COMMUNICATION-AWARE HIGHER ORDER MOBILE COVERAGE COTROL

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ECE-699: Network Control

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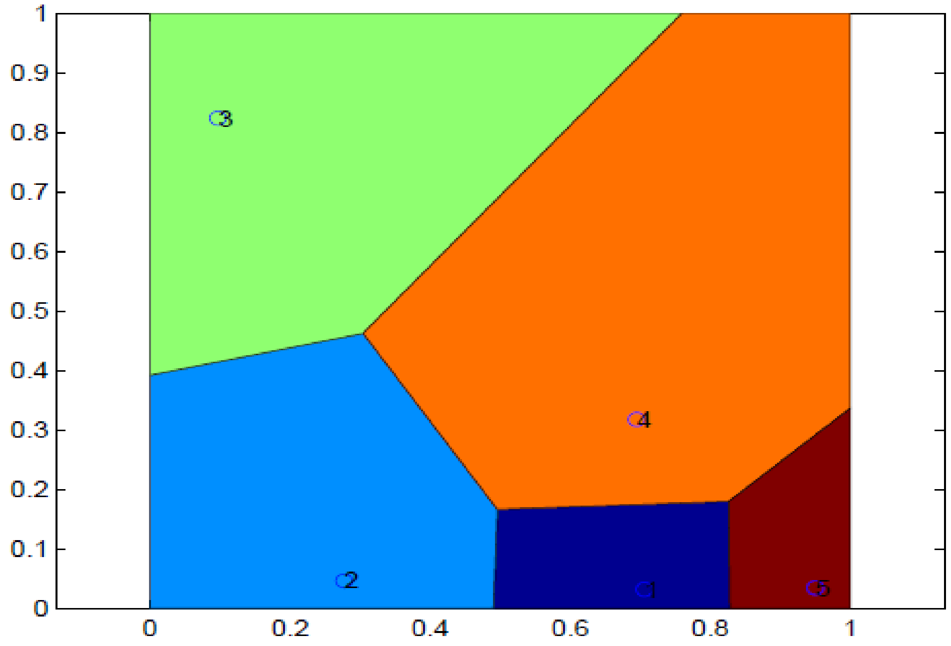
WHAT IS COVERAGE CONTROL…?

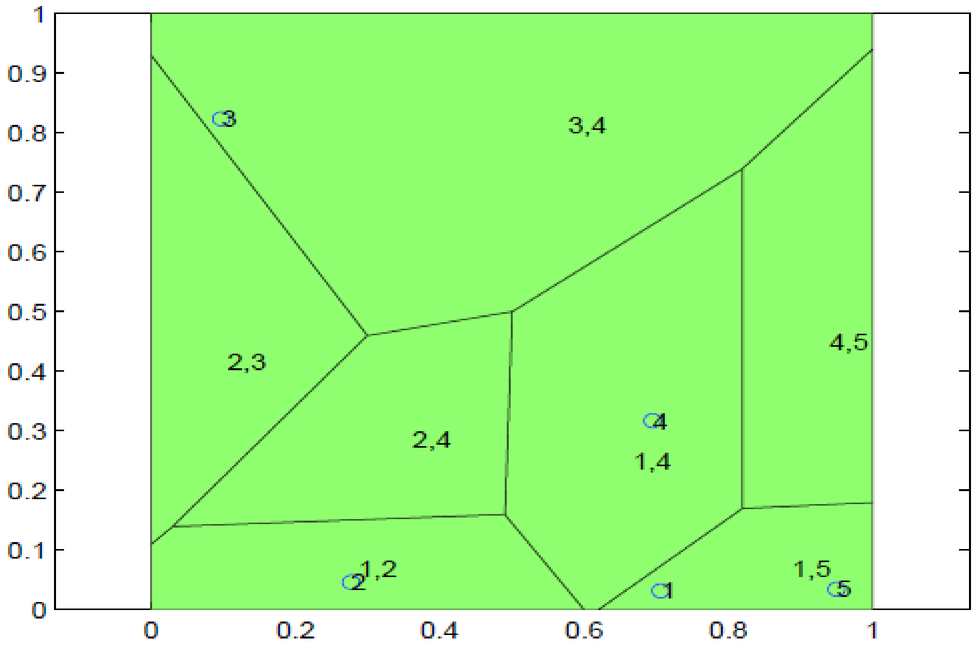
- As technology is advancing, the probability of deploying a large group of autonomous vehicles/agents is becoming possible.
- Fundamental problem- 'Where to place each vehicle/agent?'
- Maximum quality of service from each sensor/agent/vehicle.
- Coverage performance function in terms of quality of service of a mobile sensing agent and the density of occurrence events detected by the sensors/agents.
- Optimal positioning of agents to cover an area in a way that some predefined coverage performance function can be optimized.

VORONOI PARTITIONS

❖ Usually solved by using the geometry tool of Voronoi partitions.



Fig(1): One agent responsible for single cell



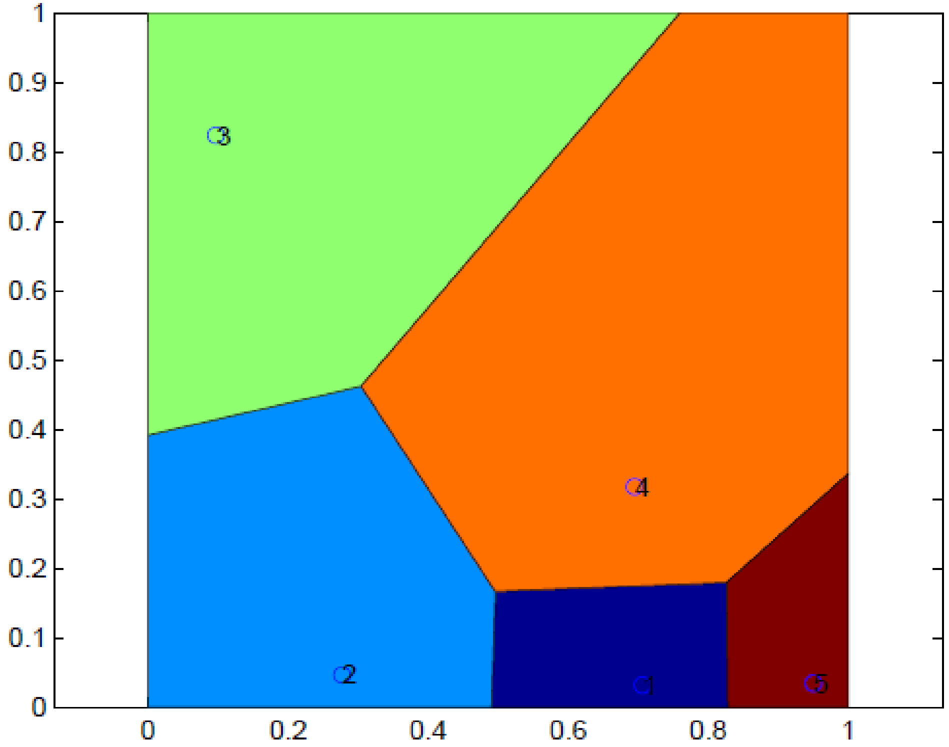
Fig(2): Two agents responsible for single cell

VORONOI PARTITIONS…….(cont.)

Let $(p_1, \dots, p_n) \in S^n$ denote the positions of n points.

The Voronoi partition $V(P) = \{V_1, \dots, V_n\}$ generated by $(p_1, \dots, p_n)$ is

$$V_i = \{q \in S \mid \| q - p_i \| \le \| q - p_j \|, \forall j \neq i\}$$

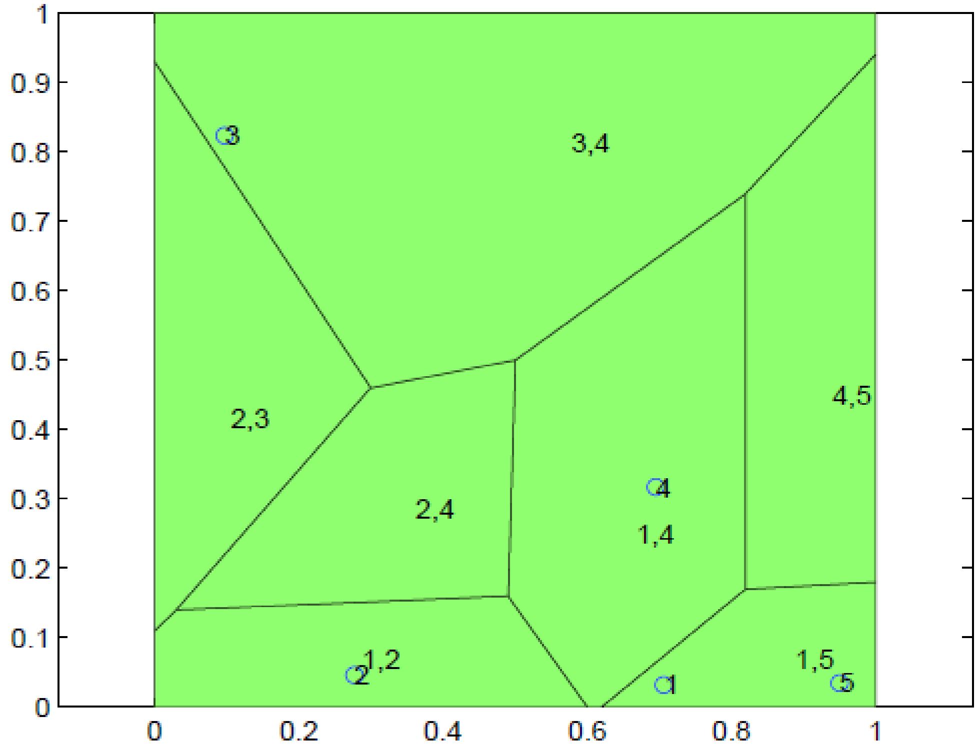


VORONOI PARTITIONS…….(cont.)

Let $(p_1, \dots, p_n) \in S^n$ denote the positions of n points. Suppose $'\tau'$ is a subset of sensor/agents position set with cardinality 2.

The Voronoi partition $V(P) = \{V_1, \dots, V_n\}$ generated by $(p_1, \dots, p_n)$ is

$$V(\tau) = \{q \in S | \forall v \in \tau, \forall w \in P \setminus \tau \, \| (q, v) \| \leq \| (q, w) \|, |\tau| = 2\}$$



WHY WE NEED COVERAGE/OPTIMAL POSITIONING..?

- **Environmental Monitoring** : Autonomous Oceanographic Sampling network in which Autonomous under water vehicles(AUV) measures temperature, currents and other distributed oceanographic signals. They communicate and coordinate their motion in response to evolving data
- **Servicing**: Emergency response system like 911, where the police cars need to be deployed optimally by themselves throughout the city to minimize the expected time to service a call.
- **Data Collection**: Reconnaissance by scout robots in the military where they need to be optimally deployed to sense data and report the findings.
- **Surveillance**: Border patrol by UAVs where they are optimally positioned in order to maximize the probability of detection.

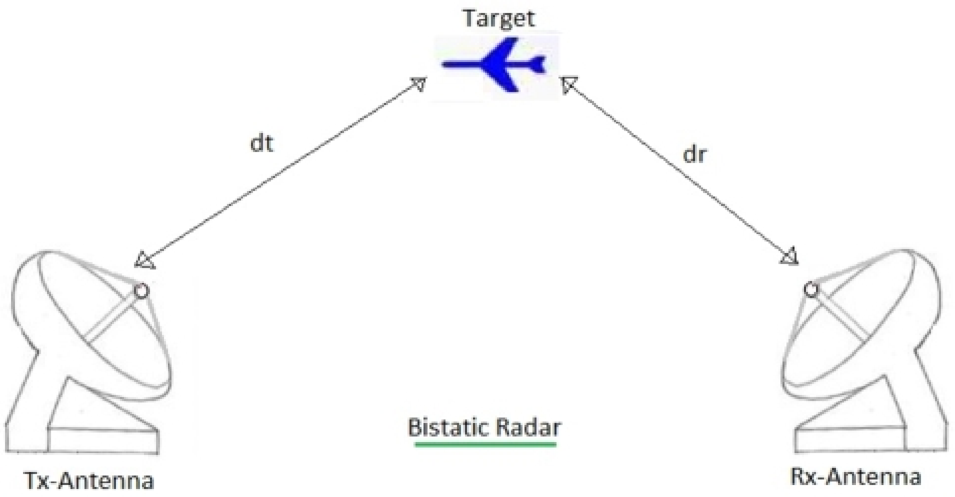
WHY HIGHER ORDER COVERAGE..?

❖ Robustness.

❖ Some real world applications requires more than one sensor/robot/agent to monitor a location.

❖ **Bi-static radar**: Deployment of the bi-static radar requires placement of transmitter and receiver at different locations but reasonably close to a potential target to have satisfactory detection.

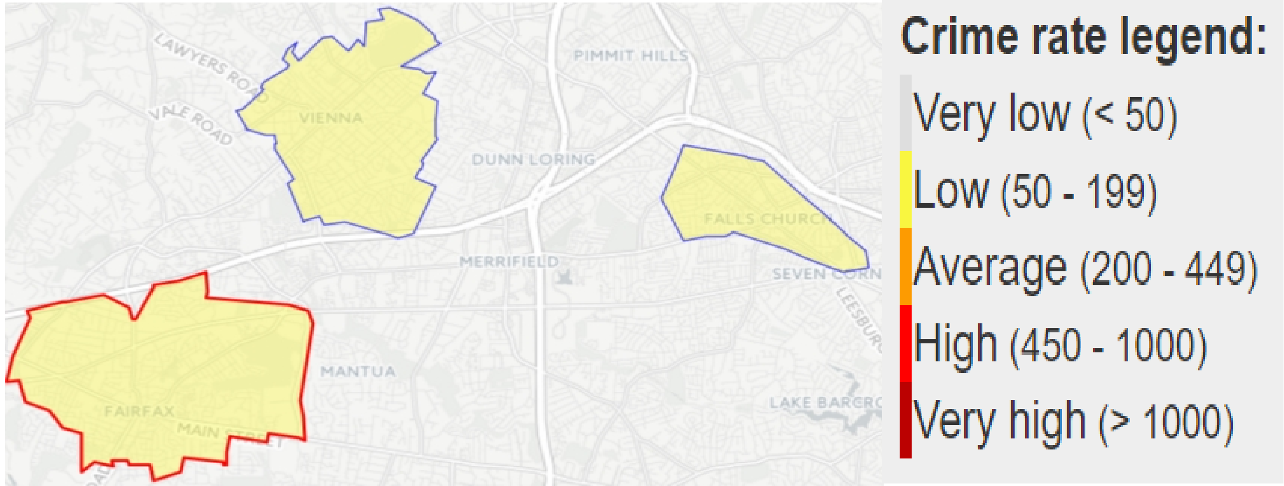
❖ **Geolocalization:** At least three TDOA sensors at different noncollinear locations are required to localize an event.



PROBLEM FORMULATION

Objective: Given n sensors/robots/agents moving in an environment, we want to achieve *higher order optimal coverage* in which more than one sensor is responsible for single cell or partition.

$\dot{p}_i = u_i, \qquad \|u_i\|_2 \le v_{max}$



$\phi: \mathbb{R}^d \rightarrow \mathbb{R}_{\geq 0}$ crime density

Minimize $H(p_1, \dots, p_n) = \int_Q \min_{(i,j)\in C} f\big(\|q, p_i\|, \|q, p_j\|\big)\phi(q)\, dq$

where $C = \{i, j \mid i, j \in \{1, \dots, n\}, i < j\}$

WHAT IS $f(\cdot,\cdot)$...??

- $f(\cdot,\cdot)$ indicates the measurement quality of a point q by a pair of agents.
- Generally the quality of sensing at a point, for e.g. q, decreases with increase in its distance from the sensing agent.
- $f(\cdot,\cdot) \to f\left(\|q,p_i\|,\|q,p_j\|\right)$
- $f(\cdot,\cdot)$ should be differentiable and also should have following properties:

1) $\frac{\partial}{\partial\|q,p_i\|} f(\|q,p_i\|,\|q,p_j\|) \geq 0$

2) $\frac{\partial}{\partial\|q,p_j\|} f(\|q,p_i\|,\|q,p_j\|) \geq 0$, and

3) $f(\|q,p_i\|,\|q,p_j\|) = f(\|q,p_j\|,\|q,p_i\|)$

- Some possible $f(\cdot,\cdot)$s are:
  - $\|q,p_i\| + \|q,p_j\|$
  - $\|q,p_i\|^2 + \|q,p_j\|^2$
  - $(\|q,p_i\|^n + \|q,p_j\|^n)^{\frac{1}{n}}$

CONTINUOUS-COMMUNICATION CONTROL SOLUTION

- To minimize the performance function, we use a gradient- descent control law for each sensor.

$$u_i = -\frac{\partial H}{\partial p_i} \quad \Rightarrow \quad \dot{p}_i = -\frac{\partial H}{\partial p_i}$$

- Using higher order Voronoi partition, the performance function can be transformed as

$$H(p_1, \dots, p_n) = \int_Q \min_{(i,j)\in C} f\big(\|q, p_i\|, \|q, p_j\|\big)\phi(q)\, dq$$

$$= \sum_{\forall T_{ij}\subset S} \int_{V_{T_{ij}}} f\big(\|q, p_i\|, \|q, p_j\|\big)\phi(q)\, dq$$

Where T_{ij} means $p_i, p_j \in T_{ij}$ and $(i, j) \in C$

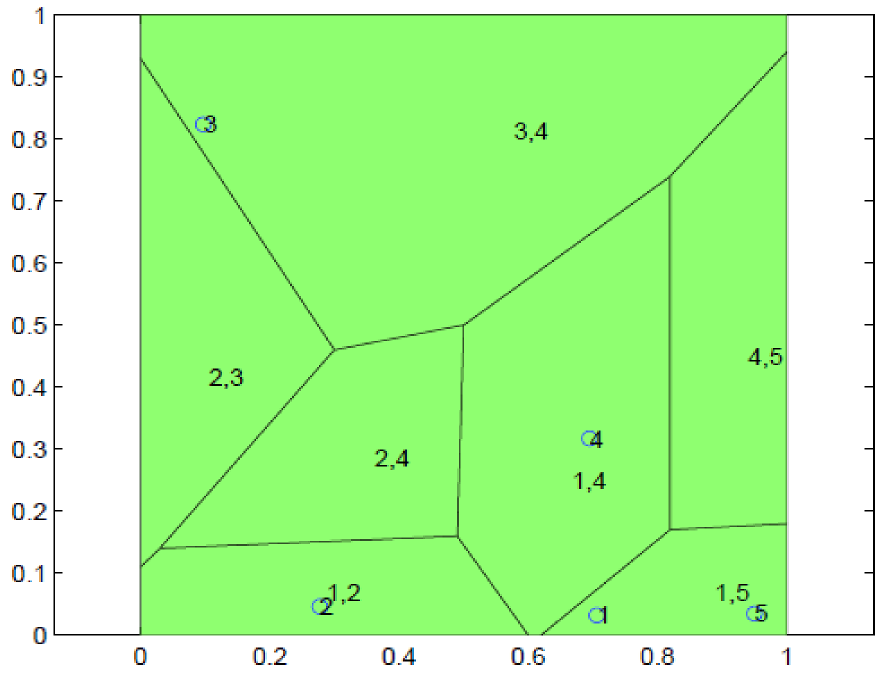
- Implementation of above controller requires each agent to know the position of its neighboring agents.

CONTINUOUS-COMMUNICATION CONTROL SOLUTION

- For a Voronoi cell ($V_{\mathcal{T}_{ij}}$) the Centroid ($C_{V_{\mathcal{T}_{ij}}}$) and mass ($M_{V_{\mathcal{T}_{ij}}}$) are:

$$C_{V_{\mathcal{T}_{ij}}} = \frac{\int_{V_{\mathcal{T}_{ij}}} q\phi(q)dq}{\int_{V_{\mathcal{T}_{ij}}} \phi(q)dq} \qquad M_{V_{\mathcal{T}_{ij}}} = \int_{V_{\mathcal{T}_{ij}}} \phi(q)dq$$

- $$\frac{\partial H}{\partial p_i} = \sum_{\mathcal{T}_{ij}\in\mathcal{P}_i} \int_{V_{\mathcal{T}_{ij}}} \frac{\partial}{\partial p_i}\frac{1}{2}(\|q,p_i\|^2 + \|q,p_j\|^2)\phi(q)dq$$
$$= -\sum_{\mathcal{T}_{ij}\in\mathcal{P}_i} \int_{V_{\mathcal{T}_{ij}}} (q-p_i)\phi(q)dq$$
$$= \sum_{\mathcal{T}_{ij}\in\mathcal{P}_i} -M_{V_{\mathcal{T}_{ij}}}(C_{V_{\mathcal{T}_{ij}}} - p_i)$$

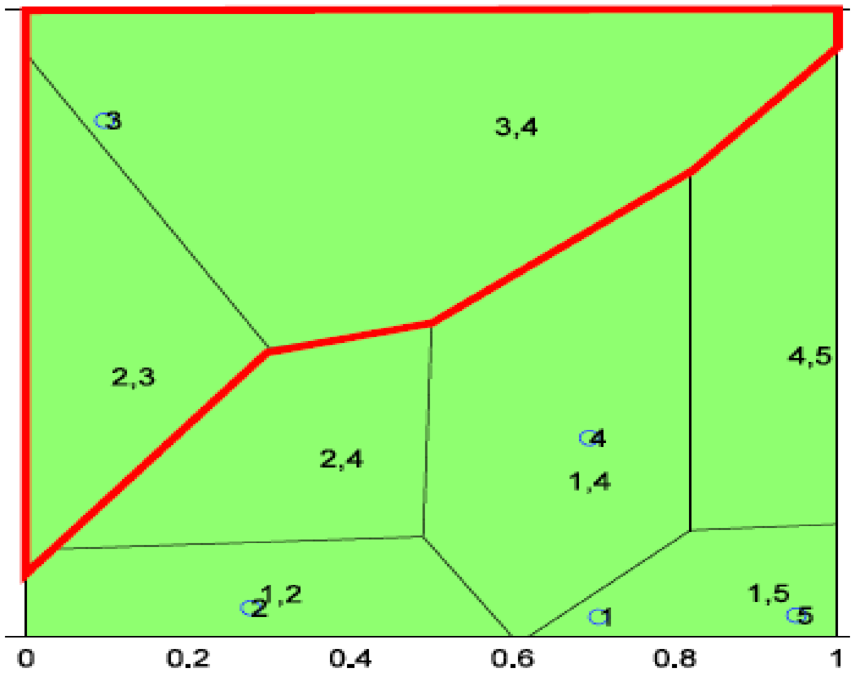


CONTINUOUS-COMMUNICATION CONTROL SOLUTION

- For W_i, the Centroid (C_{W_i}) and mass (M_{W_i}) are:

$$C_{W_i} = \frac{\sum_{\mathcal{T}_{ij} \in \mathcal{P}_i} M_{V_{\mathcal{T}_{ij}}} C_{V_{\mathcal{T}_{ij}}}}{\sum_{\mathcal{T}_{ij} \in \mathcal{P}_i} M_{V_{\mathcal{T}_{ij}}}} \qquad M_{W_i} = \sum_{\mathcal{T}_{ij} \in \mathcal{P}_i} M_{V_{\mathcal{T}_{ij}}}$$

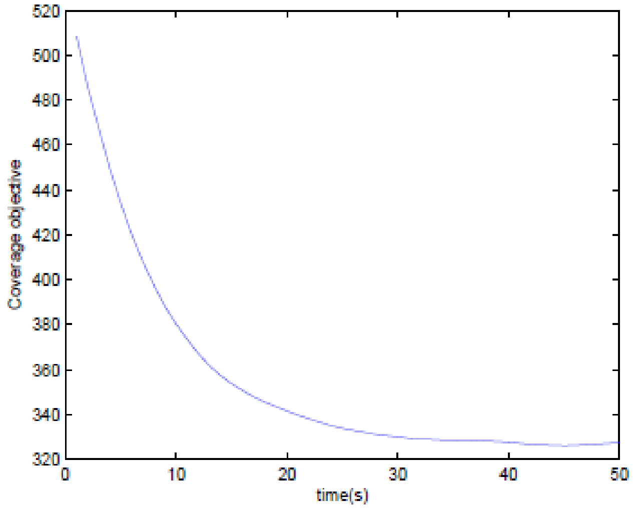
- $$\frac{\partial H}{\partial p_i} = \sum_{\mathcal{T}_{ij} \in \mathcal{P}_i} -M_{V_{\mathcal{T}_{ij}}} (C_{V_{\mathcal{T}_{ij}}} - p_i)$$
$$= -C_{W_i} \sum_{\mathcal{T}_{ij} \in \mathcal{P}_i} M_{V_{\mathcal{T}_{ij}}} + p_i \sum_{\mathcal{T}_{ij} \in \mathcal{P}_i} M_{V_{\mathcal{T}_{ij}}}$$
$$= -M_{W_i} (C_{W_i} - p_i)$$



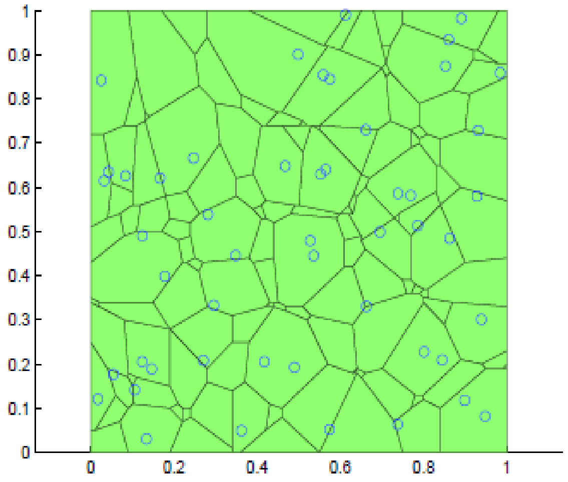
STABILITY & CONVERGENCE

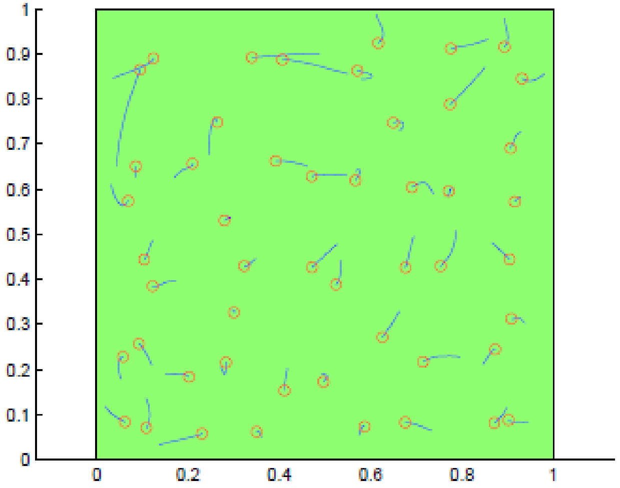
❖ Lyapunov stability theory:

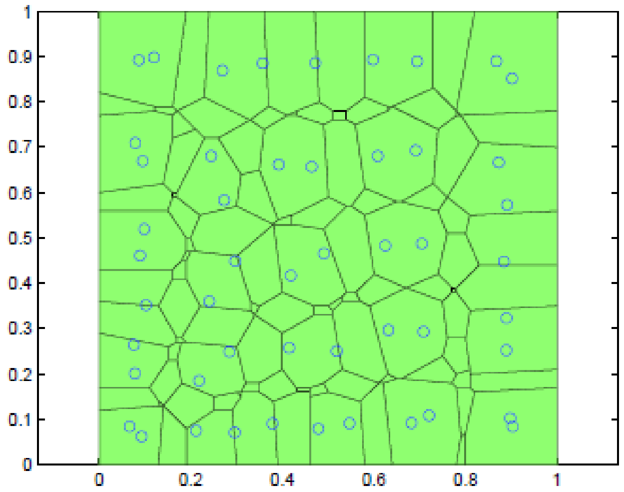
$$\begin{aligned}\frac{dH_i}{dt} &= \frac{\partial H}{\partial p_i} * \dot{p_i} \\ &= -M_{W_i}(C_{W_i} - p_i) * M_{W_i}(C_{W_i} - p_i) \\ &= -[M_{W_i}(C_{W_i} - p_i)]^2 \\ &\leq 0\end{aligned}$$



❖ The above is true for all i and $\frac{dH}{dt} = 0, \forall i\ p_i = C_{W_i}$

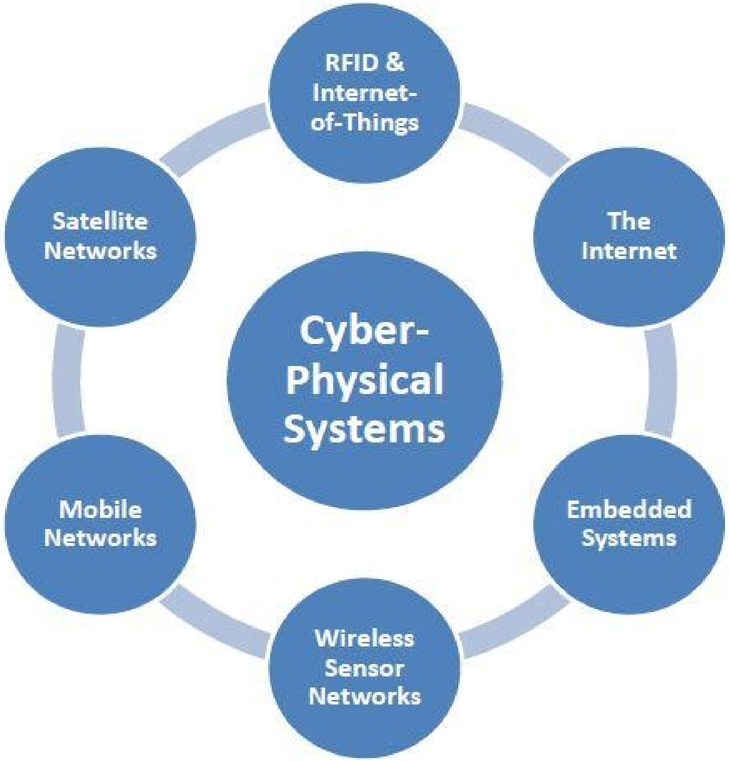






WHAT'S WRONG…???

- ❖ Assumed that agents have continuous access to state information about their neighbors.
- ❖ Assumed that agents can update their control signals continuously.
- ❖ Not a fair assumptions in the context of cyber-physical systems.
- ❖ Digital controllers can neither update actuator signals continuously nor can agents be in constant communication with others.



COMMUNICATION-AWARE CONTROL SOLUTION

- Minimizing the communication usage and still guaranteeing the optimal positioning of the agents
- Agents should have the liberty to choose when to communicate and when to update the control signal.
- Trading computation and decision making at the agent level for less communication, sensing, or actuator effort while still be able to achieve the optimal placement.
- Design a naturally self-triggered strategy that endows agents with sufficient level of autonomy so that they can decide when to take actions based on the local information available to them on their neighboring agents.

COMMUNICATION-AWARE CONTROL SOLUTION

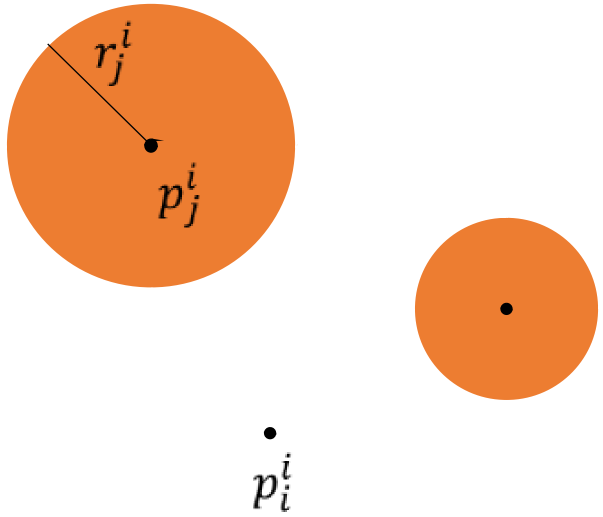
- Suppose agents move at maximum speed v_{max}.
- All agents maintain the reachable sets on others.
- Each agent i maintains a data structure

$$D^i = \left((p_1^i, r_1^i), \ldots, (p_n^i, r_n^i)\right)$$

where p_j^i : last known location agent j to agent i

r_j^i : maximum distance travelled by agent j since the last info

$p_i^i = p_i$ and $r_i^i = 0$



CONTROL INPUT

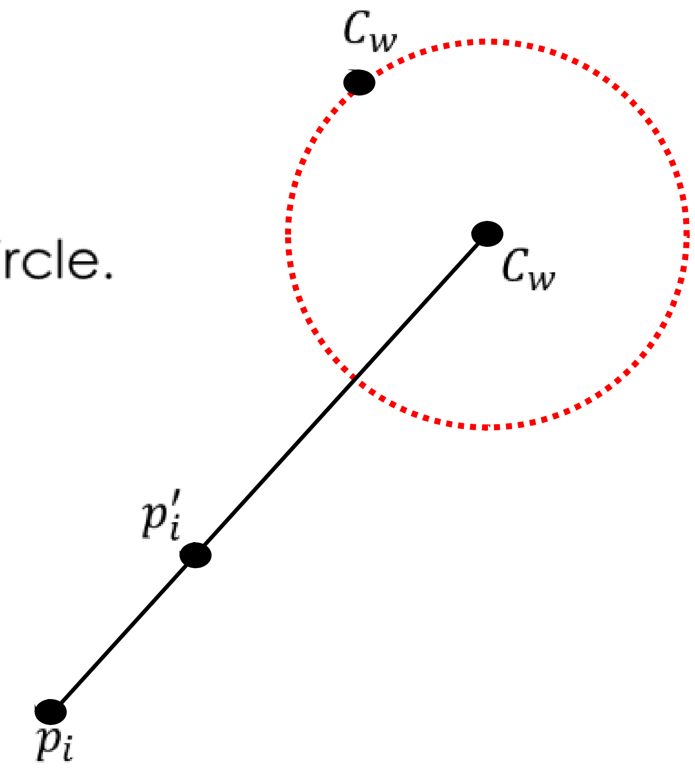
- $C_{w_i}^{init}$: initial centroid position of W_i.
- $C_{w_i}^{true}$: true centroid position of W_i

$$u_i = v_{max} \frac{C_{w_i}^{init} - p_i}{\left\| C_{w_i}^{init} - p_i \right\|_2}$$

- Agent can move towards $C_{w_i}^{init}$ as long as it is outside of the circle.
- The agent can move from p_i to p_i' while ensuring that

$$\left\| p_i' - C_{w_i}^{init} \right\| \geq \left\| C_{w_i}^{true} - C_{w_i}^{init} \right\|$$
$$\geq \text{bound}\left(\left\| C_{w_i}^{true} - C_{w_i}^{init} \right\| \right)$$

- As time elapses without communication the bound increases.
- Condition can be invalid if the agent is close to $C_{w_i}^{init}$



SELF-TRIGGERED CENTROID ALGORITHM

Initialization: Execute W_i cell computation

At each time step agent $i \in \{1, \dots, n\}$ performs the following steps:

1. Extract the neighboring agents information from the data structure D^i
2. Compute $q = C_{w_i}^{init}$ and $r = \text{bound}(\|C_{w_i}^{true} - C_{w_i}^{init}\|)$
3. **If** $r \geq \|q - p_i\|$ then
4. reset D^i by running W_i cell computation
5. extract the neighboring agents information from the updated D^i
6. Set $q = C_{w_i}^{init}$ and compute $r = \text{bound}(\|C_{w_i}^{true} - C_{w_i}^{init}\|)$
7. **end if**
8. Move the agent i by $v_{max}\Delta t$ towards $C_{w_i}^{init}$ or to the nearest point on $\bar{B}_{(q,r)}$
9. Update the agent i's position in D^i
10. Increase the radii of other agents in D^i by $v_{max}\Delta t$

CONCLUSIONS

- Considered a 2^{nd} order Voronoi coverage control problem.
- Introduced the higher order Voronoi partition concept in the coverage performance function.
- Analyzed the performance function, the controller design and the controller performance.
- Highlighted the drawbacks in the approach.
- Proposed Self-triggered centroid algorithm which combined an update law to determine when old information needs to be refreshed and a motion control law that uses this information to decide how to best move.

THINGS TO BE DONE…!

- Expression for the "bound($\| C_{w_i}^{true} - C_{w_i}^{init} \|$)" need to be obtained.
- W_i cell computation algorithm need to be developed.
- Analytically establish the convergence and stability of the self-triggered centroid algorithm.
- Simulations have to be performed to illustrate substantial communication savings of the self-triggered centroid algorithm.

FUTURE WORK

- ❖ Extension of results to even higher order Voronoi based coverage control.
- ❖ Obstacle avoidance
- ❖ Communication and control co-design for the robust optimal deployment

THANK YOU!!!!