns - Log



Part 2 – Stationarity

Q2.1 – A time series is said to have a unit root if the lag coefficient (ϕ) is equal to one. If the time series appears to have a unit root, it is not covariance stationary. Under the condition of stationarity, it must be true that the autoregressive coefficient is smaller than 1, and shocks are, therefore, transitory. If otherwise, the process will invariantly shift upwards over time by taking the total value of the previous observation. Thus, the null hypothesis should be defined as $\phi = 1$, signifying a unit root in the time series, which implies the time series is non-stationary, against the alternative hypothesis H_a where ϕ is lower than 1, suggesting that there is no unit root in the time series and the time series is therefore stationary. Formally, both hypotheses are shown as follows:

$$H_0: \phi = 1$$
$$H_a: \phi < 1$$

Q2.2 – The rejection or non-rejection of the null hypothesis relies upon the t-statistic of the OLS estimate of the autoregressive coefficient (sometimes referred to as tau-statistic). In our case, the test statistic should be defined as:

$$t_\phi = \frac{\phi - 1}{SE(\phi)}$$

It must be pointed out that the Dickey-Fuller test has its own distribution under the null hypothesis of a unit root. Therefore, the distribution of this statistic does not follow Student's t-distribution, so we must use the critical value provided by the Dickey-Fuller distribution to test the hypothesis at our desired level of significance. If we can reject the null hypothesis, the time series does not have a unit root and is stationary.

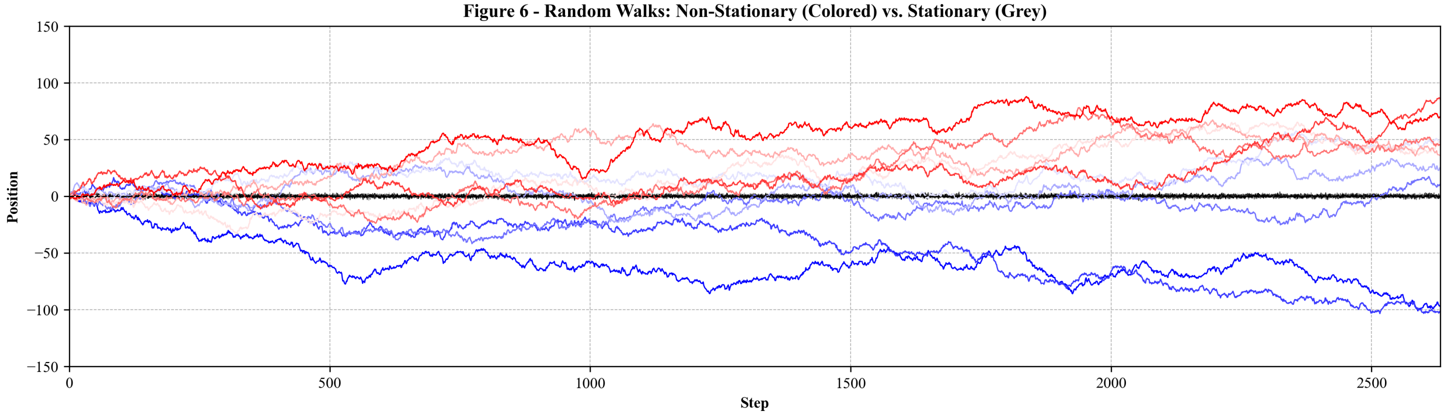
Q2.3 – Now, by taking the first-order difference, our model becomes:

$$p_t - p_{t-1} = \mu + \phi p_{t-1} - p_{t-1} + \varepsilon_t$$
$$p_t - p_{t-1} = \mu + (\phi - 1)p_{t-1} + \varepsilon_t$$
$$r_t = \mu + (\phi - 1)p_{t-1} + \varepsilon_t$$

Hence, our null and alternative hypothesis transforms to $H_0: (\phi - 1) = 0$, against $H_a = (\phi - 1) < 0$, which is equivalent to our initial hypothesis. Since this transformation does not change the underlying logic of the test, results in both scenarios should be almost identical.

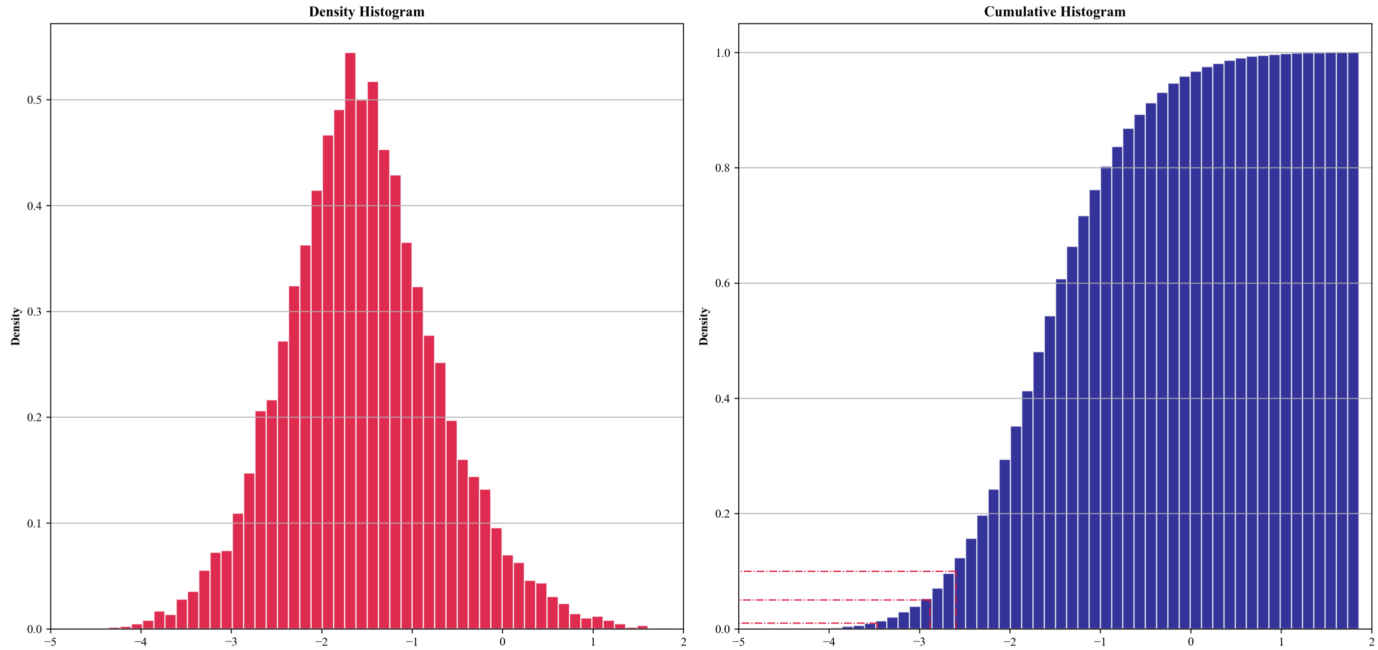
Q2.4 – By simulating a set of random walks, we can determine the distribution of the Dickey-Fuller statistic. We generate a set of random walks that we know are not stationary. At times, these series might appear stationary; at other times, they will explode. By determining the distribution of the Dickey-Fuller statistic, we can find a threshold value for the test statistic, below which we can say that the series is not stationary at our desired level of confidence. Figure 6 demonstrates some of the non-stationary random walks, oscillating around true stationary processes at the 0 line while others diverge. This is precisely the reason why we must simulate and test a large number of random walks.

Figure 6 - Random Walks: Non-Stationary (Colored) vs. Stationary (Grey)



Q2.5 – The histogram in Figure 7 represents the possible values of the DF statistic for a non-stationary random walk, which appears to be normally distributed. Plotting the observed values on a cumulative histogram gives us a visual aid in understanding the critical values of the DF test. Suppose for instance, that our test gave us a value of -3. The histogram allows us to see that a considerable number of time series present a value of -3 or less. The cumulative histogram in Figure 7 lets us immediately determine the probability of observing a value as low or lower than 3 for a non-stationary process - around 3%. Therefore, if we want to proclaim that a process is stationary and observe a DF statistic of -3, then there is a 3% chance that we are observing a non-stationary process.

Figure 7 - Dickey-Fuller Statistic Distribution



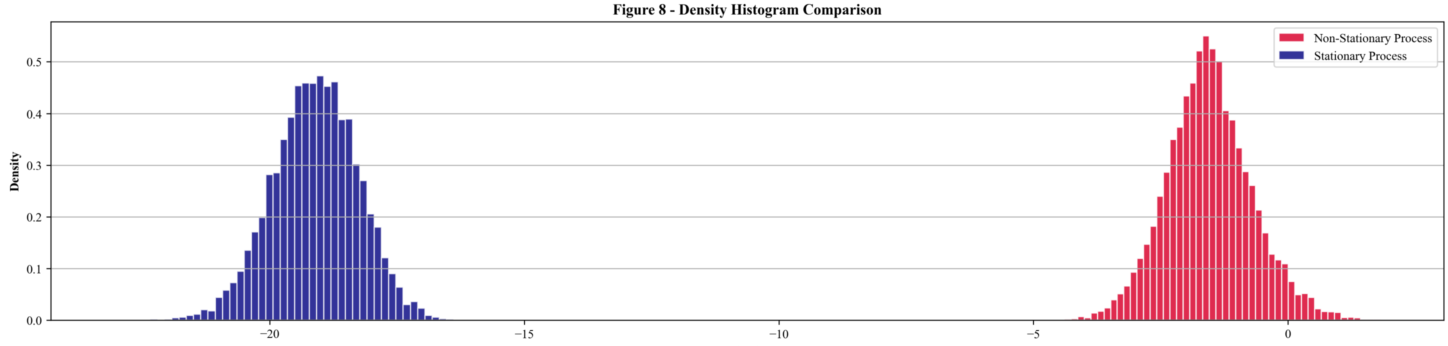
Q2.6 – Our simulated critical values for the DF test shown in Table 1 are slightly lower than those presented by Fuller, 1976.

Confidence Level	Critical Value
90%	-2.58
95%	-2.89
99%	-3.47
Sample Length: 2632	
Number of Iterations: 10000	

Table 1: Stationarity Test Critical Values: Dickey-Fuller

Q2.7 – Simulating an AR(1) process with a coefficient of 0.2, we expect it to be more stationary than a random walk, therefore reducing the observed values of the DF statistic. The series becomes more stationary since the autoregressive coefficient of 0.2 causes consecutive observations to be pulled toward the mean. This increased stationarity is captured by a leftward shift in the histogram.

Figure 8 - Density Histogram Comparison



Q2.8 – To test whether the log-price series has a unit root and determine the p-values, we use the critical values and the cumulative distribution of the simulated Dickey-Fuller statistic from question 2.5. Table 2[1] aggregates the five hotels' test statistics, p-values, and the critical value at a 95% confidence level. Clearly, all the test statistics are far too high; thus, we fail to reject the null hypothesis, implying that all the log-price series are not stationary.

Company	**HLT**	**MAR**	**IHG**	**BKNG**	**HYATT**
DF-stat	-0.568	-1.391	-1.803	-1.269	-1.48
p-val	0.875	0.584	0.379	0.642	0.539
Critical Value (95%)			-2.89		
H_0 rejected?	F	F	F	F	F

Table 2: Stationarity Test Results

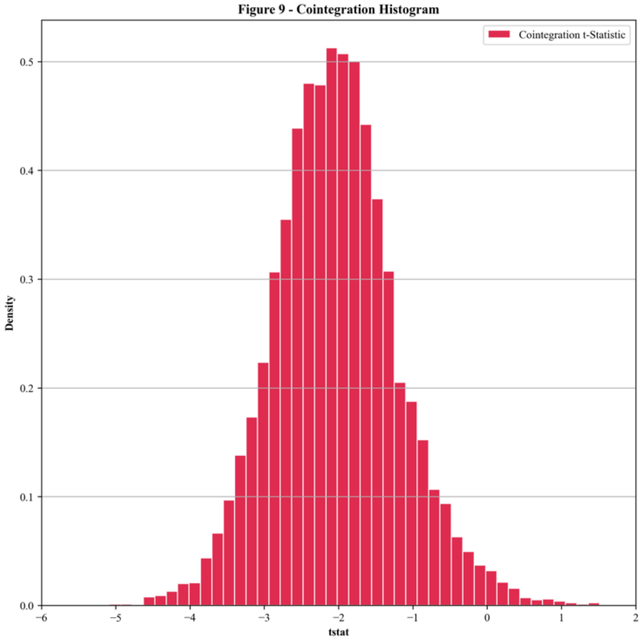
Q2.9 – A linear combination of two non-stationary series can result in a stationary series, whilst a linear combination of a stationary and a non-stationary series, in principle, cannot. A pair of series is then said to be cointegrated when their linear combination results in a stationary series. Since all of our series exhibit non-stationarity, we can potentially implement a pair-trading strategy using cointegration. Nevertheless, we must test whether cointegration exists between any two assets by testing whether the residuals of an OLS regression of the linear combination of their prices are stationary and whether this stationarity is statistically significant.

[1] For the reason of conciseness, the five hotels are referred as: HILTON→HLT ; MARRIOTT →MAR ; ICTL → ICTL ; BOOKING → BKNG ; HYATT → HYATT. These labels are inherently used throughout the whole report.

Part 3 – Cointegration

Q3.1 – The histogram in Figure 9 illustrates the distribution of the test statistic for the N=10’000 cointegration test of two non-stationary random walks. The wide dispersion of the test statistics demonstrates the dangers of spurious regressions, where, at times, we may observe statistically significant relationships between random processes that are inherently independent. Hence, the new lower critical values account for such relations and prevent us from falsely rejecting the null of no cointegration. The corresponding critical values are reported in Table 3.

Figure 9 - Cointegration Histogram



Confidence Level	Critical Value
90%	-3.0475
95%	-3.3391
99%	-3.9320
Sample Length: 500	
Number of Iterations: 10000	

Table 3: Cointegration Test Critical Values: Phillips-Ouliaris

Q3.2 – Table 4 demonstrates the t-statistic values, and the parameter estimates of the cointegration test for every pairwise relation of assets in our dataset. The stars indicate any cointegration relationships that are statistically significant at a 5% significance level or lower, whereas bold text highlights the strongest cointegration relation for a given asset.

	HLT → MAR	**HLT → IHG**	**HLT → BKNG**	**HLT → HYATT**
τ-stat	−2.7037	**−3.4250**∗∗∗	−2.8038	−2.6893
p-val	0.1995	0.0408	0.1661	0.2045
$\hat{\alpha}$	−0.5820	−1.5675	−5.3816	−1.5690
$\hat{\beta}$	1.0623	1.5042	1.3902	1.4103
	MAR → HLT	**MAR → IHG**	**MAR → BKNG**	**MAR → HYATT**
τ-stat	−3.0890	**−5.5000**∗∗∗	−3.9798∗∗∗	−3.3347
p-val	0.0920	0.0001	0.0090	0.0506
$\hat{\alpha}$	0.9745	−1.0184	−4.3861	−0.7146
$\hat{\beta}$	0.8447	1.4388	1.2148	1.2770
	IHG → HLT	**IHG → MAR**	**IHG → BKNG**	**IHG → HYATT**
τ-stat	−3.8621∗∗∗	***−5.6155***∗∗∗	−5.5910∗∗∗	−3.7886∗∗∗
p-val	0.0114	0.0001	0.0001	0.0146
$\hat{\alpha}$	1.6701	1.0204	−2.0739	0.6391
$\hat{\beta}$	0.5224	0.6285	0.8086	0.7866
	BKNG → HLT	**BKNG → MAR**	**BKNG → IHG**	**BKNG → HYATT**
τ-stat	−3.0311	−3.8933∗∗∗	**−5.4199**∗∗∗	−3.0489
p-val	0.1041	0.0109	0.0001	0.0999
$\hat{\alpha}$	4.8080	4.1574	3.1984	3.6572
$\hat{\beta}$	0.6057	0.7069	1.0772	0.9015
	HYATT → HLT	**HYATT → MAR**	**HYATT → IHG**	**HYATT → BKNG**
τ-stat	−3.0287	−3.2844	**−3.6190**∗∗∗	−3.1490
p-val	0.1054	0.0575	0.0230	0.0803
$\hat{\alpha}$	1.6431	1.0886	0.4820	−1.8448
$\hat{\beta}$	0.5888	0.6706	0.9456	0.8136

Table 4: Pairwise Cointegration Test Results

Q3.3 – From Table 4, we can observe that all $\hat{\beta}$ are positive, indicating a positive linear relation between asset pairs for any given combinations. This could be explained by the fact that the companies we are considering are all operating in the same industry and are therefore affected by similar external influences on their stock prices. The regression coefficient $\hat{\alpha}$ represents the difference in price level between asset pairs. Naturally, for any non-zero $\hat{\alpha}$, the sign is opposite when we revert the regression within the same asset pair, for instance, $\hat{\alpha}_{IHG \to MAR} = 1.0204$ and $\hat{\alpha}_{HLT \to HYATT} = -1.0184$.

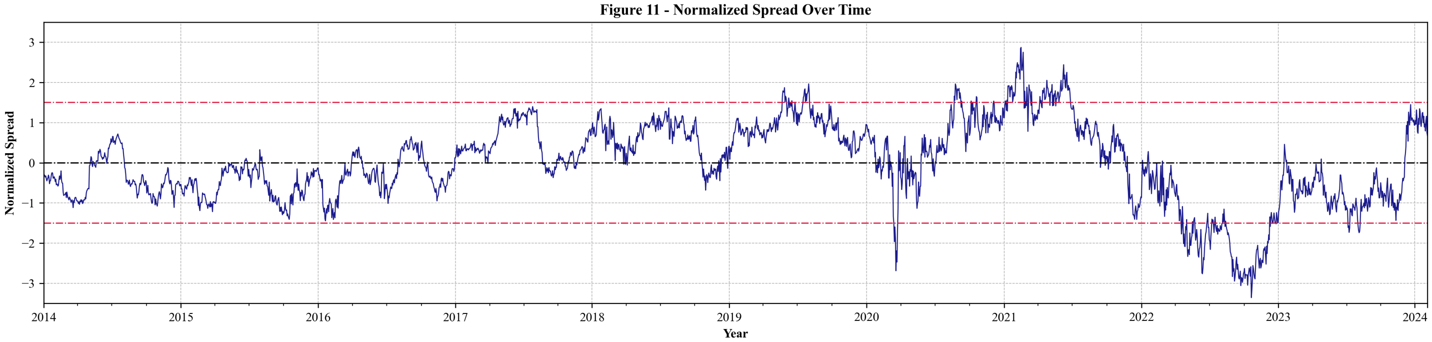
Q3.4 & Q3.5 – Among all pairs analyzed, the IHG to MAR relation stands out as the most cointegrated, as shown in Figure 10 because the linear combination of the prices of these two assets is the most stationary. Intuitively, their prices exhibit the most synchronized fluctuations, displaying the highest level of consistency out of all asset pairs in our sample. This implies that once we see deviations away from this synchronized movement, we will be able to engage in pair trading by constructing a long-short portfolio.

Part 4 - Pair Trading

Q4.1 – Given that the asset pair A and B are cointegrated, the spread between the two assets should be stationary, which implies that the spread is invariantly mean-reverting around its mean (or 0 if the spread is normalized). Hence, the divergence between the stock price of A and B is only temporary and is expected to be corrected over time. If the spread is significantly larger than 0, it implies that asset A is overvalued, and asset B is undervalued compared to what the linear model would suggest. Since the difference between the prices of assets A and B will eventually convert to its mean value, a statistical arbitrage strategy can be proposed to hold a short position on asset A and a long position on asset B.

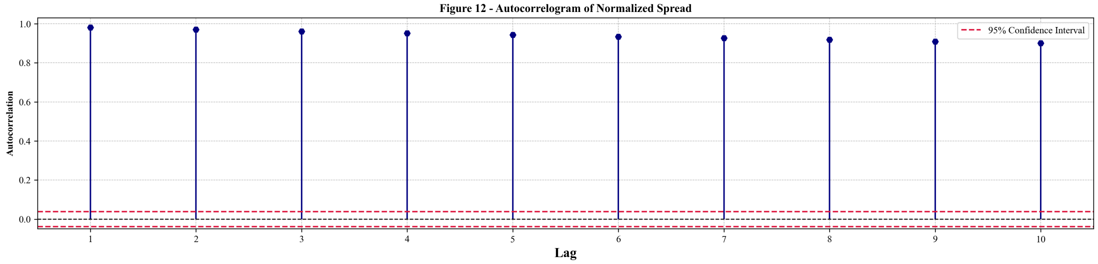
Q4.2 – By normalizing the spread shown in Figure 11, we obtain a unitless measure of divergence whose scale is unaffected by the price level of the underlying asset pair. Furthermore, units on the y-axis now directly translate to the number of standard deviations away from the mean value of the spread.

Figure 11 - Normalized Spread Over Time



Q4.3 – The autocorrelogram in Figure 12 shows the Pearson correlation between the normalized spread and its N-shifted self, where N is the number of lags, i.e., the order N autocorrelation. We can see that the spread is highly autocorrelated for all the 10 orders of lag shown, far above the threshold of statistical significance of 5%. These observations are coherent with the results of the Ljung-Box test, which tests whether the first N serial correlations are all jointly equal to zero.

Figure 12 - Autocorrelogram of Normalized Spread



The Ljung-Box test statistic value for our normalized spread is 23325.23, far beyond the critical value of 18.31 at a 95% confidence level, suggesting the probability of randomly observing such a strong autocorrelation is very low. A statistically significant high-order autocorrelation indicates that the spread is predictable and persistent. Predictability is useful in the context of a pair trading strategy since it allows us to estimate future values of the spread based on historical data. By estimating the future levels of the spread, we can take advantage of its mean-reverting nature by selecting the best time to enter or close a certain position. Furthermore, a persistent spread is important for risk management, especially if we take highly leveraged positions that could be particularly sensitive to sudden market movements.

Q4.4 – A self-financing strategy would require that all trades and adjustments be made solely with the assets currently within the portfolio, without the need for additional cash injections or withdrawals. This is not the case in our pair-trading strategy. For instance, if a signal 1 appears in the time series, the position $Q_1 = [-1 \quad \beta]'$ should be implemented. If it were a self-financing strategy, the value of a short position in asset A should equal that of a long position in asset B. Mathematically, it should result in $P_A \times (-1) + P_B \times \beta = 0$. This equation indicates a very restrictive situation, which is true only if $\beta = P_A / P_B$ when you open the position at $\tilde{z}_t > \tilde{z}_t^{in}$.

It is also worth mentioning that the parameter alpha is not the main target to examine if a strategy is self-financing. In the defined linear relation $P_t^A = \alpha + \beta P_t^B + z_t$, the intercept shifts the whole relation upwards or downwards. However, it does not impact the dynamics of our wealth. In the defined spread $z_t = P_t^A - \alpha - \beta P_t^B$. α affects the mean around which the cointegrated process oscillates. Although it is useful for setting up a threshold for triggering the trading process when the signal appears, the spread is normalized as $\tilde{z}_t$ and thus not of concern here.

Q4.5 – Assuming that investors can trade without delay and friction, the profit per unit of the spread when the position is closed at $\tilde{z}_t = 0$, should be equivalent to the amount spent on short-selling asset A and buying asset B when $\tilde{z}_t = \tilde{z}^{in}$. When the signal just appears, the cost (payoff) is $P_A \times (-1) + P_B \times \beta$, expressing P_t^A in terms of cointegrated relation, we have $-1(\alpha + \beta P_t^B + z_t) + \beta P_t^B$. Since the normalized spread $\tilde{z}_t = z_t/\sigma(z_t)$, we can further expand the payoff to $-\left(\alpha + \beta P_t^B + \tilde{z}_t \sigma_{z_t}\right) + \beta P_t^B$. It can be simplified to $-\alpha - \tilde{z}_t \sigma(z_t)$. When the normalized spread equals 0 ($\tilde{z}_t = 0$), we close our position, and the position's value will have increased by the decreased magnitude of the spread. Which is the initial value of the spread $\tilde{z}^{in}$ with $payoff = -\left(-\alpha - \tilde{z}^{in} \times \sigma(\tilde{z}_t)\right) = \alpha + \tilde{z}^{in}\sigma(\tilde{z}_t)$. Given that in the absence of friction and delays, α does not impact the payoff function because we are effectively buying and selling at the equilibrium price, the profit function can be further simplified to $\tilde{z}^{in}\sigma(z_t)$ at $\tilde{z}_t = 0$.

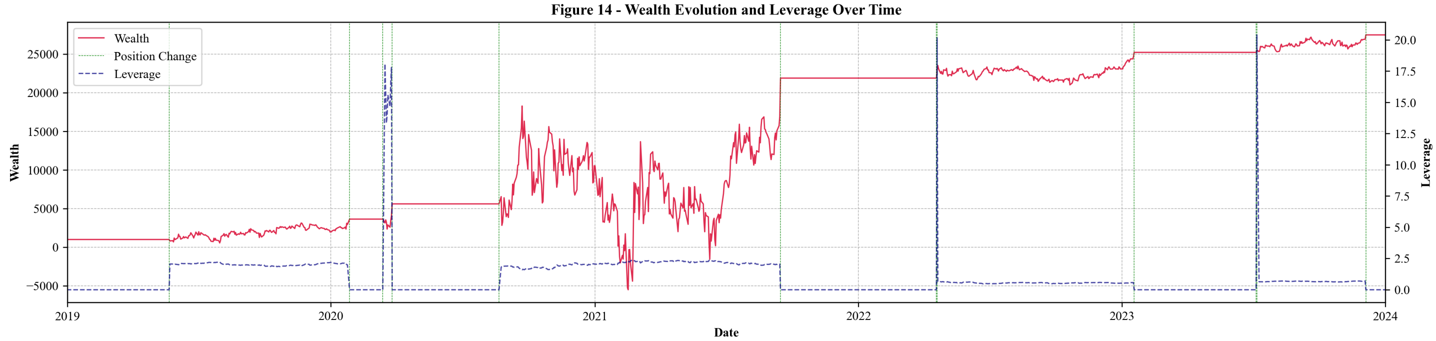
Q4.6 – The choice of $\tilde{z}^{in}$ should be considered with caution. On the one hand, if we pick a $\tilde{z}^{in}$ too low, there is a risk of overtrading. This means that the trading strategy will most probably react to noise rather than a profitable spread and may lead to increased exposure to the risk that the spread does not revert as expected. As a result, any further increase of the spread will not benefit us, since our capital will be trapped in the low threshold position. Furthermore, if the spread continues to widen, having entered into a trade too early may lead to substantial losses as we have to finance our growing short position. On the other hand, by setting $\tilde{z}^{in}$ too high, we risk under-trading. The spread will cross the threshold less frequently, causing the strategy to mistakenly ignore smaller fluctuations in the spread that are, in fact, profitable. The trade-off between under/over-trading is directly linked to the overall payoff of our strategy. Ideally, we must pick a threshold that is sensitive enough to react to profitable spread fluctuations but not so sensitive that it generates signals for inconsequential ones. It is worth conducting regular revisions and back-testing to set an appropriate threshold.

Q4.7 – Figure 13[2] shows that this is an overall profitable strategy to implement in our sample. There was not a single trade before 2019 as the spread never reached the signal requirement $\tilde{z}^{in}$ = 1.5. Overall, we traded 12 times, and our wealth more than doubled, from \$1000 to \$2817.19 by the end of the whole period. The final wealth level is also the maximum wealth level. The minimum wealth level amounted to \$955.63, observed during the first trading period in 2019.

[2] From Figure 13 – 15, the timeframe is adjusted to start from 2019 to have a clearer image as there was no trade before 2019

Q4.8 – As shown in Figure 14, when setting the maximum leverage to 20, the number of trades remained the same at 12 times. We generated a much higher profit; the final and maximum wealth level in theory is 27472.09. However, the minimum wealth level is -5501.44, which occurred during 2021. Since we do not inject any capital after we start a position, this implies that we will go bankrupt. In reality, our position would be forced to liquidate, and our wealth pinned at 0. Therefore, trading with 20 leverage would not be profitable overall because we would have defaulted before reaching the maximum final wealth level. However, for the purposes of our analysis, we intentionally did not implement a condition that would force us into liquidation so that we could see how our wealth would evolve over time had we injected more capital.

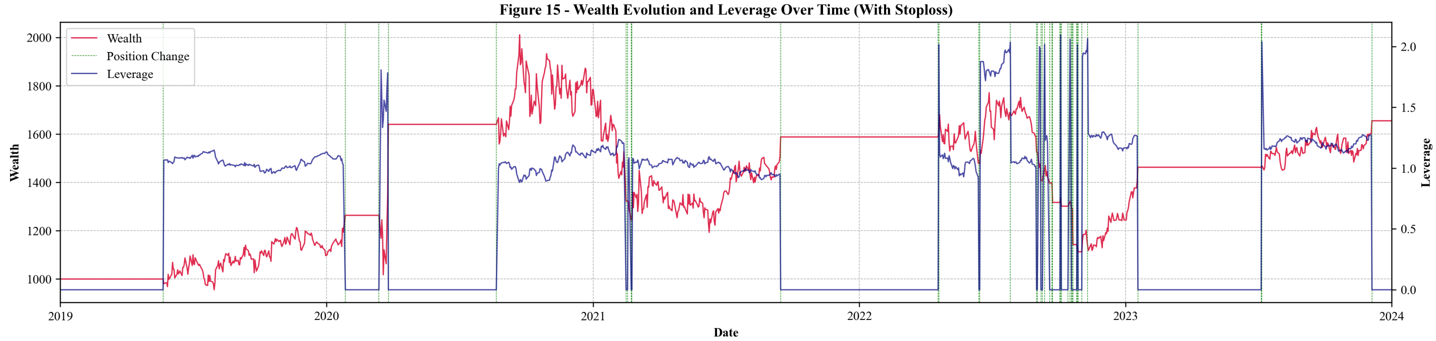
Figure 14 - Wealth Evolution and Leverage Over Time



Q4.9 – The stop-loss rule helps the investor to moderate the loss of their position to a certain threshold when the spread moves against their anticipation. We design the AR(1) process to estimate $\tilde{z}_{t+1}$: $\tilde{z}_{t+1} = \phi_0 + \phi_1 \tilde{z}_t + \varepsilon_{t+1}$. We assume the residual (ε) follows a normal distribution. Therefore, the probability of the next period spread is bigger than the stop-loss spread threshold should also be examined under normal distribution. The result for $\Pr(\tilde{z}_{t+1} > \tilde{z}^{stop}) = 7.38\%$

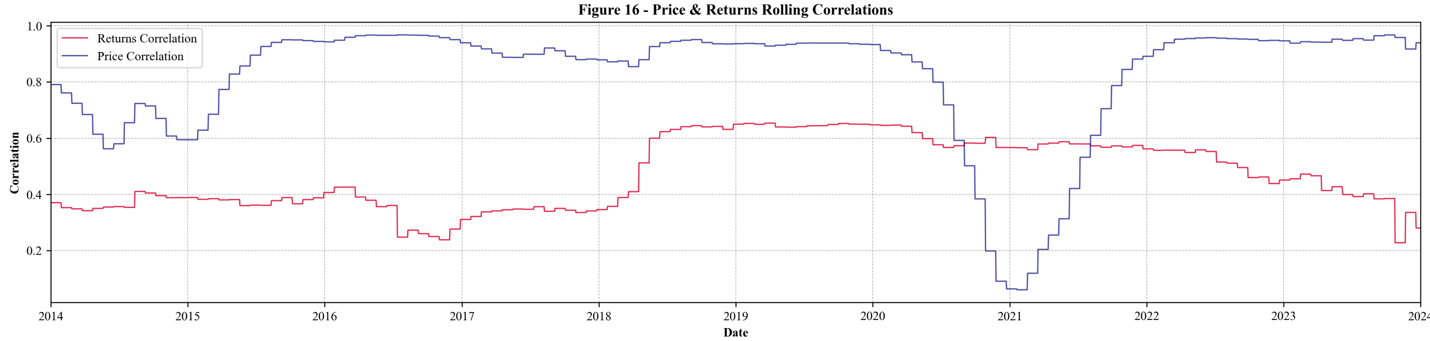
Q4.10 – Figure 15 shows a trading strategy with the stop-loss protection being implemented, we observe much more frequent trading, having traded 40 times over the sample. The maximum and final wealth levels have decreased compared to Q4.7 to $1655.26, and the minimum wealth level is unchanged. We can conclude that implementing a stop-loss strategy in our sample turned out to be unnecessary. This is because, by the time we enter a period when the stop-loss activates in 2022, we have amassed enough equity wealth to absorb our losses and are better off not selling our position. Hence, not only setting a stop-loss protection at 2.75 ($\tilde{z}^{stop} = 2.75$) does not prevent us from bearing large losses, but it also limits the performance of some profitable positions.

Figure 15 - Wealth Evolution and Leverage Over Time (With Stoploss)



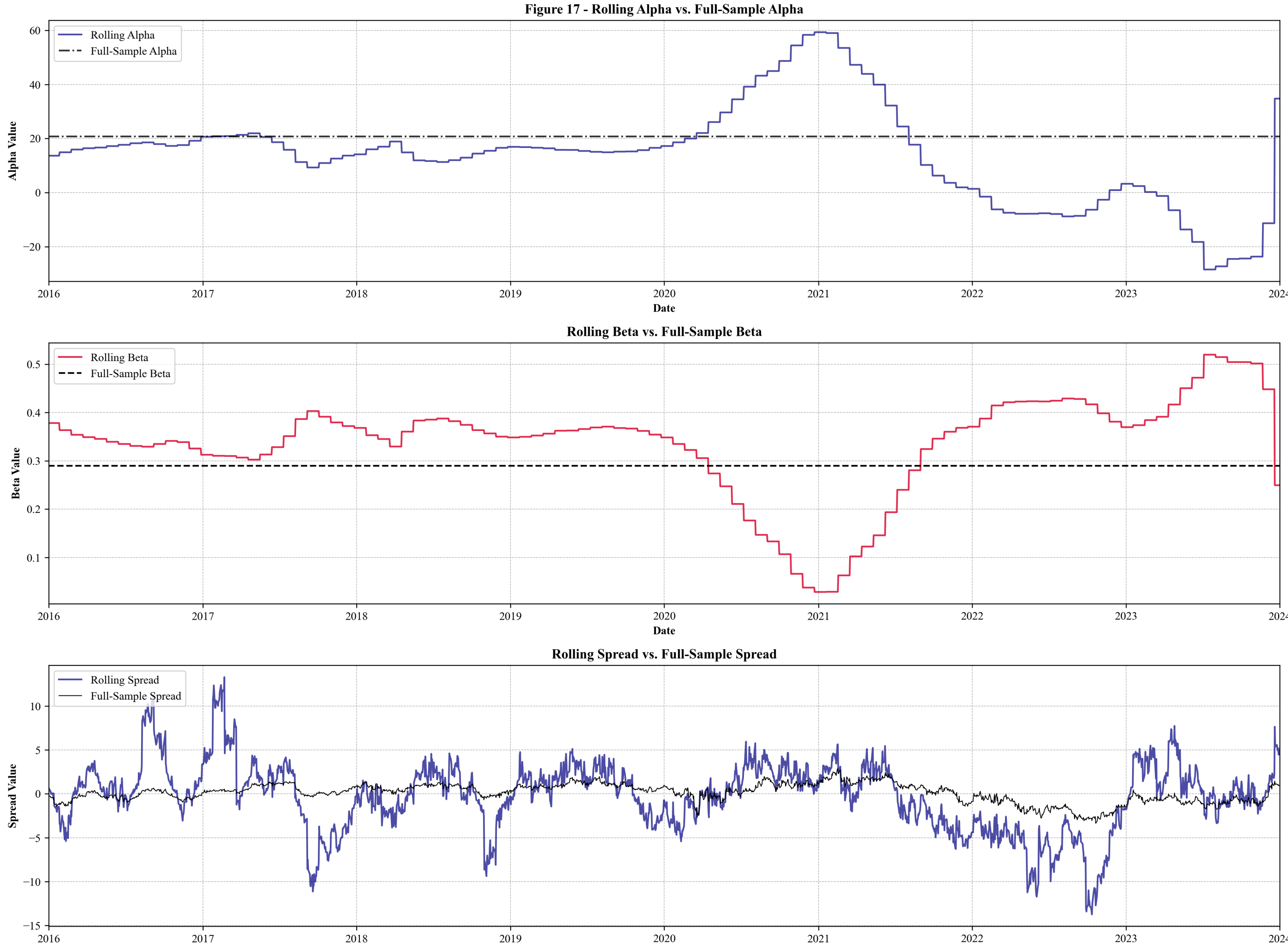
Q4.11 – As shown in Figure 16, the correlation of returns hovers around 0.4 until it increases to its new steady state of 0.6. This increase is likely attributable to the companies becoming more aligned in their business practices, which homogenized their exposure external market factors that drive their returns. The price correlation remains relatively high and stable throughout most of the sample, except for minor fluctuations prior to 2016 and a dramatic drop, with a subsequent recovery from 2020 to 2022. This breakdown of price correlation is in high likelihood related to the COVID-19 pandemic. As hotel businesses struggled worldwide, their stock prices fell at different rates in absolute terms since they started at different initial levels. The fact that the returns correlation remained stable in that period suggests that the asset prices were falling at a similar rate in relative terms, which is further supported by the recovery phase that saw these assets regain value in a similarly aligned manner.

Figure 16 - Price & Returns Rolling Correlations



Q4.12 – The rolling parameter estimates are more volatile as expected. Alpha captures the difference between the price levels of the two assets, while beta captures the sensitivity of asset A's price to changes in asset B's price. The Figure 17[3] shows that the rolling alpha and beta move synchronously in opposing directions. This inverse relationship suggests that when the price levels of asset A and B are further apart, asset A becomes less sensitive to the price movements of asset B and vice-versa. This makes sense mathematically, as an increase in the price of asset B now results in a more minor change in the price of asset A, thereby reducing the estimate of beta. Furthermore, the spread calculated based on rolling parameters is significantly more volatile and sees much larger positive and negative values. This observation suggests that a rolling window of 20 days tends to overstate the spread's volatility.

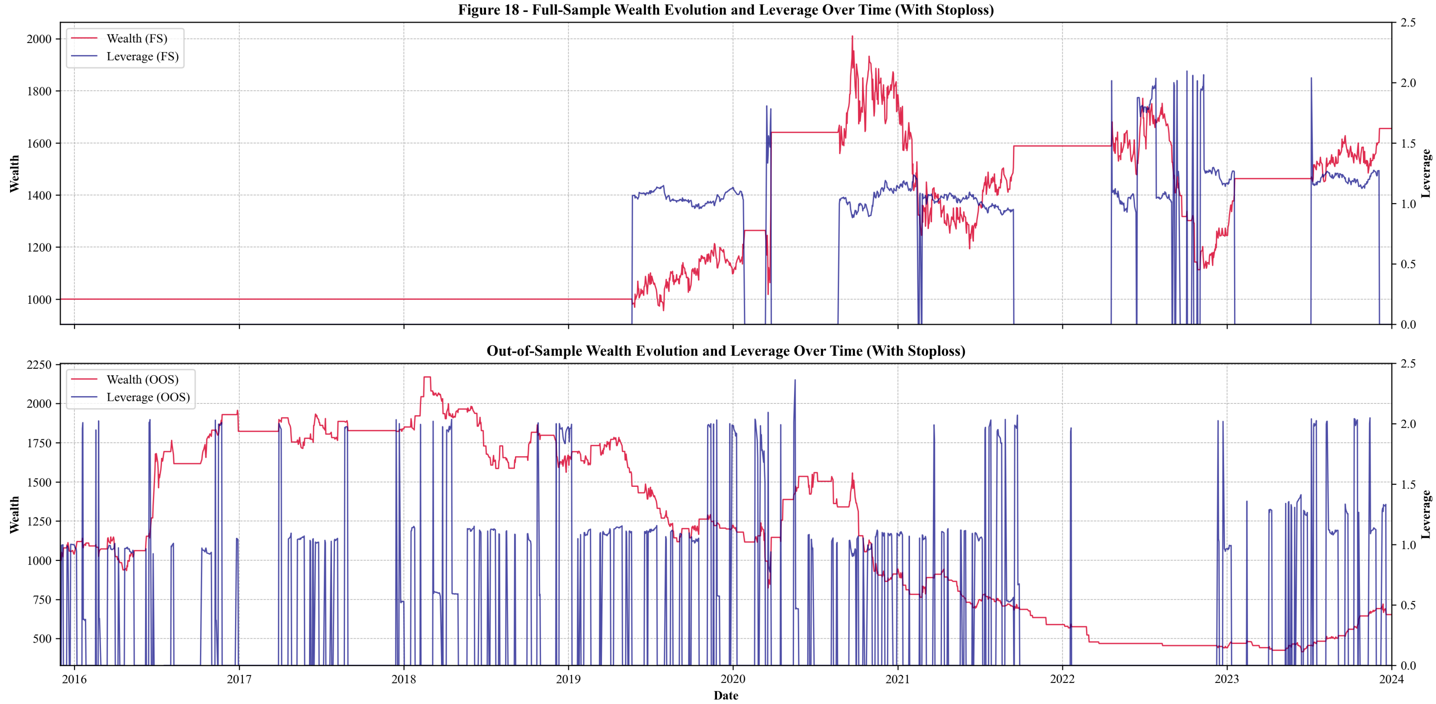
Figure 17 - Rolling Alpha vs. Full-Sample Alpha



[3] For Figures 17, 18, 20, the estimation of rolling parameters requires 2 years of in-sample data , thus the first date shown is 2016.

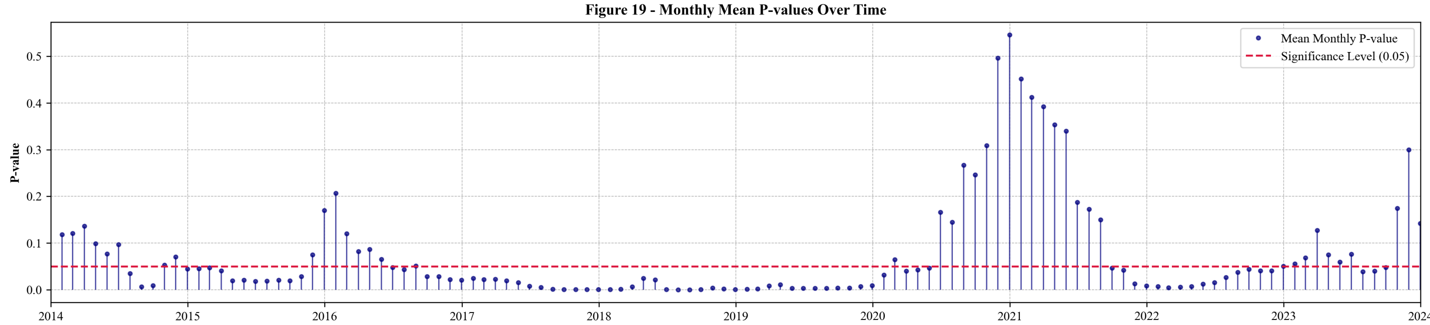
Q4.13 – Unlike the trading strategy implemented in the case of the full sample, the trading starts immediately. Using the rolling window parameter estimates with the same threshold values for opening and closing positions resulted in less-than-ideal performance, with a final wealth of $663.46 that gradually decreased from its maximum of $2169.93 in 2018, recovering slightly from the minimum of $416.69 in 2023. The new strategy causes us to trade frequently, with 377 trades made. The frequent trading is likely the result of our $\tilde{z}^{in}$ and $\tilde{z}^{stop}$ not being well adapted to the more volatile normalized spread that comes with rolling parameter estimation.

Figure 18 - Full-Sample Wealth Evolution and Leverage Over Time (With Stoploss)

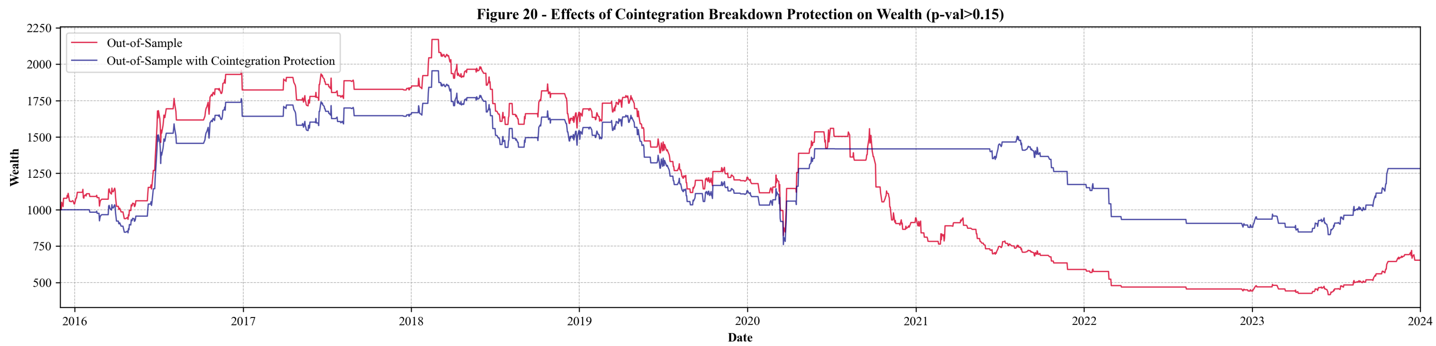


Q4.14 – In figure 19, The assets appear cointegrated for most of the sample, but there are several subsamples when that is not the case. Cointegration is not statistically significant at a 5% confidence level for any subsample where the vertical lines cross the horizontal marker. Our pair-trading strategy is predicated on the presence of statistically significant cointegration between the two assets, the absence of which suggests that either cointegration no longer exists or that we are in a period of extreme divergence away from the typical spread. If cointegration indeed broke down, we can no longer expect to benefit from the mean-reverting spread of the two assets. If, on the other hand, the apparent lack of cointegration is only temporary, we are likely to be in a dangerous position by shorting one of the assets. In any case, we are exposed to more risk during these periods.

Figure 19 - Monthly Mean P-values Over Time



Q4.15 – Figure 20 shows that closing our positions when cointegration appears to break down results in improved performance towards the end of the sample, with the final wealth significantly higher at $1302.42. Additionally, the strategy resulted in 298 trades with a minimum wealth that was higher at $760.7, but a maximum wealth that was lower at $1954.69 compared to Q4.13. We can see that for most of the sample, the trading strategy with cointegration breakdown protection performs slightly worse because we do not initially invest in the position that turned out to be lucrative, but towards the end, we are better off not having invested in a position that turned out to be not profitable. The implications of implementing such a protection mechanism in a pair-trading strategy are clear, by being conservative, the strategy sacrifices early gains for improved stability and protects us from loss later on.



Q4.16 – Our findings do make it seem like taking advantage of the statistical properties of asset prices is a lucrative endeavor. However, our out-of-sample performance was not remarkable, and that is without considering trading fees and margin calls. Furthermore, we did not conduct a thorough assessment of the risks associated with these strategies. Real investors implementing strategies involving short positions must be wary of market liquidity since an illiquid market might not provide an opportunity to go short or even cover an existing short position at a reasonable cost. This is an even more pertinent concern for institutional investors such as hedge funds, whose trades materially affect market dynamics.