

# Introduction to R

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## **Further information:**

<https://www.math.ethz.ch/sfs>

Scripts, Slides, Exercises and Datasets in  
<ftp://stat.ethz.ch/U/sfs/RKurs/R.Intro/>

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In this section we will have a look at ...

- ... R & RStudio

- ... some shortcuts

- ... basic syntax

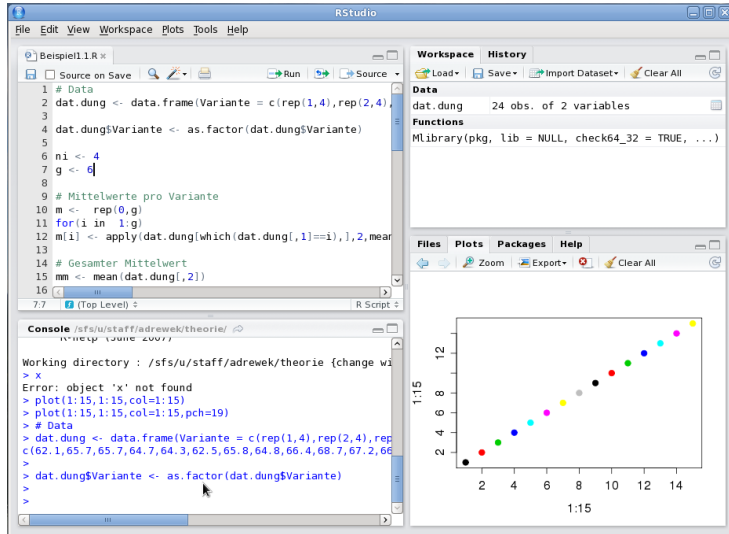
- ... where to find help

# What is R?

R is...

- **free**
- **open-source**
- **flexible**
- **expandable** with thousands of **add-on packages**:  
<http://cran.r-project.org/web/views/>
- **widely used** both in academia and industry
- teaser: <http://shiny.rstudio.com/gallery/>

# RStudio: Integrated Development Environment (IDE) for R



- Write your code in a script file inside the [source code editor](#)
- Execute R code by sending lines of code from editor to [R console](#)
- Add comments using the hashtag `#` (enables automatic sectioning of code in RStudio)
  - ▶ No lines of code are lost when R is shut down or crashes
  - ▶ Assures reproducible code/coding
  - ▶ Makes it easy to share your code with colleagues or reviewers

- **Send Code to Console:** Ctrl+Enter (Mac: Cmd+Enter) or Menu Button Run
- **Code Completion:** RStudio supports the automatic completion of code using the Tab key.
- **Previous Commands:** recall previous commands using the arrow key Up or Ctrl+Up
- **Console Title Bar:** current working directory and key to interrupt R during a long computation
- **Clear Console** with Ctrl+L
- **Assignments (<-)** with Alt+Minus



# R and RStudio: Basics

## R as a calculator

```
> sqrt(2)
```

```
[1] 1.4142
```

```
> x <- 3
```

```
> y <- x^2
```

```
> x + y
```

```
[1] 12
```

```
> sin(2 * pi)
```

```
[1] -2.4493e-16
```

## Creating vectors

```
> c(1, 5, 80)
```

```
[1] 1 5 80
```

```
> 2:11
```

```
[1] 2 3 4 5 6 7 8 9 10 11
```

```
> a <- c(1, 6, 10, 22, 7, 13)
```

```
> mean(a)
```

```
[1] 9.8333
```

```
> sum(a)
```

```
[1] 59
```

# R and RStudio: Basics

## Creating matrices and data frames

```
> matrix()
```

```
      [,1]  
[1,]   NA
```

```
> m <- matrix(1:6, nrow=3, ncol=2, byrow = TRUE)  
> m
```

```
      [,1] [,2]  
[1,]     1     2  
[2,]     3     4  
[3,]     5     6
```

```
> data.frame()
```

```
data frame with 0 columns and 0 rows
```

```
> df <- data.frame(Name = c("Ich","Du","Er"),  
+                 Geschlecht = as.factor(c(0,1,0)), Alter = c(21,47,33))  
> df
```

```
  Name Geschlecht Alter  
1  Ich           0    21  
2  Du            1    47  
3  Er            0    33
```

# R and RStudio: details

In the R console, you see the **prompt**: >

You type a command, get a result and a new prompt.

```
> 3 + 4
```

```
[1] 7
```

An incomplete statement can be **continued** on the next line

```
> 3 * (4 +  
+      2)
```

```
[1] 18
```

An incomplete statement is indicated by the **prompt**: +

- Check that prompt is > after error message before further code execution

# R and RStudio: details

An **R statement** may consist of

- an assignment:

```
> t.a <- 3 * (4 + 2)
> t.b <- t.a + 2.5
```

stores the result of the calculation under t.a resp. t.b.

- a name of an object: displays object

```
> t.a
[1] 18
```

- a call to a function: numerical or graphical result

```
> mean(c(t.a,t.b))
[1] 19.25
> mn <- mean(c(t.a,t.b))
```

A function is called by its name followed by ().

# Assignment and Workspace

- Everything in R is an object and has a certain **name** like `t.a`, `mean`, `mn`.
- R stores objects in your workspace
  - > `t.a <- 3 * (4 + 2)`
- ATTENTION: Overwriting an object in R throughs no warning
  - > `t.a <- t.b^2`
  - > `t.a`
  - [1] 420.25

# Where to Find Help?

- Help about any function `?foo`
- If you have any question, google 'how do I ... with R'.
  - ▶ huge community
  - ▶ already asked by somebody else
- Very useful and helpful Q&A website: <http://stackexchange.com/>
- Cheat Sheet for Base R  
<https://www.rstudio.com/resources/cheatsheets/>
- R Reference Card  
<https://cran.r-project.org/doc/contrib/Baggott-refcard-v2.pdf>
- **Learning by doing** is particularly true for programming

# Import Data Sets

In this section we will learn how to ...

- ... `import` data sets

- ... change the `working directory`



# Import Data: Website

We have a look at a data set with results of several disciplines.

Importing data from a website

```
> url <- "http://stat.ethz.ch/Teaching/Datasets/WBL/sport.dat"  
> d.sport <- read.table(url, header = TRUE)  
> head(d.sport)
```

	weit	kugel	hoch	disc	stab	speer	punkte
OBRIEN	7.57	15.66	207	48.78	500	66.90	8824
BUSEMANN	8.07	13.60	204	45.04	480	66.86	8706
DVORAK	7.60	15.82	198	46.28	470	70.16	8664
FRITZ	7.77	15.31	204	49.84	510	65.70	8644
HAMALAINEN	7.48	16.32	198	49.62	500	57.66	8613
NOOL	7.88	14.01	201	42.98	540	65.48	8543

# Import Data: Working Directory

Let's set the working directory - the folder from which to operate (e.g., save and load from). Use:

```
> getwd() ## print the current working directory  
> setwd("/userdata/Documents/Rcourse/")
```

Or alternatively in RStudio use 'Session' → 'Set Working Directory' → 'Choose Directory...'

# Import Data: File

Importing data in R is easy

- Different ways depending on the format (csv, txt, xlsx, etc.).
- **Alternative:** use the 'Import Dataset' tool in RStudio (upper-right panel)

```
> d.sport <- read.table(file = "sport.dat", header = TRUE)
> head(d.sport)
```

# Save Data or Write Data to a File

To save or write data to a file:

- Text-files:

```
> write.table(x, file = "xy.txt", sep = " ")
```

where `x` is the data object to be stored and `xy.txt`

- Excel-files: Use CSV

```
> write.csv(...)
```

```
> write.csv2(...)
```

In this section we will have a look at ...

- ... R objects

- ... how indexing works

# Introductory Example: R Objects

## R Objects:

- data frame: most essential data structure in R

```
> str(d.sport)
```

```
'data.frame':      15 obs. of  7 variables:
 $ weit   : num  7.57 8.07 7.6 7.77 7.48 7.88 7.64 7.61 7.27 7.49 ...
 $ kugel  : num  15.7 13.6 15.8 15.3 16.3 ...
 $ hoch   : int  207 204 198 204 198 201 195 213 207 204 ...
 $ disc   : num  48.8 45 46.3 49.8 49.6 ...
 $ stab   : int  500 480 470 510 500 540 540 520 470 470 ...
 $ speer  : num  66.9 66.9 70.2 65.7 57.7 ...
 $ punkte : int  8824 8706 8664 8644 8613 8543 8422 8318 8307 8300 ...
```

- vector, e.g. a column of the data set d.sport

```
> kugel <- d.sport$kugel
```

```
> str(kugel)
```

```
num [1:15] 15.7 13.6 15.8 15.3 16.3 ...
```

```
> participant <- rownames(d.sport)
```

```
> str(participant)
```

```
chr [1:15] "OBRIEN" "BUSEMANN" "DVORAK" "FRITZ" "HAMALAINEN" "NOOL" ...
```

# Introductory Example: Select Elements I

Select elements:

```
> participant[4]
```

```
[1] "FRITZ"
```

```
> d.sport[c(3, 6, 4), c(1:3, 7)]
```

	weit	kugel	hoch	punkte
DVORAK	7.60	15.82	198	8664
NOOL	7.88	14.01	201	8543
FRITZ	7.77	15.31	204	8644

```
> d.sport["FRITZ", ]
```

	weit	kugel	hoch	disc	stab	speer	punkte
FRITZ	7.77	15.31	204	49.84	510	65.7	8644

# Accessing Parts of an Object

Goal is to look at or use a part of your object

To access only part of an object, use `[ ]`:

- for vectors: `myvector[x]`
- for two-dimensional objects, e.g. data frames or matrices:  
`mydata.frame[x, y]`

Specify the indices by a vector (e.g. `c(1, 2, 6)`) and separate the indices of different dimensions by commas



# Introductory Example: Select Elements II

Let's play around with the indexing of a data frame: **two-dimensional object!**

```
> d.sport[ , ]  
> c(1, 3, 7)  
> 1:10  
> d.sport[1:10 , ]  
> d.sport[-c( 1, 3, 7), ] # negative indices are excluded  
> d.sport[ , 2:3]  
> d.sport[c(1, 3, 6), 2:3]
```

In this section we will have a look at ...

- ... basic function calls
- ... mandatory and optional arguments
- ... R packages

# Introductory Example: Functions

Example function calls:

```
> mean(d.sport$kugel)
```

```
[1] 15.199
```

```
> quantile(d.sport$kugel)
```

```
 0%   25%   50%   75%  100%  
13.53 14.60 15.31 15.74 16.97
```

```
> quantile(d.sport$kugel, probs = c(0.75, 0.9))
```

```
 75%   90%  
15.740 16.674
```

Check out the function's help file:

```
> ?mean
```

- Functions consist of **mandatory** and **optional arguments**:

```
mean(x, trim = 0, na.rm = FALSE, ...)
```

**x**: mandatory argument

**trim**: optional argument, default is 0

**na.rm**: optional argument, default is FALSE

# Function Call

The arguments of a function have a defined order and each argument has its own unique name.

```
> mean(x = d.sport$kugel, na.rm = TRUE)
```

You can either use the names of the arguments, or place the values in the correct order (or a mix of both):

```
> mean(d.sport$kugel, ,TRUE)
```

Example functions with no mandatory arguments: `matrix()`, `vector()`, `array()`, `list()`

```
> ?matrix
```

# Useful Functions

Useful functions (look for help by typing `?str`):

- `str()`
- `nrow()` and `ncol()`
- `dim()`
- `summary()`
- `apply()`
- `head()` and `tail()`

see also R Reference card

By default, R only provides a basic set of functions. Additional functions (and datasets) are obtained by loading additional **add-on packages**:

- Install and load:

```
> install.packages("MASS") # install onto computer once  
> require(MASS) # for every R session. Alternatively:  
> library(MASS)
```

- Online resources:

- ▶ list of all packages: <http://cran.r-project.org/web/packages/>
- ▶ by topic: <http://cran.r-project.org/web/views/>
- ▶ ask Google

In this section we will have a look at ...

... missing values



# Missing Values

Very common with real data. Let's fake the situation for the d.sport data.

```
> d.sport.NA <- d.sport
> d.sport.NA[2, "kugel"] <- NA
> d.sport.NA[3, "hoch"] <- -999
```

In R, missing values are coded as NA (not available) and are treated in a special way, e.g. `is.na()`:

```
> is.na(d.sport.NA) # one logical value per element
> sum(is.na(d.sport.NA)) # adds up the TRUE elements
```

```
[1] 1
```

```
> which(is.na(d.sport.NA), arr.ind = TRUE) # where are the NA's?
```

```
      row col
BUSEMANN  2  2
```

# Missing Values

Specify missing values while reading in the data, or afterwards:

```
> d.sport.NA[d.sport.NA == -999] <- NA  
> ## alternatively, while reading in the data:  
> d.sport <- read.csv("sport.dat", header = TRUE, na.strings = c("NA", "-999"))
```

Many functions have an argument to handle missing values, e.g. `na.rm`, `na.omit`:

```
> sum(d.sport.NA$kugel)  
> sum(d.sport.NA$kugel, na.rm = TRUE)
```

Drop observations (rows) that contain an NA:

```
> na.omit(d.sport.NA)
```

In this section we will have a look at ...

- ... basic [plot functions](#)

- ... [alternative R packages](#) for graphics

# Iris Data

R comes with built-in data sets, e.g. `iris`:

```
> data(iris) # now it shows up in the RStudio environment
> iris
```

It has the following structure

```
> str(iris)

'data.frame':      150 obs. of  5 variables:
 $ Sepal.Length: num  5.1 4.9 4.7 4.6 5 5.4 4.6 5 4.4 4.9 ...
 $ Sepal.Width : num  3.5 3 3.2 3.1 3.6 3.9 3.4 3.4 2.9 3.1 ...
 $ Petal.Length: num  1.4 1.4 1.3 1.5 1.4 1.7 1.4 1.5 1.4 1.5 ...
 $ Petal.Width : num  0.2 0.2 0.2 0.2 0.2 0.4 0.3 0.2 0.2 0.1 ...
 $ Species      : Factor w/ 3 levels "setosa","versicolor",...: 1 1 1 1 1 1 1 1 1 1
```

A factor is another class of values in R, representing categorical values with different “levels”. Internally it is coded with numbers (that are used to build dummy-variables in statistical models).

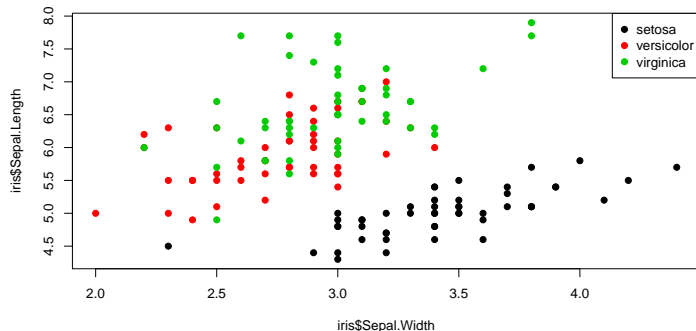
# The plot Function

The `plot` function has only one mandatory argument which is `x`.

- second most important argument: `y`
- many optional arguments (`col`, `pch`, `main`, `cex`,...)
- check `?plot` and google!
- use function `par` (`?par`) to set or query graphical parameters

# The plot Function

```
> # High-level plotting function: opens a plot
> plot(x = iris$Sepal.Width, y = iris$Sepal.Length,
+      col = iris[, "Species"], pch = 19)
> # Low-level plotting functions: add to an existing plot
> legend("topright", legend = levels(iris[, "Species"]),
+      pch = 19, col = 1:3) # add the legend
```



# The plot Function

Let's look at the previous function call step-by-step:

- execute only parts
- omit arguments one-by-one

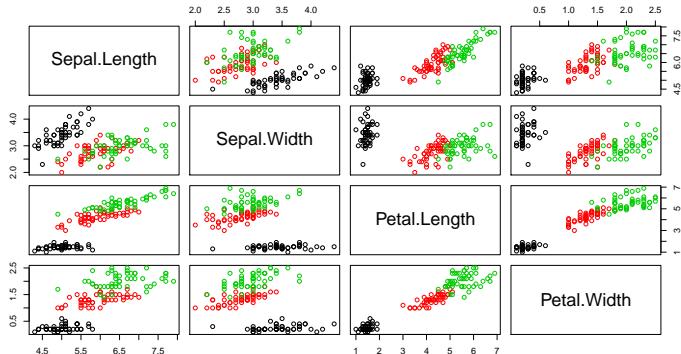
Alternatively one could use formula notation:

```
> # same plot as on the previous slide without a legend  
> plot(Sepal.Length ~ Sepal.Width,  
+       col = Species, pch = 19, data = iris)
```



# Which plot did we look at just before?

```
> pairs(iris[, -5], col = iris[, 5])
```



# Arguments of plot

Statement	Meaning
type	Style of drawing (single points, lines etc.)
log	logarithmic scale
xlim	range of $x$ -coordinates
ylim	range of $y$ -coordinates
pch	Plotting character
col	Coloring points
lty	line type
lwd	line width
main	main title (appears above the plot)
xlab	label of $x$ -axis
ylab	label of $y$ -axis

# The three categories of graphics functions

Three categories of *R graphics functions*:

- **High-level plotting functions** such as `plot()`  
*to generate a new graphical display.*
- **Low-level plotting functions** such as `legend()`  
*to add further graphical elements to an existing plot.*
- **Interactive functions** such as `identify()`  
*to amend or collect information interactively from a plot.*

# Low-level Plotting Functions

Statement	Meaning
<code>points(x, y, pch = 1)</code>	Draws points pictured as pch.
<code>text(x, y, text)</code>	Writes text at coordinate (x, y).
<code>lines(x, y, lty = 1)</code>	Adds a line to graph.
<code>abline(a, b)</code>	Adds a line with intercept a and slope b.
<code>abline(h = y, v = x)</code>	Horizontal and vertical lines.
<code>legend(x, y, text, lty, pch)</code>	Creates a legend.

# Useful Plot Functions

Useful functions (look for help by typing `?foo`):

- `plot`, `pairs`, `interaction.plot`
- `boxplot`, `hist`
- `plot3d`

# Graphical Output

It is easy to export R graphics. Remember to specify your working directory!

```
> pdf(file = "iris_plot.pdf") # open the graphics device
> plot(Sepal.Length ~ Sepal.Width, data = iris)
> ## add anything else you want in your plots, and when you are done, use:
> dev.off() # close the graphics device
```

Several plots in one graphical window

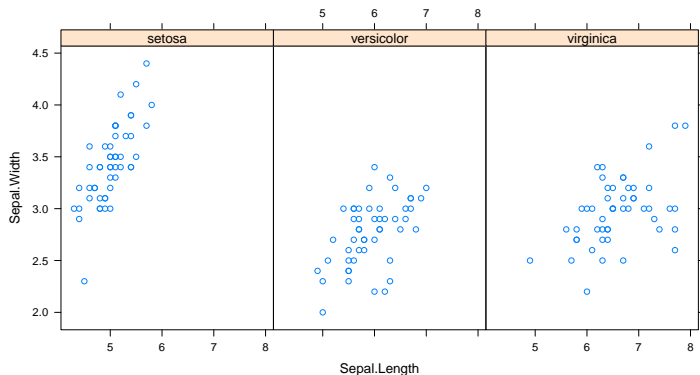
```
> par(mfrow=c(3, 2))
```

splits the graphical window into 3 rows and 2 columns.

# Newer Graphics: lattice

lattice package functions: excel at repeating graphs for various groups

```
> library(lattice)  
> xyplot(Sepal.Width ~ Sepal.Length | Species, data = iris)
```

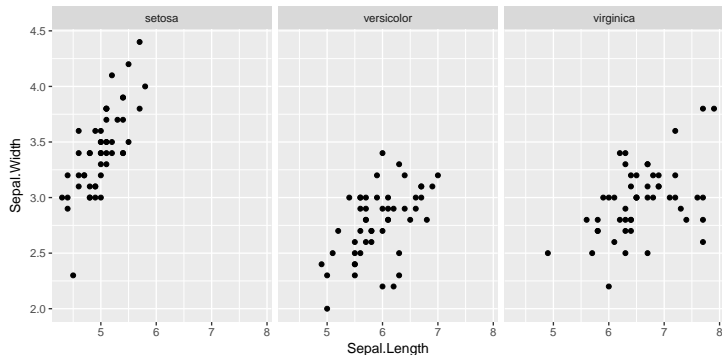


See <http://www.statmethods.net/advgraphs/trellis.html> for more information.

# Newer Graphics: ggplot2

ggplot2 package: very flexible, based on grammar of graphics

```
> library(ggplot2)
> ggplot(data = iris, aes(x = Sepal.Length, y = Sepal.Width)) +
+   facet_grid(facets = ~ Species) + geom_point()
```



More information can be found on <http://ggplot2.org>.



# Hypothesis Tests

In this section we will have a look at ...

- ... t-Test and Wilcoxon rank-sum test

- ... Chi-square test

# Hypothesis Tests - Approach

Hypothesis testing in 6 steps:

- ➊ Declare model by which data were generate (e.g. population is normally distributed, large sample size and  $\sigma$  not known)
- ➋ Define **null hypothesis**  $H_0^*$  and **alternative hypothesis**  $H_A^{**}$ 
  - ▶ *\*The statement being tested in a test of [statistical] significance. Test the strength of the evidence against the null hypothesis.*
  - ▶ *\*\* The statement that is hoped or expected to be true instead of the null hypothesis.*
- ➌ Choose the level of significance  $\alpha$
- ➍ Determine critical values for the level of significance  $\alpha$  and degrees of freedom  $df = (n - 1)$
- ➎ Define and calculate test statistic, e.g. one-sample test,  $t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}}$
- ➏ Compare the test statistic to the critical values and make decision: **reject or fail to reject**  $H_0$

# Hypothesis Tests - an Example

Is the sepal length of versicolor different to that of virginica? Let's use a t-Test and a Wilcoxon rank-sum test.

We first subset the iris data set:

```
> testdata <- iris[iris$Species != "setosa", c("Sepal.Length", "Species")]
> testdata$Species <- droplevels(testdata$Species)
> str(testdata) ## prepare and check data

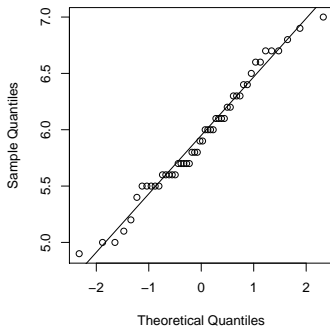
'data.frame':      100 obs. of  2 variables:
 $ Sepal.Length: num  7 6.4 6.9 5.5 6.5 5.7 6.3 4.9 6.6 5.2 ...
 $ Species      : Factor w/ 2 levels "versicolor","virginica": 1 1 1 1 1 1 1 1 1 1
```

# Hypothesis Tests - an Example

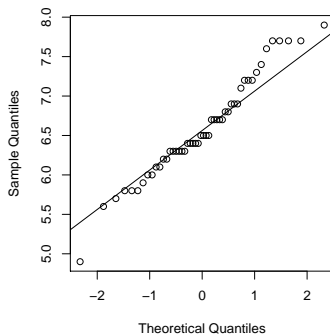
Check normality assumption of t-Test using QQ-Plot:

```
> versi.id <- testdata$Species == "versicolor"  
> par(mfrow=c(1,2))  
> qqnorm(testdata$Sepal.Length[versi.id]); qqline(testdata$Sepal.Length[versi.id])  
> qqnorm(testdata$Sepal.Length[!versi.id]); qqline(testdata$Sepal.Length[!versi.id])
```

Normal Q-Q Plot



Normal Q-Q Plot



# Hypothesis Tests - an Example

## Two sample t-test:

```
> versi.id <- testdata$Species == "versicolor"
> t.test(x = testdata$Sepal.Length[versi.id],
+       y = testdata$Sepal.Length[!versi.id],
+       var.equal = TRUE )
```

### Two Sample t-test

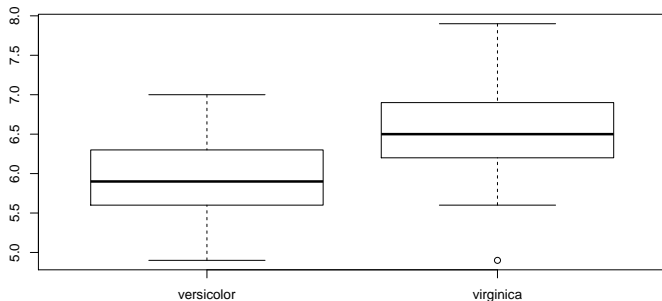
```
data: testdata$Sepal.Length[versi.id] and testdata$Sepal.Length[!versi.id]
t = -5.63, df = 98, p-value = 1.7e-07
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.88185 -0.42215
sample estimates:
mean of x mean of y
  5.936    6.588
```

t-test rejects the null hypothesis at 5% significance level. Don't forget to visually check the normality assumptions (QQ-Plot).

# Hypothesis Tests - an Example

Check assumption of Wilcoxon Rank Test:  
same distribution, just a location shift

```
> boxplot(testdata$Sepal.Length ~ testdata$Species)
```



# Hypothesis Tests - an Example

Now, perform the test

```
> versi.id <- testdata$Species == "versicolor"
> wilcox.test(x = testdata$Sepal.Length[versi.id],
+            y = testdata$Sepal.Length[!versi.id])
```

Wilcoxon rank sum test with continuity correction

data: testdata\$Sepal.Length[versi.id] and testdata\$Sepal.Length[!versi.id]

W = 526, p-value = 5.9e-07

alternative hypothesis: true location shift is not equal to 0

Wilcoxon Rank Test also rejects the null hypothesis at 5% significance level. Wilcoxon Rank Test is the preferred test for a two-samples statistical test!

# Hypothesis Tests

How to proceed:

- Formulate null and alternative hypotheses
- Choose appropriate test
- Collect data, i.e. do an experiment
- Look at data:  
`plot()`, `pairs()`, `hist()`, `boxplot()`
- Validate assumptions for test
- Carry out the test and interpret result



# Hypothesis Tests

	1 sample / 2 dep. samples	2 indep. samples
parametric	t-Test → normality	t-Test → normality (& equal variance)
non-param.	Wilcoxon Test (signed rank) → symmetric distribution	Wilcoxon Test (rank sum) → location shift

# Hypothesis Tests - Chi-Squared Test of Independence

Hypothesis:

$H_0$  : Independence of education and marriage status

$H_A$  : Dependence of education and marriage status

Example:

Education	Married once	Married > 1	Total
College	550	61	611
No College	681	144	825
Total	1231	205	1436

# Chi-Squared Test of Independence

```
> url <- "http://stat.ethz.ch/Teaching/Datasets/edu.txt"
> d.edu <- read.table(url, header = TRUE)
```

## Cross-tables in R:

- Count number of cases with same value:

```
> table(d.edu[, "Married"])
```

Married	more	Married	once
205		1231	

- Cross-table

```
> table(d.edu[, "Education"], d.edu[, "Married"])
```

	Married	more	Married	once
College	61		550	
No College	144		681	

# Chi-Squared Test of Independence

Now we perform the Chi-squared test:

```
> chisq.test(d.edu[, "Education"], d.edu[, "Married"])
```

Pearson's Chi-squared test with Yates' continuity correction

data: d.edu[, "Education"] and d.edu[, "Married"]

X-squared = 15.4, df = 1, p-value = 8.7e-05

Result: Reject  $H_0$ , i.e. education and marriage are dependent.

In this section we will have a look at ...

- ... correlation

- ... why you should plot your data

# Correlation

Load a new data set:

```
> url <- "http://stat.ethz.ch/Teaching/Datasets/basischOhneNA.dat"
> d.basisch <- read.table(url, header = TRUE)
```

Look at the structure of the object:

```
> str(d.basisch)
```

```
'data.frame':      123 obs. of  4 variables:
 $ ph      : num  7.33 7.69 7.9 8.14 7.62 ...
 $ l.sar   : num  0.0969 0.4393 1 1.316 0.0607 ...
 $ height: num  5.91 5.2 4.4 4.5 6.05 6 5.35 5.55 4.95 5.2 ...
 $ h.quad: num  34.9 27 19.4 20.2 36.6 ...
```

- ph: pH value of soil around the tree
- l.sar: log(sodium absorption ratio)
- height: height of tree
- h.quad: square of height of tree

# Correlation

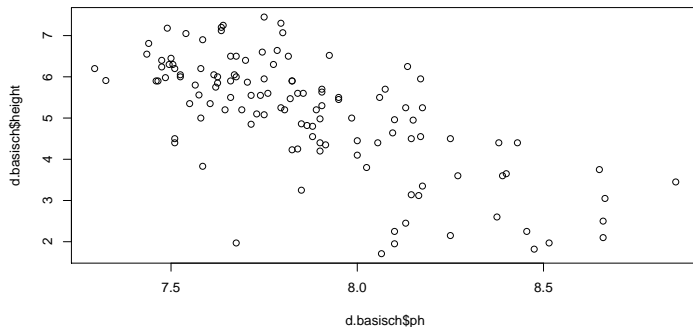
Calculate the (Pearson) correlation of ph and height

```
> cor(d.basisch$ph, d.basisch$height)
```

```
[1] -0.69257
```

Corresponding plot:

```
> plot(d.basisch$ph, d.basisch$height)
```



# Correlation

All plots show two variables with a correlation of 0.7!

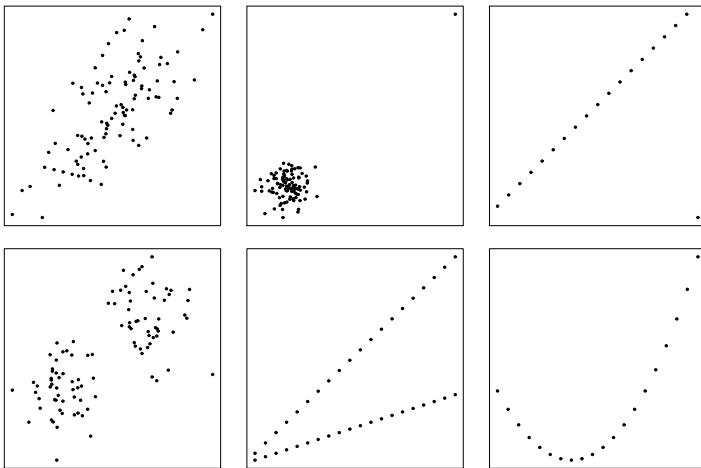


Figure: from the script Regression by Prof. H.R. Künsch



In this section we will have a look at ...

- ... fitting a **simple linear regression** model
- ... checking **model assumptions**
- ... fitting a **multiple linear regression** model

We continue working with the data set `d.basisch` from the correlation section.

Response variable:

- `height` or `h.quad`: Height of trees or squared height, respectively.

Possible explanatory variables:

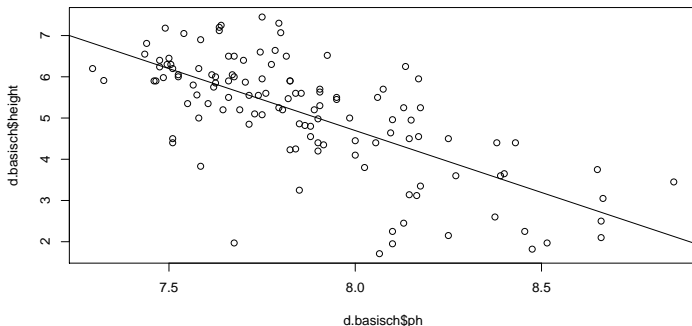
- `ph`: PH-Values of soil,
- `l.sar`:  $\log(\text{sodium absorption ratio})$

# Simple Linear Regression

The simple linear regression model is

$$Y_i = \alpha + \beta x_i + E_i, \quad E_i \stackrel{i.i.d}{\sim} \mathcal{N}(0, \sigma^2)$$

Let us pick the variable `ph` as the explanatory variable.



# Simple Linear Regression

Fit to data using `lm`:

```
> fit <- lm(formula = height ~ ph, data = d.basisch)
```

Show a summary of fit:

```
> summary(fit)
```

Call:

```
lm(formula = height ~ ph, data = d.basisch)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.7020	-0.5471	0.0874	0.6663	2.0033

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	28.7227	2.2395	12.82	<2e-16 ***
ph	-3.0034	0.2844	-10.56	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.008 on 121 degrees of freedom

Multiple R-squared: 0.4797, Adjusted R-squared: 0.4754

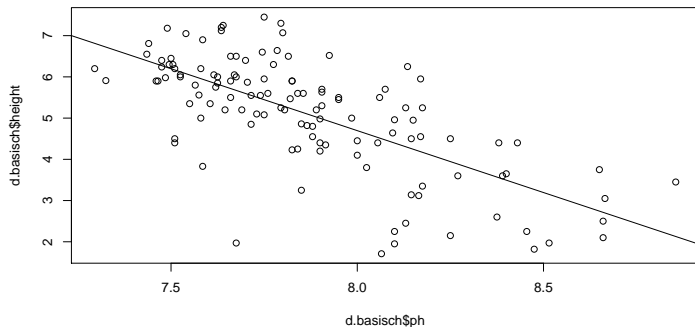
F-statistic: 111.5 on 1 and 121 DF, p-value: < 2.2e-16

# Simple Linear Regression

Estimated Equation:  $\text{height} = 28.7 - 3.0 \text{ pH}$

Drawing line into scatterplot:

```
> plot(d.basisch$ph, d.basisch$height)
> abline(fit)
```



# Simple Linear Regression - Residual Analysis

Diagnostics plots are straightforward:

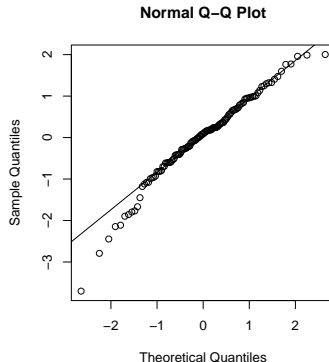
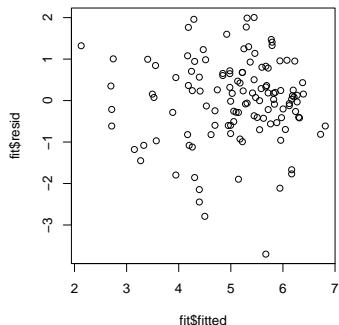
```
> par(mfrow = c(2, 2))  
> plot(fit)
```

- Tukey-Anscombe plot (is the variance of the errors  $E_i$  constant? Is the regression function correct?)
- Q-Q plot (are the errors  $E_i$  normally distributed?)
- Scale Location plot (similar to Tukey-Anscombe plot.)
- Leverage plot (what points have a strong influence on the fit?)

# Simple Linear Regression

Residual plots by hand:

```
> par(mfrow = c(1,2))  
> plot(fit$fitted,fit$resid)      # Tukey-Anscombe  
> qqnorm(fit$resid)             # Quantil-Quantil plot  
> qqline(fit$resid)             # adds the diagonal line
```



# Simple Linear Regression

Some Tukey-Anscombe plots:

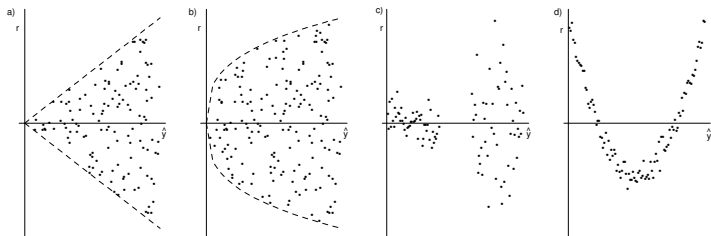


Figure: from the lecture notes "Regression" by Prof. H.R. Künsch



# Multiple Linear Regression

Expand the simple linear model to more than one explanatory variable.

$$Y_i = \alpha + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + E_i, \quad E_i \stackrel{i.i.d}{\sim} \mathcal{N}(0, \sigma^2)$$

Fit the model with `lm`:

```
> fitm <- lm(height ~ ph + l.sar, data = d.basisch)
```

# Multiple Linear Regression

```
> summary(fitm)
```

Call:

```
lm(formula = height ~ ph + l.sar, data = d.basisch)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.1314	-0.4911	0.0849	0.6488	2.4754

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	26.9466	2.7445	9.818	< 2e-16 ***
ph	-2.7558	0.3603	-7.649	5.6e-12 ***
l.sar	-0.2519	0.2255	-1.117	0.266

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.007 on 120 degrees of freedom

Multiple R-squared: 0.485, Adjusted R-squared: 0.4764

F-statistic: 56.51 on 2 and 120 DF, p-value: < 2.2e-16

# Multiple Linear Regression

## Residual analysis:

- Look at the same plots as for simple linear regression.
- It may help to plot the explanatory variables against the residuals.

```
> plot(fitm)
```

```
> plot(d.basisch$ph, fitm$resid)
```

```
> plot(d.basisch$l.sar, fitm$resid)
```

# Going further

In this section we will briefly look at ...

... how to **report** your results

# Report your results

**Knitr**: awesome tool to [embed your R code and results](#) (including tables and plots) in a nicely looking document.

<http://yihui.name/knitr/>

**Rmarkdown** can be used to produce reports in any of these formats:

- PDF
- HTML
- slides
- Word document (if your collaborators require it)

<http://rmarkdown.rstudio.com>

**Book** about reproducible research:

<http://christophergandrud.github.io/RepResR-RStudio/>

Useful [cheatsheets for R](#) can be found here:

<https://www.rstudio.com/resources/cheatsheets/>