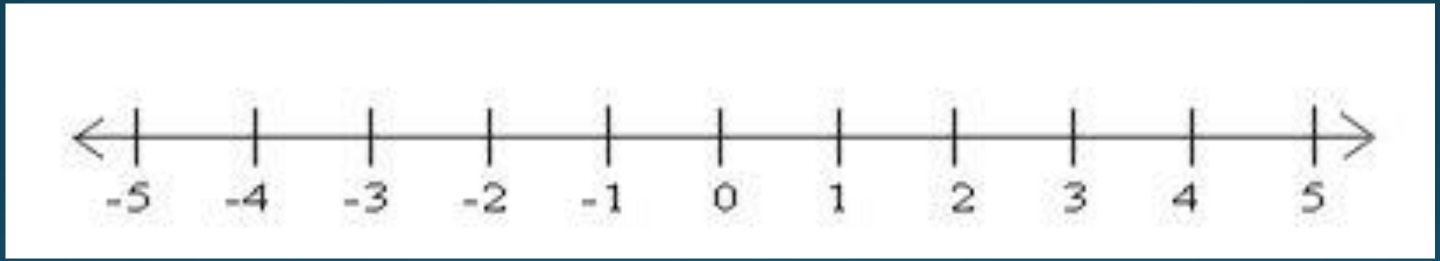
NUMBER SYSTEM

Types of Numbers

- **Number line**: A number line is line where all the numbers are allocated their positions. The origin of the number line starts from zero and it continues to infinity, on either side.



Positive Numbers: Numbers which are to the right of zero are said to be positive numbers.

For example 1, 3 ,1.2 , 2.6 , 7 etc.

Negative Numbers: Numbers which are to the left of zero are said to be negative numbers.

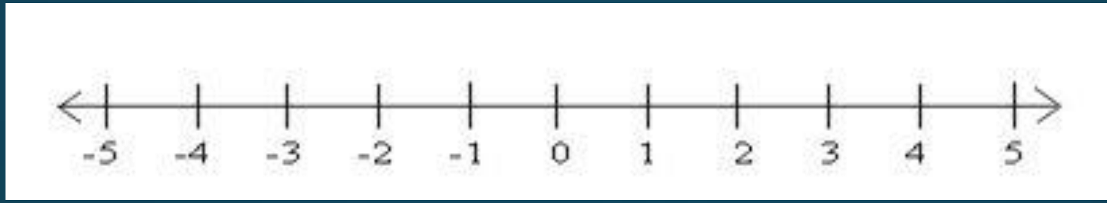
For example -1, -5 , -7.2 , -2.5 , -9 etc.

Counting Numbers: Numbers which are well managed on the number line with the difference of 1. The smallest counting number on the number line is 1.

- **Natural Numbers:** Numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11…and so on are called natural numbers. The lowest natural number is 1.

- **Whole Numbers:** All counting numbers together with zero from the set of whole numbers
  Example:- 0, 1, 2, 3, 4, ------ are whole number.

- **Integers:** All counting numbers, 0 and -ve of counting numbers are called integers.
  Example:- -∞---------, -3, -2, -1, 0, 1, 2, 3, -------∞



Types of Integers-

a. **Positive Integers-** 1,2,3,4,……
b. **Negative Integers-** ……,-4,-3,-2,-1
c. **Zero-** Neither positive nor negative
d. **Non-positive integers-** Set of 0 and negative integers
e. **Non-negative integers-** Set of 0 and positive integers

- **Prime numbers:** The numbers, which have exactly two factors, namely 1 and the number itself.
- For example 2, 3, 5, 7 etc
- All prime numbers greater than 3 can be expressed in the form of 6n+1 or 6n+5

- Composite Numbers- Numbers which have more than 2 factors.

Practice Question

Question:- What is the remainder when any prime number greater than 3 is divided by 6?

A. 1
B. 5
C. Both A and B
D. Neither A nor B

Divisibility Rules

A number is divisible by

- **2** If the last digit is even.
- **3** If the sum of the digits is divisible by 3.
- **4** If the last two digits of the number divisible by 4.
- **5** If the last digit is a 5 or a 0.
- **6** If the number is divisible by both 3 and 2.
- **8** If the last three digits form a number divisible by 8.
- **9** If the sum of the digits is divisible by 9.
- **10** If the last digit of number is 0.

- **Rule of 11:-** If the difference between sum of digits in even places and the sum of the digits in odd places is 0 or divisible by 11.
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  Example: 365167484
- (3+5+6+4+4) – (6+1+7+8) = 0
- ∴ 365167484 is divisible by 11.
- **12** If the number is divisible by both 3 and 4.

Any other numbers can be written in terms of the numbers whose divisibility is already known.

Example: $15 = 3 \times 5$

$18 = 2 \times 9$

$33 = 3 \times 11$

Note: The numbers expressed should be co-prime (i.e., the HCF of the two numbers should be 1)

Example: $40 = 4 \times 10$ is wrong because HCF(4,10) is 2.

$\therefore$ $40 = 5 \times 8$ because HCF(5,8) is 1.

Practice Question

1. Which of the following numbers is divisible by 3x4?

946 (b) 947 (c) 948 (d) 949

2. The number 567xy is completely divisible by 30. The possible of x and y can be

(a) 0 and 0 (b) 1 and 0 (c) 2 and 0 (d) 0 and 1

Q3. What should come in place of x if 563x5 is divisible by 9?

Q4. What should come in place of x if 4857x is divisible by 88?

Solution

As per the divisibilty rule of 8, if **35Y** will be divisible by 8 then the whole number **1568X35Y** will be divisible by 8.
Hence, in order to divide **35Y** by 8, the value for *Y will be 2*.
Now the number will be **1568X352.**

As per the divisibilty rule of 11, the difference of (2+3+8+5 =18) and (5+X+6+1 =12+X) i.e. **18 - (12+X)** should be divisible by 11.
Hence, 6-X will be divisible by 11, only if *X will be 6.*

Hence, the value for X is 6 and for Y is 2, and the value of X +Y will be 6+2= 8.

Unit Digit Concept

- ***Note:*** *The last digit of an expression will always depend on the unit digit of the values.*

- ***Example:*** *The unit digit of 123 x 456 x 789 is ?*

	Power			
Base	1	2	3	4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6		
9	9	1		

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be o.

What is the unit digit of (123)^42?

Last digit of $(123)^{42} \approx 3^{42}$.

Now, the cyclicity of '3' is 4.

So, we're going to divide the power by 4.

$\text{Rem}\left(\frac{42}{4}\right) = 2$

$\therefore$ 2^{nd} value in the cyclicity of 3 will be the answer.

ie answer is 9.

LCM & HCF

- The greatest number that will exactly divide a, b and c is **HCF(a, b, c)**.
- The least number which is exactly divisible by a, b and c is **LCM(a, b, c)**.

FINDING THE H.C.F. OF BIG NUMBERS

For larger numbers you can use the following method:

Step 1:- Find all prime factors of both numbers.

Step 2:- Write both numbers as a multiplication of prime numbers.

Step 3:- Find which factors are repeating in both numbers and multiply them to get H.C.F

FINDING L.C.M. OF BIG NUMBERS

Step 1:- Find all the prime factors of both numbers.

Step 2:- Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included

To determine LCM of 14, 42, 21.

$$\begin{array}{r|lll} 7 & 14, & 42, & 21 \\ \hline 2 & 2, & 6, & 3 \\ \hline 3 & 1, & 3, & 3 \\ \hline & 1, & 1, & 1 \end{array}$$

$\therefore$ LCM of 14, 42, 21 = $7 \times 2 \times 3 = 42$

To determine HCF of 33, 55, 22

$$\begin{array}{l} 33\ )\ 55\ (\ 1 \\ \quad\ \ \underline{33} \\ \quad\ \ 22\ )\ 33\ (\ 1 \\ \qquad\quad \underline{22} \\ \qquad\quad 11\ )\ 22\ (\ 2 \\ \qquad\qquad\quad \underline{22} \\ \qquad\qquad\quad \times \\ \qquad\qquad\quad 11\ )\ 22\ (\ 2 \\ \qquad\qquad\qquad\quad \underline{22} \\ \qquad\qquad\qquad\quad \times \end{array}$$

$\therefore$ HCF of 33, 55, 22 = 11

Hence, Required LCM = $\dfrac{42}{11}$

Important formulae:

$$\boxed{LCM(a,b) = \frac{a \times b}{HCF(a,b)}}$$

- Product of Two numbers = LCM X HCF
- HCF of fractions = $\frac{HCF\ OF\ numerators}{LCM\ OF\ denominators}$
- LCM of fractions = $\frac{LCM\ of\ numerators}{HCF\ of\ denominators}$

Practice Question

Q) The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is:

A)308
B)310
C)312
D)None

Q) The H.C.F of 9/10, 12/25, 18/35, and 21/40 is?

A)3/1400
B)5/1400
C)7/1400
D)None

Solution

1) We know, HCF × LCM = Product of 2 nos.

$$\Rightarrow 11 \times 7700 = 275 \times x$$

$$\Rightarrow \frac{\cancel{11} \times \cancel{7700}^{308}}{\cancel{275}_{\cancel{25}}} = x$$

$$\Rightarrow 308 = x.$$

2) HCF of fractions = $\dfrac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

$$= \frac{HCF(9, 12, 18, 21)}{LCM(10, 25, 35, 40)}$$

$$= \frac{3}{1400}$$

Q. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and c in 198 seconds, all starting at the same point. After what time will they again at the starting point?

A)26mint 18 sec

B)42 mint 36 sec

C)45 mint

D)46 mint 12 sec

Solution

A complete his round in 252 seconds.

B completes his round in 308 seconds.

C completes his round in 198 seconds.

They will agian at starting together after,

LCM of 252, 308 and 198.

252 = 2 *2 *3*3*7

308 = 2 *2*7*11

198 = 2 *3*3*11

Required LCM = 2*2*3*3*7*11 = 2772 seconds = 46 minutes 12 seconds