THESIS1

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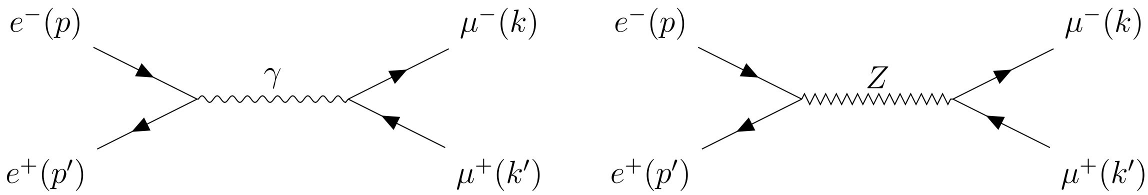
Quá trình tán xạ $e^-e^+ \to \mu^-\mu^+$



Dựa vào giản đồ, ta biết được biên độ tán xạ Feynman:

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z$$

Trong đó:

$$\begin{aligned}
\mathcal{M}_\gamma &= \left[\bar{v}^{s'}(p')(ie\gamma^\mu)u^s(p)\right]\left(\frac{-g_{\mu\nu}}{q^2}\right)\left[\bar{u}^r(k)(ie\gamma^\nu)v^{r'}(k')\right] \\
&= \frac{e^2}{q^2}\left[\bar{v}^{s'}(p')\gamma^\mu u^s(p)\right]\left[\bar{u}^r(k)\gamma_\mu v^{r'}(k')\right] \\
\mathcal{M}_Z &= \left[\bar{v}^{s'}(p')\frac{i}{2}\gamma^\mu\left(g_V - g_A\gamma^5\right)u^s(p)\right]\left(-\frac{g_{\mu\nu}}{q^2 - m_Z^2 + i\Gamma_Z m_Z}\right)\left[\bar{u}^r(k)\frac{i}{2}\gamma^\nu\left(g_V - g_A\gamma^5\right)v^{r'}(k')\right] \\
&= \frac{1}{4\left(q^2 - m_Z^2 + i\Gamma_Z m_Z\right)}\left[\bar{v}^{s'}(p')\gamma^\mu\left(g_V - g_A\gamma^5\right)u^s(p)\right]\left[\bar{u}^r(k)\gamma_\mu\left(g_V - g_A\gamma^5\right)v^{r'}(k')\right]
\end{aligned}$$

Với g_V và g_A đối với electron và muon được xác định:

$$\begin{aligned}
g_V &= -\frac{e}{2s_W c_W}\left(1 - 4s_W^2\right) \\
g_A &= -\frac{e}{2s_W c_W}
\end{aligned}$$

Lấy liên hợp phức, ta được:

$$\begin{aligned}
\mathcal{M}_\gamma^* &= \frac{e^2}{q^2}\left[\bar{v}^{r'}(k')\gamma_\nu u^r(k)\right]\left[\bar{u}^s(p')\gamma^\nu v^{s'}(p')\right] \\
\mathcal{M}_Z^* &= \frac{1}{4\left(q^2 - m_Z^2 - i\Gamma_Z m_Z\right)}\left[\bar{v}^{r'}(k')\gamma_\nu\left(g_V - g_A\gamma^5\right)u^r(k)\right]\left[\bar{u}^s(p)\gamma^\nu\left(g_V - g_A\gamma^5\right)v^{s'}(p')\right]
\end{aligned}$$

Khi nhân biên độ Feynman với liên hợp phức, ta thu được bình phương biên độ tán xạ:

$$|\mathcal{M}|^2 = \mathcal{M}_\gamma\mathcal{M}_\gamma^* + \mathcal{M}_\gamma\mathcal{M}_Z^* + \mathcal{M}_Z\mathcal{M}_\gamma^* + \mathcal{M}_Z\mathcal{M}_Z^*$$

Thành phần đầu tiên đã được xử lý ở bài trước:

$$\mathcal{M}_\gamma\mathcal{M}_\gamma^* = \frac{8e^4}{q^4}\left[\left(p'\cdot k\right)\left(p\cdot k'\right) + \left(p\cdot k\right)\left(p'\cdot k'\right) + m_\mu^2\left(p\cdot p'\right)\right]$$

Thành phần thứ hai được xử lý:

$$\mathcal{M}_\gamma\mathcal{M}_Z^* = \frac{e^2}{q^2}\left[\bar{v}^{s'}(p')\gamma^\mu u^s(p)\right]\left[\bar{u}^r(k)\gamma_\mu v^{r'}(k')\right]$$

$$\frac{1}{4\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left[\bar{v}^{r'}(k')\gamma_\nu\left(g_V-g_A\gamma^5\right)u^r(k)\right]\left[\bar{u}^s(p)\gamma^\nu\left(g_V-g_A\gamma^5\right)v^{s'}(p')\right]$$

$$=\frac{e^2}{q^2 4\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left[\bar{v}^{s'}(p')\gamma^\mu u^s(p)\right]\left[\bar{u}^s(p)\gamma^\nu\left(g_V-g_A\gamma^5\right)v^{s'}(p')\right]$$

$$\times\left[\bar{u}^r(k)\gamma_\mu v^{r'}(k')\right]\left[\bar{v}^{r'}(k')\gamma_\nu\left(g_V-g_A\gamma^5\right)u^r(k)\right]$$

$$=\frac{e^2}{16q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\left(g_V-g_A\gamma^5\right)\right]\mathrm{Tr}\left[\left(\not{k}+m_\mu\right)\gamma_\mu\left(\not{k}'-m_\mu\right)\gamma_\nu\left(g_V-g_A\gamma^5\right)\right]$$

$$=\frac{e^2}{16q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left\{g_V\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\right]-g_A\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\gamma^5\right]\right\}$$

$$\times\left\{g_V\mathrm{Tr}\left[\not{k}\gamma_\mu\not{k}'\gamma_\nu\right]-g_A\mathrm{Tr}\left[\not{k}\gamma_\mu\not{k}'\gamma_\nu\gamma^5\right]-m_\mu^2 g_V\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\right]\right\}$$

$$=\frac{e^2}{16q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left\{g_V\left(p'_\rho p_\sigma\right)\mathrm{Tr}\left[\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\right]-g_A\left(p'_\rho p_\sigma\right)\mathrm{Tr}\left[\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu\gamma^5\right]\right\}$$

$$\times\left\{g_V\left(k^\rho k'^\sigma\right)\mathrm{Tr}\left[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu\right]-g_A\left(k^\rho k'^\sigma\right)\mathrm{Tr}\left[\gamma_\rho\gamma_\mu\gamma_\sigma\gamma_\nu\gamma^5\right]-m_\mu^2 g_V\mathrm{Tr}\left(\gamma_\mu\gamma_\nu\right)\right\}$$

$$=\frac{e^2}{16q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left[4g_V\left(p'_\rho p_\sigma\right)\left(g^{\rho\mu}g^{\sigma\nu}+g^{\rho\nu}g^{\mu\sigma}-g^{\rho\sigma}g^{\mu\nu}\right)+4g_A\left(p'_\rho p_\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\right]$$

$$\times\left[4g_V\left(k^\rho k'^\sigma\right)\left(g_{\rho\mu}g_{\sigma\nu}+g_{\rho\nu}g_{\mu\sigma}-g_{\rho\sigma}g_{\mu\nu}\right)+4g_A\left(k^\rho k'^\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)-4m_\mu^2 g_V g_{\mu\nu}\right]$$

$$=\frac{e^2}{16q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left[4g_V\left(p'^\mu p^\nu+p'^\nu p^\mu-g^{\mu\nu}\left(p'\cdot p\right)\right)+4g_A\left(p'_\rho p_\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\right]$$

$$\times\left[4g_V\left(k_\mu k'_\nu+k_\nu k'_\mu-g_{\mu\nu}\left(k\cdot k'\right)\right)+4g_A\left(k^\rho k'^\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)-4m_\mu^2 g_V g_{\mu\nu}\right]$$

$$=\frac{e^2}{q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left\{g_V^2\left[p'^\mu p^\nu+p'^\nu p^\mu-g^{\mu\nu}\left(p'\cdot p\right)\right]\left[k_\mu k'_\nu+k_\nu k'_\mu-g_{\mu\nu}\left(k\cdot k'\right)\right]\right.$$

$$\left.+g_A^2\left(p'_\rho p_\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\left(k^\rho k'^\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)-m_\mu^2 g_V^2 g_{\mu\nu}\left[p'^\mu p^\nu+p'^\nu p^\mu-g^{\mu\nu}\left(p'\cdot p\right)\right]\right\}$$

$$=\frac{e^2}{q^2\left(q^2-m_Z^2-i\Gamma_Z m_Z\right)}\left[2g_V^2\left(p'\cdot k\right)\left(p\cdot k'\right)+2g_V^2\left(p\cdot k\right)\left(p'\cdot k'\right)\right.$$

$$\left.+2g_A^2\left(p'\cdot k\right)\left(p\cdot k'\right)-2g_A^2\left(p\cdot k\right)\left(p'\cdot k'\right)+2m_\mu^2 g_V^2\left(p\cdot p'\right)\right]$$

Tương tự cho thành phần thứ ba:

$$\mathcal{M}_Z\mathcal{M}_\gamma^*=\frac{e^2}{q^2\left(q^2-m_Z^2+i\Gamma_Z m_Z\right)}\left[2g_V^2\left(p'\cdot k\right)\left(p\cdot k'\right)+2g_V^2\left(p\cdot k\right)\left(p'\cdot k'\right)\right.$$
$$\left.+2g_A^2\left(p'\cdot k\right)\left(p\cdot k'\right)-2g_A^2\left(p\cdot k\right)\left(p'\cdot k'\right)+2m_\mu^2 g_V^2\left(p\cdot p'\right)\right]$$

Thành phần thứ tư được xử lý:

$$\begin{aligned}
\mathcal{M}_Z\mathcal{M}_Z^* &= \frac{1}{16\left(q^2-m_Z^2\right)^2+\Gamma_Z^2 m_Z^2}\left[\bar{v}^{s'}(p')\gamma^\mu\left(g_V-g_A\gamma^5\right)u^s(p)\right]\left[\bar{u}^r(k)\gamma_\mu\left(g_V-g_A\gamma^5\right)v^{r'}(k')\right]\\
&\times\left[\bar{v}^{r'}(k')\gamma_\nu\left(g_V-g_A\gamma^5\right)u^r(k)\right]\left[\bar{u}^s(p)\gamma^\nu\left(g_V-g_A\gamma^5\right)v^{s'}(p')\right]\\
&= \frac{1}{16\left(q^2-m_Z^2\right)^2+\Gamma_Z^2 m_Z^2}\left[\bar{v}^{s'}(p')\gamma^\mu\left(g_V-g_A\gamma^5\right)u^s(p)\right]\left[\bar{u}^s(p)\gamma^\nu\left(g_V-g_A\gamma^5\right)v^{s'}(p')\right]\\
&\times\left[\bar{u}^r(k)\gamma_\mu\left(g_V-g_A\gamma^5\right)v^{r'}(k')\right]\left[\bar{v}^{r'}(k')\gamma_\nu\left(g_V-g_A\gamma^5\right)u^r(k)\right]\\
&= \frac{1}{64\left[\left(q^2-m_Z^2\right)^2+\Gamma_Z^2 m_Z^2\right]}\mathrm{Tr}\left[\not{p}'\gamma^\mu\left(g_V-g_A\gamma^5\right)\not{p}\gamma^\nu\left(g_V-g_A\gamma^5\right)\right]\\
&\times\mathrm{Tr}\left[\left(\not{k}+m_\mu\right)\gamma_\mu\left(g_V-g_A\gamma^5\right)\left(\not{k}'-m_\mu\right)\gamma_\nu\left(g_V-g_A\gamma^5\right)\right]
\end{aligned}$$

Xử lý Trace đầu tiên:

$$\begin{aligned}
&\mathrm{Tr}\left[\not{p}'\gamma^\mu\left(g_V-g_A\gamma^5\right)\not{p}\gamma^\nu\left(g_V-g_A\gamma^5\right)\right]\\
=&\mathrm{Tr}\left[\left(\not{p}'\gamma^\mu g_V-\not{p}'\gamma^\mu g_A\gamma^5\right)\left(\not{p}\gamma^\nu g_V-\not{p}\gamma^\nu g_A\gamma^5\right)\right]\\
=&g_V^2\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\right]-g_Vg_A\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\gamma^5\right]-g_Vg_A\mathrm{Tr}\left[\not{p}'\gamma^\mu\gamma^5\not{p}\gamma^\nu\right]+g_A^2\mathrm{Tr}\left[\not{p}'\gamma^\mu\gamma^5\not{p}\gamma^\nu\gamma^5\right]\\
=&g_V^2\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\right]-g_Vg_A\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\gamma^5\right]-g_Vg_A\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\gamma^5\right]+g_A^2\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\gamma^5\gamma^5\right]\\
=&\left(g_V^2+g_A^2\right)\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\right]-2g_Vg_A\mathrm{Tr}\left[\not{p}'\gamma^\mu\not{p}\gamma^\nu\gamma^5\right]\\
=&4\left(g_V^2+g_A^2\right)\left(p'_\rho p_\sigma\right)\left(g^{\rho\mu}g^{\sigma\nu}+g^{\rho\nu}g^{\mu\sigma}-g^{\rho\sigma}g^{\mu\nu}\right)+8g_Vg_A\left(p'_\rho p_\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\\
=&4\left(g_V^2+g_A^2\right)\left[p'^\mu p^\nu+p'^\nu p^\mu-g^{\mu\nu}\left(p'\cdot p\right)\right]+8g_Vg_A\left(p'_\rho p_\sigma\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)
\end{aligned}$$

Xử lý Trace thứ hai:

$$\begin{aligned}
&\mathrm{Tr}\left[\left(\not{k}+m_\mu\right)\gamma_\mu\left(g_V-g_A\gamma^5\right)\left(\not{k}'-m_\mu\right)\gamma_\nu\left(g_V-g_A\gamma^5\right)\right]\\
=&\mathrm{Tr}\left[\left(\not{k}+m_\mu\right)\left(g_V\gamma_\mu-g_A\gamma_\mu\gamma^5\right)\left(\not{k}'-m_\mu\right)\left(g_V\gamma_\nu-g_A\gamma_\nu\gamma^5\right)\right]\\
=&\mathrm{Tr}\left[\left(\not{k}g_V\gamma_\mu-\not{k}g_A\gamma_\mu\gamma^5+m_\mu g_V\gamma_\mu-m_\mu g_A\gamma_\mu\gamma^5\right)\left(\not{k}'g_V\gamma_\nu-\not{k}'g_A\gamma_\nu\gamma^5-m_\mu g_V\gamma_\nu+m_\mu g_A\gamma_\nu\gamma^5\right)\right]\\
=&g_V^2\mathrm{Tr}\left[\not{k}\gamma_\mu\not{k}'\gamma_\nu\right]-g_Vg_A\mathrm{Tr}\left[\not{k}\gamma_\mu\not{k}'\gamma_\nu\gamma^5\right]-g_Vg_A\mathrm{Tr}\left[\not{k}\gamma_\mu\gamma^5\not{k}'\gamma_\nu\right]+g_A^2\mathrm{Tr}\left[\not{k}\gamma_\mu\gamma^5\not{k}'\gamma_\nu\gamma^5\right]\\
&-m_\mu^2g_V^2\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\right]+m_\mu^2g_A^2\mathrm{Tr}\left[\gamma_\mu\gamma^5\gamma_\nu\gamma^5\right]\\
=&\left(g_V^2+g_A^2\right)\mathrm{Tr}\left[\not{k}\gamma_\mu\not{k}'\gamma_\nu\right]-2g_Vg_A\mathrm{Tr}\left[\not{k}\gamma_\mu\not{k}'\gamma_\nu\gamma^5\right]-m_\mu^2\left(g_V^2+g_A^2\right)\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\right]\\
=&4\left(g_V^2+g_A^2\right)\left(k^\rho k'^\sigma\right)\left(g_{\rho\mu}g_{\sigma\nu}+g_{\rho\nu}g_{\mu\sigma}-g_{\rho\sigma}g_{\mu\nu}\right)+8g_Vg_A\left(k^\rho k'^\sigma\right)\left(i\epsilon_{\rho\mu\sigma\nu}\right)-4m_\mu^2g_{\mu\nu}\left(g_V^2-g_A^2\right)\\
=&4\left(g_V^2+g_A^2\right)\left[k_\mu k'_\nu+k_\nu k'_\mu-g_{\mu\nu}\left(k\cdot k'\right)\right]+8g_Vg_A\left(k^\rho k'^\sigma\right)\left(i\epsilon_{\rho\mu\sigma\nu}\right)-4m_\mu^2g_{\mu\nu}\left(g_V^2-g_A^2\right)
\end{aligned}$$

Nhân 2 Trace lại, ta được:

$$
\begin{aligned}
&\left\{4\left(g_V^2+g_A^2\right)\left[p'^{\mu}p^{\nu}+p'^{\nu}p^{\mu}-g^{\mu\nu}\left(p'\cdot p\right)\right]+8g_Vg_A\left(p'_{\rho}p_{\sigma}\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\right\}\\
&\times\left\{4\left(g_V^2+g_A^2\right)\left[k_{\mu}k'_{\nu}+k_{\nu}k'_{\mu}-g_{\mu\nu}\left(k\cdot k'\right)\right]+8g_Vg_A\left(k^{\rho}k'^{\sigma}\right)\left(i\epsilon_{\rho\mu\sigma\nu}\right)-4m_{\mu}^2g_{\mu\nu}\left(g_V^2-g_A^2\right)\right\}\\
=&16\left(g_V^2+g_A^2\right)^2\left[p'^{\mu}p^{\nu}+p'^{\nu}p^{\mu}-g^{\mu\nu}\left(p'\cdot p\right)\right]\left[k_{\mu}k'_{\nu}+k_{\nu}k'_{\mu}-g_{\mu\nu}\left(k\cdot k'\right)\right]\\
&+32\left(g_V^2+g_A^2\right)g_Vg_A\left[p'^{\mu}p^{\nu}+p'^{\nu}p^{\mu}-g^{\mu\nu}\left(p'\cdot p\right)\right]\left(k^{\rho}k'^{\sigma}\right)\left(i\epsilon_{\rho\mu\sigma\nu}\right)\\
&-16m_{\mu}^2\left(g_V^2+g_A^2\right)\left(g_V^2-g_A^2\right)g_{\mu\nu}\left[p'^{\mu}p^{\nu}+p'^{\nu}p^{\mu}-g^{\mu\nu}\left(p'\cdot p\right)\right]\\
&+32\left(g_V^2+g_A^2\right)g_Vg_A\left(p'_{\rho}p_{\sigma}\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\left[k_{\mu}k'_{\nu}+k_{\nu}k'_{\mu}-g_{\mu\nu}\left(k\cdot k'\right)\right]\\
&+64g_V^2g_A^2\left(p'_{\rho}p_{\sigma}\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\left(k^{\rho}k'^{\sigma}\right)\left(i\epsilon_{\rho\mu\sigma\nu}\right)\\
&-32m_{\mu}^2\left(g_V^2-g_A^2\right)g_Vg_Ag_{\mu\nu}\left(p'_{\rho}p_{\sigma}\right)\left(i\epsilon^{\rho\mu\sigma\nu}\right)\\
=&32\left(g_V^2+g_A^2\right)^2\left(p'\cdot k\right)\left(p\cdot k'\right)+32\left(g_V^2+g_A^2\right)^2\left(p\cdot k\right)\left(p'\cdot k'\right)+32m_{\mu}^2\left(g_V^2+g_A^2\right)\left(g_V^2-g_A^2\right)\left(p\cdot p'\right)\\
&+128\left(p'\cdot k\right)\left(p\cdot k'\right)-128\left(p\cdot k\right)\left(p'\cdot k'\right)\\
=&32g_V^4\left(p'\cdot k\right)\left(p\cdot k'\right)+32g_V^4\left(p\cdot k\right)\left(p'\cdot k'\right)+32g_A^4\left(p'\cdot k\right)\left(p\cdot k'\right)+32g_A^4\left(p\cdot k\right)\left(p'\cdot k'\right)\\
&+192g_V^2g_A^2\left(p'\cdot k\right)\left(p\cdot k'\right)-64g_V^2g_A^2\left(p\cdot k\right)\left(p'\cdot k'\right)+32m_{\mu}^2g_V^4\left(p\cdot p'\right)-32m_{\mu}^2g_A^4\left(p\cdot p'\right)
\end{aligned}
$$

Cuối cùng thay vào, ta thu được:

$$
\begin{aligned}
\mathcal{M}_Z\mathcal{M}_Z^* =& \frac{1}{64\left[\left(q^2-m_Z^2\right)^2+\Gamma_Z^2m_Z^2\right]}\left[32g_V^4\left(p'\cdot k\right)\left(p\cdot k'\right)+32g_V^4\left(p\cdot k\right)\left(p'\cdot k'\right)\right.\\
&+32g_A^4\left(p'\cdot k\right)\left(p\cdot k'\right)+32g_A^4\left(p\cdot k\right)\left(p'\cdot k'\right)+192g_V^2g_A^2\left(p'\cdot k\right)\left(p\cdot k'\right)-64g_V^2g_A^2\left(p\cdot k\right)\left(p'\cdot k'\right)\\
&\left.+32m_{\mu}^2g_V^4\left(p\cdot p'\right)-32m_{\mu}^2g_A^4\left(p\cdot p'\right)\right]\\
=&\frac{1}{\left(q^2-m_Z^2\right)^2+\Gamma_Z^2m_Z^2}\left[\frac{1}{2}g_V^4\left(p'\cdot k\right)\left(p\cdot k'\right)+\frac{1}{2}g_V^4\left(p\cdot k\right)\left(p'\cdot k'\right)\right.\\
&+\frac{1}{2}g_A^4\left(p'\cdot k\right)\left(p\cdot k'\right)+\frac{1}{2}g_A^4\left(p\cdot k\right)\left(p'\cdot k'\right)+3g_V^2g_A^2\left(p'\cdot k\right)\left(p\cdot k'\right)-g_V^2g_A^2\left(p\cdot k\right)\left(p'\cdot k'\right)\\
&\left.+\frac{1}{2}m_{\mu}^2g_V^4\left(p\cdot p'\right)-\frac{1}{2}m_{\mu}^2g_A^4\left(p\cdot p'\right)\right]
\end{aligned}
$$

Hệ quy chiếu khối tâm:

$$
\begin{cases}
p\cdot k=p'\cdot k'=E^2-\vec{p}\vec{k}=E^2-E\left|\vec{k}\right|\cos\theta\\
p\cdot k'=p'\cdot k=E^2-\vec{p}\vec{k}=E^2+E\left|\vec{k}\right|\cos\theta\\
q^2=\left(p+p'\right)^2=4E^2=s\\
p\cdot p'=2E^2\\
k\cdot k'=E^2+\left|\vec{k}\right|^2
\end{cases}
$$

Thay vào biên độ tán xạ Feynman, ta được:

$$
\mathcal{M}_{\gamma}\mathcal{M}_{\gamma}^*=\frac{8e^4}{s}\left[\left(E^2+E\left|\vec{k}\right|\cos\theta\right)\left(E^2+E\left|\vec{k}\right|\cos\theta\right)+\left(E^2-E\left|\vec{k}\right|\cos\theta\right)\left(E^2-E\left|\vec{k}\right|\cos\theta\right)+m_{\mu}^2\left(2E^2\right)\right]
$$

$$= \frac{16e^4}{s}\left[2E^4 + 2E^2\left|\vec{k}\right|^2\cos^2\theta + 2m_\mu^2 E^2\right]$$
$$= \frac{16e^4E^2}{s}\left[E^2 + \left|\vec{k}\right|^2\cos^2\theta + m_\mu^2\right]$$

$$\begin{aligned}\mathcal{M}_\gamma\mathcal{M}_Z^* + \mathcal{M}_Z\mathcal{M}_\gamma^* &= \frac{2e^2\left(s-m_Z^2\right)}{s\left[\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right]}\left[2g_V^2\left(E^2 + E\left|\vec{k}\right|\cos\theta\right)^2 + 2g_V^2\left(E^2 - E\left|\vec{k}\right|\cos\theta\right)^2\right. \\ &\left. +2g_A^2\left(E^2 + E\left|\vec{k}\right|\cos\theta\right)^2 - 2g_A^2\left(E^2 - E\left|\vec{k}\right|\cos\theta\right)^2 + 4m_\mu^2 g_V^2 E^2\right] \\ &= \frac{4e^2\left(s-m_Z^2\right)}{s\left[\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right]}\left[2g_V^2\left(E^4 + E^2\left|\vec{k}\right|^2\cos^2\theta + m_\mu^2 E^2\right) + 4g_A^2 E^3\left|\vec{k}\right|\cos\theta\right] \\ &= \frac{8e^2\left(s-m_Z^2\right)E^2}{s\left[\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right]}\left[g_V^2\left(E^2 + \left|\vec{k}\right|^2\cos^2\theta + m_\mu^2\right) + 2g_A^2 E\left|\vec{k}\right|\cos\theta\right]\end{aligned}$$

$$\begin{aligned}\mathcal{M}_Z\mathcal{M}_Z^* &= \frac{1}{\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2}\left[\frac{1}{2}g_V^4\left(E^2 + E\left|\vec{k}\right|\cos\theta\right)^2 + \frac{1}{2}g_V^4\left(E^2 - E\left|\vec{k}\right|\cos\theta\right)^2\right. \\ &+\frac{1}{2}g_A^4\left(E^2 + E\left|\vec{k}\right|\cos\theta\right)^2 + \frac{1}{2}g_A^4\left(E^2 - E\left|\vec{k}\right|\cos\theta\right)^2 + 3g_V^2g_A^2\left(E^2 + E\left|\vec{k}\right|\cos\theta\right)^2 \\ &\left.-g_V^2g_A^2\left(E^2 - E\left|\vec{k}\right|\cos\theta\right)^2 + m_\mu^2 g_V^4 E^2 - m_\mu^2 g_A^4 E^2\right] \\ &= \frac{1}{\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2}\left[g_V^4 E^2\left(E^2 + \left|\vec{k}\right|^2\cos^2\theta\right) + g_A^4 E^2\left(E^2 + \left|\vec{k}\right|^2\cos^2\theta\right)\right. \\ &\left.+2g_V^2g_A^2E^2\left(E^2 + \left|\vec{k}\right|^2\cos^2\theta\right) + 8g_V^2g_A^2E^3\left|\vec{k}\right|\cos\theta + m_\mu^2\left(g_V^4 - g_A^4\right)E^2\right] \\ &= \frac{E^2}{\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2}\left[\left(g_V^2 + g_A^2\right)^2\left(E^2 + \left|\vec{k}\right|^2\cos^2\theta\right) + 8g_V^2g_A^2E\left|\vec{k}\right|\cos\theta + m_\mu^2\left(g_V^4 - g_A^4\right)\right]\end{aligned}$$

Vì bình phương biên độ tán đã rất dài nên ta sẽ thu gọn nó lại bằng cách đặt:

$$\mu = \frac{m_\mu^2}{E^2}$$
$$\chi_0(s) = \frac{s}{4e^2\left(s - m_Z^2 + i\Gamma_Z m_Z\right)}$$

Biến đổi $\chi_0(s)$ cho phù hợp với bài:

$$\begin{aligned}\mathbf{1}/\left|\chi_0\right|^2 = \chi_0\chi_0^* &= \frac{s}{4e^2\left(s - m_Z^2 + i\Gamma_Z m_Z\right)}\cdot\frac{s}{4e^2\left(s - m_Z^2 - i\Gamma_Z m_Z\right)} \\ &= \frac{s^2}{16e^4\left[\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2\right]}\end{aligned}$$
$$\Rightarrow \frac{16e^4}{s^2}\left|\chi_0\right|^2 = \frac{1}{\left(s-m_Z^2\right)^2 + \Gamma_Z^2 m_Z^2}$$

$$\begin{aligned}
\mathbf{2}/\chi_0(s) &= \frac{s}{4e^2\left(s-m_Z^2+i\Gamma_Z m_Z\right)} \\
&= \frac{s}{4e^2}\left[\frac{s-m_Z^2}{(s-m_Z^2)^2+\Gamma_Z^2 m_Z^2} - i\frac{\Gamma_Z m_Z}{(s-m_Z^2)^2+\Gamma_Z^2 m_Z^2}\right] \\
\Rightarrow \frac{4e^2}{s}\operatorname{Re}\chi_0 &= \frac{s-m_Z^2}{(s-m_Z^2)^2+\Gamma_Z^2 m_Z^2}
\end{aligned}$$

Áp dụng vào, ta thu được:

$$\begin{aligned}
\mathcal{M}_\gamma \mathcal{M}_\gamma^* &= \frac{16e^4E^2}{s}\left(E^2+m_\mu^2+\left|\vec{k}\right|^2\cos^2\theta\right) \\
\mathcal{M}_\gamma \mathcal{M}_Z^* + \mathcal{M}_Z \mathcal{M}_\gamma^* &= \frac{16e^4E^2}{s^2}\operatorname{Re}\chi_0\left[2g_V^2\left(E^2+m_\mu^2+\left|\vec{k}\right|^2\cos^2\theta\right)+4g_A^2E\left|\vec{k}\right|\cos\theta\right] \\
\mathcal{M}_Z \mathcal{M}_Z^* &= \frac{16e^4E^2}{s^2}|\chi_0|^2\left[\left(g_V^2+g_A^2\right)^2\left(E^2+\left|\vec{k}\right|^2\cos^2\theta\right)+m_\mu^2\left(g_V^4-g_A^4\right)+8g_V^2g_A^2E\left|\vec{k}\right|\cos\theta\right]
\end{aligned}$$

Vì khi tính biên độ tán xạ biến sẽ là $\cos\theta$. Do đó, ta sẽ gom lại gọn hơn nữa các hằng số lại và để $\cos\theta$ là biến đồng thời gom m_μ lại thành 1 cụm để dễ dàng xét trường hợp $m_\mu = 0$.

$$\begin{aligned}
G_1(s) &= |\chi_0|^2\left(g_V^2+g_A^2\right)^2+2g_V^2\operatorname{Re}\{\chi_0\}+1 \\
G_2(s) &= 2g_A^2g_V^2|\chi_0|^2+g_A^2\operatorname{Re}\chi_0 \\
G_3(s) &= |\chi_0|^2\left[\left(g_V^2+g_A^2\right)^2+(g_V^4-g_A^4)\mu\right]+2g_V^2\operatorname{Re}\chi_0\,(\mu+1)+\mu+1
\end{aligned}$$

Cuối cùng bình phương biên độ tán xạ có dạng:

$$|\mathcal{M}|^2 = \frac{16e^4E^2}{s^2}\left[G_1(s)\left|\vec{k}\right|^2\cos^2\theta+4EG_2(s)\left|\vec{k}\right|\cos\theta+G_3(s)E^2\right]$$

Vi phân tiết diện tán xạ:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{4E^2}\frac{\left|\vec{k}\right|}{|\vec{p}|}|\mathcal{M}|^2 = \frac{e^4\left|\vec{k}\right|}{16\pi^2Es^2}\left[G_1(s)\left|\vec{k}\right|^2\cos^2\theta+4EG_2(s)\left|\vec{k}\right|\cos\theta+G_3(s)E^2\right]$$

Đặt $\alpha = \dfrac{e^2}{4\pi}$:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\left|\vec{k}\right|}{Es^2}\left[G_1(s)\left|\vec{k}\right|^2\cos^2\theta+4EG_2(s)\left|\vec{k}\right|\cos\theta+G_3(s)E^2\right]$$

Tiết diện tán xạ toàn phần:

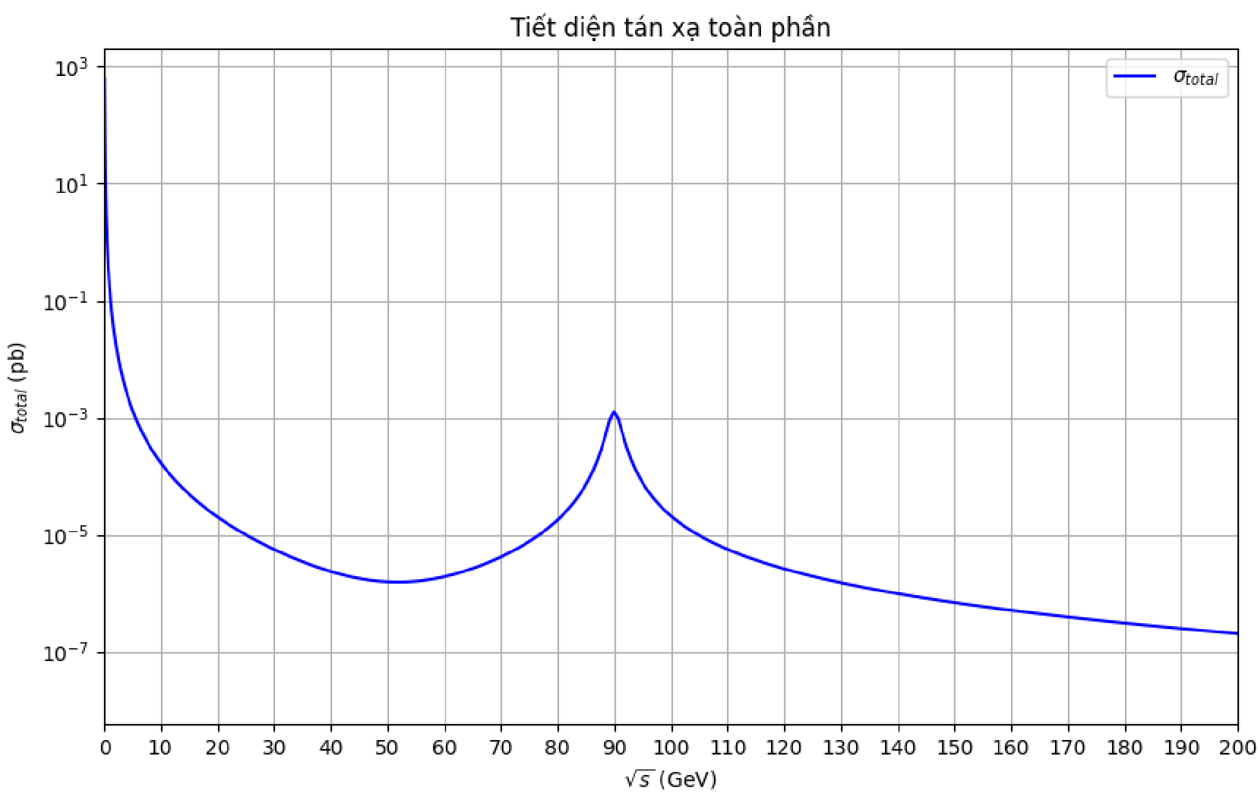
$$\begin{aligned}
\sigma_{total} = \int\frac{d\sigma}{d\Omega}d\Omega &= \int_{-1}^{1}\frac{2\pi\alpha^2\left|\vec{k}\right|}{Es^2}\left[G_1(s)\left|\vec{k}\right|^2\cos^2\theta+4EG_2(s)\left|\vec{k}\right|\cos\theta+G_3(s)E^2\right]d(\cos\theta) \\
&= \frac{2\pi\alpha^2\left|\vec{k}\right|}{Es^2}\left[G_1(s)\left|\vec{k}\right|^2\frac{\cos^3\theta}{3}+4EG_2(s)\frac{\cos^2}{2}+G_3(s)E^2\cos\theta\right]_{-1}^{1}
\end{aligned}$$

$$= \frac{2\pi\alpha^2\left|\vec{k}\right|}{Es^2}\left[\frac{2}{3}G_1(s)\left|\vec{k}\right|^2 + 2G_3(s)E^2\right]$$

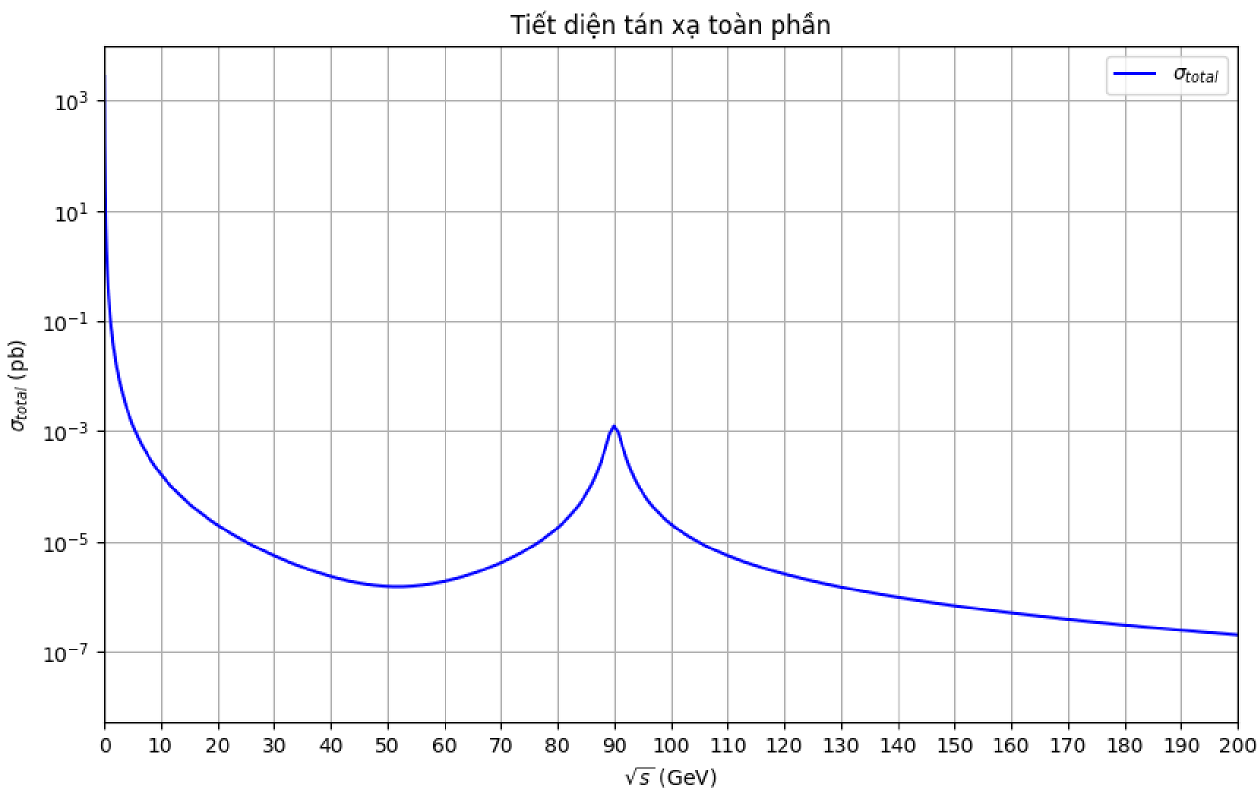
$$= \frac{4\pi\alpha^2\left|\vec{k}\right|}{Es^2}\left[G_1(s)\frac{\left|\vec{k}\right|^2}{3} + G_3(s)E^2\right]$$

Nếu $m_\mu = 0$ thì ta chỉ cần bỏ số hạng phụ thuốc vào m_μ là μ bên trong $G_3(s)$

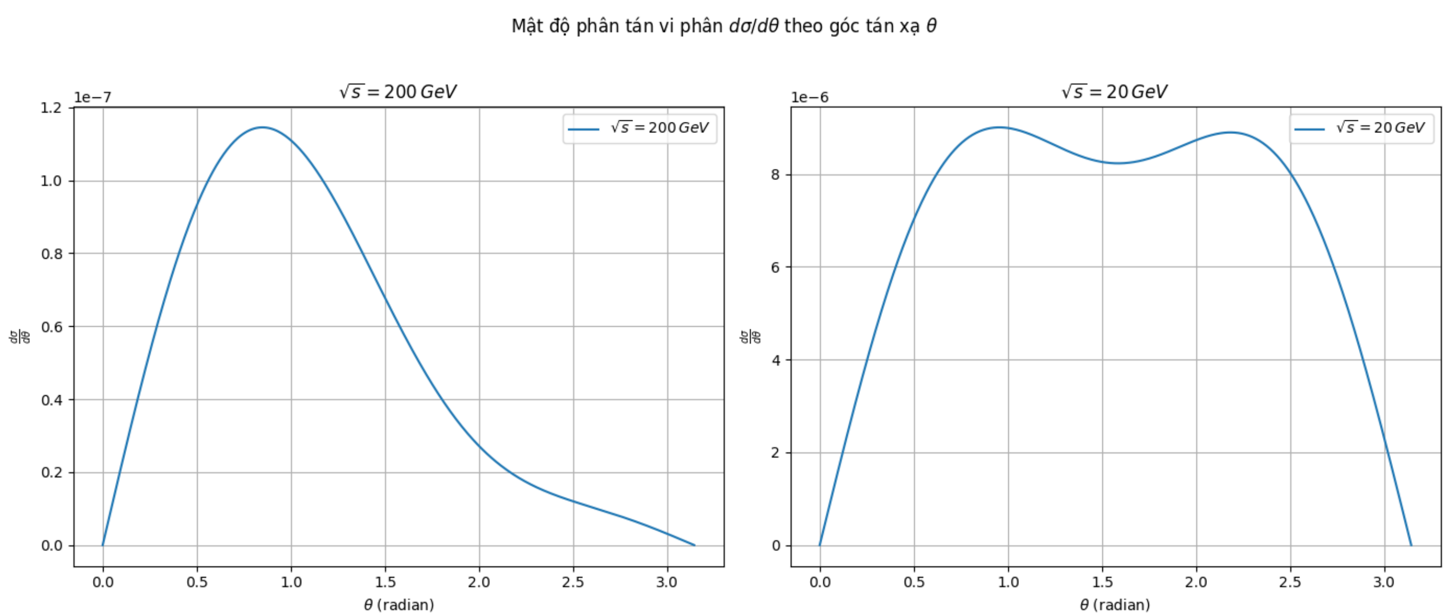
Hình kết quả:



Hình 1: Tiết diện tán xạ toàn phần với $m_\mu = 0.105\ GeV$



Hình 2: Tiết diện tán xạ toàn phần với $m_\mu = 0\ GeV$



Hình 3: Vi phân tiết diện tán xạ