NTA UGC-NET

Computer Science & Applications

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Eduncle.com
MPA 44, 2nd Floor, Rangbari Main Road,
Mahaveer Nagar 2nd, Kota
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Fuzzy System and Genetic Algorithms

1. FUZZY SETS

Sets are defined as collection of objects. An element x belonging to a set A is defined as $x \in A$, an element that is not a member in A, denoted as $x \notin A$.

A characteristic function/membership functions $\mu_A(x)$:

A membership function is a set/crisp set is an element in the universe having value 1 or 0. That is, for every $x \in u$,

$$\mu_A(x) = \{ 1 \text{ for } x \in A$$
$$0 \text{ for } x \notin A \}$$

Eg.: Let u = {0, 1, 2, 3, 4, 5}

A = {2, 4, 5}

So, $\mu_A(0) = 0$ $\quad u_A(2) = 1$

$u_A(1) = 0$ $\quad u_A(4) = 1$

as, $0, 1 \notin A$ but $2, 4 \in A$

In crisp set the membership taken value 1 or 0.

That is $\mu_A(x) = \{0, 1\}$

Notion of Fuzziness

The concept of fuzziness was introduced by Zadeh Fuzziness includes imprecision uncertainty and degree of truthfulness of values. Fuzzy logic is a superset of boolean logic that has been extended to handle the concept of partial truth or value between 'completely true' and 'completely false.'

Linguistic Variables are used for system input and output and are represented by words such as ‘size’, ‘age’ and temperature.

A fuzzy set is created to describe linguistic variable in more detail. The linguistic variable e.g. cold may have categories of ‘cold’, ‘very cold’, ‘moderate’, ‘warm’ and ‘very hot’. The membership is then developed for each member in the set.

So, A fuzzy set A is defined as,

$$A = \{(x, u_A(x)) \mid x \in A, \mu_A(x) \in [0, 1]\}$$

or $$A = \left\{ \frac{\sum x}{u_A(x)} \mid x \in A, \mu_A(x) \in [0,1] \right\}$$

Here, membership takes value is the interval [0, 1] the range between 0 and 1 is defined as membership grade or degree of membership.

Example, Let S be set of people

A fuzzy subset YOUNG is defined

$$\text{young}(x) = \{ 1, \text{ if age } (x) \le 20$$

$$\frac{30 - \text{age}(x)}{10}, \text{ if } 20 < \text{age } (x) \le 30$$

$$0 \quad , \text{ if age } (x) > 30 \}$$

So, if

- John age is 10.

  Since age < 20 $u_A(x) = 1$

- Edwin age 21

  $20 < \text{age} \leq 30$ So, $\frac{30-20}{10} = \frac{9}{10} = 0.9$

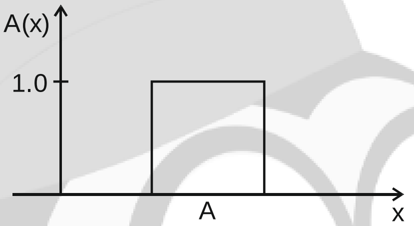
  $\mu_{youg}(x) = 0.9$

That is, 'Edwin is YOUNG' degree of truth is 0.9

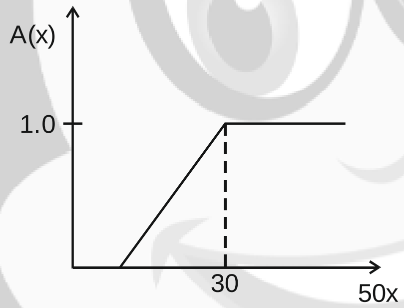
Crisp Set vs Fuzzy Set

The membership function of a crisp set can be 0 or 1.

$$A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & x \notin A \end{cases}$$



In fuzzy set, membership is between 0 to 1. If temperature can be in between 0 and 50. The temperature above 30°C is considered 'high'. An example, of membership can be given as



Ex. Assume a fuzzy set A = {0.1/1 + 0.4/3 + 0.6/4 + 1.0/5 + 1.0/6 + 0.6/7 + 0.5/8 + 0.2/10}

OR

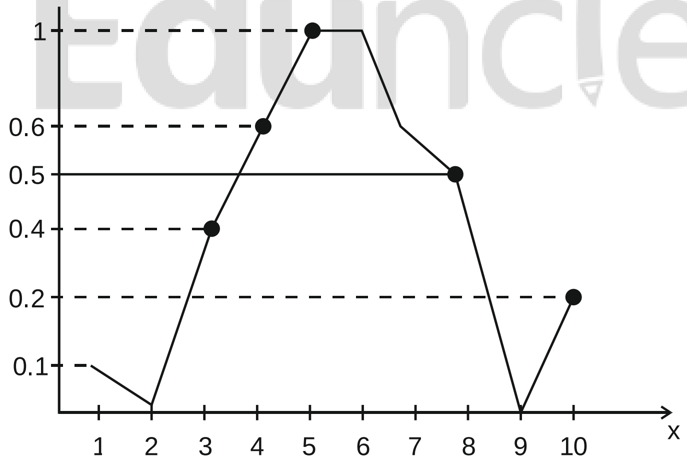
A = {(0.1, 1) (0.4, 3) (0.6, 4) (1.0, 5) (1.0, 6) (0.6, 7) (0.5, 8) (0.2, 10)} defined in

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(a) Sketch the membership function of this fuzzy set

(b) Which elements have lowest and highest membership degrees.

Sol.



The elements with highest membership are, 5 and 6 with $\mu_A(x) = 1$.

Lowest membership is of elements 2 and 9, $\mu_A(x) = 0$

Types of Membership Function

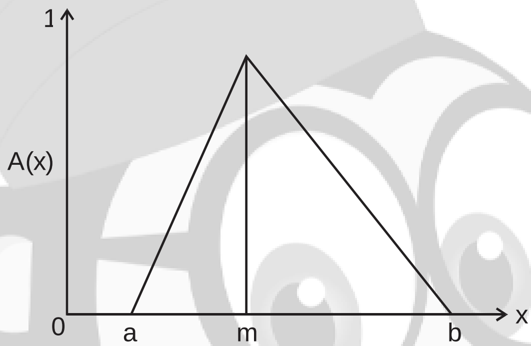
Zadeh has proposed a series of membership function.

(a) **Triangular :** Defined by its lower limit a, its upper limit b and the model value m so that a < m < b.

The value b – m is called margin when it is equal to m – a

$$A(x) = \begin{cases} 0 & \text{if } x <= a \\ (x-a)/(m-a) & \text{if } x \in (a, m) \\ (b-x)/(b-m) & \text{if } x \in (b, m) \\ 1 & \text{if } x \geq 0 \end{cases}$$

It is represented by

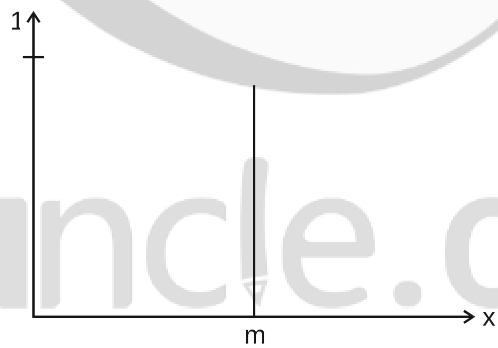


Triangular Fuzzy Sets

(b) **Singleton :** It takes value 0 is all the universe of discourse except in the point m, where it takes value 1.

It is the representation of a crisp value.

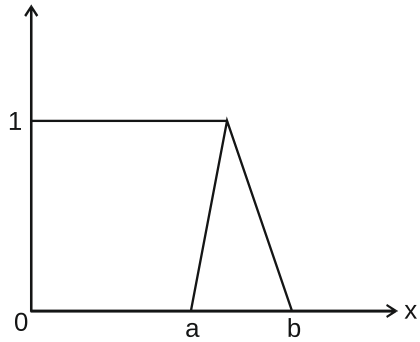
$$SG(x) = \begin{cases} 0 & \text{if } x \neq m \\ 1 & \text{if } x = m \end{cases}$$



Singleton Fuzzy Sets

(c) **L-function :** The function is defined by two parameters a and b in the following way :

$$L(x) = \begin{cases} 1 & \text{if } x <= a \\ \left(\dfrac{a-x}{b-a}\right) & \text{if } a < x <= b \\ 0 & \text{if } x > b \end{cases}$$

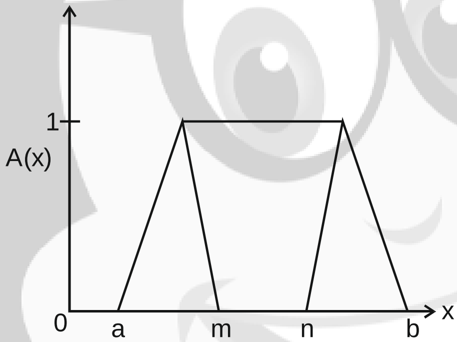


L-fuzzy Set

(d) Trapezoid Function

Defined by upper and lower limit (b, a)

$$A(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{x-a}{m-a} & \text{if } x \in [a, m] \\ 1 & \text{if } x \in [m, n] \\ \dfrac{b-x}{b-n} & \text{if } x \in [n, b] \\ 0 & \text{if } x > b \end{cases}$$



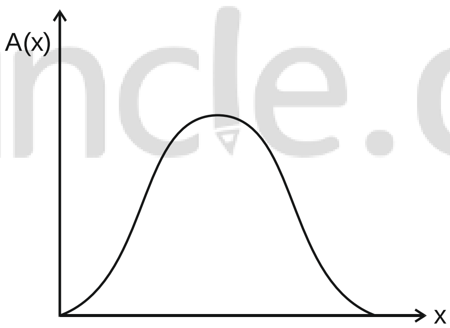
The function is easy to define, represent and simple to calculate.

(e) Gaussian Function

It is represented as,

$$A(x) = e^{-k}\,(x - m)^2$$

where $k > 0$



Gaussian Membership Function

(f) Exponential-like Function

It is given by,

$$A(x) = \frac{1}{1+k(x-m)^2}, k > 1$$

or

$$A(x) = \frac{k(x-m)^2}{1+k(x-m)^2}, k > 0$$

Fuzzification

Fuzzification is the process of changing a scalar value into a fuzzy value. This is achieved by using different types of fuzzifiers (membership function). Example, the water temperature in bath tub is to be controlled, instead of using crisp value of temperature an impressive set of water temperature is defined as ‘cool’ ‘warm’ and ‘hot’. Then, degree of membership is decide like temperature ν = 92°C belongs to μ = 0.6 to ‘warm’ and 0.4 to ‘hot’, while μ = 0 to ‘cool’.

Defuzzification

Defuzzification means the conversion from fuzzy set to crisp set.

Example : Suppose T_{High} denotes a fuzzy set representing that temperature is high.

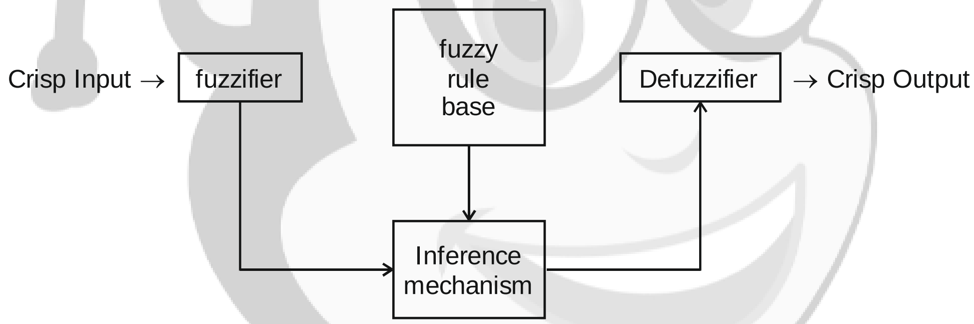
T_{High} is given as :

T_{High} = {(15, 0.1) (20, 0.4) (20, 0.45) (30, 0.55) (35, 0.65) (40, 0.7) (45, 0.85) (50, 0.9)}

What crisp value implies high temperature?

This is answered by defuzzification technique

General Structure of a Fuzzy System



Defuzzification Methods

There are a number of defuzzification methods :

1. Lambda – Out Method
2. Weighted Average Method
3. Maxima Methods
4. Centroid Methods

1. Lambda-Out Method/Alpha-Cut Method

It is used to derive crisp value of a fuzzy set.

- In this method, a fuzzy set A is transformed into a crisp set A_λ for a given value of λ $(0 \le \lambda \le 1)$

  $A_\lambda = \{x \mid \mu_A(x) \ge \lambda\}$

Example : Let A be a fuzzy set

$A = \{(x_1, 0.9)\ (x_2, 0.5)\ (x_3, 0.2)\ (x_4, 0.3)\}$

$A_{0.6} = \{x_1\}$ as $\mu_A(x_1) \ge 0.6$

$A_{0.2} = \{x_3, x_4, x_2, x_1\}$

Example : Two fuzzy sets P and Q are defined on x as follows :

$\mu(x)$	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8

Find the following :

(A) $P_{0.2}$, $P_{0.3}$

(B) $(P \cup Q)_{0.6}$

(C) $(P \cup \bar{P})_{0.8}$

(D) $(P \cap Q)_{0.4}$

Solution.

(a) $P_{0.2} = \{x_2, x_3, x_4, x_5\}$

$P_{0.3} = \{x_3, x_4, x_5\}$

(b) $(P \cup Q)_{0.6}$

$P \cup Q =$

x_1	x_2	x_3	x_4	x_5
0.9	0.6	0.7	0.5	0.8

$(P \cup Q)_{0.6} = \{x_1, x_2, x_3, x_5\}$

(c) $(P \cup \bar{P})_{0.8}$

	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
$\bar{P}$	0.9	0.8	0.3	0.5	0.6
$P \cup \bar{P}$	0.9	0.8	0.7	0.5	0.6

$(P \cup \bar{P})_{0.8} = \{x_1, x_2\}$

	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8
$P \cap Q$	0.1	0.2	0.3	0.2	0.4

$(P \cap Q)_{0.4} = \{x_5\}$

The Lamda-cut method for a fuzzy set can also be applied to fuzzy relation.

Example : For a fuzzy relation R

$$R = \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.5 & 0.9 & 0.6 \\ 0.4 & 0.8 & 0.7 \end{bmatrix}$$

The value of λ-cut at 0, 0.2, 0.9, 0.5 are :

$$R_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad R_{0.2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_{0.9} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R_{0.5} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Some Properties of λ-cut Sets

If A and B are two fuzzy sets defined with the same universe of discourse then,

1. $(A \cup B)_\lambda = A_\lambda \cup B_\lambda$
2. $(A \cap B)_\lambda = A_\lambda \cap B_\lambda$
3. $(\bar{A})_\lambda \neq \bar{A}_\lambda$ except for $\lambda = 0.5$
4. For any $\lambda \leq a$ where a varies between 0 and 1, it is true that $A_a \subseteq A_\lambda$, where the value of A_0 is the universe of discourse.

Some rules applies to fuzzy relations

Other Methods

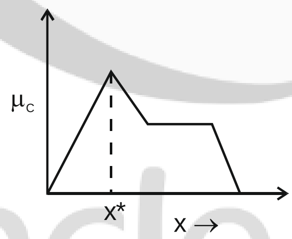
1. Maxima Methods
   (a) Height Method
   (b) First of maxima (FOM)
   (c) Last of maximum (LOM)
   (d) Mean of maxima (MOM)
2. Centroid Methods
   (a) Centre of Gravity Method (COG)
   (b) Centre of Sum Method (COS)
   (c) Centre of Area Method (COA)
3. Weighted Average Method

Maxima Methods

(a) Height Method

The method is based on max-membership principle, and defined as follows :

$$\mu_c(x^*) \geq \mu_c(x), \ \forall \ x \in X$$

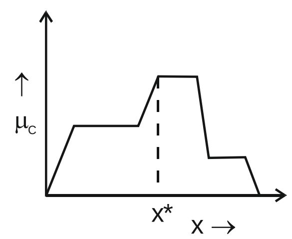


Note : x* is height of output fuzzy set C.

This method is applicable when height is unique.

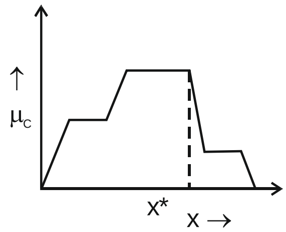
(b) First of Maxima (FOM)

FOM : $x^* = \min (x \mid c(x) = \max_w c\{w\})$



(c) Last of Maxima (LOM)

LOM : $x^* = \max (x \mid c(x) = \max_w c\{w\})$



(d) Mean of Maxima Method (MOM) : Take average of all elements that have maximum membership.

$$x_i^* = \frac{\sum_{x_i \in M} (x_i)}{(M)}$$

Where $M = \{x_i \mid \mu(x_i) = h(c)\}$ where h(c) is height of fuzzy set c.

Example : Suppose a fuzzy set Young is defined as :

young = {(15, 0.5) (20, 0.8) (25, 0.8) (30, 0.5) (35, 0.3)}

The crisp value according to :

- FOM Method

  20 is young age

- LOM

  25 is young age

- MOM Method

$$x^* = \frac{20+25}{2} = 22.5$$

A person of 22.5 yrs old is treated as young.

Centroid Methods

These methods given better result than the maxima methods but are computationly expensive than those methods.

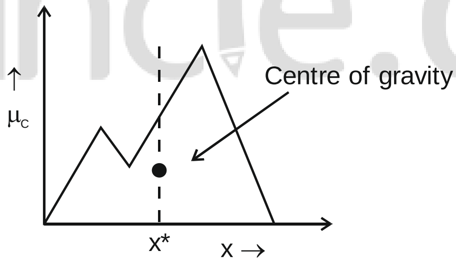
(a) Centre of Gravity (COG Method)

The basic principle in COG method is to find the point x where a vertical line would slice the aggregate into two equal masses.

The COG can be expressed as :

$(x, \mu_C(x)\ d(x))$

Graphically,



Note :

1. x* is the x-coordinate of centre of gravity.

2. $\int \mu_C(x)\, d(x)$ denote the area of region bounded by the curve μ_C.

3. If μ_C is defined with a discrete membership function, the COG can be stated as :

$$x^* = \frac{\sum x_i \mu_C(x_i)}{\sum \mu_C(x_i)}, \text{ for } i = 1 \text{ to } n$$

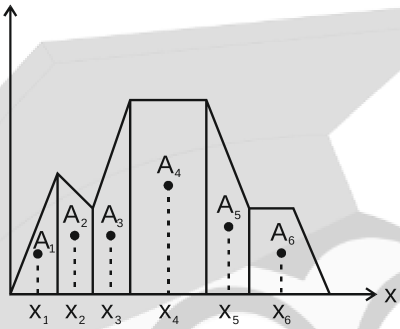
$x_i \rightarrow$ sample element

$n \rightarrow$ number of samples in fuzzy set C.

A Geometrical Method of Calculation

Steps :

Divide the entire region into a number of small regular regions (e.g. triangles, trapezoids etc)



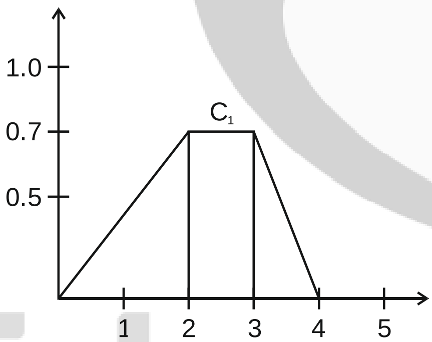
Let A_i and x_i denote the area and centre of gravity of the ith portion.

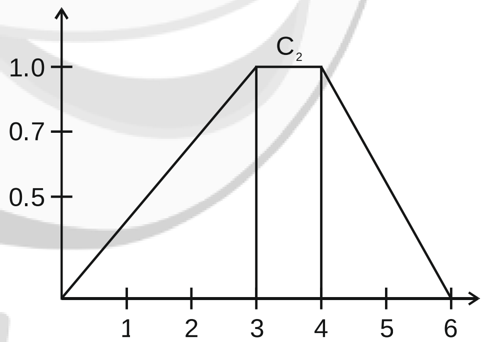
Then x* according to centre of gravity is,

$$x^* = \frac{\sum_{i=1}^{n} x_i (A_i)}{\sum_{i=1}^{n} A_i}$$

where n is the number of smaller geometrical components.

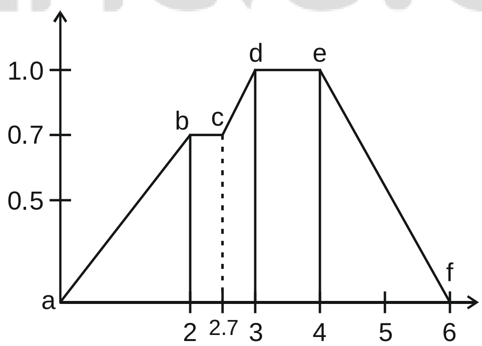
Example : Let the two fuzzy sets be the given by the graphs :





Then union of both fuzzy sets is given by,

$$C = C_1 \cup C_2$$



$$\mu_C(x) = \begin{cases} 0.35x & 0 \le x < 2 \\ 0.7 & 2 \le x < 2.7 \\ x-2 & 2.7 \le x < 3 \\ 1 & 3 \le x < 4 \\ (-0.5x+3) & 4 \le x \le 6 \end{cases}$$

This is computed as,

For A_1 :

$$y - 0 = \frac{0.7}{2}(x - 0) \Rightarrow \text{or } y = 0.35x$$

For A_2 :

$$y = 0.7$$

For A_3 :

$$y - 0 = \frac{1-0}{3-2}(x - 2) \Rightarrow y = x - 2$$

For A_4 :

$$y = 1$$

For A_5 :

$$y - 1 = \frac{0-1}{6-4}(x - 4) \Rightarrow y = -0.5x + 3$$

Thus, $x^* = \dfrac{\int x.\mu_C(x)dx}{\int u_c(x)d(x)}$

Numerator :

$$\int_0^2 0.35x^2dx + \int_2^{2.7} 0.7x^2dx + \int_{2.7}^3 \left(x^2 - 2x\right)dx \int_3^4 x\,dx + \int_4^6 \left(-0.5x^2 + 3x\right)dx = 10.98$$

Denominator :

$$D = \int_0^2 0.35x\ dx + \int_2^{2.7} 0.7\,x\ dx + \int_{2.7}^3 (x-2)dx + \int_3^4 dx + \int_4^6 \left(-0.5x + 3\right)dx = 3.445$$

Thus, $x^* = \dfrac{10.98}{3.445} = 3.187$

(II) Centroid Method : Centre of Sum (COS) Method

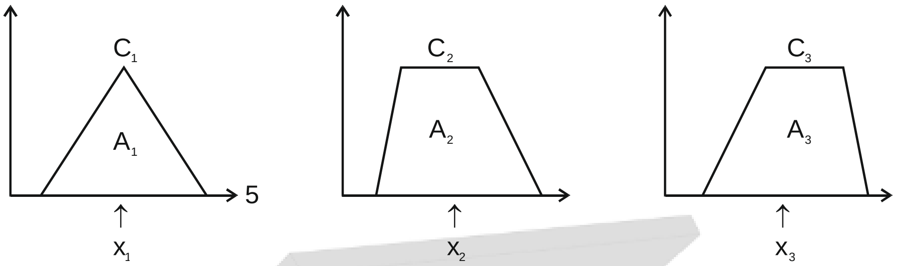
This method is computationally less expensive.

If the output fuzzy set $C = C_1 \cup C_2 \cup \ldots C_n$ then the crisp value according to COS is defined as:

$$x^* = \frac{\sum_{i=1}^{n} x_i\ A_{c_i}}{\sum_{i=1}^{n} A_{c_i}}$$

Here, A_{c_i} denotes the area of region bounded by fuzzy set c_i and x_i is the geometric centre of the area A_{c_i}.

Graphically

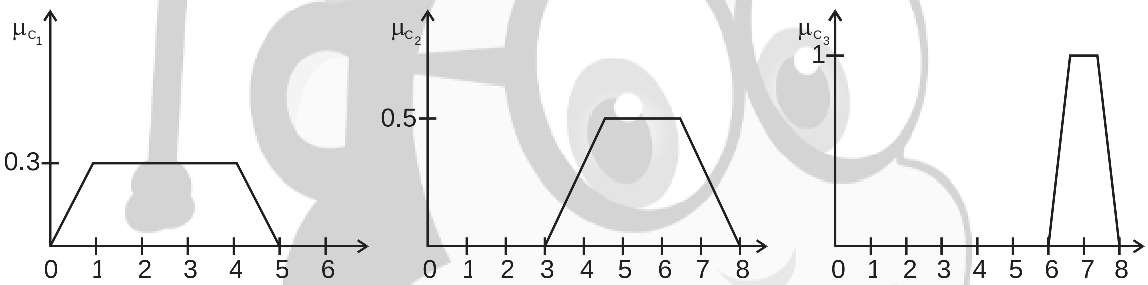


Here, no need to find the combined graph. The A_i and x_i can be calculated individually and x can be obtained.

Note :

- CoG method, the overlapping area is counted once whereas in CoS the overlapping area is counted twice.
- In CoS, we use the centre of area.

Ex.



In this case

$$A_{C_1} = \frac{1}{2} \times 0.3 \times (3 + 5),\ x_1 = 2.5$$

$$A_{C_2} = \frac{1}{2} \times 0.5 \times (4 + 2),\ x_2 = 5$$

$$A_{C_3} = \frac{1}{2} \times 1.0 \times (3 + 1),\ x_3 = 6.5$$

Thus,

$$x^* = \frac{\frac{1}{2}\times 0.3\times(3+5)\times 2.5+\frac{1}{2}\times 0.5\times(4+2)\times 5+\frac{1}{2}\times 1.0\times(3+1)\times 6.5}{\frac{1}{2}\times 0.3\times(3+5)+\frac{1}{2}\times 0.5\times(4+2)+\frac{1}{2}\times 1.0\times(3+1)}$$

$$\boxed{x^* = 4.9}$$

III. Centroid Method : Centre of largest area

Simplest Method

If the fuzzy set has two subregions, then the centre of gravity of the sub region with the largest area can be used to calculate the defuzzified value.

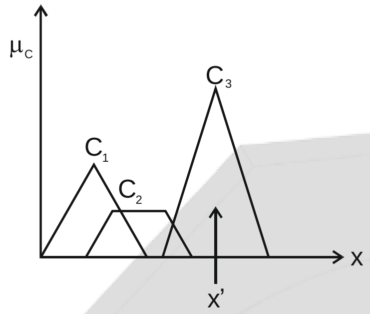
Mathematically,

$$x^* = \frac{\int \mu_{C_m}(x) \,.\, x' \, d(x)}{\int \mu_{C_m}(x)\, dx}$$

Here,

C_m is the region with largest area x' is centre of gravity of C_m

Graphically,

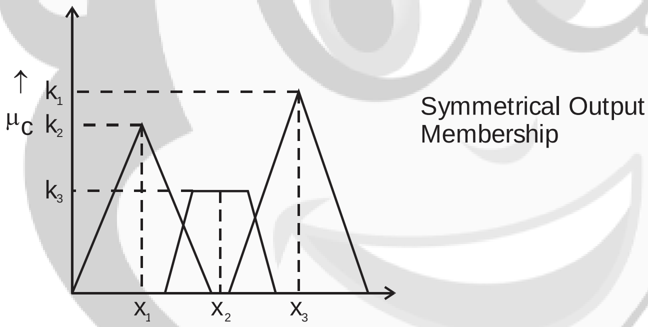


$C_m = C_3$.

Weighted Average Method

- This method is also alternatively called "Sugeno defuzzification" method
- The method can be used only for symmetrical output membership functions.
- That is if fuzzy membership is symmetrical in shape

Example,

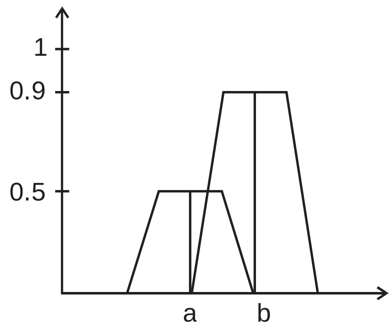


Middle value of each area $\rightarrow$ x_1, x_2, x_3

$$x^* = \frac{\sum_{i=1}^{n} \mu_{C_i}(x_i).x_i}{\sum_{i=1}^{n} \mu_{C_i}(x_i)}$$

where, c_1, c_2, ..., c_n are output fuzzy sets (x_i) is the value where middle of the fuzzy set L_i is observed

Example : Consider two function shown in figure below



Find the crisp value according to weighted average method, (where a = 10, b = 20)

Solution. According to weighted average method :

$$Z^* = \frac{\sum \mu_C(\bar{z}).\bar{z}}{\sum \mu_C(\bar{z})}$$

So,

$$Z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

$$= \frac{10 \times 0.5 + 20 \times 0.9}{1.4}$$

$$= \frac{5 + 18}{1.4}$$

$Z^* = 16.42$

Fuzzy Operations

1. **Union ($A \cup B$)**

   $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$

   Example : $A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

   $B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$

   $C = A \cup B$

   $= \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.5)\}$

2. **Intersection ($A \cap B$)**

   $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$

   $A = \{(x_1, 0{,}5), (x_2, 0.1), (x_3, 0.4)\}$

   $B = \{(x_1, 0{,}2), (x_2, 0.3), (x_3, 0.5)\}$

   $C = A \cap B$

   $= \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.4)\}$

3. **Complement (A^C)**

   $\mu_A{}^C(x) = 1 - \mu_A(x)$

   Example : $A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

   $C = A^C = \{(x_1, 0.5), (x_2, 0.9), (x_3, 0.6)\}$

   Few more operations

4. **Algebraic Product / Vector Product A.B**

   $\mu_{A.B}(x) = \mu_A(x) \cdot \mu_B(x)$

5. **Scalar Product**

   $\mu_{\alpha A}(x) = \alpha \times \mu_A(x)$

6. **Sum (A + B)**

   $\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$

7. **Difference** ($A - B = A \cap B^C$)

   $\mu_{A-B}(x) = \mu_{A \cap B}{}^C(x)$

8. **Disjunction Sum**

   $A \oplus B = (A^C \cap B) \cup (A \cap B^C)$

9. Bounded Sum

$$|A(x) \oplus B(x)| = \mu_{|A(x)\oplus B(x)|} = \min\,[1,\ \mu_A(x) + \mu_B(x)]$$

10. Bounded Difference

$$|A(x) \ominus B(x)| = \mu_{A(x)\ominus B(x)} = \max\,\{0,\ \mu_A(x) + \mu_B(x) - 1\}$$

11. Equality (A = B)

$$\mu_A(x) = \mu_B(x)$$

12. Power of Fuzzy Set A^α

$\alpha \to$ constant

$\mu_A{}^\alpha(x) = [\mu_A(x)]^\alpha$

- If $\alpha < 1$, then it is called dialation
- If $\alpha > 1$ it is called concentration

13. Cartesian Product (A × B)

If A is a fuzzy set defined over a universe of discoverse x and B is a fuzzy set defined over a universe of discoverse y

then, $\mu_{A\times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$

Example : $A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$

$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$

$$A \times B = \begin{array}{c c} & \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix} \end{array}$$

Example : Consider two fuzzy sets :

$$\tilde{A} = \left\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\right\}$$

$$\tilde{B} = \left\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\right\}$$

The following operation

(a) $$\tilde{A} \cup \tilde{B} = \left\{\frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5}\right\}$$

(b) $$\tilde{A} \cap \tilde{B} = \left\{\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5}\right\}$$

(c) $$\tilde{A}^c = \left\{\frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5}\right\}$$

(d) $$\tilde{B}^c = \left\{\frac{0.5}{2} + \frac{0.3}{.3} + \frac{0.8}{4} + \frac{0.6}{5}\right\}$$

(e) $$A - B = \left\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\right\} \cap \left\{\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}\right\}$$

$$A \cap \overline{B} = \left\{ \frac{0.5}{2} + \frac{0.3}{3} + \frac{0.3}{4} + \frac{0.2}{5} \right\}$$

(f) $$A.B = \left\{ \frac{0.5}{2} + \frac{0.35}{3} + \frac{0.06}{4} + \frac{0.08}{5} \right\}$$

Example : Let two fuzzy set A and B be :

$$A = \left\{ \frac{1}{1} + \frac{0.5}{1.5} + \frac{0.3}{2.0} + \frac{0.4}{2.5} \right\}$$

$$B = \left\{ \frac{0.4}{1} + \frac{0.2}{1.5} + \frac{0.7}{2.0} + \frac{0.1}{2.5} \right\}$$

(a) Algebraic Sum $\mu_{A+B}(x) = [\mu_A(x) + \mu_B(x)] - [\mu_A(x) \cdot \mu_B(x)]$

So, $$\mu_{A+B}(x) = \left\{ \frac{1}{1} + \frac{0.6}{1.5} + \frac{0.79}{2.0} + \frac{0.46}{2.5} \right\}$$

(b) Algebraic product

$$\mu_{A.B}(x) = \left\{ \frac{0.4}{1} + \frac{0.1}{1.5} + \frac{0.21}{2.0} + \frac{0.04}{2.5} \right\}$$

(c) Bounded sum $\mu_{A \oplus B} = \min [1, \mu_A(x) + \mu_B(x)]$

$$\mu_{A \oplus B}(x) = \left\{ \frac{1}{1} + \frac{0.7}{1.5} + \frac{1}{2.0} + \frac{0.5}{2.5} \right\}$$

(d) Bounded Difference

$$\mu_{A \odot B}(x) = \max \{0, \mu_A(x) - \mu_B(x)\}$$

$$\mu_{A \odot B} = \left\{ \frac{0.6}{1} + \frac{0.3}{1.5} + \frac{0}{2.0} + \frac{0.3}{2.5} \right\}$$

Properties of Fuzzy Sets

Fuzzy sets follow the properties :

1. Commutativity

   $A \cap B = B \cap A$

   $A \cup B = B \cup A$

2. Associativity

   $A \cup (B \cup C) = (A \cup B) \cup C$

   $A \cap (B \cap C) = (A \cap B) \cap C$

3. Distributivity

   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. Idempotence

   $A \cup A = A$

   $A \cap A = \phi$

$A \cup \phi = A$

$A \cap \phi = \phi$

5. Transitivity

If $A \subseteq B$; $B \subseteq C$ then $A \subseteq C$

6. Involution

$(A^C)^C = A$

7. Demorgan's law

$(A \cap B)^C = A^C \cup B^C$

$(A \cup B)^C = A^C \cap B^C$

Q. If two fuzzy sets A and B are given with membership functions

$\mu_A(x) = \{0.2, 0.4, 0.8, 0.5, 0.1\}$

$\mu_B(x) = \{0.1, 0.3, 0.6, 0.3, 0.2\}$

Then for $A \cap B$ value of μ will be :

(A) {0.9, 0.7, 0.4, 0.8, 0.9} (B) {0.2, 0.4, 0.8, 0.5, 0.2}

(C) {0.1, 0.3, 0.6, 0.3, 0.1} (D) {0.7, 0.3, 0.4, 0.2, 0.7}

Sol. (C) $\mu.(A \cap B) = \min(\mu_A(x), \mu_B(x))$

So, $\mu_A(x) = 0.2, 0.4, 0.8, 0.5, 0.1$

$\mu_B(x) = 0.1, 0.3, 0.6, 0.3, 0.2$

$\min(\mu_A(x), \mu_B(x)) = 0.1, 0.3, 0.6, 0.3, 0.1$

Correct option is (C)

Q. Consider a fuzzy set A defined on the interval X = [0, 10] by membership

$$\mu_A(x) = \frac{x}{x+2}$$

Then the α-cut corresponding to $\alpha = 0.5$ will be :

(A) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} (B) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(C) {2, 3, 4, 5, 6, 7, 8, 9, 10} (D) { }

Sol. (C) $\mu_A(0) = \frac{0}{0+2} = \frac{0}{2} = 0$

$\mu_A(1) = \frac{1}{1+2} = \frac{1}{3} = 0.33$ $\mu_A(2) = \frac{2}{2+2} = \frac{2}{4} = 0.5$

$\mu_A(3) = \frac{3}{3+2} = 0.6$

$\mu_A(4) = \frac{4}{4+2} = 0.66$

$\mu_A(5) = \frac{5}{5+2} = 0.71$

$\mu_A(6) = \frac{6}{6+2} = 0.75$

$$\mu_A(7) = \frac{7}{7+2} = 0.77$$

$$\mu_A(8) = \frac{8}{8+2} = 0.8$$

$$\mu_A(9) = \frac{9}{9+2} = 0.81$$

$$\mu_A(10) = \frac{10}{10+2} = 0.83$$

α-cut mean $\mu_A(x) \geq 0.5$

That is for {2, 3, 4, 5, 6, 7, 8, 9, 10}

Correct option is (C)

Linguistic Variable

A linguistic variable can be regarded either as a variable whose value is a fuzzy number or as a variable whose values are defined in linguistic terms.

- A linguistic variable is characterised by a quintuple.

  (x, T(x), u, G, M)

  in which,
- x is name of variable
- T(x) is the term set of x, that is set of names of linguistic values of x
- G is a syntatic rule for generating the names of values of x.
- M is syntatic rule for associating each value with its meaning.

Ex. Speed : linguistic variable name.

Term set T = {slow, moderate, fast, very slow, more or less fast ...}

Where each term T is in universe of discourse U = [0,100]

Syntalic Rule

Slow Speed as $\rightarrow$ Speed below 40 mph

Moderate Speed as $\rightarrow$ Speed close to 55 mph

Fast as $\rightarrow$ Speed above 70 mph.

Some commonly used linguistic terms $\rightarrow$

It fuzzy set A has membership function μ_A

Then

(a) Very A = $(\mu_A)^2$

(b) Somewhat A or more or less A = $\sqrt{\mu_A}$

(c) Extremely A = $(\mu_A)^8$

Some Concepts Related to Fuzzy Sets

1. **Support :** The support of a fuzzy set A is the set of all points x in X

Such that $\mu_A(x) > 0$

Support (A) = $\{x \mid \mu_A(x) > 0\}$

2. **Core :** Core(A) = $\{x \mid \mu_A(x) = 1\}$

Set of all points whose membership is 1.

3. Crossover Points

Crossover (A) = $\{x \mid \mu_A(x) = 0.5\}$

Set of all points whose membership is 0.5

4. Normality : A fuzzy set A is called normal if its core is non-empty. In other words, we can find a point $x \in X$ such that $\mu_A(x) = 1$.

5. Fuzzy Singleton : A fuzzy set whose support is a single point in X is called a fuzzy singleton.

6. α-cut and Strong α-cut

α-cut (A_α)

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$$

Set of all points whose membership is greater than equal to α.

Strong α-cut (A_α^+)

$$A_\alpha^+ = \{x \mid \mu_A(x) > \alpha\}$$

Set of all points whose membership is greater than α.

7. Fuzzy set boundary

$$= \{\min(\mu_A(x)), \max(\mu_A(x))\}$$

8. Height of a Fuzzy Set

is $\max(\mu_A(x))$

9. Bandwidth of fuzzy set

$$\{x \mid \mu_A(x) \geq 0.5\}$$

Set of all points whose membership is greater than 0.5

10. Convex and Concave Fuzzy Sets

- A fuzzy set A is convex if its membership function is such that

  $A[\lambda x_1 + (1 - \lambda)x_2] \geq \min[A(x_1), A(x_2)]$

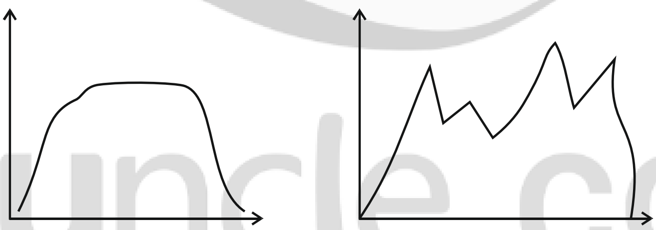
  For any $x_1, x_2 \in X$

  $\lambda \in [0, 1]$

- A fuzzy set is concave if the corresponding membership function is such that

  $A[\lambda x_1 + (1 - \lambda)x_2] \leq \max[A(x_1), A(x_2)]$ for any $x_1, x_2 \in X$

  $\lambda \in [0, 1]$



Convex Fuzzy Set Concave Fuzzy Set

11. Cardinality of Fuzzy set

Cardinality of fuzzy set A, with finite universe X, is defined as,

Card(A) or $|A| = \Sigma u_A(x)\ x \in X$

Example : Let A be a fuzzy set A = {1, 2, 3, 4, 5}

$\mu_A(x) = \{(1, 0)\ (2, 0.2)\ (3, 0.4)\ (4, 0.5)\ (5, 1)\}$

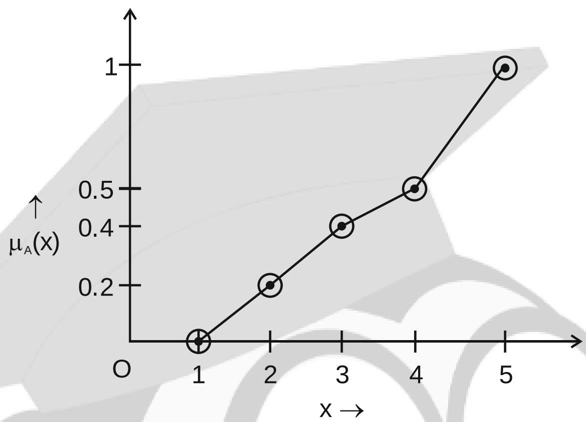
Then,

- Core (A) = {5} as it $\mu_A(5) = 1$

- Crosspoint (A) = {4}
- Support (A) = {2, 3, 4, 5}
- $A_{0.4}$ = {3, 4, 5}
- $A^{+}_{0.4}$ = {4, 5}
- Height of fuzzy set = 1
- Cardinality of fuzzy set

  = 0 + 0.2 + 0.4 + 0.5 + 1

  |A| = 2.1



Fuzzy Set

Example : Let a fuzzy set of temperature being cold is :

$$\text{hot} = \left\{\frac{1}{20} + \frac{0.5}{30} + \frac{0.3}{40} + \frac{0.2}{50}\right\}$$

Find the membership values for :

(a) Temperature being very cold

(b) Temperature being somewhat cold

Solution. Temp being very cold $(\mu_A)^2$

$$\left\{\frac{1}{20} + \frac{0.25}{30} + \frac{0.09}{40} + \frac{0.04}{50}\right\}$$

Temperature being somewhat cold $\sqrt{\mu_A}$

$$\left\{\frac{1}{20} + \frac{0.707}{30} + \frac{0.54}{40} + \frac{0.44}{50}\right\}$$

Example : A fuzzy set A on R is ____________ iff $A(\lambda x_1 + (1 - \lambda) x_2) \geq \min [A(x_1), A(x_2)]$ $\forall$ x_1 x_2 $\in$ R and $\lambda \in [0, 1]$

Solution. If $A(\lambda x_1 + (1 - \lambda) x_2) \geq \min (A(x_1), A(x_2))$ $\forall$ $x_1 \in x_2 \in R$

Then fuzzy set is called convex fuzzy set

Correct option is (C)

Relations

Crisp

Ordered Pairs (A × B) Suppose A and B are two (crisp) sets. The Cartesian product denoted as A × B is a collection of ordered pairs such that,

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Properties

- $A \times B \neq B \times A$ as $A \times B$ provides mapping from A to B.
- Number of elements in $A \times B$ is $|A \times B| = |A| \times |B|$

A relation R is a subset of Cartesian product $A \times B$ i.e.,

$$R \subseteq A \times B$$

Example : Consider two crisp sets A and B

$A = \{1, 2, 3, 4\}$ $B = \{3, 5, 7\}$

Then, $A \times B = \{(1, 3)\ (1, 5)\ (1, 7)\ (2, 3)\ (2, 5)\ (2, 7)\ (3, 3)\ (3, 5)\ (3, 7)\ (4, 3)\ (4, 5)\ (4, 7)\}$

Let Relation R be $R = \{(a, b) \mid b = a + 1 \text{ such that } a \in A, b \in B\}$

Here $R = \{(2, 3)\ (4, 5)\}$

It can be represented as, in matrix form :

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} 3 & 5 & 7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Operation on Crisp Relations

Suppose R(x, y) and S(x, y) are two relation defined over two crisp sets $x \in A$ and $y \in B$.

(a) Union : $R(x, y) \cup S(x, y) = \max(R(x, y), S(x, y))$

(b) Intersection : $R(x, y) \cap S(x, y) = \min(R(x, y), S(x, y))$

(c) Complementation : $R\overline{(x, y)} = 1 - S(x, y)$

Example : Let two relations R and S be :

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So, $$R \cup S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R \cap S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\overline{R} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\bar{S} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(d) Composition of Relations RoS

Given R is a relation on X, Y and S is another relation on Y, Z. Then RoS is defined as,

$RoS = \{(x, y) \mid (x, y) \in R \text{ and } (y, z) \in S \text{ and } \forall y \in y\}$

Composition of relation is mathematically, determined by 'Max–Min composition'

Given two relation matrices R and S, the max-min composition is defined as,

$T = RoS$

$T(x, 2) = \max\{\min \{R(x, y), S(y, z) \text{ and } \forall y \in Y\}\}$

Example : Given X = {1, 3, 5} Y = {1, 3, 5}

$R = \{(x, y) \mid y = x + 2\}$

$S = \{(x, y) \mid x \leq y\}$

Here, R and S is on X × Y

Thus, we have R = {(1, 3) (3, 5)}

S = {(1, 3) (1, 5) (3, 5)}

$$R = \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ 1 \\ 3 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

For T = RoS

- T_{11} max {min (R_{11}, S_{11}) min (R_{13}, R_{31}) min (R_{15}, S_{51})}
  max {min(0, 0) min(1, 0) min (0, 0)}
  max {0, 0, 0} = 0
- T_{12} max {min(0, 1), (1, 0), (0, 0)}
  max {0, 0, 0} = 0
- T_{13} max {min (0, 1), min (1, 1), min (0, 0)}
  max {0, 1, 0} = 1
  Similarly, finding this way

$$T = RoS \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Fuzzy Relations

Fuzzy relation is a fuzzy set defined on the cartesian product of crisp set $X_1, X_2, ..., X_n$. Here, n tuples $(X_1, X_2, ..., X_n)$ may have varying degree of memberships within the relationship. The membership values indicate the strength of the relation between the tuples.

Example : Let X = {typhoid, viral, cold}

Y = {running nose, hightemp, shivering}

be two sets.

The fuzzy relation R is defined as :

$$R = \begin{array}{r} \\ \\ \text{typhoid} \\ \text{viral} \\ \text{cold} \end{array} \begin{array}{c} \begin{array}{ccc} \text{running nose} & \text{high – temp} & \text{Shivering} \end{array} \\ \begin{bmatrix} 0.1 & 0.9 & 0.8 \\ 0.2 & 0.9 & 0.7 \\ 0.9 & 0.4 & 0.6 \end{bmatrix} \end{array}$$

Operations on Fuzzy Relations

(a) **Fuzzy Cartesian Product :** Suppose A and B be two fuzzy set on universe of discourse X with $\mu_A(x) \mid x \in X$ and y with $\mu_B(y) \mid y \in Y$. Then R = A × B Cx × y; where R has its membership function given by.

$$\mu_R(x, y) = \mu_{A\times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$$

Example : Let a fuzzy relations be defined on two fuzzy sets.

$A = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\}$

$B = \{(b_1, 0.5), (b_2, 0.6)\}$

$$R = A \times B = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{bmatrix} \min(0.2,0.5) & \min(0.2,0.6) \\ \min(0.7,0.5) & \min(0.7,0.6) \\ \min(0.4,0.5) & \min(0.4,0.6) \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

(b) **Union :** $\mu_{R\cup S}(a, b) = \max\{\mu_R(a, b), \mu_S(a, b)\}$

(c) **Intersection :** $\mu_{R\cap S}(a, b) = \min\{\mu_R(a, b), \mu_S(a, b)\}$

(d) **Complementation :** $\mu_{\bar{R}}(a, b) = 1 - \mu_R(a, b)$

(e) **Composition :** T = RoS

$$\mu_{RoS} = \max_{y \in Y}\{\min(\mu_R(x, y), \mu_S(y, z)\}$$

Example : Let $X = (x_1, x_2, x_3)$, $Y = (y_1, y_2)$, $Z = (z_1, z_2, z_3)$

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array} \qquad S = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 & 0.7 \\ 0.5 & 0.8 & 0.9 \end{bmatrix} \end{array}$$

For RoS

$$\mu_{RoS}(x_1, y_1) = \max\{\min(\mu_R(x_1, y_1), \mu_S(y_1, z_1)), \min(\mu_R(x_1, y_2), \mu_S(y_2, z_1))\}$$

$= \max\{\min(0.5, 0.6), \min(0.1, 0.5)\}$

$= \max\{0.5, 0.1\} = 0.5$

Similarly,

$$RoS = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.5 \\ 0.5 & 0.8 & 0.9 \\ 0.6 & 0.6 & 0.7 \end{bmatrix} \end{array}$$

Example : If Relation $R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array}$ $S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.5 \\ 0.3 & 0.4 \end{bmatrix} \end{array}$

Then $R \cup S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.2 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array}$

$$R \cap S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.5 \\ 0.3 & 0.4 \end{bmatrix} \end{array}$$

$$\bar{R} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.9 \\ 0.8 & 0.1 \\ 0.2 & 0.4 \end{bmatrix} \end{array}$$

Example : Consider the following two sets P and D, which represent a set of paddy plants and a set of plant diseases. More precisely.

P = $\{P_1, P_2, P_3, P_4\}$ a set of 4 varieties of paddy plants

D = $\{D_1, D_2, D_3, D_4\}$ of the four various diseases affecting the plants.

In addition to these consider another Set S = $\{S_1\ S_2\ S_3\ S_4\}$ be common symptoms of the diseases.

Let R be a relation on P × D, representing which plant is susceptible to which diseases, which is stated as,

$$R = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{bmatrix} 0.6 & 0.6 & 0.9 & 0.8 \\ 0.1 & 0.2 & 0.9 & 0.8 \\ 0.9 & 0.3 & 0.4 & 0.8 \\ 0.9 & 0.8 & 0.4 & 0.2 \end{bmatrix} \end{array}$$

Also, consider T be another relation on D × S,

$$S = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.7 & 0.9 \\ 1.0 & 1.0 & 1.4 & 0.6 \\ 0.0 & 0.0 & 0.5 & 0.9 \\ 0.9 & 1.0 & 0.8 & 0.2 \end{bmatrix} \end{array}$$

Obtain the association of plants with different symptom on disease using min max composition.

Solution. $$\text{RoS} = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array}\begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.7 & 0.9 \end{bmatrix} \end{array}$$

Example : Let X = R^T = y (the positive real line) and

R = X × Y = 'y is much greater than x'

The membership function of $\mu_R(x, y)$ is defined as :

$$\mu_R(x,y) = \begin{cases} \dfrac{y-x}{4} & \text{if } y > x \\ 0 & \text{if } y \le x \end{cases}$$

Suppose X = {3, 4, 5} and Y = {3, 4, 5, 6, 7}

Find the μ_R

Solution. X = {3, 4, 5} Y = {3, 4, 5, 6, 7}

$$R = \begin{array}{c} \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 0.25 & 0.5 & 0.75 & 1.0 \\ 0 & 0 & 0.25 & 0.5 & 0.75 \\ 0 & 0 & 0 & 0.25 & 0.5 \end{bmatrix} \end{array}$$

Fuzzy Rule Based System

Fuzzy Rules

A fuzzy implication (also known as fuzzy if then rule, fuzzy rule) assumes that form :

If x is A then y is B

where A and B are two linguistic variables defined by fuzzy sets A and B on universe of discourse X and Y.

- x is A called antecedent or premise
- y is B called consequence or conclusion

Example :

If pressure is high then temperature is low.

The fuzzy implication is denoted as R : A → B. It represents a binary fuzzy relation R on the cartesian product of A × B.

Suppose P and T are two universe of discourse representing pressure and temperature respectively as follows :

P = {1, 2, 3, 4} and T = {10, 15, 20, 25, 30, 35, 40, 45, 50}

Let linguistic variables high temperature and low pressure are given as :

T_{High} = {(20, 0.2) (25, 0.4) (30, 0.6) (35, 0.6) (40, 0.7) (45, 0.8) (50, 0.8)}

P_{Low} = {(1, 0.8) (2, 0.8) (3, 0.6) (4, 0.4)}

Then fuzzy implication if temperature is high then pressure is low can be defined as :

R : $T_{High} \rightarrow P_{Low}$ is given cartesian product T × P

$$R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 20 & 0.2 & 0.2 & 0.2 & 0.2 \\ 25 & 0.4 & 0.4 & 0.4 & 0.4 \\ 30 & 0.6 & 0.6 & 0.6 & 0.4 \\ 35 & 0.6 & 0.6 & 0.6 & 0.4 \\ 40 & 0.7 & 0.7 & 0.6 & 0.4 \\ 45 & 0.8 & 0.8 & 0.6 & 0.4 \\ 50 & 0.8 & 0.8 & 0.6 & 0.4 \end{array}$$

So, if temperature is 40 what about low pressure?

$$\text{Temperature } 40 \rightarrow \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \end{array}$$
$$\text{Temperature } 40 \rightarrow [0.7 \quad 0.7 \quad 0.6 \quad 0.4]$$

i.e., {(1, 0.7) (2, 0.7) (3, 0.6) (4, 0.4)}

In generate there are two ways to compute $A \rightarrow B$ fuzzy rule,

(a) A coupled with B

(b) A entails B

The result may differ

1. Interpretation as A coupled with B

$$R : A \rightarrow B = A \times B = \int_{X \times Y} \mu_A(x) * \mu_B(y) / (x, y);$$

where * is called a T-norm operator

Most commonly used T-norm operator

- Minimum : T_{min} (a, b) = min(a, b) = $a \wedge b$
- Algebraic product : T_{ap} (a, b) = ab
- Bounded product : T_{bp} (a, b) = $O \vee (a + b - 1)$
- Drastic product : $T_{dp} = \begin{cases} a & \text{if } b = 1 \\ b & \text{if } a = 1 \\ 0 & \text{if } a, b < 1 \end{cases}$

Here $a = \mu_A(x)$

$b = \mu_B(y)$

Based on the T-norm operator, the fuzzy rule $R : A \rightarrow B$ is defined with two-dimensional membership function.

Few implication of R : A → B

(a) Min operator : This is called mamdani Rule

$$R_m = A \times B = \int_{X \times Y} \mu_A(x) \wedge \mu_B(y) | (x, y) \text{ or } f_{min}(a, b) = a \wedge b$$

(b) Algebraic product operator : This is called Larsen Rule

$$R_{ap} = A \times B = \int_{X \times Y} \mu_A(x) \,.\, \mu_B(y) | (x, y) \text{ or } f_{ap}(a, b) = ab$$

(c) Bounded product operator :

$$R_{bp} = A \times B = \int_{X\times Y} \mu_A(x) \odot \mu_B(y) \,|\, (x,y)$$

$$= \int 0 \vee (\mu_A(x) + \mu_B(y) - 1) \,|\, (x,y)$$

or f_{bp} (a, b) = $\underline{0 \vee (a + b - 1)}$

(d) Drastic product operator :

$$R_{dp} = A \times B = \int_{X\times Y} \mu_A(x) \,\hat{\bullet}\, \mu_B(y) \,|\, (x,y)$$

or $$f_{dp}(a,b) = \begin{cases} a & \text{if } b = 1 \\ b & \text{if } a = 1 \\ 0 & \text{if otherwise} \end{cases}$$

2. Interpretation of A Entails B

There are three main ways to interpret this :

(a) Material Implication

$$R : A \to B = \bar{A} \cup B$$

(b) Propositional Calculus

$$R : A \to B = \bar{A} \cup (A \cup B)$$

(c) Extended Propositional Calculus

$$R : A \to B = (\bar{A} \cap \bar{B}) \cup B$$

Note : Rule derived may be different

Implication of Above Rules

1. Zadeh's Arithmetic Rule : (Material Implication)

$$R_{Za} = \bar{A} \cup B = \int_{X\times Y} 1 \wedge (1 - \mu_A(x) + \mu_B(y)) \,|\, (x,y)$$

or

$$\boxed{f_{za}(a,b) = 1 \wedge (1 - a + b)}$$

2. Zadeh's max-min rule (Propositional Calculus)

$$R_{mn} = \bar{A} \cup (A \cap B) = \int_{X\times Y} (1 - \mu_A(x)) \vee (\mu_A(x) \wedge \mu_B(y)) \,|\, (x,y)$$

or

$$\boxed{f_{mm}(a,b) = (1 - a) \vee (a \wedge b)}$$

3. Boolean Fuzzy Rule

$$R_{bf} = \bar{A} \cup B = \int_{X\times Y} (1 - \mu_A(x)) \vee \mu_B(x) \,|\, (x,y)$$

or

$$\boxed{f_{bf}(a,b) = (1 - a) \vee b}$$

4. Gognen's Fuzzy Rule

$$R_{gf} = \int_{X \times Y} \mu_B(x) * \mu_B(y) | (x,y)$$

$$\text{where } a * b = \begin{cases} 1 & \text{if } a \le b \\ \dfrac{b}{a} & \text{if } a > b \end{cases}$$

Example : Zadeh's max-min rule

If x is a then y is B, then Zadeh's max-min rule can be written equivalently as,

$R_{mm} = (A \times B) \cup (\bar{A} \times Y)$ {y is used so that union can be applied}

Here y is the universe of discourse with membership values for all y $\in$ Y is 1, that is, $\mu_Y(y) = 1 \ \forall \ y \in Y$;

Suppose X = {a, b, c, d} Y = {1, 2, 3, 4}

A = {(a, 0.0) (a, 0.8) (c, 0.6) (d, 1.0)}

B = {(1, 0.2) (2, 1.0) (3, 0.8) (4, 0.0)} are two fuzzy sets.

Then $R_{mm} = (A \times B) \cup (\bar{A} \times Y)$

$$A \times B = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\bar{A} \times Y = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Here Y = {(1, 1) (2, 1) (3, 1) (4, 1)}

$$R_{mm} = (A \times B) \cup (\bar{A} \times Y)$$

$$R : A \to B = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.4 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

Example : If X is A then y is B else y is C is equivalent to

$$R = (A \times B) \cup (\bar{A} \times C)$$

Let X = {a, b, c, d} Y = {1, 2, 3, 4}

A = {(a, 0.0), (b, 0.8), (c, 0.6), (d, 1.0)}

B = {(1, 0.2), (2, 1.0), (3, 0.8), (4, 0.0)}

C = {(1, 0.0), (2, 0.4), (3, 1.0), (4, 0.8)}

Then the relation :

If x is A then y is B else y is C.

$$A \times B = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.2 & 0.8 & 0.8 & 0 \\ 0.2 & 0.6 & 0.6 & 0 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

$$\bar{A} \times C = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1.0 & 0.8 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.4 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Then

$$R = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 1.0 & 0.8 \\ 0.2 & 0.8 & 0.8 & 0.2 \\ 0.2 & 0.6 & 0.6 & 0.4 \\ 0.2 & 1.0 & 0.8 & 0 \end{bmatrix} \end{array}$$

Example : If we have a fuzzy rule

"If temperature is high then humidity is fairly high"

T – Universe of discoverse of temperature

H – Universe of discoverse of humidity.

High is denoted as $A \subseteq T$ fairly high is denoted as $B \subseteq H$.

Then rule is,

If t is A then h is B i.e. R(t, h) : R(t) $\rightarrow$ R(h)

Then values of A and B are given as,

t	20	30	40
$\mu_A(t)$	0.1	0.5	0.9

h	20	50	70	90
$\mu_B(x)$	0.2	0.5	0.7	1

Use memdani rule to find the rule $R_C(t, h)$

Solution. According to mamdani rule,

$$R_C(t, h) = A \times B = \int \mu_A(t) \wedge \mu_B(t)(t, h)$$

t \ n	20	50	70	90
20	0.1	0.1	0.1	0.1
30	0.2	0.5	0.5	0.5
40	0.2	0.5	0.7	0.9

Example : Consider the proposition below :

If it is north of wisconsin, then it is cold John lives is the FAR north of wisconsin

∴ It is very cold.

The fuzzy sets are :

$$\text{North} = \frac{0.1}{\text{Madisson}} + \frac{0.5}{\text{Dells}} + \frac{0.7}{\text{Greenbay}} + \frac{1}{\text{Superior}}$$

$$\text{Cold} = \frac{1}{20} + \frac{0.9}{35} + \frac{0.4}{50} + \frac{0.2}{65}$$

If north (A) then cold (B). Find the rule matrix using Zaden's arithmetic rule.

Solution.

North / Cold	1	0.9	0.4	0.2
0.1	1	1	1	1
0.5	1	1	0.9	0.7
0.7	1	1	0.7	0.5
1	1	0.9	0.4	0.2

The matrix is formed using,

$\mu_{north}\ \phi_{cold}$ (u, v) = 1 $\wedge$ {1 – north (u) + cold (v)}

Fuzzy Inferences

This technique is used to find more fuzzy rules from a given set of fuzzy rules

We know in proportional logic,

1. Modus Ponens

$$\begin{array}{l} p \\ \underline{p \to q} \\ \underline{\therefore \quad q} \end{array}$$

2. Modus Tollens

$$\begin{array}{r} p \to q \\ \underline{\neg q} \\ \underline{\therefore \ \neg q} \end{array}$$

3. Chain Rule

$$\begin{array}{r} p \to q \\ \underline{q \to r} \\ \underline{\therefore \ p \to r} \end{array}$$

Similar concept is followed in fuzzy logic to infer a fuzzy rule from a given set of fuzzy rules

Inferring Procedures in Fuzzy Logic

(a) Generalized Modus Ponens (GMP)

If x is A then y is B

x is A'

∴ y is B'

(b) Generalized Modus Tollens (GMT)

If x is A then y is B

y is B'

∴ x is A'

Procedure

Here A, B, A', B' are fuzzy sets.

To compute the membership function A' and B' the max-min composition of fuzzy sets B' and A', respectively with R(x, y) [known as implication relation] is used.

Thus,

- B' = A'oR(x, y)

  $\mu_B(y) = \max[\min(\mu_A'(x), \mu_R(x, y)]$

- A' = B'oR(x, y)

  $\mu_A(x) = \max[\min(\mu_B'(y), \mu_R(x, y)]$

Here, O is composition operation.

Example : (GMP)

P : If x is A then y is B.

Let us consider two sets of variables x and y be

$X = \{x_1 \ \ x_2 \ \ x_3\} \quad Y = \{y_1 \ \ y_2\}$

Let us consider :

$A = \{(x, 0.5)\ (x_2, 1)\ (x_3, 0.6)\}$; $A' = \{(x, 0.6)\ (x_2, 0.9)\ (x_3, 0.7)\}$

$B = \{(y_1, 1)\ (y_2, 0.4)\}$

GMP Says that :

If x is A then y is B $\Rightarrow$ R : A $\to$ B

$$\frac{x \text{ is } A'}{y \text{ is } B'} \quad \Rightarrow \quad A' = \{(x_2, 0.6)(x_2, 0.9)(x_3, 0.7)\} \text{ (given)}$$

Solution. B' = A'oR(x, y)

$$R(x,y) = \max\{A \times B,\ \bar{A} \times Y\}$$

$$A \times B = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.5 & 0.4 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix} \end{array} \qquad \bar{A} \times Y = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

So, $R(x,y) = (A \times B) \cup (\bar{A} \times Y)$

$$= \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

$A' = \{(x_1\ 0.6), (x_2\ 0.9), (x_3\ 0.7)\}$

$$B' = [0.6 \quad 0.9 \quad 0.7] \bullet \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}$$

$$= [0.9 \quad 0.5]$$

So, y is B' where B' = {(y, 0.9), (y_2 0.5)}

Example : GMT

If x is A then y is B given where,

A = {(x_1, 0.5) (x_2, 1) (x_3, 0.6)}

B = {(y_1, 1) (y_2, 0.4)}

y is B' is given

B' = {(y_1, 0.9) (y_2, 0.7)}

From above conclude x is A'

Solution.

1. $R(x, y) = (A \times B) \cup (\bar{A} \times Y)$

$$R(x,y) = \begin{array}{c c} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix} \end{array}$$

2. A' = B'oR(x, y)

$$A' = [0.9 \; 0.7] \cdot \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}$$

$$= [0.5 \quad 0.9 \quad 0.6]$$

3. x is A'

A' = {(x_1, 0.5) (x_2, 0.9) (x_3, 0.6)}

Example : Consider two premise :

1. If temperature is high then rotation is slow
2. Temperature is very high

Deduce Rotation is quite slow. Where,

X = {30, 40, 50, 60, 70, 80, 90, 100} be set of temperatures.

Y = {10, 20, 30, 40, 50, 60} be set of rotations per minute.

The fuzzy set high (H), very high (VH), slow (S), quite slow (QS) are given below :

H = {(70, 1), (80, 1), (90, 0.3)}

VH = {(80, 0.6), (90, 0.9), (100, 1)}

S = {(30, 0.8), (40, 1.0), (50, 0.6)}

QS = {(10, 1), (20, 0.8), (30, 0.5)}

Solution. Here, generalized modus ponens (GMP) need to be applied.

1. If temperature is high then rotation is slow

$$R = (H \times S) \cup (\bar{H} \times Y)$$

2. Temperature is very high, rotation is quite slow

$$\boxed{QS = VH \text{ o } R(x, y)}$$

$$H \times S = \begin{bmatrix} 0.8 & 1.0 & 0.3 \\ 0.8 & 1.0 & 0.6 \\ 0.3 & 0.3 & 0.3 \end{bmatrix}$$

$$\bar{H} \times Y = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.7 & 0.7 & 0.7 \end{bmatrix}$$

$$\bar{R} = \begin{bmatrix} 0.8 & 1.0 & 0.3 \\ 0.8 & 1.0 & 0.6 \\ 0.7 & 0.7 & 0.7 \end{bmatrix}$$

$$QS = \begin{bmatrix} 0.6 & 0.9 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 1.0 & 0.3 \\ 0.8 & 1.0 & 0.6 \\ 0.7 & 0.7 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.8 & 0.9 & 0.7 \end{bmatrix}$$

So, QS = {(10, 0.8), (20, 0.9), (30, 0.7)}

Example : Let set of values of variables x and y be x = $\{x_1 \quad x_2 \quad x_3\}$ y = $\{y_1 \quad y_2\}$

Assume that a proposition 'if x is A, then y is B' is given where $A = \frac{5}{x_1} + \frac{1}{x_2} + \frac{6}{x_3}$ and $B = \frac{1}{y_1} + \frac{0.4}{y_2}$.

The fact that 'x is A' is given where $A' = \frac{0.6}{x_1} + \frac{0.9}{x_2} + \frac{0.7}{x_3}$. What is the conclusion 'y is B'.

Solution. We will use generalized modus ponens

$$R = \frac{1}{x_1}, y + 0.9x_1, y_2 + \frac{1}{x_2} + \frac{4}{x_2}, y_2 + \frac{1}{x_3}, y_1 + \frac{0.8}{x_3}, y_2$$

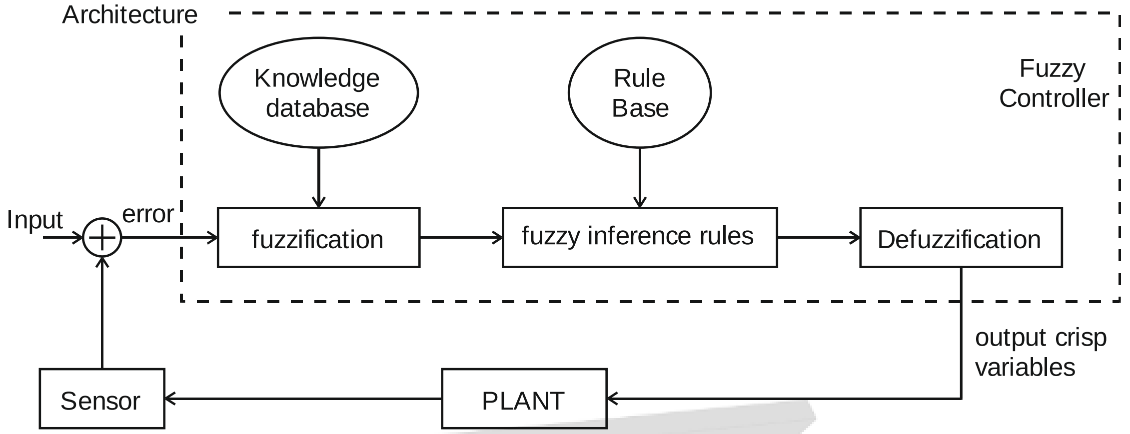
$B'(y_1)$ = max(min(0.6, 1), min(0.9, 1), min(0.7, 1)] = 0.9

$B'(y_2)$ = max(min(0.6, 0.9), min(0.9, 0.4), min(0.7, 0.8)] = 0.7

So, y is B' = $\frac{0.9}{y_1} + \frac{0.7}{y_2}$

Fuzzy Control System

Fuzzy logic is applied in various control application. Example include controlling your room temperature, anti braking system used in vehicles, control on traffic lights etc.



Major Components of FLC

(a) **Fuzzifier :** The role of fuzzifier is to convert crisp input values of fuzzy values.

(b) **Fuzzy Knowledge Base :** It stores knowledge about all the input and output fuzzy relationships. It also has membership functions.

(c) **Fuzzy Rule Base :** It stores the knowledge about operation of the process of domain.

(d) **Inference Engine :** It acts as kernel of any FLC. It stimulates human decisions by performing approximate reasoning.

(e) **Defuzzifier :** The role is to convert fuzzy value into crisp values.

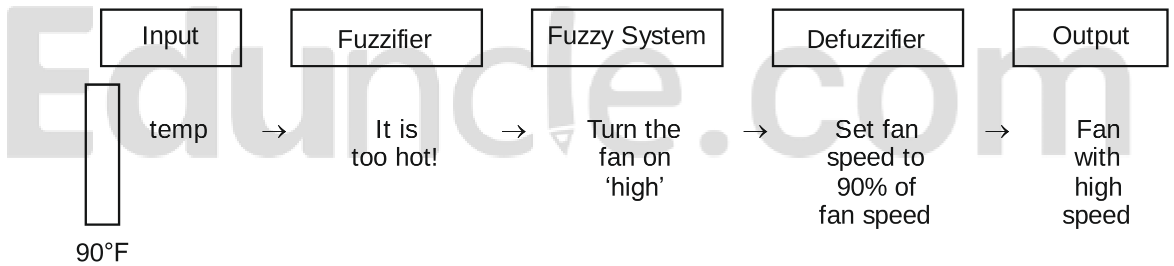
Advantage of Fuzzy Control System

1. Cheaper
2. Robust
3. Customizable
4. Emulate human dedutive thinking
5. Reliability
6. Efficiency

Disadvantage

1. Requires a lot of data
2. Useful in case of moderate historical data
3. Needs human expertise
4. Needs regular updating of rules.

Fuzzy Control System Example : Consider an example of fuzzy controlled motor based fan.



So, to build a fuzzy controller

1. Pick the linguistic variables

   Example : Let temperature (x) be input, motor speed (y) be output

2. Pick the fuzzy sets

   Define fuzzy subsets of the x and y

3. Pick the fuzzy rule

   Associate output to input

So, for the given example of fuzzy controlled motor fan

Step 1 : Assign input and output variables.

X be temp in Fahrenheit

Y be motor speed of fan

Step 2 : Pick fuzzy sets

Define linguistic terms

On X : 5 linguistic terms : – [Temperature]

Cold, Cool, Just Right, Warm, Hot

On Y [Motor Speed] : 5 linguistic terms :

Stop, Slow, Medium, Fast, Blast

Step 3 : Assign input to output

Eg., If temperature is cold then motor speed is stop

It temperature is cool then motor speed is slow

So, if input is temperature $\Rightarrow$ 65°F

Then temperature is just right motor speed is medium, Defuzzify the motor speed of medium and it will be converted to 30% speed.

Computer Science and Applications

Model Solved Paper

Time : 2hrs. **Maximum Marks : 200**

Note : This paper contains hundred (100) objective type questions, each question carries two (2) marks. Attempt all the questions.

1. Let R = $\{r_0, r_{60}, r_{120}, r_{180}, r_{240}, r_{300}\}$.
where r_θ denotes rotation on a plane by θ degrees
Let □ be an operation defined as,

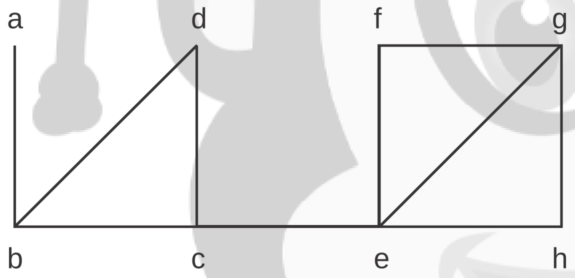
$$r_{\theta_1} \square r_{\theta_2} = r_{\theta_1+\theta_2}$$

Then (R, □) is –

(A) Abelian group (B) A monoid but not group

(C) A semigroup but not a monoid (D) None

2. For the graph given below -

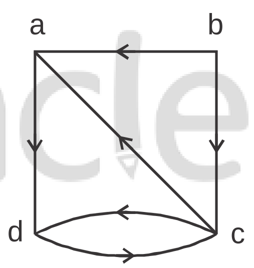


The number of cut vertices and cut edges are —

(A) 2, 2 (B) 3, 2

(C) 2, 3 (D) 3, 3

3. The transitive closure of the graph —



(A) (a,a) (a,c) (c,a) (c,c) (b,a) (b,d) (d,d)

(B) (a,a) (a,c) (a,d) (b,a) (b,c) (b,d) (c,a) (c,c) (c,d) (d,a) (d,c) (d,d)

(C) (a,a) (a,c) (a,d) (b,a) (b,c) (b,d) (c,a)

(D) (a,a) (a,c) (a,d) (b,a) (b,c) (b,d) (c,a) (c,c) (c,d) (d,a)

4. Let p, q and r be propositions —

$(p \rightarrow q) \rightarrow r$ be contradiction. Then the expression $(r \rightarrow p) \rightarrow q$ is —

(A) tautology
(B) Always true when p is false
(C) contradiction
(D) Always true when q is true

5. Consider the logic sentence —

$F : \forall x\ (\exists y\ R(x, y))$

which of the following sentence below are implied by F?

I. $\exists y\ (\exists x\ R(x, y))$
II. $\exists y\ (\forall x\ R(x, y))$
III. $\forall y\ (\exists x\ R(x, y))$
IV. $\neg\ \exists x\ (\forall y\ \neg R(x, y))$

(A) IV only
(B) I and IV only
(C) II only
(D) II and III only

6. If you make a mistake in choosing the pivot row in the simplex method, the solution in the next tableau.

(A) will be nonbasic
(B) will be infeasible
(C) will have a worse objective value
(D) None of these

7. Solve the following assignment problem shown in Table using Hungarian method. The matrix entries are processing time of each man in hours.

$$\begin{array}{cc} & \begin{array}{ccccc} & & & \text{men} & \\ 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} & \begin{bmatrix} 20 & 15 & 18 & 20 & 25 \\ 18 & 20 & 12 & 14 & 15 \\ 21 & 23 & 25 & 27 & 25 \\ 17 & 18 & 21 & 23 & 20 \\ 18 & 18 & 16 & 19 & 20 \end{bmatrix} \end{array}$$

(A) 85
(B) 86
(C) 87
(D) 88

8. Which method is/are used for the solution of assignment problem :-

(A) Hungarian Method
(B) Simplex Method
(C) Transportation Method
(D) All of the above

9. Consider the following conditions :-

(a) The solution must be feasible
(b) The number of positive allocations must be equal to $m + n - 1$
(c) All the positive allocations must be in independent positions.

(A) (a) and (b) only
(B) (a) and (c) only
(C) (b) and (c) only
(D) (a), (b) and (c)

10. Consider the following transportation problem:

Factories	Stores				
	I	II	III	IV	Supply
A	4	6	8	13	50
B	13	11	10	8	70
C	14	4	10	13	30
D	9	11	13	8	50
Demand	25	35	105	20	

The transportation cost in the initial basic feasible solution of the above transportation problem using VAM is

(A) 1460 (B) 1480

(C) 1465 (D) 1525

11. The two numbers given below are multiplied using the Booth's algorithm.

Multiplicand : 0101 1010 1110 1110

Multiplier : 0111 0111 1011 1101

How many additions/Subtractions are required for the multiplication of the above two numbers ?

(A) 6 (B) 8

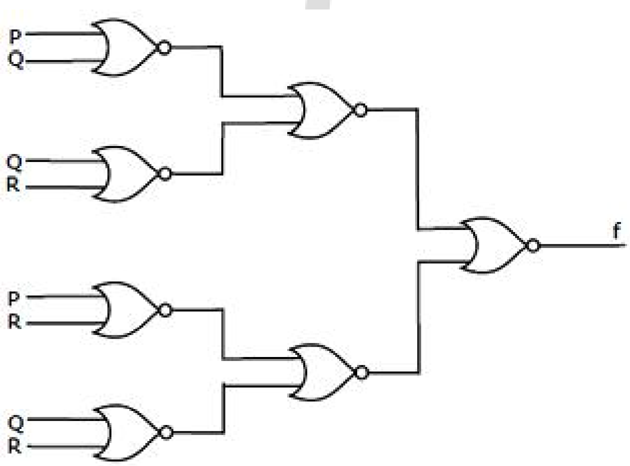
(C) 10 (D) 12

12. A positive edge-triggered D flip-flop is connected to a positive edge-triggered JK flipflop as follows. The Q output of the D flip-flop is connected to both the J and K inputs of the JK flip-flop, while the Q output of the JK flip-flop is connected to the input of the D flip-flop. Initially, the output of the D flip-flop is set to logic one and the output of the JK flip-flop is cleared. Which one of the following is the bit sequence (including the initial state) generated at the Q output of the JK flip-flop when the flip-flops are connected to a free-running common clock? Assume that J = K = 1 is the toggle mode and J = K = 0 is the state-holding mode of the JK flip-flop. Both the flip-flops have nonzero propagation delays.

(A) 0110110… (B) 0100100…

(C) 011101110… (D) 011001100...

13. What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below ?



(A) $\overline{Q+R}$ (B) $\overline{P+Q}$

(C) $\overline{P+R}$ (D) $\overline{P+Q+R}$

14. In a look-ahead carry generator, the carry generate function G_i and the carry propagate function P_i for inputs A_i and B_i are given by :

$P_i = A_i \oplus B_i$

and $G_i = A_iB_i$

The expressions for the sum bit S_i and the carry bit C_{i+1} of the look-ahead carry adder are given by :

$S_i = P_i \oplus C_i$

and $C_{i+1} = G_i + P_iC_i$

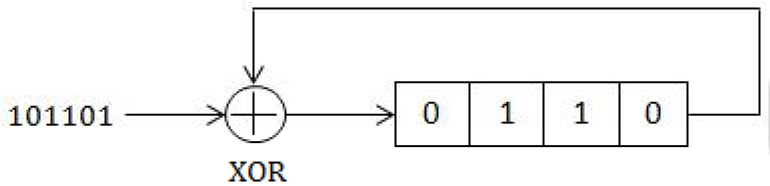
where C_0 is the input carry.

Consider a two-level logic implementation of the look-ahead carry generator. Assume that all P_i and G_i are available for the carry generator circuit and that the AND and OR gates can have any number of inputs. The number of AND gates and OR gates needed to implement the look-ahead carry generator for a 4-bit adder with S_3, S_2, S_1, S_0 and C_4 as its outputs are respectively :

(A) 6, 3 (B) 10, 4

(C) 6, 4 (D) 10, 5

15. What is the final value stored in the linear feedback shift register if the input is 101101 ?



(A) 0110 (B) 1011

(C) 1101 (D) 1111

16. An instruction set of a processor has 125 signals which can be divided into 5 groups of mutually exclusive signals as follows: Group 1:20 signals, Group 2:70 signals, Group 3:2 signals, Group 4:10 signals, Group 5:23 signals. How many bits of the control words can be saved by using vertical micro programming over horizontal microprogramming?

(A) 0 (B) 103

(C) 22 (D) 55

17. An instruction pipeline consist of 4 stages – Fetch (F), Decode field (D), Execute (E) and Result Write (W). The five instructions in a certain instruction sequence need these stages for the different number of clock cycles as shown by the table below.

No of cycles needed for

Instruction	F	D	E	W
1	1	2	1	1
2	1	2	2	1
3	2	1	3	2
4	1	3	2	1
5	1	2	1	2

Find the number of clock cycles needed to perform the 5 instructions.

(A) 12 (B) 14

(C) 15 (D) 10

18. An instruction requires four stages to execute : stage 1 (instruction fetch) requires 35 ns, stage 2 (instruction decode) = 10 ns, stage 3 (instruction execute) = 15 ns and stage 4 (store results) = 15 ns. An instruction must proceed through the stages in sequence. What is the minimum asynchronous time for any single instruction to complete?

(A) 70 (B) 68

(C) 73 (D) 75

19. Consider a machine with a byte addressable main memory of 2^{20} bytes, block size of 16 bytes and a direct mapped Cache having 2^{12} cache lines. Let the addresses of two consecutive bytes in main memory be $(E201F)_{16}$ and $(E201F)_{20}$. What are the tag and cache line address (in hex) for main memory address $(E201F)_{16}$?

(A) E, 201 (B) F, 201

(C) E, E20 (D) 2, 01F

20. Consider a machine with a byte addressable main memory of 232 bytes divided into block of size 16 bytes. Assume that a direct mapped cache having 1024 cache lines is used with the same machine. The size of the tag field In bits is ______.

(A) 4 (B) 10

(C) 18 (D) 16

21. The following program fragment

```
for (i = 1; i<5; ++i)
if (i == 3) continue;
else printf ("%d", i);
```

result in the printing of

(A) 1 2 4 5 (B) 1 2 4

(C) 2 4 5 (D) None of the above

22. The following program fragment

```
int k = –7;
printf ("%d", 0 < !k);
```

(A) prints 0 (B) prints a non-zero value

(C) is illegal (D) prints an unpredictable value

23. Reusability is a desirable feature of a language as it

(A) decrease the testing time

(B) lowers the maintenance cost

(C) reduces the compilation time

(D) Both (A) and (B)

24. What is the output of this program?

```
#include <iostream.h>
using namespace std;
int main()
{
    int a = 5, b = 10, c = 15;
    int * arr[ ] = { &a, &b, &c };
    cout << arr[1];
    return 0;
}
```

(A) 5 (B) 10

(C) 15 (D) It will return some random number.

25.

```
#include <iostream.h>
int main()
{
    try
        {
          try
            {
             throw 20;
            }
             catch (int n)
            {
              cout << "Inner catch n ";
              throw;
         }
       }
    Catch (int x)
       {
            cout <<" Outer Catch n";
       }
    return 0;
     }
```

(A) Outer Catch (B) Inner Catch

(C) Inner Catch, Outer Catch (D) Compile Error

26. All attribute declarations begin with the keyword ____________ followed by the element name, attribute name, attribute type, and default data information.

(A) XML (B) SGML

(C) ATTLIST (D) HTML

27. Which entity is not for punctuation character?

(A) &quot (B) &para

(C) &not (D) &acute

28. Consider the following mark-up and answer the question that follow.

<source src="html_5.mp4" type="video/mp4">

<source src="html_5.ogv" type="video/ogg">

What is the need to add multiple file formats for the same file?

(A) To provide fallback support

(B) To address the media support problem

(C) To provide fallback support & address the media support problem

(D) To provide foul back support

29. Consider the markup and answer the question that follow.

<? controls autobuffer autoplay>

</?>

Which HTML5 element can replace the "?" ?

(A) <audio> (B) <video>

(C) <media> (D) both <audio> & <video>

30. In which part of the HTML metadata is contained?

(A) body tag (B) html tag

(C) head tag (D) title tag

31. Give the principal vanishing point along with their direction for the standard perspective transformation.

(A) only one in the direction of k.

(B) Two in the direction of i and j

(C) Three in the direction I and j and k

(D) only in the direction j and k

32. The procedure that increases the number of intensity levels for each pixel to total number of sub pixels is

(A) Area-sampling (B) Anti-aliasing

(C) Super-sampling procedure (D) None

33. Which device is used to input two-dimensional coordinates by activating a hand cursor on a flat surface?

(A) Graphic tablet (B) Data tablet

(C) Only (B) (D) Both (A) and (B)

34. What is the use of voice system?

(A) To initiate graphics operation (B) To enter data

(C) Neither (A) nor (B) (D) Both (A) and (B)

35. Which is the best line algorithm to balance the processing load among the processors?

(A) Parallel line algorithm
(B) DDA line algorithm
(C) Bresenham's line algorithm
(D) Position Bresenham's line algorithm

36. Highest Normal form achieved by a R(ABCDEF) will FD :

$\{A \rightarrow CD,\ B \rightarrow E,\ E \rightarrow F\}$ is :

(A) 1NF (B) 2NF
(C) 3NF (D) BCNF

37. R(ABCDEF) be a relation with set of FD's = F = $\{A \rightarrow B\ C \rightarrow D\ E \rightarrow F\}$. Find 2NF decomposition for R

(A) (ABC) (DE) (ACF) (B) (ACE) (BD) (ACF)
(C) (AB) (CD) (EF) (ACE) (D) (AB) (BC) (CDEF)

38. Consider the schedule :

$R_1(x)\ R_1(y)\ W_2(x)\ R_2(z)\ W_4(y)\ R_4(y)\ R_3(x)\ W_3(z)\ W_2(z)\ W_1(y)$

The above schedule :

(A) Conflict Serializable
(B) View Serializable with order <T1, T2, T4, T3> but not conflict
(C) View Serializable with order <T1, T2 T3, T4>
(D) Neither view nor conflict serializable

39. Calculate order of internal and leaf node for a B^+ tree with block size 512 B, Block pointer 7B, Record pointer 8B and key field 10B.

(A) 30, 29 (B) 31, 30
(C) 29, 28 (D) 32, 31

40. Adequate normal form is :

(A) First normal form (B) Second normal form
(C) Third normal form (D) Fourth normal form

41. Data transformation includes which of the following ?

(A) A process to change data from a detailed level to a summary level
(B) A process to change data from a summary level to a detailed level
(C) Joining data from one source into various sources of data
(D) Separating data from one source into various sources of data

42. _______ is a process of taking operational data from one or more sources and mapping it, field by field, onto a new data structure in the data warehouse

(A) Transformation (B) Cleansing.
(C) Integration (D) Scrubbing

43. Some of the columns of a relation are at different sites is which one of the following ?

(A) Data Replication (B) Horizontal Partitioning

(C) Vertical Partitioning (D) Horizontal and Vertical Partitioning

44. What do you mean by a hard margin ?

(A) The SVM allows very low error in classification

(B) The SVM allows high amount of error in classification

(C) None of the above

(D) All the above

45. The minimum time complexity for training an SVM is O(n2). According to this fact, what sizes of datasets are not best suited for SVM's ?

(A) Large datasets (B) Small datasets

(C) Medium sized datasets (D) Size does not matter

46. The best way to compare the different implementations of symbol table is to compare the time required to

(A) Add a new name

(B) Make an enquiry

(C) Add a new name and make an enquiry

(D) All of the mentioned

47. Which compiling option can be used for finding which part of the program is consuming most of the processing time?

(A) -f (B) -g

(C) -p (D) -c

48. Developing software ___________ means a major commitment of time, money and resources.

(A) In-house (B) In-sync

(C) On-date (D) On-duration

49. A type of middleware that allows for between the built-in applications and the real-time OS?

(A) Firmware (B) Database middleware

(C) Portals (D) Embedded Middleware

50. The instructions that tell the assembler what to do are.

(A) Executable instructions (B) Pseudo-ops

(C) Logical instructions (D) Macros

51. Which one of the following option is false ?

(A) An executing instance of a program is called a process while a thread is a subset of process

(B) Threads have considerable overhead while process have no overhead

(C) Execution of processes are independent while execution of threads are dependent

(D) New process require duplication of parent process while new threads are easily created

52. At a particular time, the value of counting semaphore is 10, it will become 7 after :

(a) 3V operations

(b) 3P operations

(c) 5V operations and 2P operations

(d) 2V operations and 5P operations

Which of the following option is correct?

(A) Only (b) (B) Only (d)

(C) Both (b) and (d) (D) None of these

53. Consider a set of 5 processes whose arrival time, CPU time needed and priority are given below.

Process	Arrival Time	CPU Time	Priority
P1	0	10	5
P2	0	5	2
P3	2	3	1
P4	5	20	4
P5	10	2	3

Smaller the number, higher the priority.

If the CPU scheduling algorithm is priority scheduling without preemption, the average waiting time will be –

(A) 12.8 ms (B) 11.8 ms

(C) 10.8 ms (D) 9.8 ms

54. Determine the number of page fault when references to pages occur in order –1, 2, 4, 5, 2, 1, 2, 4. Assume that main memory can accommodate 3 pages and the main memory already has page 1 and 2, with page 1 having thought earlier than page 2. Assume LRU is used.

(A) 3 (B) 4

(C) 5 (D) None of these

55. In a system there are 3 resources E, F and G. Four processes P0, P1, P2 and P3 execute concurrently,. Consider the state of system with allocation and Max matrix below, and in which 3 instances of E and 3 instances of F are only resource available.

Allocation

	E	F	G
P0	1	0	1
P1	1	1	2
P2	1	0	3
P3	2	0	0

Max

	E	F	G
P0	4	3	1
P1	2	1	4
P2	1	3	3
P3	5	4	1

From the perspective of deadlock avoidance, which is true ?

(A) The system is in safe state

(B) System is not safe, but would be safe it one more instance of E were available.

(C) System is not safe, but would be safe if one more instance of F were available

(D) System is not safe, but would be safe it one more instance of G were available

56. A company needs to develop a strategy for software product development for which it has a choice of two programming languages L1 and L2. The number of lines of code (LOC) developed using L2 is estimated to be twice the LOC developed with L1. The product will have to be maintained for five years. Various parameters for the company are given in the table below.

Parameter	Language L 1	Language L 2
Man years needed for Development	LOC/10000	LOC/10000
Development cost per man Year	Rs. 10,00,000	Rs. 7,50,000
Maintenances time	5year	5 year
Cost of maintenance per year	Rs. 10,00,000	Rs. 50,000

Total cost of the project includes cost of development and maintenance. What is the LOC for L1 for which the cost of the project using L1 is equal to the cost of the project using L2?

(A) 4000 (B) 5000

(C) 4333 (D) 4667

57. Which of the following requirement specifications can be validated?

(S1) If the system fails during any operation, there should not be any loss of data

(S2) The system must provide reasonable performance even under maximum load conditions

(S3) The software executable must be deployable under MS Windows 95, 2000 and XP

(S4) User interface windows must fit on a standard monitor's screen

(A) S4 and S3 (B) S4 and S2

(C) S3 and S1 (D) S2 and S1

58. Activities which ensure that the software that has been built, is traceable to customer requirement is covered as part of

(A) Verification (B) Validation

(C) Maintenance (D) Modeling

59. Choose the correct option according to given below statement.

Statement 1 : Umbrella activities are independent of any one framework activity and occur throughout the process.

Statement 2 : Software quality assurance, software configuration management are umbrella activity.

Statement 3 : Software quality assurance, software configuration management are not umbrella activity.

(A) Only statement 1 is correct.

(B) Statement 1 and statement 2 are correct.

(C) Only statement 3 is correct.

(D) Statement 1 and statement 3 are correct.

60. What is Software ?

(A) Set of computer programs, procedures and possibly associated document concerned with the operation of data processing.

(B) A set of compiler instructions

(C) A mathematical formula

(D) None of above

61. Consider the following code. Hence, int Queue is an integer queue.

```
void fun (int n)
{
int queue q = new int Queue () ;
 q. enqueue (0);
 q. enqueue (1) ;
for (i = 0; i <n ; i++)
{
int a = q.dequeue ();
int b = q.dequeue ();
 q. enqueue (b);
 q. enqueue (a+b);
print (a) ;
}
}
```

(A) Print number 0 to (n-1)

(B) Print numbers from (n-1) to 0

(C) Print first n fibbonacci numbers

(D) Print first n fibbonacci numbers in reverse

62. A BST is generated by inserting the following 50,15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24

The number of nodes in left subtree and right subtree of root is -

(A) 4,7 (B) 7,4

(C) 8,3 (D) 3,8

63. The keys 12, 18, 13, 2,3, 23,5, and 15 are inserted into an initially empty hash table of length 10 using open addressing with hash function h(K) = k mod 10 and linear probing. What is the resultant hash table?

0	
1	
2	2
3	23
4	
5	15
6	
7	
8	
9	

(A)

0	
1	
2	12
3	13
4	
5	5
6	
7	
8	18
9	

(B)

0	
1	
2	12
3	13
4	2
5	3
6	23
7	5
8	18
9	15

(C)

0	
1	
2	12, 2
3	13, 3, 23
4	
5	5, 15
6	
7	
8	18
9	

(D)

(A) A (B) B

(C) C (D) D

64. Given two BST, B_1 having in elements, B_2 having m elements what is the time complexity of the best known algorithm to merge these trees for from another balanced binary tree. Containing (min) elements?

(A) 0 (m+n) (B) 0 (mlogn)

(C) 0 (nlogm) (D) 0 $(m^2 + n^2)$

65. Consider a 13 element hash table for which f(key) key mod 13 is used with integer keys. Assuming linear probing is used for collision resolution at what location key 103 will be inserted, if the keys 661, 182, 24 and 103 are inserted in that order

(A) 0 (B) 1

(C) 11 (D) 12

66. Consider the following –

L_1 : Regular (reg)

L_2 : Context free (cf)

L_3 : Recursive (rec)

L_4 : Recursive enumerable (r.e.)

Which of the following is/are true ?

(I) $L_3' \cup L_4$ is recursively enumerable (II) $L_2 \cup L_3$ is recursive

(III) $L_1^* \cup L_2$ is context free (IV) $L_1 \cup L_{2'}$ is context free

(A) (I) (B) (I) and (III)

(C) (I) and (IV) (D) (I), (II), (III)

67. What is the number of steps required to derive the string ((() ()) ())

$S \rightarrow SS$

$S \rightarrow (S)$

$S \rightarrow \in$

(A) 10 (B) 15

(C) 12 (D) 16

68. The number of states in minimized DFA for a language λ that accepts strings whose second last symbol is 1 for Σ = {0, 1} are

(A) 8 (B) 6

(C) 5 (D) 3

69. What is the number of final states in minimized DFA which accepts complement of the language 'L' where, {L = ω | ω contains even a's and each string ending with b over Σ = {a, b}}

(A) 1 (B) 3

(C) 2 (D) 4

70. The NFA has 6 states and out of these 6 states, 2 states are final states. If this NFA is converted to equivalent DFA, what will be maximum number of final states in converted DFA –

(A) 48 (B) 64

(C) 16 (D) 60

71. Match the following :

	List-I		**List-II**
(a)	Physical Layer	(i)	Flow and error control
(b)	Application Layer	(ii)	Transmission Mode
(c)	Transport Layer	(iii)	IGMP
(d)	Network Layer	(iv)	Gateway

Codes :

	(a)	**(b)**	**(c)**	**(d)**
(A)	(i)	(ii)	(iii)	(iv)
(B)	(i)	(iv)	(iii)	(ii)
(C)	(ii)	(iv)	(i)	(iii)
(D)	(ii)	(iii)	(iv)	(i)

72. Which is true ?

S1 : Transport layer is responsible for NIC to NIC delivery.

S2 : TCP allow only fixed size packets.

S3 : TCP ensure inorder delivery of message.

S4 : UDP performs limited error checking.

(A) S1, S3 (B) S3, S4

(C) S2, S3, S4 (D) S1, S3, S4

73. Say there is a community of n people. So, for what value of 'n', the total number of keys required using symmetric and asymmetric will be equal.

(A) 2 (B) 3

(C) 5 (D) 6

74. Error correcting code has 5 codes :

01001, 10101, 00001, 10000, 11001

Maximum number of bit error. Which can be detected is :

(A) 0 (B) 1

(C) 2 (D) 3

75. In cryptography, the following uses transposition ciphers and keyword in LAYER. Encrypt the following message –

WELCOME TO NETWORK SECURITY !

(A) WMEKREETSILTWETCOOCYONRU!

(B) EETSLCOOCYWMEKRONRU! LTWET

(C) LTWETONRU! WMEKRCOOCYEETSI

(D) ONRU! COOCYLTWETEET SIWMEKR

76. Which of the following does not come under subsystem of GSM architecture?

(A) BSS (B) NSS

(C) OSS (D) Channel

77. _________ manages the switching function in GSM.

(A) BSS (B) NSS

(C) OSS (D) MSC

78. _________ supports the operation and maintenance of GSM.

(A) BSS (B) NSS

(C) OSS (D) MSC

79. If D is the distance between co-channel cells and R be the cell radius, co-channel reuse ratio is given by _________

(A) D*R (B) D^2/R

(C) D/R (D) D/R^2

80. Frequency reuse factor for CDMA system is _________

(A) One (B) Two

(C) Zero (D) Ten

81. To Remove the redundant rule matching attempts in the forward chaining?

(A) Decremental forward chaining

(B) Incremental forward chaining

(C) Data complexity

(D) None of the mentioned

82. The values of alpha-beta search get updated when___________?

(A) Along the path of search (B) Initial state itself

(C) At the end (D) None of the mentioned

83. Define Weak AI

(A) The embodiment of human intellectual capabilities within a computer.

(B) A set of computer programs that produce output that would be considered to reflect intelligence if it were generated by humans.

(C) The study of mental faculties through the use of mental models implemented on a computer.

(D) All of the above

84. Searching using query on Internet is, use of which of the following type of agent

(A) Offline agent (B) Online agent

(C) Both Offline & Online agent (D) Goal Based & Online agent

85. General games solution strategy involves

(A) Single-agent

(B) Multi-agent

(C) Neither Single-agent nor Multi-agent

(D) Single-agent or Multi-agent

86. What was the name of the first model which can perform weighted sum of inputs?

(A) McCulloch-Pitts neuron model (B) Marvin Minsky neuron model

(C) Hopfield model of neuron (D) none of the mentioned

87. Who developed the first learning machine in which, connection strengths could be adapted automatically?

(A) McCulloch-pitts (B) Marvin Minsky

(C) Hopfield (D) none of the mentioned

88. Which of the following techniques can be used for the purpose of keyword normalization, the process of converting a keyword into its base from?

(1) Lemmatization (2) Levenshtein

(3) Stemming (4) Soundex

(A) 1 and 2 (B) 2 and 4

(C) 1 and 3 (D) 1, 2 and 3

89. What are affine transformations?

(A) addition of bias term (-1) which results in arbitrary rotation, scaling, translation of input pattern.

(B) addition of bias term (+1) which results in arbitrary rotation, scaling, translation of input pattern.

(C) addition of bias term (-1) or (+1) which results in arbitrary rotation, scaling, translation of input pattern.

(B) none of the mentioned

90. Which of the following statements is NOT true for a self-organizing map (SOFM)?

(A) The size of the neighbourhood is decreased during training.

(B) The units are arranged in a regular geometric pattern such as a square or ring.

(C) The weights of the winning unit k are adapted by $\Delta wk = \eta(x - wk)$, where x is the input vector.

(D) The weights of the neighbours j of the winning unit are adapted by $\Delta wj = \eta\, j\, (x - wj)$, where $\eta\, j > \eta$ and $j \neq k$.

91. The crossover points of a membership function are defined as the elements in the universe for which a particular fuzzy set has values equal to

(A) infinite (B) 1

(C) 0 (D) 0.5

92. A realtor wants to classify the house he offers to his clients based on number of rooms in it.

Let X = {1, 2, 3, 4 ... 10} be number of bedrooms in a house.

A be fuzzy set of comfortable type of house for a 4-person family

A = {(1, 2) (2, 5) (3, 0.8) (4, 1) (5, 0.7) (6, 0.3)}

The support (A) is :

(A) {4} (B) {3, 4, 5}

(C) {1, 2, 3, 4, 5, 6} (D) {2, 3, 4, 5, 6}

93. Consider two fuzzy set

$\bar{A} = \{(2,1)(3,0.5)\}$

$\bar{B} = \{(3,1)(4,0.5)\}$

Find $\mu_{A\oplus B}(z) = v_{z=x+y}(m_A(x) \wedge \mu_B(y))$

(A) (2, 1) (3, 0.5) (7, 0.5)
(B) (5, 0.5) (6, 1) (7, 1)
(C) (5, 1) (6, 0.5) (7, 0.5)
(D) (2, 0.5) (3, 0.5) (4, 0.5)

94. Let $\bar{A}_1$ be a fuzzy set 'John is Young'

where young $\in$ [0, 80]

$\mu_{\bar{A}_1}(30) = 0.9$

What is the membership of '30' in the fuzzy set 'John is very young', where very young $\in$ [0, 80]

(A) 0.5
(B) 1
(C) 0.8
(D) 0.4

95. Consider a fuzzy set old as defined below

old={(20,0),(30,0.2),(40,0.4),(50,0.6),(60,0.8),(70,1),(80,1)}. Then the alpha-cut for alpha=0.4 for the set old will be

(A) {(40,0.3)}
(B) {50,60,70,80}
(C) {(20,0.1),(30,0.2)}
(D) {(20,0),(30,0),(40,1),(50,1),(60,1),(70,1),(80,1)}

96. A ________ point of a fuzzy set A is a point $x \in X$ at which μA(x)=0.5

(A) Core
(B) Support
(C) Crossover
(D) α-cut

97. What is the value of following recurrence :

$T(n) = T(n/4) + T(n/2) + cn^2$

$T(1) = c$

$T(0) = 0$

(A) $O(n^3)$
(B) $O(n^2)$
(C) $O(n^2 \text{ Logn})$
(D) O(nLogn)

98. Consider the following recurrence :

$$T(n) = 2T(\lceil \sqrt{n} \rceil) + 1, \ T(1) = 1$$

Which one is true?

(A) $T(n) = \theta(\log \log n)$
(B) $T(n) = \theta(\log n)$
(C) $T(n) = \theta(\sqrt{n})$
(D) $T(n) = \theta(n)$

99. The running time of an algorithm

if n < = 3 then T(n) = n

else T(n) = T(n/3) + cn

Which one represents the time complexity?

(A) $\theta(n)$ (B) $\theta(n \log n)$

(C) $\theta(n^2)$ (D) $\theta(n^2 \log n)$

100. Consider the following recurrence :

$$T(n) = T\left(\sqrt{n}\right) + \theta(\log \log n)$$

What is the value of recurrence?

(A) $\theta((\log \log n)^2)$ (B) $\theta(\log \log n)$

(C) $\theta(n)$ (D) $\theta(\log \log \log n)$

ANSWER KEY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	B	B	D	B	B	B	D	D	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	A	B	A	B	C	D	A	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	A	D	D	C	C	D	C	D	C
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A	C	D	D	A	A	C	D	A	C
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	C	B	A	A	D	C	A	D	A
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
B	C	C	B	A	B	C	B	B	A
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
C	B	C	A	B	D	A	D	C	A
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
C	B	C	A	B	D	B	C	C	A
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
B	A	C	D	D	A	B	C	A	D
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
D	C	C	C	D	C	B	B	A	A

SOLUTIONS

1. (A) Here 0° and 360° is same

For abelian group. The following properties should be met -

a) closure - for all a, b, $\in$ A the result of operation $a * b \in A$

Here,

Let r_0, $r_{60} \in$ set R

$r_0 \square r_{60} = r_{0+60} = r_{60} \in R$

Closed.

b) Associativity -

For a, b, c $\in$ A $\quad a * (b * c) = (a * b) * c$

i.e □ is associative operation

as, $(r_0 \square r_{60}) \square r_{120} = r_0 \square (r_{60} \square r_{120})$

L.H.S. R.H.S.

$r_{60} \square r_{120}$ $\quad$ $r_0 \square r_{180}$

r_{180} (same) r_{180}

c) Identity element -

r_0 is the identity element

as,

$a * e = a$ for all a $\in$ A'

In this,

$$\left.\begin{array}{l} r_{60} \square r_0 = r_{60} \\ r_{180} \square r_{180} = r_{180} \end{array}\right\}$$

d) Inverse element -

For an inverse to exist

$a * a^{-1} = e$

So, $r_{60} \square r_{(180-60)} = r_0$

i.e $r_{60} \square r_{120}$

$= r_{180}$

$= r_0$ (identity)

Thus, for any element r_θ

$r_{360-\theta}$ is the inverse

e) Commutativity -

For all a, b $\in$ A $\quad a * b = b * a$

i.e

$r_0 \square r_{60} = r_{60} \square r_0$

<u>L.H.S</u> <u>R.H.S</u>

r_{60} $\quad$ r_{60}

same

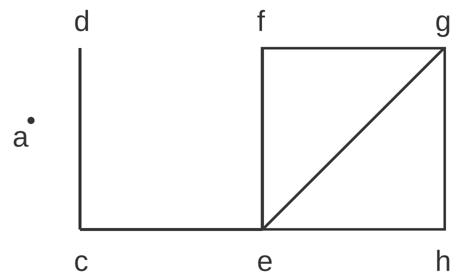
So, given (R, □) is an abelian group.

Correct option is A.

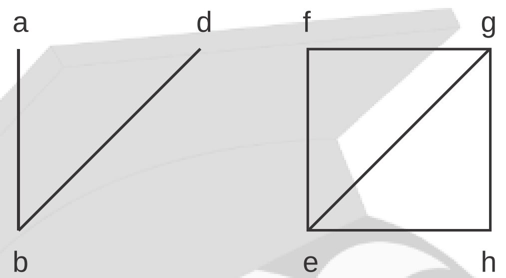
2. (B) Cut vertex means removal of the vertices disconnects the graph.

Here cut vertex are b, c, e

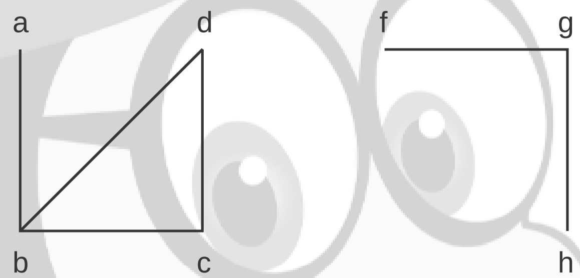
<u>Removal of b</u>



Removal of c



Removal of e

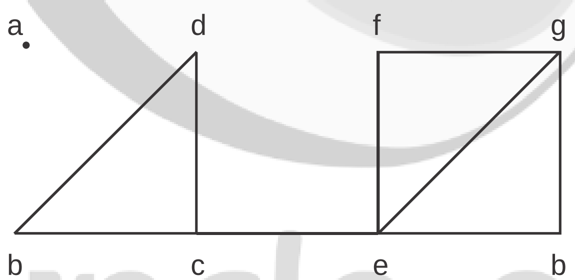


Thus, three cut vertices

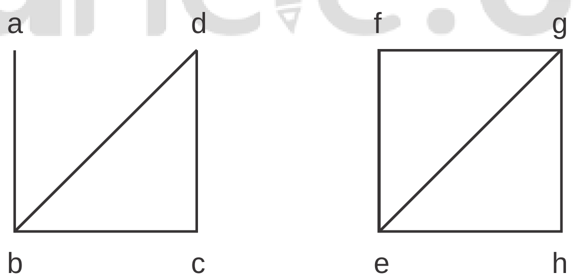
<u>Cut edges</u> — Removal of minimum number of edges that disconnect the graph.

Here {a, b} and {c, e} are such edges

<u>Removal of {a, b}</u>



Removal of {c, e}



Thus, 2 cut edges

Correct option is (B).

3. (B) Transitive closure is the smallest transitive relation R' on given relation R

Here the adjacency matrix

$$w_0 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

w_1 has (i, j) the entry is 1 if there is a path i to j that has only 1 intermediate vertex.

Ex.- path b to d i.e b, a, d

$$w_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Similarly,

$w_2 = w_1$

$$w_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

w_4 has (i, j)th entry is 1 if there is a path i to j with 4 intermediate vertices.

$$w_4 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

i.e (a,a) (a,c) (a,d) (b,a) (b,c) (b,d)

(c,a) (c,c) (c,d) (d,a) (d,c) (d,d)

Correct option is (B).

4. (D) $(p \to q) \to r = f$

$\therefore$ $(p \to q) \to r = 0$

i.e $(p' + q)' + r = 0$ (as $p \to q = p' + q$)

$\Rightarrow$ $pq' + r = 0$

This is possible if $pq' = 0$ and $r = 0$

$pq' = 0$ if $p = 0$ or $q' = 0$ $(q = 1)$

For $(r \to p) \to q$

$= (r' + p)' + q$

$= rp' + q$

But $r = 0$

So, $(r \to p) \to q = 0 + q = q$

So, $(r \to p) \to q$ is true whenever q is true

Correct option is (D).

5. (B) I. $\forall x\, \exists y\, R(x,y) \to \exists y(\exists x\, R(x, y))$ is true

as $\exists y\, (\exists x\, R(x, y)) \equiv \exists x(\exists y\, R(x, y))$

II. $\forall x\, \exists y\, R(x,y) \to \exists y(\forall x\, R(x, y))$ is false

Since $\exists y$ outside is stronger than $\forall x$

III. $\forall x\, \exists y\, R(x, y) \to \forall y \exists x\, R(x, y)$ is false

as $R(x, y) \neq R(y, x)$ might not be symmetric

IV. $\forall x\, \exists y\, R(x,y) \to \neg(\exists x\, \forall y \neg R(x, y))$ is true

Since, $\neg(\exists x\, \forall y\, \neg R(x, y) \equiv \forall x\, \exists y\, R(x, y)$

So, it will reduce to $\forall\, x\, \exists\, y\, R(x, y) \to \forall x\, \exists y R(x, y)$

Which is always true

Correct option is (B).

6. (B) A linear program is infeasible if there exists no solution that satisfies all of the constraints. In other words no feasible solution can be constructed. Infeasibility occur in specifying some of the constraints in your model, or from some wrong numbers in your date.

7. (B) **Row wise reduction : -**

$$\text{Job}\quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \text{men} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 5 & 0 & 3 & 5 & 10 \\ 6 & 8 & 0 & 2 & 3 \\ 0 & 2 & 4 & 6 & 4 \\ 0 & 1 & 4 & 6 & 3 \\ 2 & 2 & 0 & 3 & 4 \end{bmatrix} \end{array}$$

Reduce the matrix by selecting the smallest value in row and subtracting from other values in corresponding row

Column wise reduction:- Reduce the matrix by selecting the smallest value in each column and subtracting from other value in corresponding column.

$$\text{Job}\quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \text{men} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 5 & 0 & 3 & 3 & 7 \\ 6 & 8 & 0 & 0 & 0 \\ 0 & 2 & 4 & 4 & 1 \\ 0 & 1 & 4 & 4 & 0 \\ 2 & 2 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Matrix with minimum number of lines drawn to cover all zeros in shown in table.

Matrix will all zero covered :-

$$\text{Job}\quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} \begin{array}{c} \text{men} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 5 & 0 & 3 & 3 & 7 \\ 6 & 8 & 0 & 0 & 0 \\ 0 & 2 & 4 & 4 & 1 \\ 0 & 1 & 4 & 4 & 0 \\ 2 & 2 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

The number of lines drawn is 5, which is equal to the order of matrix. Hence optimality is reached the optimal assignments are shown in table.

Optimal assignment :-

$$\begin{array}{ccc} & & \text{men} \longrightarrow \\ & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Job} \longrightarrow & \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \\ \text{V} \end{array} & \begin{bmatrix} 5 & \boxed{0} & 3 & 3 & 7 \\ 6 & 8 & \cancel{0} & \boxed{0} & \cancel{0} \\ \boxed{0} & 2 & 4 & 4 & 1 \\ \cancel{0} & 1 & 4 & 4 & \boxed{0} \\ 2 & 2 & \boxed{0} & 1 & 1 \end{bmatrix} \end{array}$$

Therefore, the optimal solution is

Job	Men	Time
I	2	15
II	4	14
III	1	21
IV	5	20
V	3	16
	Total time 86 hrs.	

8. (D) There are mainly four methods so called Enumeration method, simplex method, transportation method and Hungarian method for solving Assignment problem.

9. (D) The initial solution of a transportation problem is said to be non-degenerate basis feasible solution if it satisfies :

(i) The solution must be feasible i.e. it must satisfy all the supply and demand constraints.

(ii) The number of positive allocations must be equal to m + n – 1, where m is the number of rows and n is the number of column.

(iii) All the positive allocations must be in independent positions.

So, option (D) is correct.

10. (D) Here demand and supply are not equal so a dummy column (with cost 0 for all elements) is added with demand 15 to balance it. Then by Vogel's method penalties are calculated and we choose the row with highest penalties and if there is ties choose the min, next min cost etc.

	I	II	III	IV	V	VI	Supply	Penalty
A	$\boxed{25}$ 4	$\boxed{5}$ 6	$\boxed{20}$ 8	13	0	0	~~50~~ ~~15~~ 20	4 2 4 5
B	13	11	$\boxed{50}$ 10	$\boxed{20}$ 8	0	70	12-8-4	8 2 2 2
C	12	$\boxed{30}$ 4	10	13	0	30	14-4-6-4	4 6 – –
D	9	11	$\boxed{35}$ 13	8	$\boxed{15}$ 0	~~50~~ 35	14-4-6-4	8 1 1 5
Demand	25	~~35~~ 5	~~105~~ ~~85~~ 50	20	15	200		
Penalty	5 5	2 5	2 2 3	0 0 0	0 –			

→ Add fifth column with zero cost to balance the demand and supply.

Total cost = 15 × 0 + 30 × 4 + 25 × 4 + 5 × 6 + 20 × 8 + 20 × 8 + 50 × 10 + 35 × 13

= 1525

→ Consider The penalty with highest value and allocate maximum possible units to the least cost in that row/column.

11. (B) **Booth's Algorithm :** First evaluate 2's complement of given numbers, if number is negative then append 0 into LSB.

Then each pair from LSB to MSB (add 1 bit at a time):

00 = 0, 01 = + 1, 10 = – 1, 11 = 0

Booth's algorithm based on multiplier which is already given in binary representation:

0111 0111 1011 1101

Now, append 0 into LSB of

(0111 0111 1011 1101) = 0111 0111 1011 1101 0

Now, Booth's code (add 1 bit at a time, from LSB to MSB):

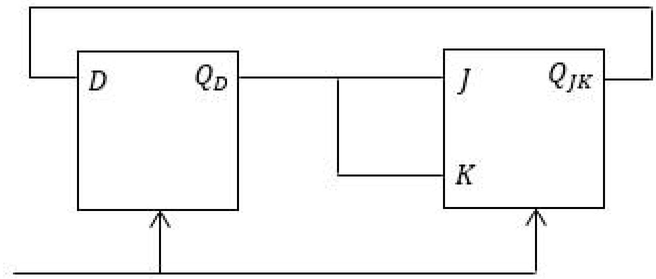
01, 11, 11, 10, 01, 11, 11, 11, 10, 01, 11, 11, 11, 10, 01, 10

+1 0 0 – 1 + 1 0 0 0 – 1 + 1 0 0 0 – 1 + 1 – 1

Here, 4 subtractions and 4 additions required

Total = 8

12. (A) The circuit for the given data is



The characteristic equations are

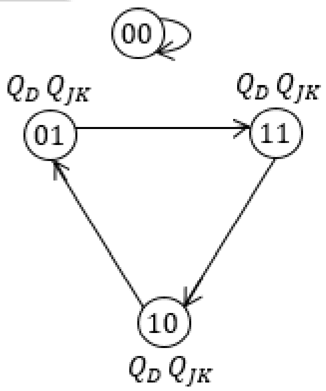
$$Q_{DN} = D = Q_{JK}$$

$$Q_{JKN} = J\bar{Q}_{JK} + \bar{K}Q_{JK}$$

$$= Q_D\bar{Q}_{JK} + \bar{Q}_D Q_{JK} \qquad (\because J = K = Q_0)$$

The state table and state transition diagram are as follows :

Q_D	Q_{JK}	Q_{DN}	Q_{JKN}
0	0	0	0
0	1	1	1
1	0	0	1
1	1	1	0



Consider Q_DQ_{JK} = 10 as initial state because in the options Q_{JK} = 0 is the initial state of JK flip-flop.

The state sequence is

$$1\ \overset{Q_{JK}}{0} \rightarrow 0\ \overset{Q_{JK}}{1} \rightarrow 1\ \overset{Q_{JK}}{1} \rightarrow 1\ \overset{Q_{JK}}{0} \rightarrow 0\ \overset{Q_{JK}}{1} \rightarrow 1\ \overset{Q_{JK}}{1}$$

$0 \rightarrow 1 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow 1$

13. (A) f = ((P'Q' + Q'R')' + (P'R' + Q'R')')'

= (P'Q' + Q'R')(P'R' + Q'R')

= (P'Q'P'R' + P'Q'Q'R' + Q'R'P'R' + Q'R'Q'R')

= (P'Q'R' + P'Q'R' + P'Q'R' + Q'R')

= (P'Q'R' + Q'R')

= (Q'R')

= (Q+R)'

14. (B) C1=G0+C0.P0

C2=G1+G0.P1+C0.P0.P1

C3=G2+G1.P2+G0.P1.P2+C0.P0.P1.P2

C4=G3+G2.P3+G1.P2.P3+G0.P1.P2.P3+C0.P0.P1.P2.P3

// read this as carry is generated in 3rd stage OR carry is generated in 2nd stage AND propagated to 3rd stage OR carry is generated in 1st stage AND carry is propagated through 2nd AND 3rd stage OR carry is generated in 0th stage AND propagated through 1st 2nd AND 3rd stage OR initial carry is propagated through 0th, 1st, 2nd AND 3rd stage.

4 OR gates are required for C1,C2,C3,C4

1 AND gate for C1

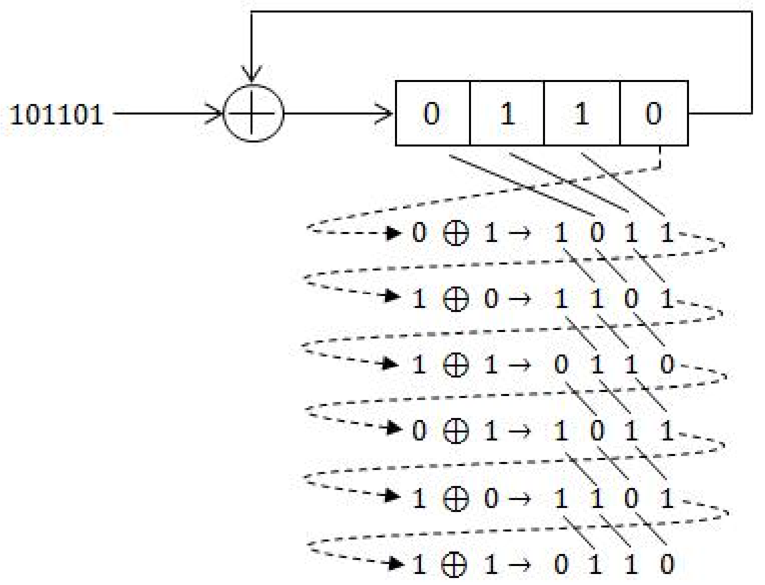
2 AND gate for C2

3 AND gate for C3

4 AND gate for C4

AND = 10

15. (A)



16. (B) In vertical microprogramming control signals are encoded form means if 64 control signals we need 6 bits in control word. In horizontal micro programming each bit is required to represent control signal, so to represent 64 control signals we require 64 bits.

So in case of vertical programming :

For Group 1:20 signals we require 5 bits

For Group 2:70 signals we require 7 bits

For Group 3:2 signals we require 1 bit

For Group 4:10 signals we require 4 bits

For Group 5:23 signals we require 5 bits

Total bits = 5 + 7 + 1 + 4 + 5 = 22 bits required in vertical programming.

In case of horizontal microprogramming we need 20 + 70 + 2 + 10 + 23 = 125 bits

Bits of the control words can be saved by using vertical micro programming over horizontal microprogramming = 125 – 22 = 103 bits

17. (C) Time space diagram :

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
F	D	D	E	W										
	F	-	D	D	E	E	W							
		F	F	-	D	-	E	E	E	W	W			
				F	-	D	D	D	-	E	E	W		
					F	-	-	-	D	D	-	E	W	W

18. (D) 35 + 10 + 15 + 15 = 75ns

19. (A) Size of main memory 220

Physical address = 20 bits

Block size = 16 bytes, number of cache lines = 212

So index part is 12 bits

In direct mapping cache, address is represent in 3 part tag, index, offset.

Offset = $\log_2$(block size) = $\log_2(16)$ = $\log_2(24)$ = 4bits

So tag field = 20 – (12 + 4) = 4bits

We have to calculate tag and cache line address for main memory address $(E201F)_{16}$.

In address $(E201F)_{16}$:

E = tag, 201 = index F = offset

20. (C) Address = 32 bits, block of size 16 bytes, 1024 cache lines

In direct mapped, cache address have three parts tag, index and offset.

Offset = $\log_2$ (block size) = $\log_2$ (16) = $\log_2$ (2^4) = 4bits

Index = $\log_2$ (1024) = $\log_2(2^{10})$ = 10 bits

So tag = 32 – (10 + 4) = 18 bits

21. (B) The use of continue statement forces the execution to skip the remainder of the current pass over the loop and initiates the next. If ' i ' is 3, printf statement will be skipped.

22. (A) k = –7. So, if 'k' is used as a Boolean variable, it will be treated as a true condition. So, !k will be false i.e., 0. So, 0 < ? !k is actually 0 < 0. Which is false. So, 0 will be printed.

23. (D) Reusable code is an already used code, as the name implies. Hence it is by free and pretested. There is no need to test it again.

24. (D) Array element cannot be address of auto variable. It can be address of static or external variables.

25. (C) The statement ‘throw’, is used to re-throw an exception. This is useful when a function can handles some part of the exception handling and then delegates the remaining part to the caller. A catch block cleans up resources of its function, and then rethrows the exception for handling else-where.

26. (C) The ATTLIST declarations identify which element types may have attributes, what type of attributes they may be, and what the default value of the attributes are. Syntax is <!ATTLIST elementName attributeName dataType default >. XML and HTML are web mark-up language used to design and create web pages. SGML stands for Standard Generalized Mark-up Language.

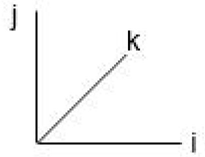
27. (D) &acute is character entity and denote small a of latin language. &quot, &para, &not are punctuation character entities. Number representation of &quot is &#34, &para is &#182, &not is &#172, &acute is &#180.

28. (C) To address the media support problem, you need to add in alternative formats to use by including several source tags.

29. (D) Autobuffer is a attribute for both <audio> and <video>, so we can replace it with either <audio> or <video>. As with the video element, you also have autobuffer and autoplay attributes for the audio element.

30. (C) Metadata is information about data. The meta tag provides metadata/meta information about the HTML document. Metadata will not be displayed on the page. Metadata is present in head. The body tag defines document's body. A title tag is an HTML element which specifies the title of a web page.

31. (A) Only one in the direction of k because, if the set of lines is parallel to one of the three principal axis, then it is called a principal vanishing point.



32. (C) The super-sampling procedure increases the number of intensity levels for each pixel to total number of sub-pixels.

33. (D) Graphic tablet are also called data tablet.

34. (D) The voice system input can be used to initiate graphics operations or to enter data.

35. (A) If there are 'n' processes then this algorithm divides it into number of partitions and generates line segments.

36. (A) R(ABCDEF)

$A \to CD$

$B \to E$

$E \to F$

Key $\to$ AB Prime $\to$ A, B Non-Prime C, D, E, F as $(AB)^+ = \{A, B, E, F, C, D\}$

Checking for 2NF

$A \to CD$ violates 2NF as A is part of key (AB) CD is non-prime.

Thus relation is in 1NF

37. (C) R(ABCDEF)

F = { $A \to B$

$C \to D$

$E \to F$ }

Key = ACE

as $(ACE)^+ = \{A, C, E, B, D, F\}$

So, prime $\to$ A, C, E

Non-prime, B, D, F

$A \to B$ violates 2NF

as A is part of key (ACE)

B is not prime

(AB) is a decomposition

- $C \to D$ violates 2NF

  C is part of key and D is not prime

  (CD) is a decomposition

- $E \to F$ violates 2NF

  as E is part of key and F is non-prime

  So, (EF) is a decomposition

But the three decomposition i.e.

(AB) (CD) (EF) do not contain key (ACE)

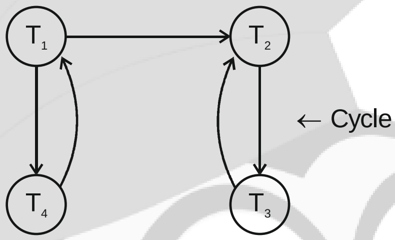
So, total 2NF decomposition are :

(AB) (CD) (EF) (ACE)

38. (D) R1(x) R1(y) W2(x) R2(z) W4(y) R4(y) R3(x) W3(z) W2(z) W1(y)

Can be written as,

T_1	T_2	T_3	T_4
r(x)			
r(y)			
	w(x)		
	r(z)		
			w(y)
			r(y)
		r(x)	
		w(z)	
	w(z)		
w(y)			



Note conflict serializable.

Since, the graph is not conflict serializable. It is also not view serializable.

39. (A) Order of B^+ tree be P

(Order 1) key + (Order – 1) Record Size + 0* Block Pointer ≤ Block Size

i.e. $(0 - 1)10 + (0 - 1)8 + 0 \times 7 \leq 512$

$10(0) - 10 + 8(0) - 8 + 70 \leq 512$

$25(0) - 18 \leq 512$

$25(0) \leq 530$

So, order of internal node= 30

leaf node is 29.

40. (C)
- 3NF is adequate normal form
- As the redundancy is less
- The schedule is guaranteed to be lossless and dependency preserving.
- 2NF drawback

  The reducing and anomalies are there
- BCNF
- Even though the redundancy in BCNF is less than 3NF but it is not considered adequate NF because the decomposition may or may not be lossy.

41. (A) Areas that are covered by Data transformation include :

(a) **Cleansing :** it is by definition transformation process in which data that violates business rules is changed to conform these rules. It is usually done by ETL programs that determine or derive correct data values and then write them into the BI target databases.

(b) **Summarization :** Values are summarized to obtain total figures which are subsequently calculated and stored at multiple levels as business fact in multidimensional fact tables.

(c) **Derivation :** New data is created from existing (detailed) source data during this process by calculations, program logic or table lookups. Some examples of derivation may be: calculating profit from income and expense items or calculating customer's age based on their date of birth and the current year.

(d) **Aggregation :** Data elements for customers may be aggregated from multiple source files and databases (e.g. Customer Master File, Sales File, Prospect File). Sometimes (in multidimensional database terminology) this term also refers to roll ups of data values.

(e) **Integration :** The expected result of this part is to have each and unique data element known by one standard name with one standard definition and approved name. Data integration forces the need to reconcile different data names and values for the same data element. So, summarization is a process to change detailed to summary data.

42. (C) Integration is a process of taking operational data from one or more sources and mapping it, field by field, onto a new data structure in the data warehouse.

The benefit of a data warehouse enables a business to perform analyses based on the data in the data warehouse. This would not be possible to do on the data available only in the source system. The reason is that the source systems may not contain corresponding data, even though the data are identically named, they may refer to different entities.

43. (B) **Horizontal Fragmentation/Partitioning :** Horizontal fragmentation groups the tuples of a table in accordance to values of one or more fields. Horizontal fragmentation should also confirm to the rule of reconstructiveness. Each horizontal fragment must have all columns of the original base table.

For example, in the student schema, if the details of all students of Computer Science Course needs to be maintained at the School of Computer Science, then the designer will horizontally fragment the database as follows :

```
CREATE COMP_STD AS
    SELECT * FROM STUDENT
    WHERE COURSE = "Computer Science";
```

44. (A) A hard margin means that an SVM is very rigid in classification and tries to work extremely well in the training set, causing overfitting.

45. (A) Datasets which have a clear classification boundary will function best with SVM’s.

46. (D) These are the different implementations of the symbol table as mentioned above.

47. (C) The -p instructs the compiler to produce codes which count the number of times each routine is called and this is useful for finding the processing time of the programs.

48. (A) Developing software in-house means the same. It is easier to carry out changes in the software if it is developed in-house.

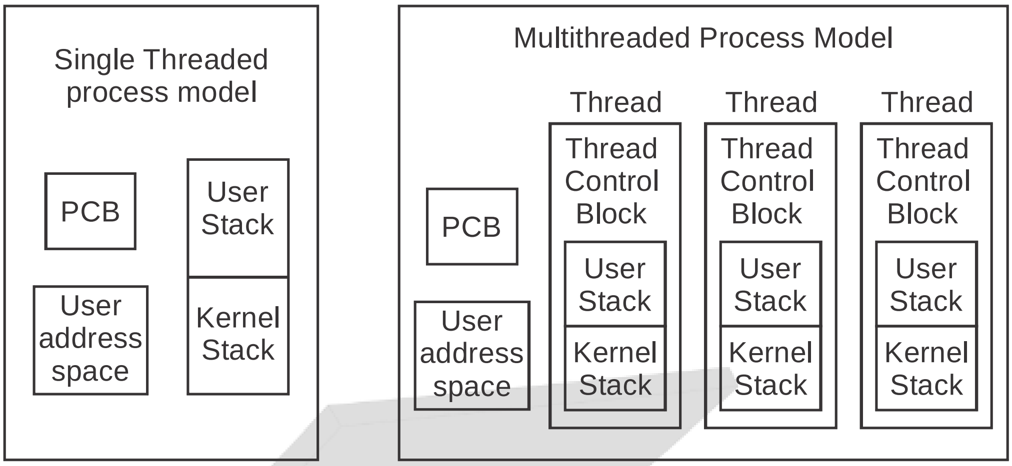
49. (D) It is called the embedded middleware since it activates the communication link between the built-in applications and the real time operating system.

50. (A) The executable instructions or simple instructions tell the processor what to do. Each instruction consists of an operation code (opcode). Each executable instruction generates one machine language instruction.

51. (B) Threads are referred to as 'light weight' process. It is like a process but is called light weight because it uses fewer resources and they also share address space.

Since the memory is shared, it is easier for the threads to communicate. Since the overhead related to thread is 'Smaller' than that of a process.

Switching between threads is faster.



Threads share address space and can be manipulated concurrently through system calls such as map and brk. Thus, overhead is less.

52. (C) P : Wait on semaphore, this decreases the semaphore value by 1.

V : Signal operation, this increases the semaphore value by 1.

Option b : S = 10

After 3P, S – 3

$\Rightarrow$ 10 – 3 = 7

Option a : S = 10

After 3V S + 3

$\Rightarrow$ 10 + 3 = 13

Option c : After 5V and 2P

S = 10 S + 5 – 2

= 10 + 5 – 2 = 13

Option d : After 2V and 5P operations

S = 10 S + 2 – 5

$\Rightarrow$ 10 + 2 – 5

= 7

Thus b and d gives value of 7

53. (C) The Gantt chart for preemptive priority scheduling will be –

P2	P3	P4	P5	P1

0 5 8 28 30 40

So,

Process	A.T	B.T	Completion time (C.T)	Turn around time (TAT) (C.T – A.T)	Waiting time (TAT – B.T)
P1	0	10	40	40	30
P2	0	5	5	5	0
P3	2	3	8	6	3
P4	5	20	28	23	3
P5	10	2	30	20	18
					54

Thus, average waiting time = $\frac{54}{5}$ = 10.8

54. (B) The reference are in order – 1, 2, 4, 5, 2, 1, 2, 4

1	2	4	5	2	1	2	4
1	1	1	2	2	5	5	1
2	2	2	4	4	2	1	2
		4	5	5	1	2	4
		P.F	P.F		P.F		P.F

i.e. stands for page fault

Thus, 4 page faults

55. (A)

	Max			Allocation			Need			Available		
	E	F	G	E	F	G	E	F	G	E	F	G
P0	4	3	1	1	0	1	3	3	0	3	3	0
P1	2	1	4	1	1	2	1	0	2			
P2	1	3	3	1	0	3	0	3	0			
P3	5	4	1	2	0	0	3	4	1			

Available (3, 3, 0) can satisfy P0 or P2

Take P0 <3, 3, 0>

Available (3, 3, 0) + (1, 0, 1) = (4, 3, 1)

Take P2 <0, 3, 0>

Available (4, 3, 1) + (1, 0, 3) = (5, 3, 4)

Take P1 <1, 0, 2>

Available (5, 4, 3) + (1, 1, 2) = (6, 4, 6)

Take P3 <3, 4, 1>

Available (6, 4, 6) + (2, 0, 0) = (8, 4, 6)

Safe Sequence : P0 $\rightarrow$ P2 $\rightarrow$ P1 $\rightarrow$ P3 or P2 $\rightarrow$ P0 $\rightarrow$ P1 $\rightarrow$ P3

56. (B) Let LOC of L1=x, so LOC of L2=2x

Now,

(x/10000)*1000000 + 5*100000 = (2x/10000)*750000 + 5*50000

Solving for x, we get x =5000

57. (C) S2: What is the meaning of reasonable performance?. The requirement should be measurable

e.g. the system should not fail while in operation for 10 hours continuously

S4: How to decide standard size of monitor. The standard size of monitor should be mentioned

58. (B) Software validation checks that the software product satisfies or fits the intended use (high-level checking), i.e., the software meets the user requirements, not as specification artifacts or as needs of those who will operate the software only; but, as the needs of all the stakeholders also.

59. (B) Any standard software process model would primarily consist of two types of activities: A set of framework activities, which are always applicable, regardless of the project type, and a set of umbrella activities, which are the non SDLC activities that span across the entire software development life cycle.

Typical activities in this category include:

1. Software project tracking and control :

Tracking and Control is the dual process of detecting when a project is drifting off-plan, and taking corrective action to bring the project back on track. But a successful project manager will also be able to tell when the plan itself is faulty, and even re-plan the project and its goals if necessary.

2. Formal technical reviews :

This includes reviewing the techniques that has been used in the project.

3. Software quality assurance :

This is very important to ensure the quality measurement of each part to ensure them.

4. Software configuration management :

In software engineering, software configuration management (SCM or S/W CM) is the task of tracking and controlling changes in the software, part of the larger cross-disciplinary field of configuration management. SCM practices include revision control and the establishment of baselines.

5. Document preparation and production :

All the project planning and other activities should be hardly copied and the production get started here.

6. Reusability management :

This includes the backing up of each part of the software project they can be corrected or any kind of support can be given to them later to update or upgrade the software at user/ time demand.

7. Measurement :

This will include all the measurement of every aspects of the software project.

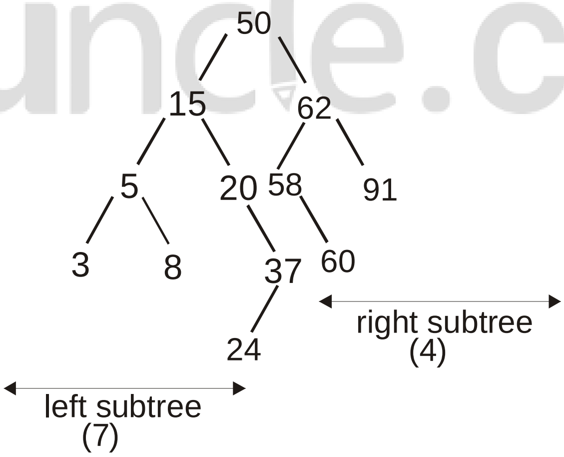
8. Risk management :

Risk management is a series of steps that help a software team to understand and manage uncertainty. It's a really good idea to identify it, assess its probability of occurrence, estimate its impact, and establish a contingency plan that - 'should the problem actually occur'.

60. (A) Computer software or just software, is a collection of computer programs and related data that provides the instructions for telling a computer what to do and how to do it.

61. (C) The function prints first n Fibonacci numbers. Note that 0 and 1 are initially there in q. In every iteration of loop sum of the two queue items in enqueued and front item is dequeued.

62. (B)



63. (C) Open addressing or closed hashing, is a method of collision resolution in hash tables. With the method a hash collision is resolved b probing, or searching through alternate locations in the array (the probe sequence) until either the target record is found, or an unused array slot is found, which indicates that there is no such key will the table. Well known probe sequence include: linear probing in which the interval between probes is fixed-often at 1. Quadratic probing in which the interval between probes increases linearly. Hence, the indices are described by quadratic function). Double hashing in which the interval between probes is fixed for each record but is computed by another hash function.

64. (A) 0 (min) as we can perform in order on both the trees and store them in two separate arrays. Now, we have two sorted arrays and we can merge in 0 (min) using standard merge algorithm and on the final sorted array we can use the BST to create the tree using recursion.

65. (B) 661 mod 13 = 11

182 mod 13 = 0

24 mod 13 = 11 collision. So, at 12

103 mod 13 = 12 collision, So, at index 1.

182	103		661	24
0	1		11	12

66. (D) S_1 : $L_3 \rightarrow$ recursive

They are closed under complementation.

$L_{3'}$ will also be recursive.

$L_{3'} \cup L_4$ is also recursive as they are closed under union.

S_2 : L_2 is context free, it will be recursive as well. $L_2 \cup L_3$ is recursive as they are closed under union

S_3 : $L_1 \rightarrow$ regular

$L_{1^*} \rightarrow$ regular

Reg $\cup$ CFL $\rightarrow$ CFL

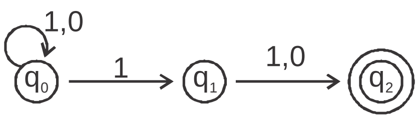
S_4 : CFL are not closed under complementation $L_{2'}$ may or may not be context free.

Not true.

67. (A) To generate ((( ) ( )) the steps required are –

1. S $\rightarrow$ (S)
2. S $\rightarrow$ (SS)
3. S $\rightarrow$ ((S) S)
4. S $\rightarrow$ ((SS) S)
5. S $\rightarrow$ (((S) S)) S)
6. S $\rightarrow$ (((S) (S)) S)
7. S $\rightarrow$ (((S) (S)) (S))   S $\rightarrow \in$
8. S $\rightarrow$ ((( ) (S)) (S)   S $\rightarrow \in$
9. S $\rightarrow$ ((( ) ( )) S)   S $\rightarrow \in$
10. S $\rightarrow$ ((( ) ( )))

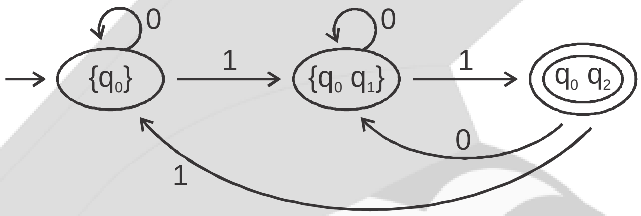
68. (D) We will first construct a NFA



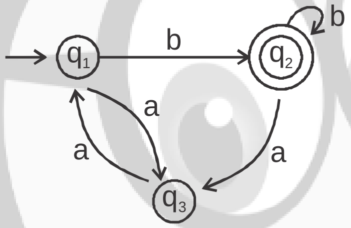
Convert NFA to DFA

δ	0	1
q_0	q_0	$(q_0\, q_1)$
q_1	q_2	q_2
q_2	ϕ	ϕ
$(q_0\, q_1)$	$(q_0\, q_2)$	$(q_1\, q_2)$
$(q_0 q_2)$	q_0	$(q_0 q_1)$
$(q_1\, q_2)$	q_2	q_2
ϕ	ϕ	ϕ

Then minimizing the above we get

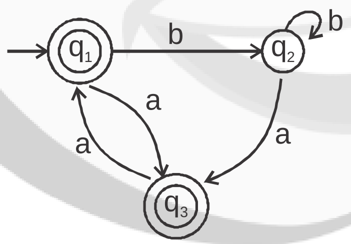


69. (C) The DFA is –



For the language L

The DFA for complement of language L is



Final states 2.

70. (A) States in NFA = 6

Maximum States in DFA = 2^6

Non-final states in NFA = 6 – 2 = 4

In DFA = 2^4

Final States in DFA

$= 2^6 - 2^4$

$= 64 - 16$

$= 48$

71. (C) Transmission mode like :

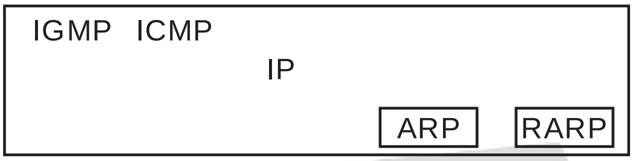
- Half Duplex
- Full Duplex

is the responsibility of physical layer.

Thus (a)-(ii).

→ IGMP (Internet Group Management Protocol) is a communications protocol used by hosts and adjacent routers on IPv4 networks to establish multicast group memberships. IGMP is an integral part of IP multicast.

IGMP is a protocol of Network Layer.



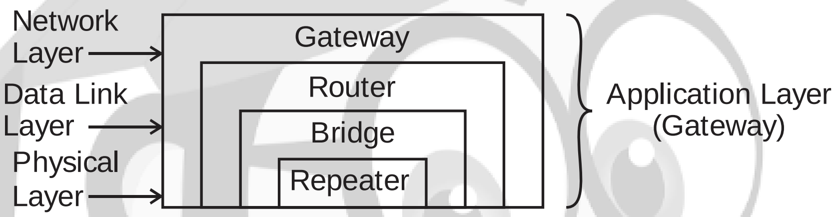
Network Layer

Thus (d)-(iii).

→ Flow and error control is responsibility of transport layer

Thus (c)-(i)

→ Gateway is an inter networking device that extends up to application layer.



Thus, (b)-(iv)

The correct option is (C).

72. (B) S1 : Transport layer is responsible for process to process delivery. The NIC address is the 48-bit number given to computer system. The MAC layer of DLL determines this address.

S2 : This is not true.

S3 : TCP is a reliable protocol and ensures in order delivery of message. S3 is true.

S4 : UDP does not perform any error checking only limited form is done by using checksum. If reliability is important then TCP is used instead of UDP. S4 is true.

73. (C) Number of Asymmetric key needed for n people

$= 2n$

Number of Symmetric key needed for n people

$$= \frac{n(n-1)}{2}$$

So, $\frac{n(n-1)}{2} = 2n$

$\Rightarrow$ $n^2 - n = 4n$

$\Rightarrow$ $n^2 - n - 4n = 0$

For option (A)

$2^2 - 2 - 4(2) = 4 - 2 - 8 \neq 0$

Option (B)

$3^2 - 3 - 4(3) = 9 - 3 - 12 \neq 0$

Option (C)

$5^2 - 5 - 4(5) = 25 - 5 - 20 = 0$

74. (A) The codes are :

01001 H(w) = 2 (Hamming Weight)

10101 H(w) = 3

00001 H(w) = 1

10000 H(w) = 1

11001 H(w) = 3

So, $d_{min} = 1$

Number of error that can be detected is 's'

Thus $d_{min} = s + 1$

$1 = s + 1$

$s = 0$

75. (B) Key Work

(3) ↓	(1) ↓	(5) ↓	(2) ↓	(4) ↓
L	A	Y	E	R
W	E	L	C	O
M	E	T	O	N
E	T	W	O	R
K	S	E	C	U
R	I	T	Y	!

Thus, EETSICOOCYWMEKRONRU! LTWET

76. (D) The GSM architecture consists of three major interconnected subsystems that interact between themselves and with the users through certain network interfaces. The subsystems are BSS (Base Station Subsystem), NSS (Network and Switching Subsystem) and OSS (Operation Support Subsystem).

77. (B) NSS (Network and Switching Subsystem) manages the switching functions of the system. It allows the MSCs to communicate with other networks such as PSTN and ISDN.

78. (C) The OSS (Operation Support Subsystem) supports the operation and maintenance of GSM. It allows system engineers to monitor, diagnose, and troubleshoot all aspects of GSM.

79. (C) Let, D is the distance between co-channel cells and R be the cell radius. Then the minimum ratio of D/R that is required to provide a tolerable level of co-channel interference is called the co-channel reuse ratio.

80. (A) The amount of out-of-cell interference determines the frequency reuse factor, f, of a CDMA cellular system. Ideally, each cell shares the same frequency and the maximum possible value of f (f=1) is achieved.

81. (B) To remove redundant rule matching attempts in the forward chaining incremental forward chaining can be used.

82. (A) Alpha-beta search updates the value of alpha and beta when it gets along and prunes the remaining branches at node.

83. (C) The study of mental faculties through the use of mental models implemented on a computer is known as weak AI.

84. (D) Searching using query on Internet is, use Goal Based & Online agent .

85. (D) Depending upon games it could be single agent (Sudoku) or multi-agent (Chess)

86. (A) McCulloch-Pitts neuron model can perform weighted sum of inputs followed by threshold logic operation.

87. (B) In 1954 Marvin Minsky developed the first learning machine in which connection strengths could be adapted automatically & efficiently.

88. (C) Lemmatisation and stemming are the technique of keyword normalization while levenshtein and soundex are technique of string matching.

89. (A) Most of the basic neural networks are composed of many units called "neurons".

These neurons perform an affine transformation followed by a nonlinear transformation on their “presynaptic neurons” affine transformation means addition of bias term (-1) which results in arbitrary rotation, scaling, translation of input pattern.

That is, they take output numbers given by several other neurons and perform produce their own output. The theory of affine transformations is important here for two reasons.

(1) our understanding of affine transformations allows us to know that an affine transformation alone would be insufficient to give neural networks the amazing flexibility they have.

(2) having part of their computation be affine prevents the entire process from being computationally slow.

90. (D) Self organizing maps are based on these principles:

- Learning by changes in synaptic weight
- A topographic organization of information i.e. Similar information is found in a similar spatial location.

Self organizing maps are a type of artificial neural network. Unlike methods like back propagation, self organizing networks are unsupervised, hence the name self organizing, for a self organising map. The weights of the neighbours j of the winning unit are adapted by

$\Delta wj = \eta\ j\ (x - wj)$, where $\Delta\eta\ j > \eta$ and $j \neq k$.

91. (D) The Crossover point of a fuzzy set is the element in U at which its membership function is 0.5.

92. (C) A = {(1, 0.2) (2, 0.5) (3, 0.8) (4, 1) (5, 0.7) (6, 0.3)}

Support (A) = $\{x \mid \mu_A(x) > 0\}$

That is for all elements 1, 2, 3, 4, 5, 6

93. (C) Here combination of x and y are

2 + 3 = (5)   2 + 4 = 6   3 + 4 = (7)

3 + 3 = 6

$\mu_{A\oplus B}(5) = v_{5=2+3}\ (\mu_A(2) \wedge \mu_B(3))$

$= v(1 \wedge 1) = 1$

$\mu_{A\oplus B}(6) = v_{\substack{6=2+4\\6=3+3}}\ (1 \wedge 0.5,\ 0.5 \wedge 1)$

$\mu_{A\oplus B}(7) = v_{7=3+4}\ (0.5,\ 0.5) = 0.5$

Thus, (5, 1) (6, 0.5) (7, 0.5)

94. (C) John is young

$\mu(30) = 0.9$

John is very young $\mu(30) = (0.9)^2 = 0.81$

95. (D) alpha-cut of a fuzzy set A will contain those elements where the membership function value is equal to or greater than alpha A. Here, alpha is given a value 0.4. Starting from (40,0.4) all the members have membership function equal or greater than 0.4. so, except (20,0) and (30,0.2) all the members are included in the alpha-cut of the fuzzy set. The only option which has 40,50,60,70, and 80 included is option D. It has (20,0) and (30,0) too. But it is already noted that any singleton where the membership function is 0 can be considered not included. So basically these two members are not part of the alpha-cut of the fuzzy set A. So the correct option is D.

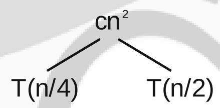
96. (C) The Crossover point of a fuzzy set is the element in U at which its membership function is 0.5

The Support of a fuzzy set F is the crisp set of all points in the Universe of Discourse U such that the membership function of F is non-zero.

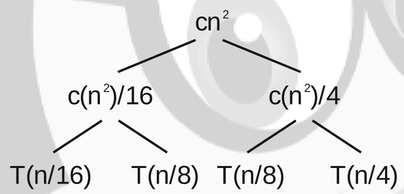
if membership is 1 then it is core of a fuzzy set alpha cut (or λ-cut) can be anything from 0 to 1 ($0 \le \lambda \le 1$)

$\mu A(x) = 0.5$

97. (B) Using recurrence tree method :



... breaking further



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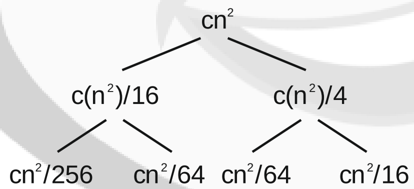

$T(n) = c(n^2 + 5(n^2)(16 + 25(n^2)/256) + ...$

The above is G.P. with ratio 5/16.

So, $T(n) = \dfrac{n^2}{1 - 5/6}$

$T(n) = O(n^2)$

98. (B) Using Master's Method.

$n = 2^m \Rightarrow m = \log n$

$T(2^m) = T(2^{m/2}) + 1$

Let $T(2^m) = S(m)$

So, $S(m) = 2S(m/2) + 1$

Using Master's Method

$S(m) = \theta(m)$

$= \theta(\log n)$

99. (A) $T(n) = cn + T(n/3)$

$= cn + cn/3 + T(n/9)$

$= cn + cn/3 + cn/9 + T(n/27)$

Taking the sum of infinite GP.

$$T(n) <= cn\left(\frac{1}{1-1/3}\right)$$

$<= 3cn/2$

or

$cn <= T(n) <= 3cn/2$

Thus, $T(n) = \theta(n)$

100. (A) Let $m = \log n$ or $n = 2^m$

The recurrence becomes,

$S(m) = S(m/2) + \theta(\log m)$

Using master's method,

$T(n) = \theta(\log \log n)^2$

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