CS7646 Project 1

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1 QUESTIONS

Project 1 has 7 questions , here is a detailed answer for each on of them supported by the code results , and also the plots produced by the code.

1.1 Question 1

Question 1 is stating the probability of winning 80$ in 1 episode (1000 bets) ,using simulation 1. Since we have unlimited bankroll , the probability of winning 1 bet is 18/38 , almost 0,474. So Theoretically, the probability of eventually reaching $80 should be very close to 100%. This is because, with the Martingale strategy, any loss is recovered by doubling the bet until a win occurs, and with 1000 bets available, the strategy has ample opportunity to hit the $80 target. In practice, the simulation should show that the probability of winning $80 within 1000 bets is extremely high, approaching nearly 100%. This is also confirmed from figure 2 on figures where the mean reach 80$ on not even 250 spins . For the simulation to don't achieve 80$ given the probabilities we would have to lose over 920 times out of 1000, but with probability 0,474 it's almost impossible to lose these many times.

1.2 Question 2

For question 2 again on simulation 1 since we have no loss the expected value of win in 1 spin is 0,474$(and since losing it gets *2 it stays the same)

Doing this *1000 it gets to 474$ expected value .The number $474 is derived from the simulation results where the Martingale strategy shows an average profit due to its nature of recovering losses and achieving target winnings frequently within 1000 bets, assuming an unlimited bankroll scenario. **!Important note our plots and simulation has a limit of 80$ so we can't get higher , but if we were**

to get the limit higher the plots would confirm the 474 number and also the simulation .So the answer is 80 with the limit and 474 without the limit.

1.3 Question 3

In Experiment 1 with the Martingale strategy and an unlimited bankroll we have the limit of 80 so even if the standard deviation start with strong fluctuations, it eventually stabilize . In more detail:

Upper and Lower Standard Deviation Lines: Yes, these lines do reach a maximum or minimum value even if It's not visible from the graphs and then stabilize. This stabilization occurs because the winnings are bounded by the $80 target and the strategy's nature of doubling bets, leading to less variability in the long run.

Convergence: The standard deviation lines do converge as the number of sequential bets increases. This convergence happens because, over many bets, the variability in winnings around the mean decreases due to the consistent achievement of the target and the reduced impact of fluctuations in individual bets.

The practical outcome is that the Martingale strategy's effect of reaching the target frequently leads to stable mean winnings with converging standard deviation lines as the number of bets grows all of this can be confirmed by figure 2 on figures and 3 where after 230 spins the sd and in general everything stabilize.

1.4 Question 4

Question 4 is the same with question 1 but this time for experiment 2 where we have a limit to what we have to lose . Winning 80$ in 1000 bets require us to don't lose all our money. So we have our initial bankroll of 256 and the probability of 18/38 of winning a single bet. If we lose and go to 2 we bet 2 to recover our money. So to win 80$ we need to win 80$ before going bankrupt and -256, we need to avoid losing all our money. One way to compute this is to run many times the 1 episode (1000sims) and calculate the number of successful episodes(80$)/total number of episodes , I did this with 2 helpful functions and computed the number 0,63% , one other way to think about it mathematically is to think what is the probability of winning 80$ without losing 8 times streak(losing all our money before recovering them) let's consider the probability p of winning as 18/38 and probability q of losing as 20/38 . let's consider p/q =1.111 the whole problem can be calculated using the probability with fixed boundaries by dividing 1-(1.111)^bankroll , with 1-(1.111)^(bankroll+target win)

The value is derived recursively, meaning it depends on the probability of winning if you were to gain or lose the next bet. Specifically, it is calculated using the probabilities of winning after increasing or decreasing your bankroll by 1 unit.

and this result in the **number 0,623** and since our 2 results align with each other we are positive the number is correct.

1.5 Question 5

Knowing the probability of around 0,63 we computed earlier this number can be calculated easily using 0,63%*80-0,37%*-256= somewhere around 44,32.

1.6 Question 6

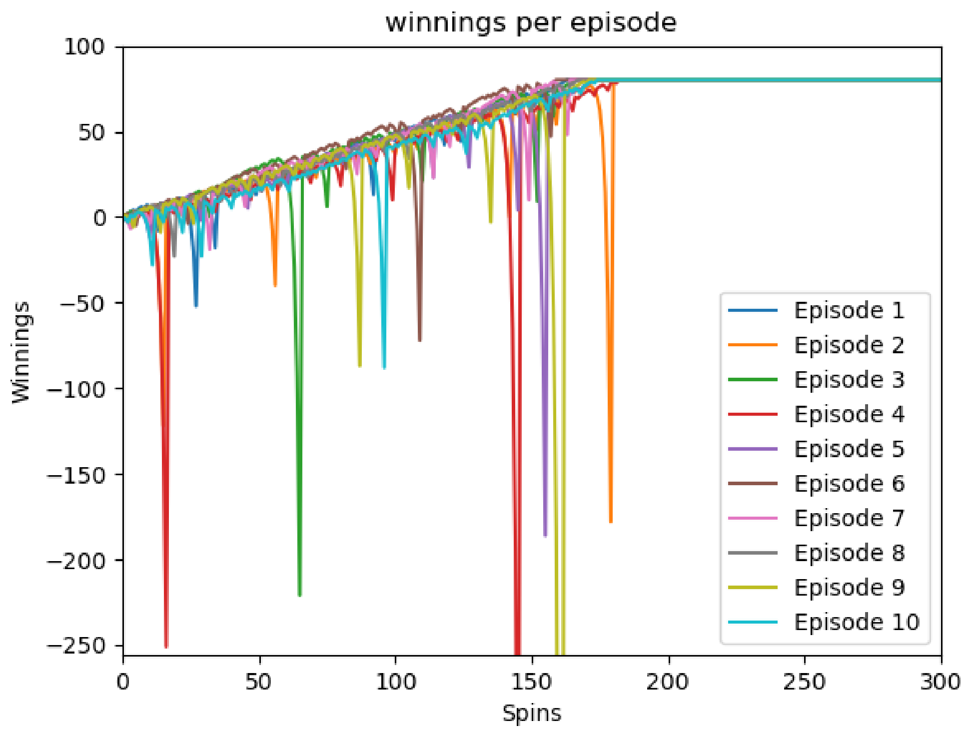
The standard deviation has limits since our bets are limited by the bounds , -256 and winning 80$, so even if on the start the standard deviation is unpredictable and fluctuating because we lose , win ,win ,lose etc, after finally we reach a winning 80$ or go bankrupt -256 , it stabilize and they converge with one another as the number of sequential bets increase from this point on(the point where we go bankrupt or reach 80$)

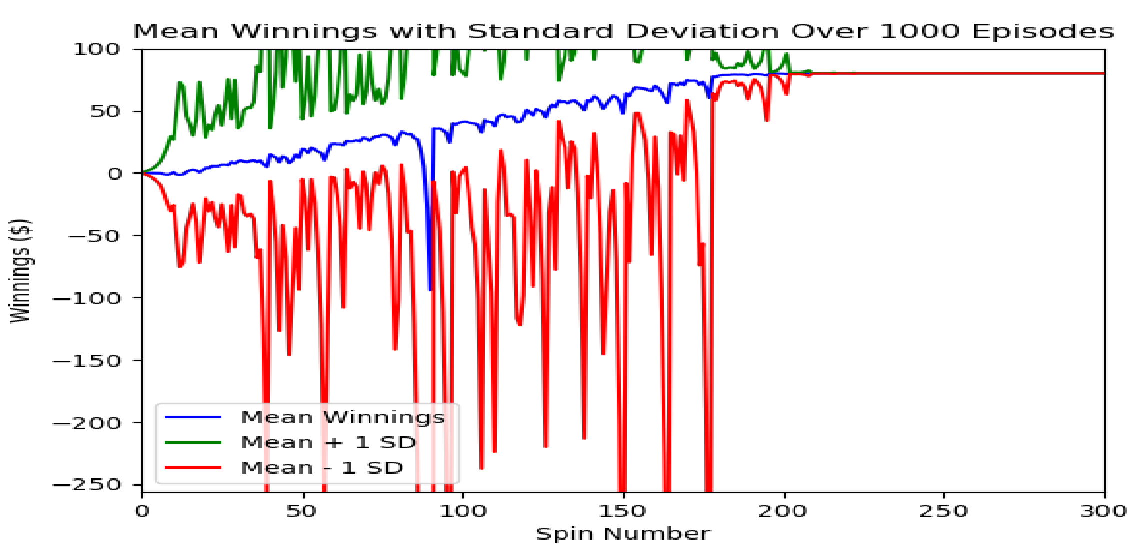
1.7 Question 7

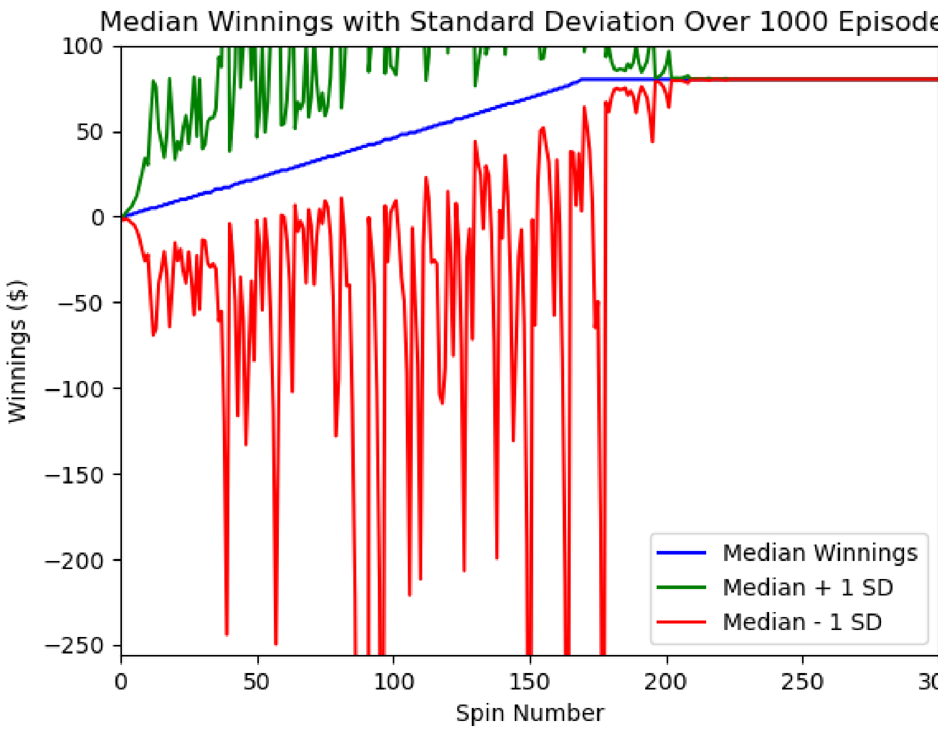
Expected value takes in account the probability ,and generalize as more episodes and outcomes we have to correctly find a type that accounts for most episodes , making an acceptable good result. Taking only one episode into account would be pretty bad in functionality , on this episode we might had won 80$ or even on 10 episodes it might happen to win every time and think that's a sure outcome while the expectations and probabilities help us to make a type and an overall good estimation of what's actually happening using average and useful stat principles.

2 FIGURES

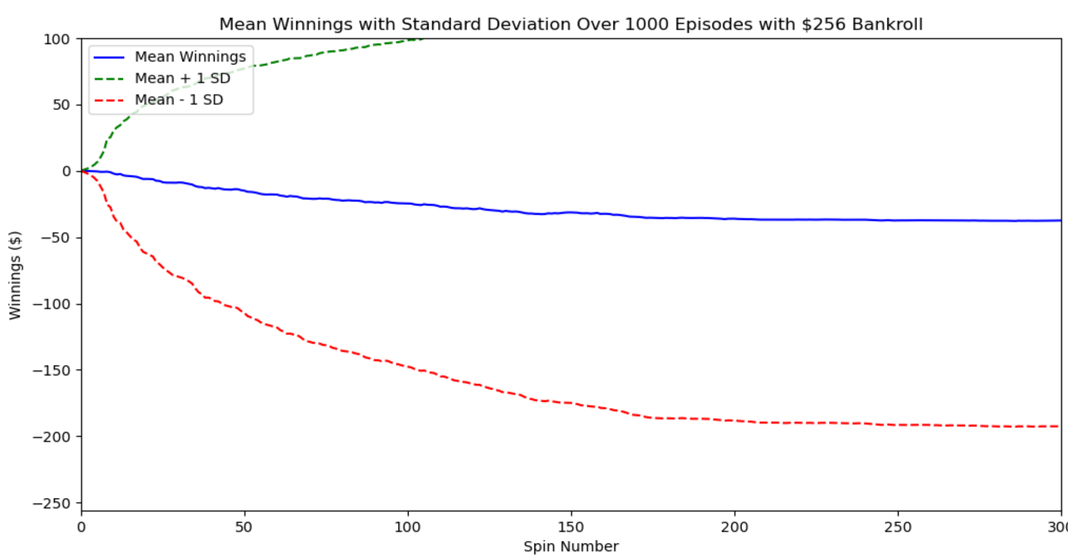
There are 5 figures required for project 1 , here they are with the appropriate descriptions.



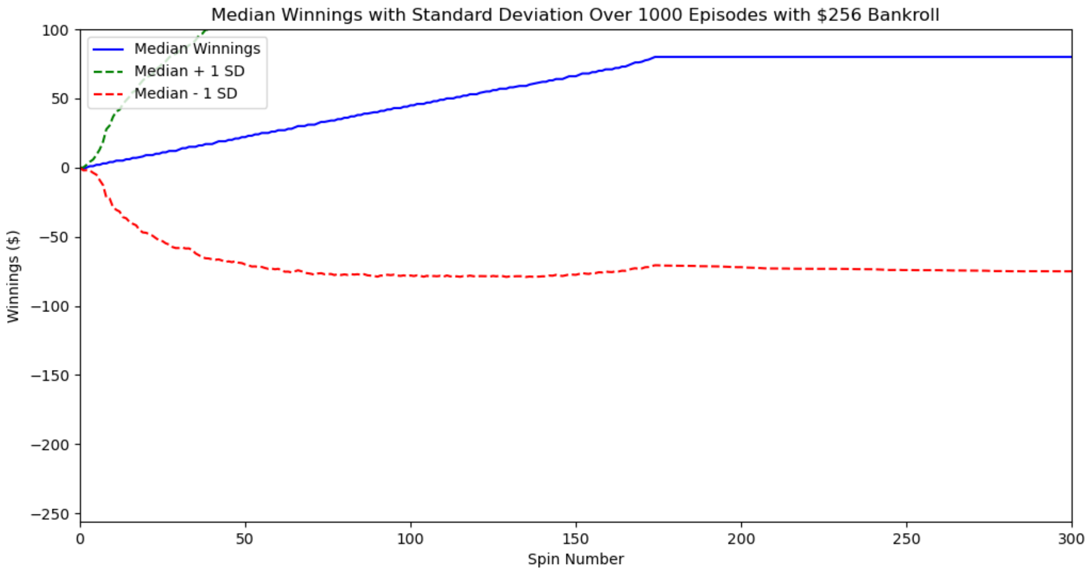




Median Winnings with Standard Deviation Over 1000 Episodes
Winnings ($)
Spin Number
Median Winnings
Median + 1 SD
Median - 1 SD



Mean Winnings with Standard Deviation Over 1000 Episodes with $256 Bankroll
Winnings ($)
Spin Number
Mean Winnings
Mean + 1 SD
Mean - 1 SD



3 REFERENCES

I didn't use many references outside of the class material . I used some references on the algorithms used and some python documentation to support my code.

1. https://en.wikipedia.org/wiki/Martingale_(betting_system)
2. Martelli, A. Ravenscroft, and S. Holden (2017), Python in a Nutshell, 3rd Edition Links to an external site.
3. James, D. Witten, T. Hastie, R. Tibshirani (2017), An Introduction to Statistical Learning (Chapter 2) Links to an external site.
4. https://medium.com/@aljohnsonex/simulating-roulette-strategies-with-python-martingale-85864c9f5dca