Real Analysis Mid-semester exam

September 2024

Full Marks : 50
Time : 1 hour 30 minutes

1. Prove that every convergent sequence of real numbers is bounded. (10)

2. State and prove squeeze theorem (of convergent sequences). (10)

3. **Define Cauchy sequence and prove that all convergent sequences of real numbers** are Cauchy sequences (You have to prove Cauchy sequences are convergent and then any convergent sequence is Cauchy). (15)

4. If $\lim_{x\to c} f(x) = l_1$ and $\lim_{x\to c} g(x) = l_2$, then show that
a) $\lim_{x\to c}(f(x)\pm g(x)) = l_1 \pm l_2$, b) $\lim_{x\to c}(f(x)g(x)) = l_1 l_2$. (5+5)

5. Consider $\{x_n\}$ is a sequence that converges to l. Show that the sequence $y_n = \frac{x_1+x_2+\ldots\ldots+x_n}{n}$ also converges to l. (5)