Digital Systems & Microcontrollers - Quiz 1 Solutions

September 2, 2024

Question 1

1. A safe has five locks, v, w, x, y, and z, all of which must be unlocked for the safe to open. The keys to the locks are distributed among five executives in the following manner:
A has keys for locks v and x;
B has keys for locks v and y;
C has keys for locks w and y;
D has keys for locks x and z;
E has keys for locks v and z.
(a) Find all the combinations of executives that can open the safe. Write an expression f(A,B,C,D,E) which specifies when the safe can be opened as a function of which executives are present. [3 M]
(b) Who is the essential executive without whom the safe cannot be opened? [1 M]
(c) Implement the function f came in (a) using AND,OR,and NOT gates [2 M]

Subpart (a):

The possible combinations are:

ABCDE, ABCD, ABCE, ACDE, BCDE, ACD, BCD, ACE, CDE

Using the above minterms, we get the expression:

$$ABCDE + ABCDE' + ABCD'E + A'BCDE + AB'CDE' + A'BCDE' + AB'CD'E + A'B'CDE$$

Which can be simplified to:

$$ABCD(E+E') + ACD'E(B+B') + AB'CD(E+E') + A'BCD(E+E') + A'B'CDE$$

$$ABCD + ACD'E + AB'CD + A'BCD + A'B'CDE$$

$$ACD(B+B') + ACD'E + A'BCD + A'B'CDE$$

$$C(AD + AD'E + A'BD + A'B'DE)$$

$$C(A(D+D'E) + A'D(B+B'E))$$

$$C(A(E+D) + A'D(E+B))$$

$$C(AE + AD + A'DE + A'BD)$$

$$C(E(D+A) + D(A+B))$$

$$C(DE + AE + AD + BD)$$

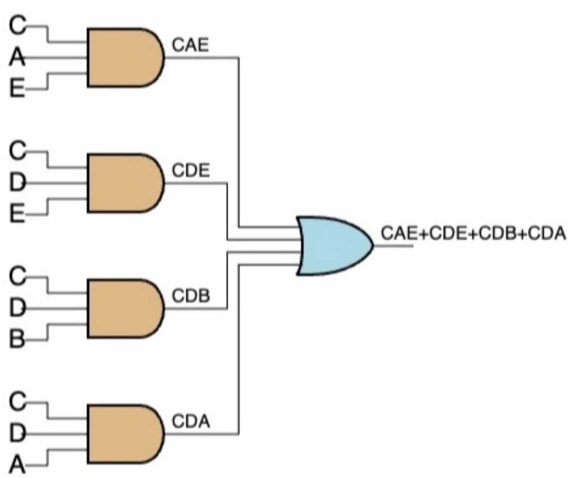
Simplification to this extent is not necessary as we have not specified in the question whether simplification of the expression is necessary.

Subpart (b):

The essential executive is C.

Subpart (c):

The below logic gate expression or any other expression equivalent to the one in Subpart (a).



Question 2

2. (a) Perform 9-17 using 2's complement representation. [2 M]
   (b) Convert given number to indicated base, $(54)_9$ to $(?)_6$. [2 M]
   (c) Find the complement of $(a+\bar{c})(a+b)(a+\bar{b}+c)$. [2 M]

(a) Subtraction using 2's Complement

To perform the subtraction $9-17$ using 2's complement representation, follow these steps:

1. **Convert the numbers to binary:**

$$9 : 00001001 \text{ (in 8-bit binary)}$$
$$17 : 00010001 \text{ (in 8-bit binary)}$$

2. **Find the 2's complement of 17:**

$$\text{1's complement of } 17 : 11101110$$
$$\text{2's complement of } 17 : 11101110 + 1 = 11101111$$

3. **Add the 2's complement of 17 to 9:**

$$00001001 + 11101111 = 11111000 \text{ (without carry)}$$

4. **Interpret the result:**
   - The result 11111000 is negative because the MSB is 1.
   - To find the magnitude, take the 2's complement of 11111000:

$$\text{1's complement of } 11111000 : 00000111$$
$$\text{2's complement of } 11111000 : 00000111 + 1 = 00001000$$

   - The magnitude is 8, so the result is -8.

Conclusion: $9-17=-8$

(b) Base Conversion

To convert the number 54_9 to base 6, follow these steps:

1. **Convert the number from base 9 to base 10:**

$$54_9 = 5 \times 9^1 + 4 \times 9^0$$

$$5 \times 9^1 = 45$$
$$4 \times 9^0 = 4$$
$$54_9 = 45 + 4 = 49_{10}$$

2. **Convert the base 10 number to base 6:**

$$\begin{aligned} 49 \div 6 &= 8 \text{ remainder } 1 \quad \text{(LSD)} \\ 8 \div 6 &= 1 \text{ remainder } 2 \\ 1 \div 6 &= 0 \text{ remainder } 1 \quad \text{(MSD)} \end{aligned}$$

$$\therefore 49_{10} = 121_6$$

Conclusion: $54_9 = 121_6$

(c) Complement of a Boolean Expression

Given the Boolean expression $(a + \overline{c})(a + b)(a + \overline{b} + c)$, we need to find its complement.

1. Start with the expression:

$$(a + \overline{c})(a + b)(a + \overline{b} + c)$$

2. Apply De Morgan's law to find the complement:

$$\overline{(a + \overline{c})(a + b)(a + \overline{b} + c)} = \overline{(a + \overline{c})} + \overline{(a + b)} + \overline{(a + \overline{b} + c)}$$

3. Complement each term individually:

$$\begin{aligned} \overline{(a + \overline{c})} &= \overline{a} \cdot c \\ \overline{(a + b)} &= \overline{a} \cdot \overline{b} \\ \overline{(a + \overline{b} + c)} &= \overline{a} \cdot b \cdot \overline{c} \end{aligned}$$

4. Combine the complements using OR:

$$\overline{(a + \overline{c})(a + b)(a + \overline{b} + c)} = (\overline{a} \cdot c) + (\overline{a} \cdot \overline{b}) + (\overline{a} \cdot b \cdot \overline{c})$$

5. Factor out the common term:

$$\overline{a} \cdot (c + \overline{b} + b \cdot \overline{c})$$

6. Simplify the expression inside the parentheses:

$$c + \overline{b} + b \cdot \overline{c} = 1$$

   Therefore:

$$\overline{(a + \overline{c})(a + b)(a + \overline{b} + c)} = \overline{a} \cdot 1 = \overline{a}$$

Final Answer: The complement of $(a + \overline{c})(a + b)(a + \overline{b} + c)$ is $\overline{a}$.

Question 3

Page 6 of 8

3. (a) Multiply in Binary Number N= 165.45 with 16. [2 M]
   (b) Can you divide N by 4 in binary easily (without doing long math)? [2 M]
   (c) Express $F = xz + \bar{x}y + \bar{z}$ in product of maxterms using Boolean algebra. [2 M]
   (d) Convert above one from maxterms to minterms using complements. [2 M]

(a) **Multiply Binary number $N = 165.45_{10}$ with 16**

First, convert the decimal number 165.45_{10} to its binary representation.

Breaking down the integer part 165_{10}:

$$165_{10} = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 10100101_2$$

Breaking down the fractional part 0.45_{10}:

$0.45_{10} = 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + 0 \times 2^{-5} + 0 \times 2^{-6} + 1 \times 2^{-7} + 1 \times 2^{-8} = 0.01110011_2 (upto 8 places)$

Thus, the complete binary representation is:

$$165.45_{10} = 10100101.01110011_2$$

Now, multiply this binary number by $16 = 2^4$, which involves shifting the binary point 4 places to the right:

$$N \times 16 = 101001010111.0011_2$$

So, the result of multiplying 165.45_{10} by 16 is:

$$165.45_{10} \times 16 = 101001010111.0011_2 = 2647.1875_{10}$$

(b) **Can you divide $N = 165.45_{10}$ by 4 in binary easily (without doing long math)?**

First, recall the binary representation of $N = 165.45_{10}$:

$$N = 10100101.01110011_2$$

To divide by 4 (which is 2^2 in binary), simply shift the binary point 2 places to the left.

Perform the division by shifting:

$$N \div 4 = 101001.0101110011_2$$

(c) **Express $F = xz + x'y + z'$ in product of maxterms using Boolean algebra**

Given function:

$$F = xz + x'y + z'$$

Step 1: We know that:

$$z' = z' + xz'$$

This is based on the identity $z' = z' \times 1$ and $1 = 1 + x$, leading to:

$$z' = z' + xz'$$

Substituting this into the function:

$$F = x(z + z') + x'y + z'$$

Step 2: Using the identity $z + z' = 1$, we simplify:

$$F = x \cdot 1 + x'y + z' = x + x'y + z'$$

Step 3: Using $x = x + xy$, we get

$$F = x + xy + x'y + z' = x + (x + x')y + z'$$

Using the identity $x + x' = 1$, we simplify:

$$F = x + y + z'$$

Final Result: The simplified expression $F = x + y + z'$ is already in the product of maxterms form:

$$F = M_1 = (x + y + z')$$

Note: You are expected to derive the product of max terms using Boolean algebra and not directly by using complements of min terms. Partial marks will be awarded in the latter case.

(d) **Convert the above function from maxterms to minterms using complements**

Given function in maxterms form:

$$F = M_1 = (x + y + z')$$

Step 1: Express the Function in Maxterms The function F in terms of maxterms is:

$$F = M_1 = (x + y + z')$$

This represents the case where $F = 0$ for the combination $(x, y, z) = (0, 0, 1)$, corresponding to the maxterm M_1.

Step 2: Use De Morgan's Theorem to Find the Complement To convert from maxterms to minterms, take the complement of the function F:

$$F' = (x + y + z')'$$

Applying **De Morgan's Theorem**:

$$F' = x' \cdot y' \cdot z$$

This expression F' corresponds to the minterm m_1, which means that F' is 1 for $(x, y, z) = (0, 0, 1)$.

Step 3: Use the Complement to Determine F Since F' is the minterm m_1, it represents the only combination for which F is 0. The original function F is true (1) for all other minterms:

$$F = m_0 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7$$