ECE 722: KALMAN FILTERING WITH APPLICATIONS SPRING 2016

PROJECT REPORT

Title: ANALYSIS ON APPLICATION OF KALMAN FILTERING FOR CONTINUOUS REAL-TIME TRACKING OF POWER SYSTEM HARMONICS

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Abstract:

The report describes a technique in measurement of harmonics in Power System, in real-time, by applying a 12 state Kalman Filter to the voltage or current samples. It also includes the analysis on performance of filter in different scenarios, like sudden change in particular harmonic level, noise content in measurements etc. Moreover, this report also discusses filter performance when different state space model of system has been considered. It also mentions the dependency of filter performance on sampling frequency. In order to test the technique, a periodic signal of known harmonic content has been generated in MATLAB and it is sampled with appropriate sampling frequency. The results in the report are obtained by using the data samples obtained in the above mentioned way. Even though the data considered is an artificial one, the results can be easily generalized to real time data.

1. Introduction

At present time increasing attention is being paid to the electromagnetic disturbances to which power systems are being subjected. The growing importance is due to the fact that large proportion of the equipment fed by the electrical system is sensitive to them. In electrical power system management, power quality is an essential factor to be taken into account.

Harmonic distortion in voltage or current signals is one of the disturbance factors. The widespread applications of electronically controlled loads have increased the harmonic distortion in the power system voltage and current waveforms. Moreover, many of power system loads, especially industrial loads, are dynamic in nature. This produces time varying amplitude for the current waveforms. Accurate measurement of harmonics is essential as it is considered as the type of disturbance which it is most necessary to control. This means imposing limitations on the emission levels of the equipment and filtering the inevitable harmonic components present.

The fast Fourier transform (FFT) and the discrete Fourier transform (DFT) are the most frequency harmonic analysis algorithms used to obtain the voltage and current frequency spectra from discrete time samples. However, misapplication of the FFT and DFT algorithms would lead to an incorrect result. Basic assumptions embodied in the application of DFT and FFT are: (i) The signal has a constant magnitude; (ii) The sampling frequency is equal to the number of samples multiplied by the fundamental frequency assumed by the algorithm; (iii) The sampling frequency is greater than twice the highest in the signal to be analyzed. When these assumptions are satisfied, the results of the DFT or FFT are accurate. There are three major pitfalls in the application of FFT; namely, aliasing, leakage, and picket-fence effect.

So, the Kalman filtering approach provides an alternative way for optimally estimating the harmonics in real-time. It provides best estimates of the magnitudes with the smallest number of samples and in the shortest time period, allowing variable parameters to be tracked with the time. By representing the signal in state variable representation and applying Kalman filter recursive algorithm, harmonics with time varying magnitudes can be tracked.

2. Modeling of a system

Kalman filter can be used to estimate the harmonic components in a power system’s voltage or current waveforms. But, the algorithm requires a state variable model for the parameters to be estimated and a measurement equation that relates the discrete measurement to the state variables. In order to obtain a state space representation of the system, Fourier series representation of the signals has been used in the below discussion and a state variable form is deduced, including all the possible spectral components which may be associated with the signal to be analyzed.

Consider a signal with a frequency ω and a magnitude of $A(t)$. Considering a reference rotating at ω, the noise-free signal may be expressed as:

$$s(t) = A(t)\cos(\omega t + \theta) = A(t)cos\theta cos\omega t - A(t)sin\theta sin\omega t \tag{1}$$

By seeing the above, let us consider X_1 be $A(t)cos\theta$ and X_2 be $A(t)sin\theta$. The X_1 and X_2 represents in-phase and quadrature-phase components respectively and referred as state variables. Moreover, the square root of sum of squares of states gives us amplitude $\{A(t)\}$ of the signal.

$$\sqrt{\{A(t)cos\theta\}^2 + \{A(t)sin\theta\}^2} = A(t) \tag{2}$$

A noise- free current or voltage signal $s(t)$ that includes 'n' harmonics may be represented by

$$s(t) = \sum_{i=1}^{n} A_i(t)\cos(i\omega t + \theta_i) \tag{3}$$

where $A_i(t)$ is the amplitude of i^{th} harmonic at time t, θ_i is the phase angle of the harmonic i and n is the harmonic number. As discussed in the equation (1) each frequency component requires two state variables, $2n$ being the total number of state variables. These state variables are defined as follows:

$$\begin{array}{ll}
x_1(t) = A_1(t)cos\theta_1 & x_2(t) = A_1(t)sin\ _1 \\
x_3(t) = A_2(t)cos\theta_2 & x_4(t) = A_2(t)sin\theta_2 \\
x_5(t) = A_3(t)cos\theta_3 & x_6(t) = A_3(t)sin\theta_3 \\
\vdots & \vdots \\
x_{2n-1}(t) = A_n(t)cos\theta_n & x_{2n}(t) = A_n(t)sin\theta_n
\end{array}$$

The state variable equation can be expressed in matrix form as

$$\boldsymbol{x_{k+1} = \phi_k x_k + w_k}$$

Where x_{k+1} is a [2n x 1] state vector at instant $(k+1)$, x_k is [2n x 1] state vector at instant k, ϕ_k is a [2n x 2n] state transition matrix and w_k represents the discrete variation of state variables due to an input white noise.

In expanded form, the state variable equation for this model can be represented as,

$$\begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{pmatrix}_{k+1} = \begin{bmatrix} 1 & 0 & . & . & 0 \\ 0 & 1 & . & . & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ . & . & . & . & . \\ 0 & . & . & . & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{pmatrix}_{k} + W_k \tag{4}$$

The measurement equation can be expressed as

$$Z_K = \boldsymbol{H_K} x_k + v_k = \begin{bmatrix} \cos(\omega k\Delta t) \\ -\sin(\omega k\Delta t) \\ \cdots . \\ \cos(n\omega k\Delta t) \\ -\sin(n\omega k\Delta t) \end{bmatrix}^T \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{pmatrix}_k + v_k \tag{5}$$

where Z_k is the measurement vector at instant k, H_k is the vector giving the ideal relation between the measurement and the state vector, v_k is the noise vector and Δt is the sampling interval. The above model can be referred as "Model-1".

As we know that the state space representation is not unique, we can represent the present system using alternative state space representation. The below is the alternative state space representation of this system.

Let us consider again the equation (1),

$$s(t) = A(t)\cos(\omega t + \theta)$$

Now, consider $x_{1(k)} = A(t_k)\cos(\omega t_k + \theta)$ and $x_{2(k)} = A(t_k)\sin(\omega t_k + \theta)$. At $t + \Delta t$, the signal may be expressed as :

$$\begin{aligned} s(t+\Delta t) &= A(t+\Delta t)\cos(\omega t + \omega\Delta t + \theta) \\ s_{k+1} &= x_{1(k)}\cos(\omega\Delta t) - x_{2k}\sin(\omega\Delta t) \\ x_{1(k+1)} &= x_{1(k)}\cos(\omega\Delta t) - x_{2k}\sin(\omega\Delta t) \end{aligned} \tag{6}$$

and also

$$x_{2(k+1)} = x_{1(k)}\sin(\omega\Delta t) + x_{2k}\cos(\omega\Delta t) \tag{7}$$

Thus, by using the equations (6) and (7), the state variable representation can be made as

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{k+1} = \begin{bmatrix} \cos(\omega\Delta t) & -\sin(\omega\Delta t) \\ \sin(\omega\Delta t) & \cos(\omega\Delta t) \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_k + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}_k \tag{8}$$

The measurement equation then becomes

$$Z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_k + v_k \tag{9}$$

By using the above equations (8) and (9), if the signal includes 'n' frequencies; the fundamental plus 'n-1' harmonics, the state variable representation can be expressed as:

$$\begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{pmatrix}_{k+1} = \begin{bmatrix} M_1 & 0 & . & . & 0 \\ 0 & M_2 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & . & M_n \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{pmatrix}_k + \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_{2n-1} \\ \alpha_{2n} \end{bmatrix} W_k \tag{10}$$

Where the submatrices M_i are shown as

$$M_i = \begin{bmatrix} \cos(i\omega\Delta t) & -\sin(i\omega\Delta t) \\ \sin(i\omega\Delta t) & \cos(i\omega\Delta t) \end{bmatrix} \tag{11}$$

The measurement equation can be expressed as

$$Z_k = [1 \quad 0 \quad \dots \quad 1 \quad 0] \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{2n-1} \\ x_{2n} \end{pmatrix}_k + v_k \tag{12}$$

The above model can be referred as "Model-2". Contrast to Model-1 this has got constant state transition and measurement matrices. However, this assumes a stationary reference. Thus, the in-phase and quadrature-phase components represent the instantaneous values of Cosine and Sine waveforms respectively.

3. Kalman filter application

The Kalman filter is an estimator used to estimate the state of a linear dynamic system perturbed by white noise using measurements that are linear functions of the system state but corrupted by additive white noise. Theoretically it has been called the linear mean squares estimator (LLSME) because it minimizes the mean-squared estimation error for a linear stochastic system using noisy linear sensors. It is recursive optimal estimator well suited for the online application.

The following are the parameters that are required in order to successfully apply Kalman filter algorithm:

- It requires state space represented model of the system. In this case we deduced two state space models of the system. Model-1 corresponds to equations (4) & (5). Model-2 corresponds to equations (10) & (12). Using either of the equations we can clearly obtain the ϕ (state transition matrix) and H (state variable and output coupling matrix).
- Initial process vector $\hat{x}_{0-}$. In our case as the Kalman filter model started with no past measurement, the initial process vector was selected to be zero.
- Initial covariance matrix (P_{0-}). The initial covariance is selected to be a diagonal matrix with diagonal elements equal to $10pu^2$.
- Noise variance (R). In this case this is selected as a constant value equivalent to $0.05pu^2$.
- State variable covariance matrix (Q). Here we will consider it as a diagonal matrix with diagonal elements equal to $0.05pu^2$.
- Measurement vector (Z). For this project as we don't have real time data, I fabricated a periodic signal, having known amount of harmonic content, by summing cosines of frequencies, which are integral multiples of fundamental frequency. The below is the mathematical representation of generated periodic signal.

$$s(t) = 1.0\cos(2\pi 50t + 30°) + 0.1\cos(3(2\pi 50)t + 210°) + 0.06\cos(5(2\pi 50)t + 180°) + 0.009\cos(9(2\pi 50)t - 145°) + 0.005\cos(11(2\pi 50)t + 30°) + 0.003\cos(13(2\pi 50)t) \quad (13)$$

Using MATLAB I have sampled the above mentioned signal, having odd harmonics except 7th till 13th, at 1600Hz sampling frequency. This gave me an output vector Z having 32 samples for each cycle of $s(t)$.

4. Results

Below is Figure (1) which shows the artificially fabricated periodic wave represented by equation (13) above. It has 3rd, 5th, 9th, 11th, & 13th harmonics.

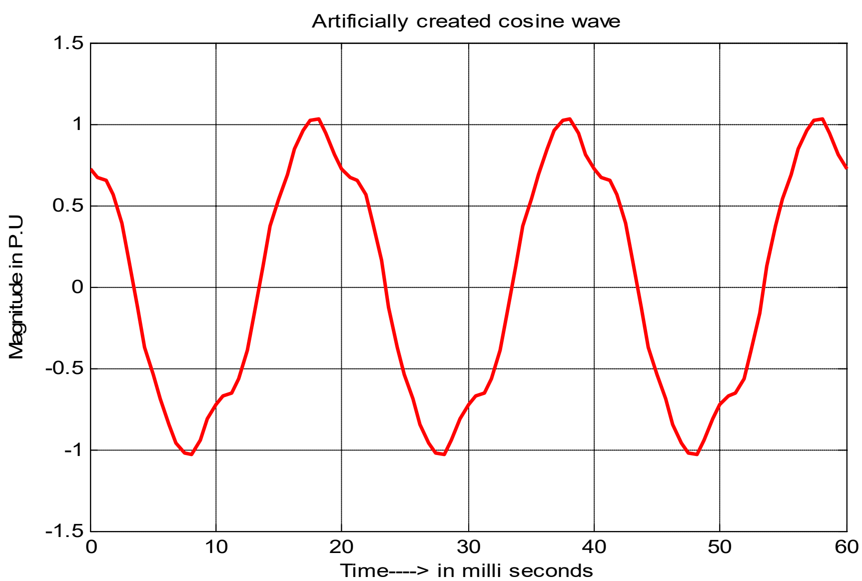
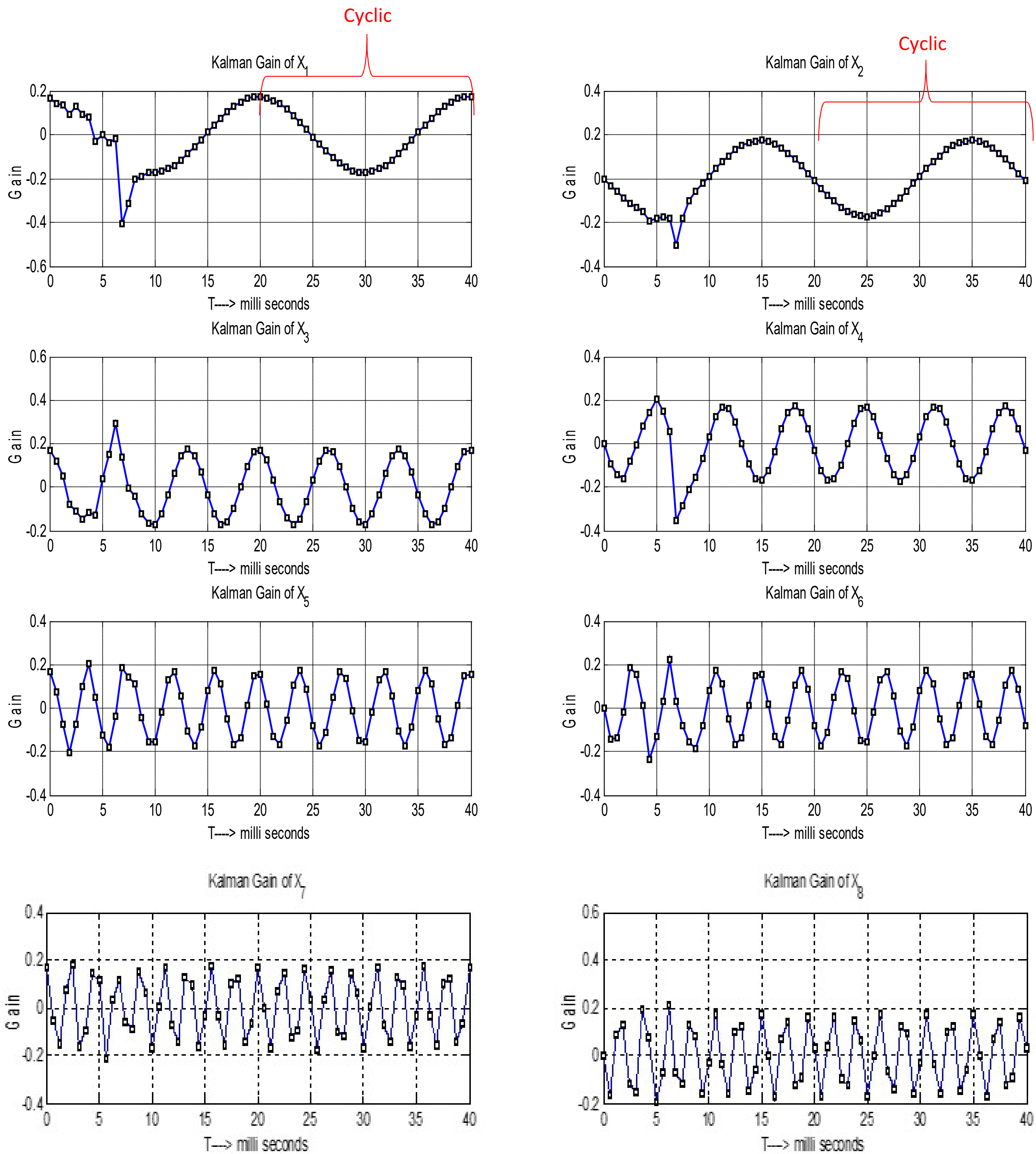


Figure (1): Waveform of $s(t)$

As our aim is to estimate 1st, 3rd, 5th, 9th, 11th, 13th, harmonics, and each harmonic needs 2 states as discussed above, so we will be using 12 state Kalman filter to estimate them all.

Model-1, described by equations (4) & (5), is considered for now to apply filtering algorithm. The state transition matrix (ϕ), would be an identity matrix. The output and state vector coupling matrix (H) would be constantly varying and periodic. So, H_k is pre calculated and stored in the memory. Based on the time k particular value of H_k is chosen and used in the computation.

The Kalman gain for each state has been computed recursively in MATLAB and have been plotted. The below, figure (2), represents the Kalman gains for each state. We can see that Kalman gain of each state takes around 9.5ms to settle. Later from that point it becomes periodic.


Cyclic
Kalman Gain of X1
G a in
T----> milli seconds
Cyclic
Kalman Gain of X2
G a in
T----> milli seconds
Kalman Gain of X3
G a in
T----> milli seconds
Kalman Gain of X4
G a in
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Kalman Gain of X5
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Kalman Gain of X8
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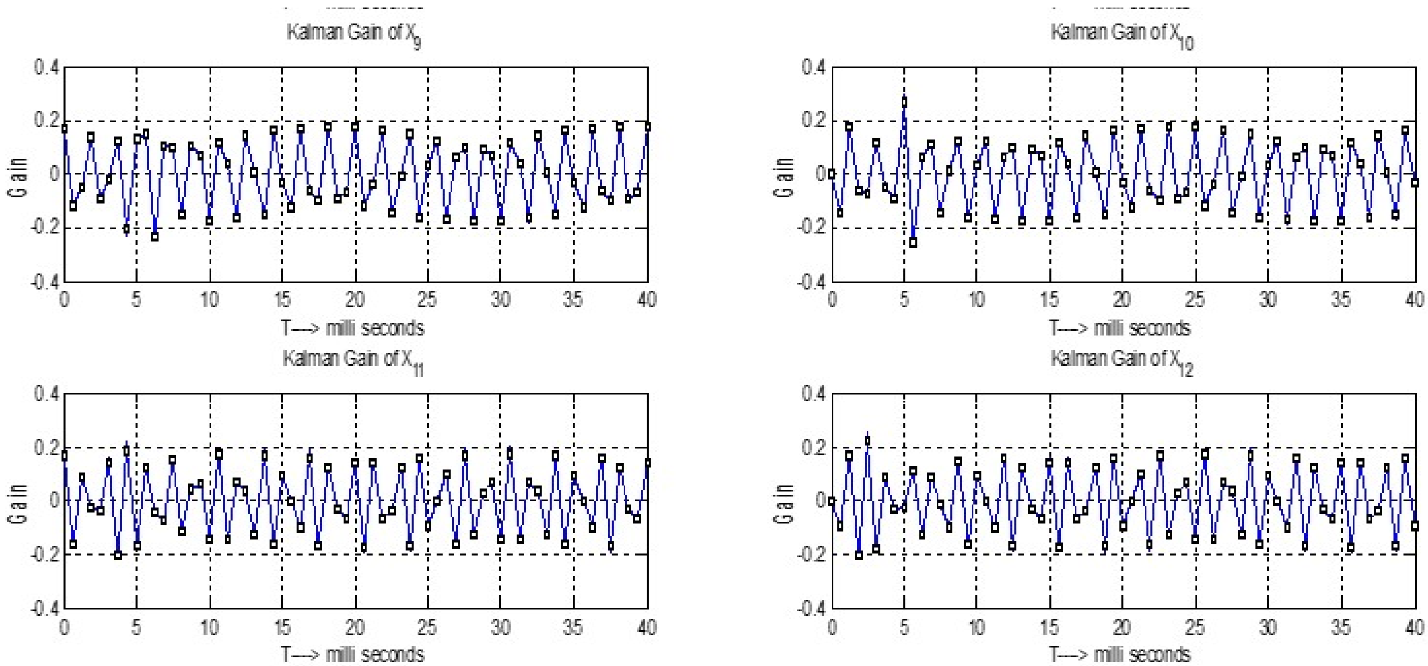
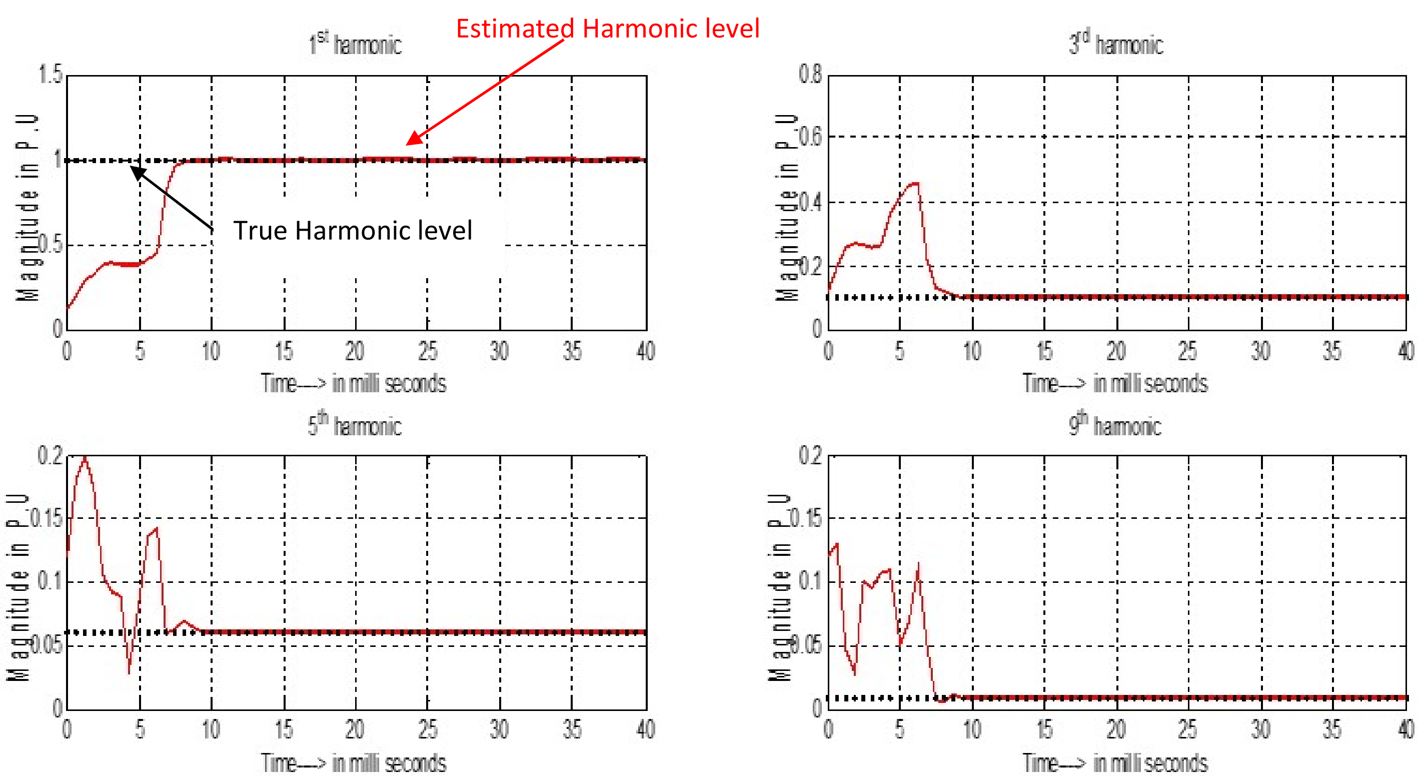


Figure (2): Kalman gains of states from X_1 to X_{12}

The below, figure (3), is the harmonic estimation by the Kalman filter. The black dotted line shows the original harmonic level and the red one is the estimated one by Kalman filter. It approximately takes 9ms from the start to correctly estimate the harmonic level.



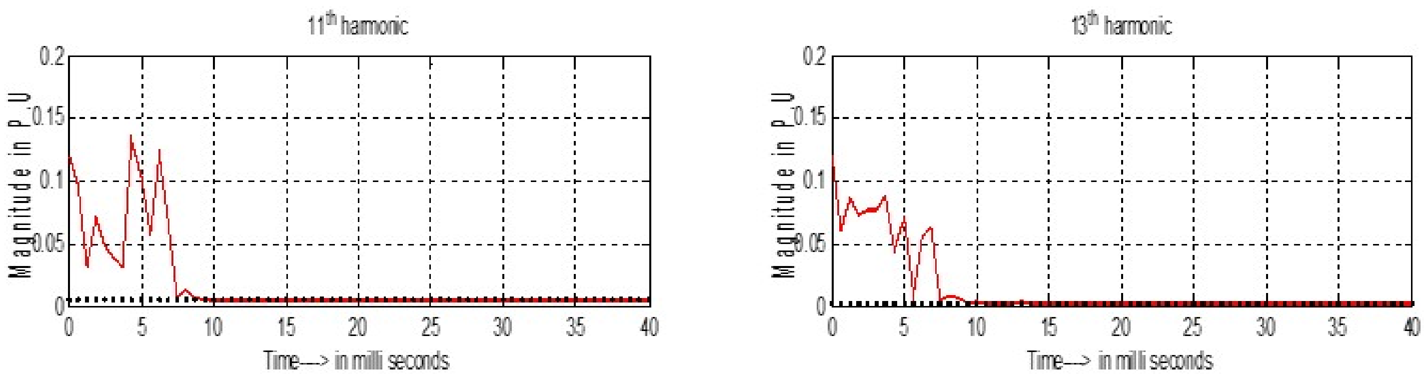
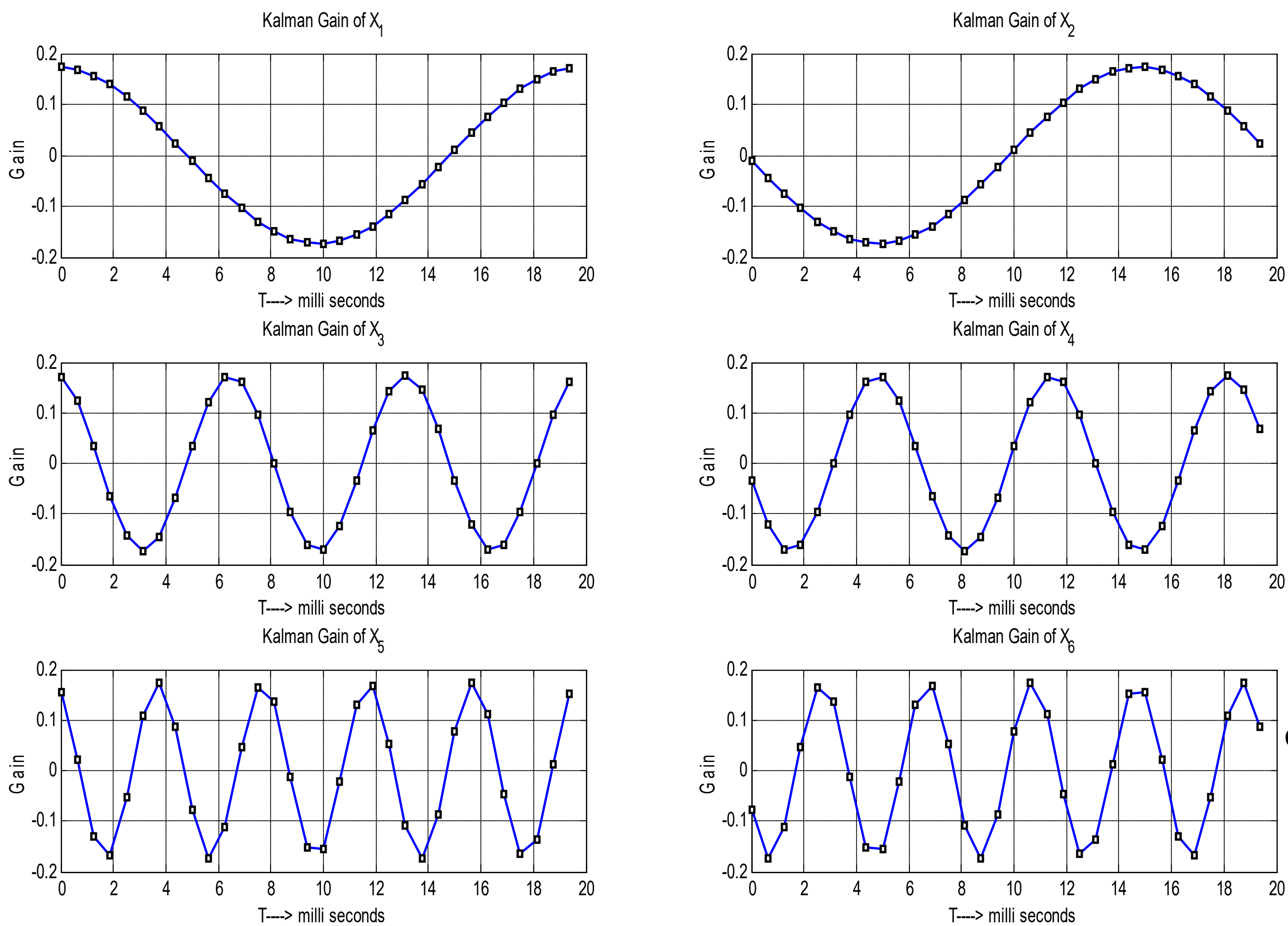


Figure (3): Harmonic levels estimated

By seeing the gains in figure (2), it is obvious that the gains of states are periodic and repeat with a time period of 20ms. So it is waste of time and resources to compute the Kalman gains of states recursively. We simply save the gains of each state and at every instant of time we can use corresponding gain cyclically. This saves lot of time and makes the algorithm very faster. It also doesn't require more computational resources compared to recursive gain computation. The below are the results and performance of filter when we use the cyclic gains



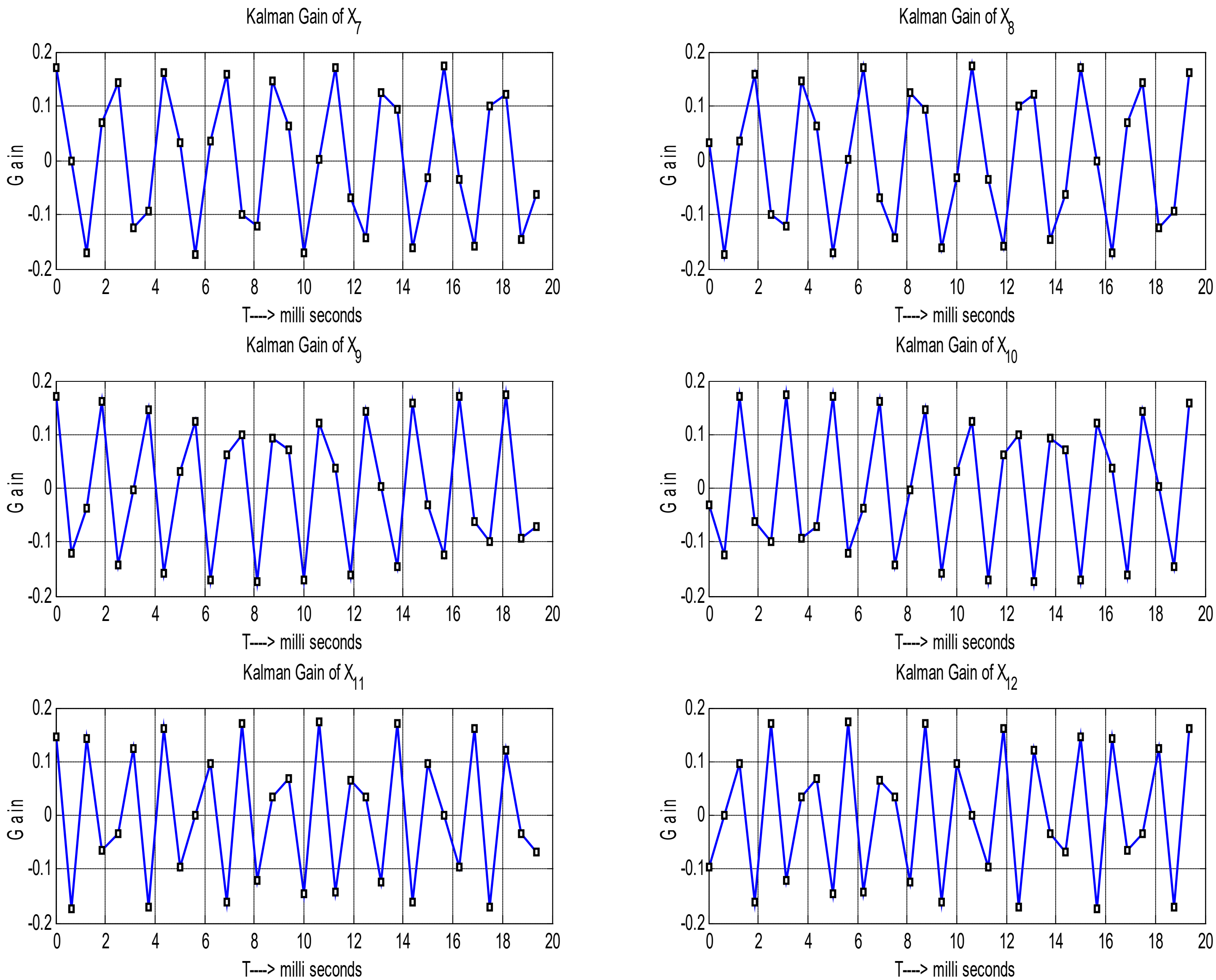


Figure (4): Cyclic Kalman gains of all states

The above, figure (4), represents the cyclic gain values. Same gains will be applied for every 20ms as long as the filter operates. These values will be stored in the memory and based on the time corresponding gains are selected from the memory for each state. This makes the computation simple and no need to bother about the calculation of Kalman gains recursively.

The below, figure (5), represents the harmonics estimated by the Kalman filter when cyclic gains are used. The usage of cyclic gains doesn't hamper the performance of the filter. It can be clearly seen from the below plots. The only downside is, its settling time is increased by 0.5ms i.e., it has become bit slower. However, 0.5ms difference is very minute and can be negligible. We can undoubtedly follow this technique. This requires less resources and less computation power compared to the recursive one.

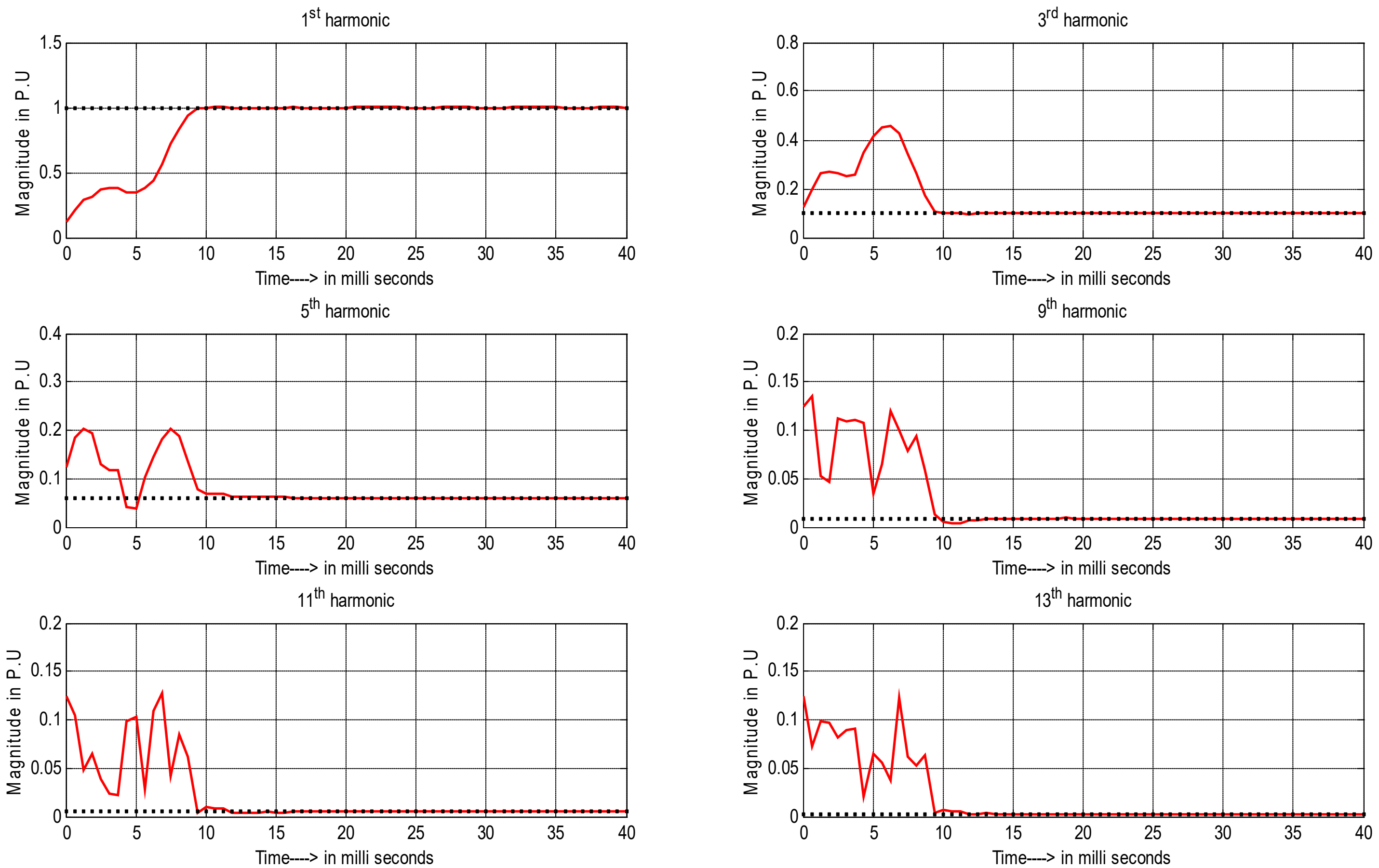


Figure (5): Estimated Harmonic levels when cyclic gains are used

So, by seeing its performance, we are convinced that the cyclic gains don't change anything. So here onwards we will be using cyclic gains to analyze the performance of the Kalman filter in different scenarios.

In our first scenario, we will induce sudden change in 5th harmonic content at the end of 2nd cycle of $s(t)$. We change the amplitude of 5th harmonic from 0.06 to 0.1. Let us see how fast the Kalman filter tracks the sudden change.

The below, figure (6), is the plot of estimation of all the harmonics. In the below plot I circled the trajectories where the Kalman filter started to track the changes. Initially due to sudden change in 5th harmonic content other trajectories also got effected but after 10ms they all settled to initial level except 5th harmonic. The 5th harmonic got settled to the newly changed one. It is clear from the plots that the Kalman filter is able to track changes in 10ms only, which is the half of the time period of $s(t)$.

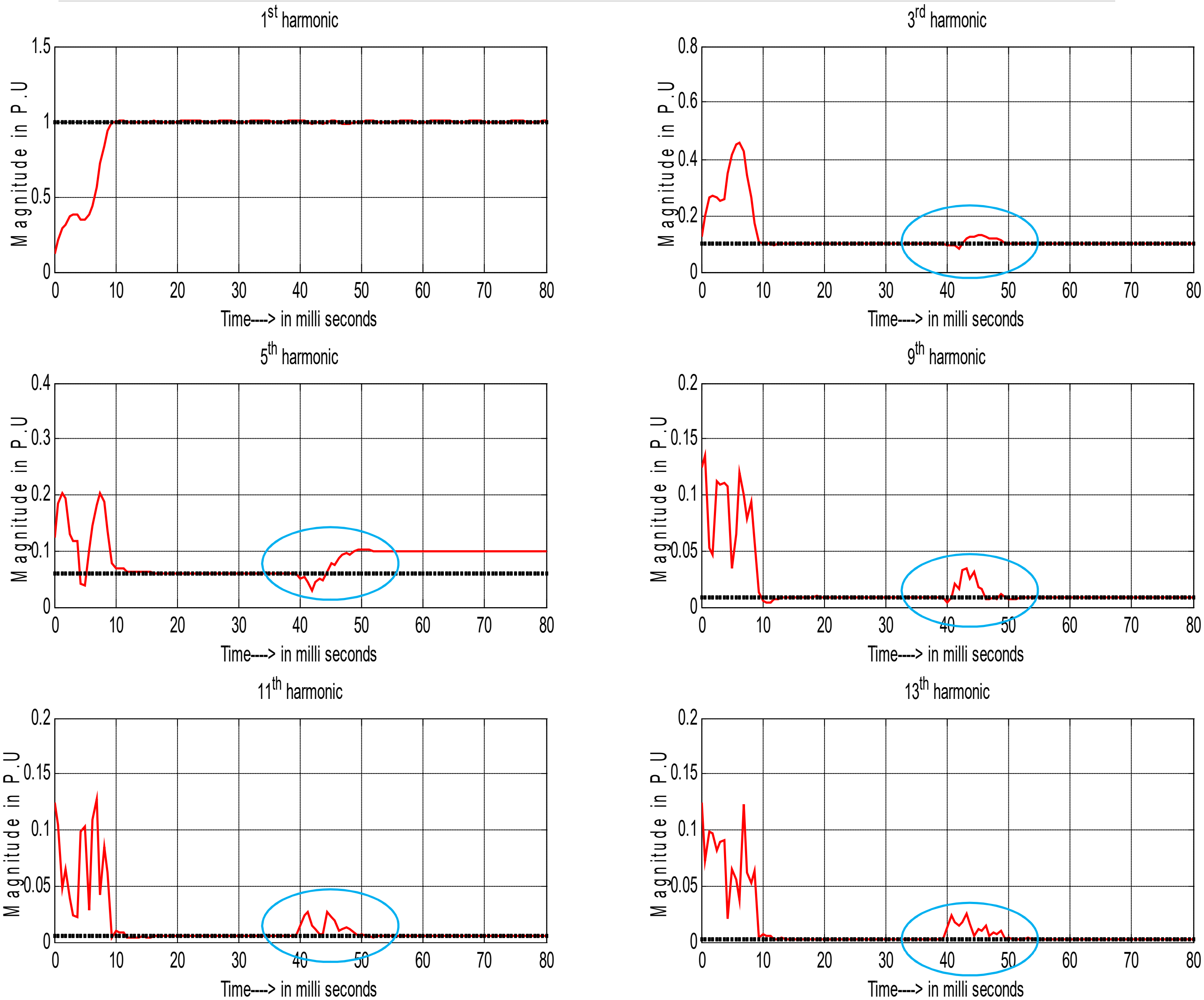


Figure (6): Estimated Harmonic levels when there is sudden change in 5th harmonic level

The next scenario is analyzing the performance of the Kalman filter based on the sampling frequency. According to sampling theorem, the signal that need to be analyzed should be sampled at the frequency at least double that of its frequency. Then only we can perfectly capture all the data of a continuous signal. So, in order to analyze, till 13th harmonic of a signal, we need to sample it at least with a sampling frequency of $2 * 13 * 50 = 1300Hz$. I tested the Kalman filter by sampling the output at frequency less than the threshold. It could track the harmonics well until the sampling frequency reduced to $900Hz$.

The below, figure (7), shows the trajectories of harmonics estimated by the Kalman filter. Even though we reduced the sampling frequency to 900Hz, only 5th and 13th harmonic were not tracked perfectly. The rest of the harmonics were tracked exactly. This makes clear that this can even work better with less samples than the required ones.

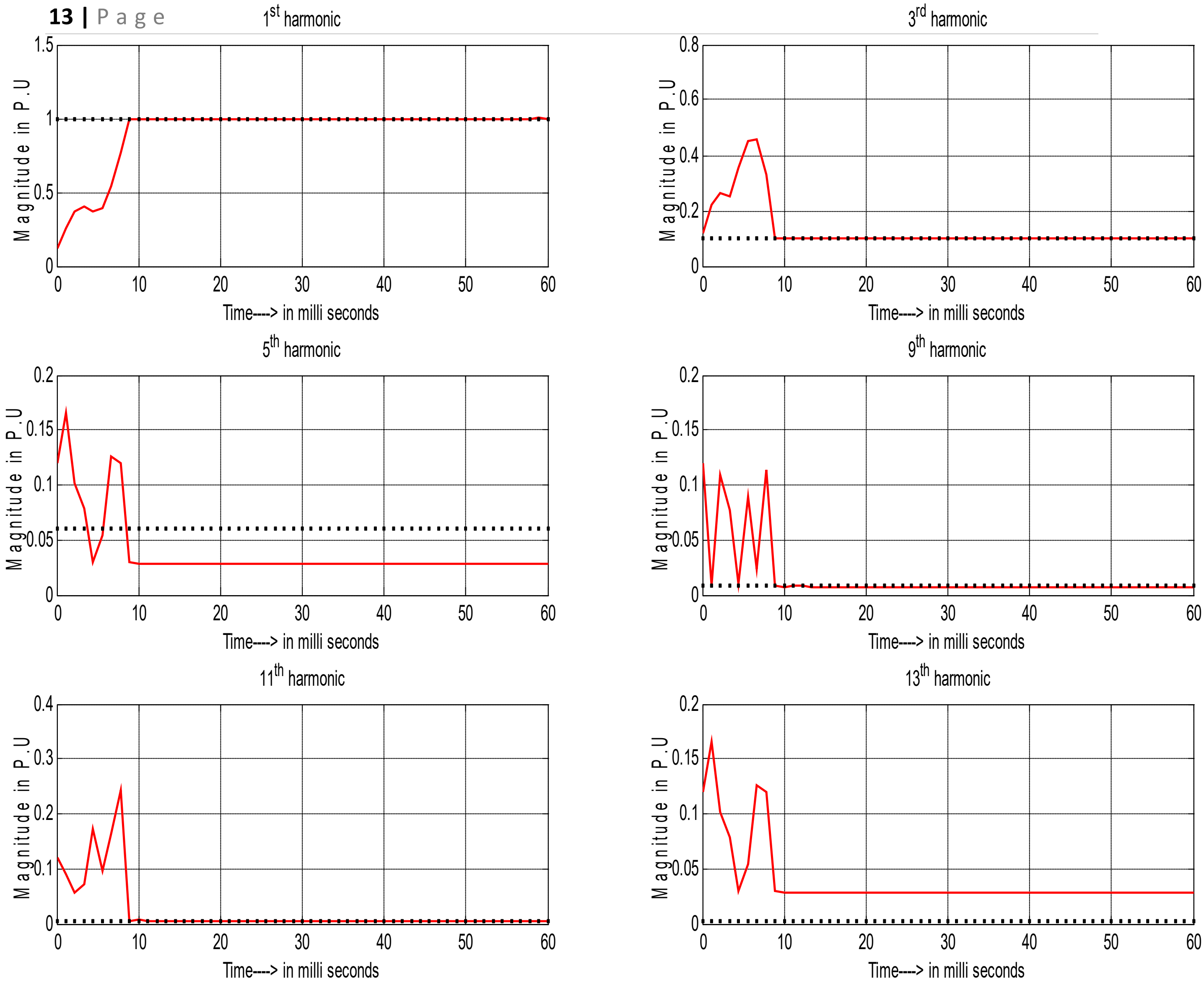
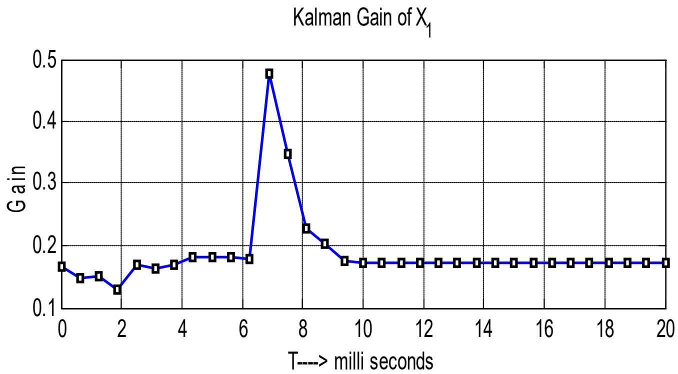


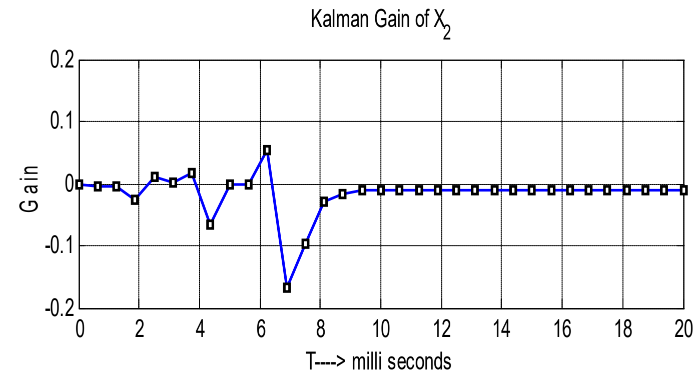
Figure (7): Estimated Harmonic levels when the sampling frequency has been reduced to 900Hz

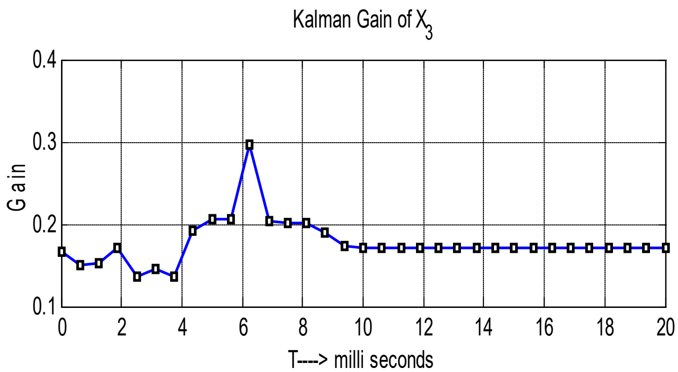
The last scenario is testing the performance of Kalman filter using a different state space representation model of a system. In the model section of this report I derived two state space representations. Till now we used Model-1 in our analysis, but for now we will use Model-2 which corresponds to equations (10) & (12).

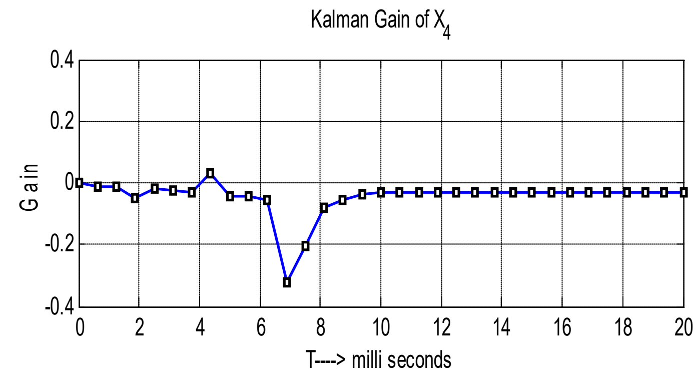
In Model-2, contrast to Model-1, H_k is a constant matrix of order [1x12] and state transition matrix (ϕ) is also a constant matrix but it has all cosines and sines.

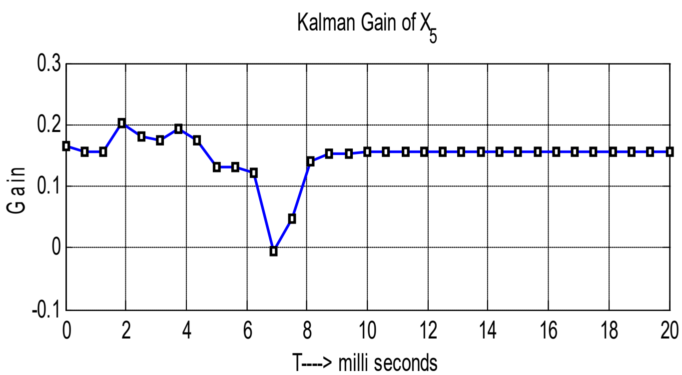
In the below, figure (8), we can see the plots of Kalman gains of all the states. It is evident that, the Kalman gains of all states settles to one fixed value in the steady state contrast to Model-1, where at the steady state the Kalman gains become periodic.

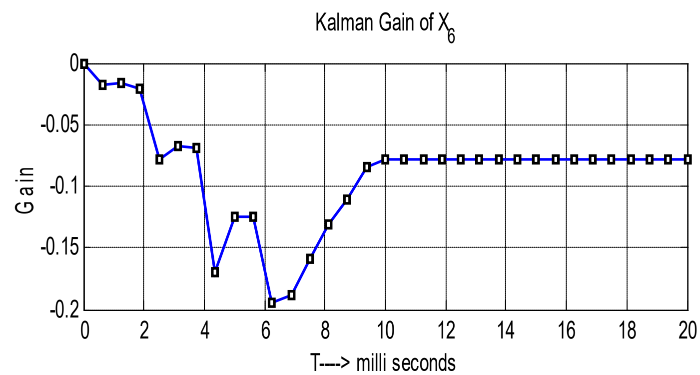

Kalman Gain of X_1
Gain
T----> milli seconds

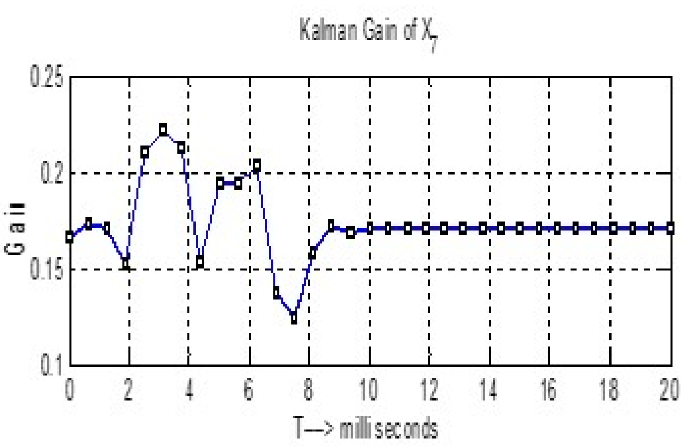

Kalman Gain of X_2
Gain
T----> milli seconds

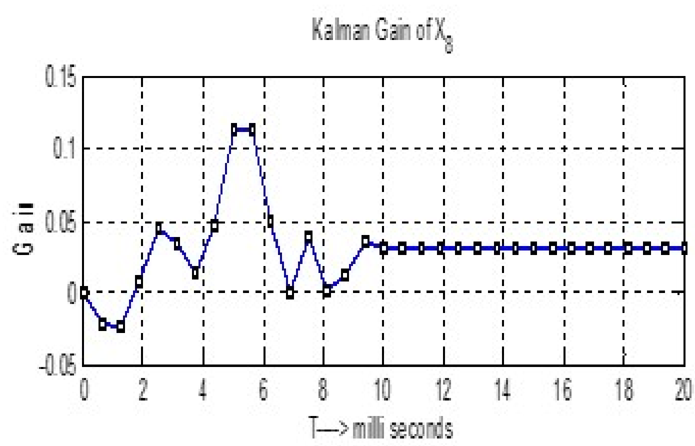

Kalman Gain of X_3
Gain
T----> milli seconds


Kalman Gain of X_4
Gain
T----> milli seconds


Kalman Gain of X_5
Gain
T----> milli seconds


Kalman Gain of X_6
Gain
T----> milli seconds


Kalman Gain of X_7
Gain
T---> milli seconds


Kalman Gain of X_8
Gain
T---> milli seconds

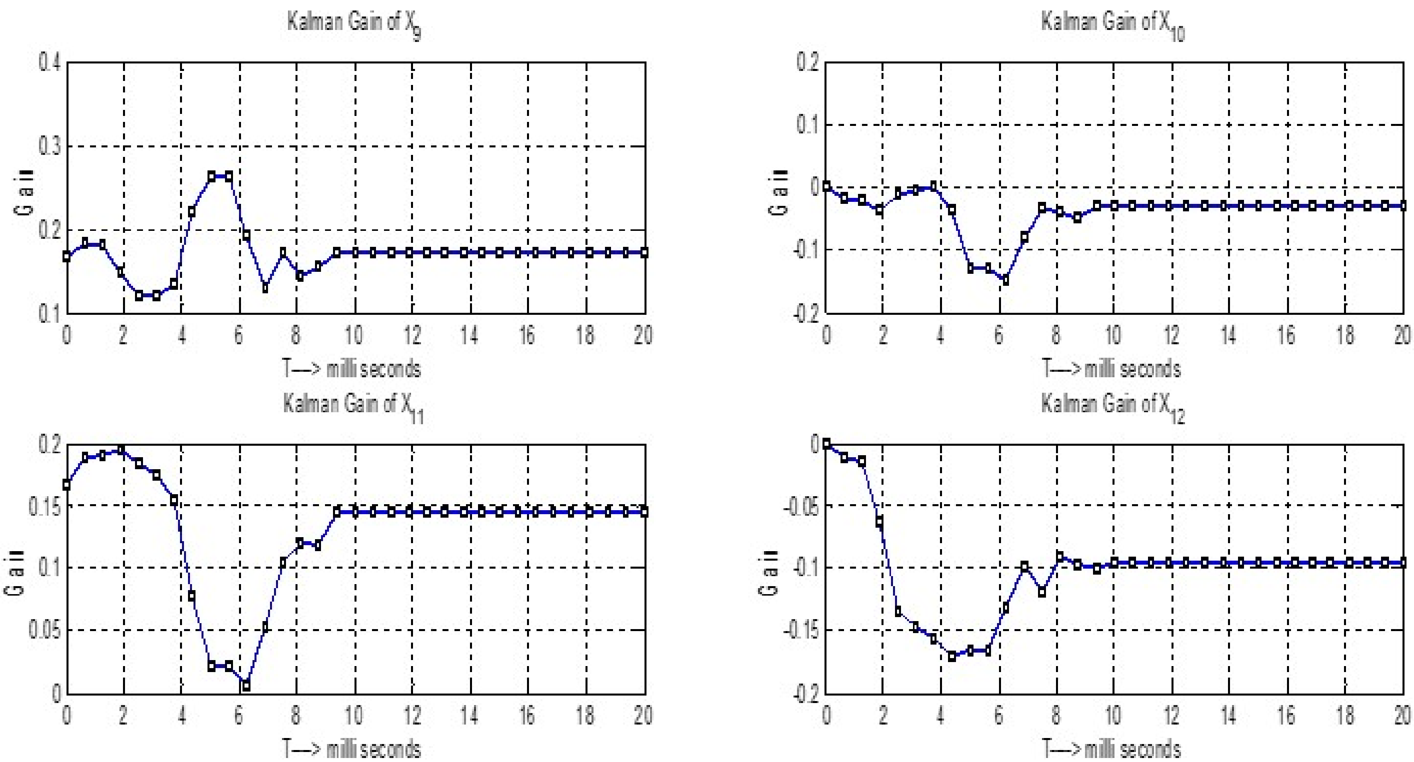


Figure (8): Kalman gains of all states when Model-2 is used

The below, Figure (9) is the plot of trajectories of harmonics by the Kalman filter when Model-2 is used for the analysis.

By carefully analyzing the below plot, it is clear that the Kalman filter is able to estimate the harmonic level of each with in 8.5ms from the start. When compared with the results of Model-1, it is a bit faster and can track the harmonics properly by 0.5ms ahead.

By simply looking at the results we can't conclude that Model-2 is superior to Model-1. Each one has different parameters and we need to choose model based on the application. For example, if we wish to implement the Kalman filter in a real-time on-line application, then computation time comes into play. We need to see which model takes less time for update and prediction of states. The update and prediction should be done before the next output sample arrives.

In Model-2, prediction may take more time because the state transition matrix has cosines and sines and it may include floating point calculation. But in Model-1, the state transition matrix is just an Identity matrix. So, the computation for the prediction may not be that intense when compared to Model-1.

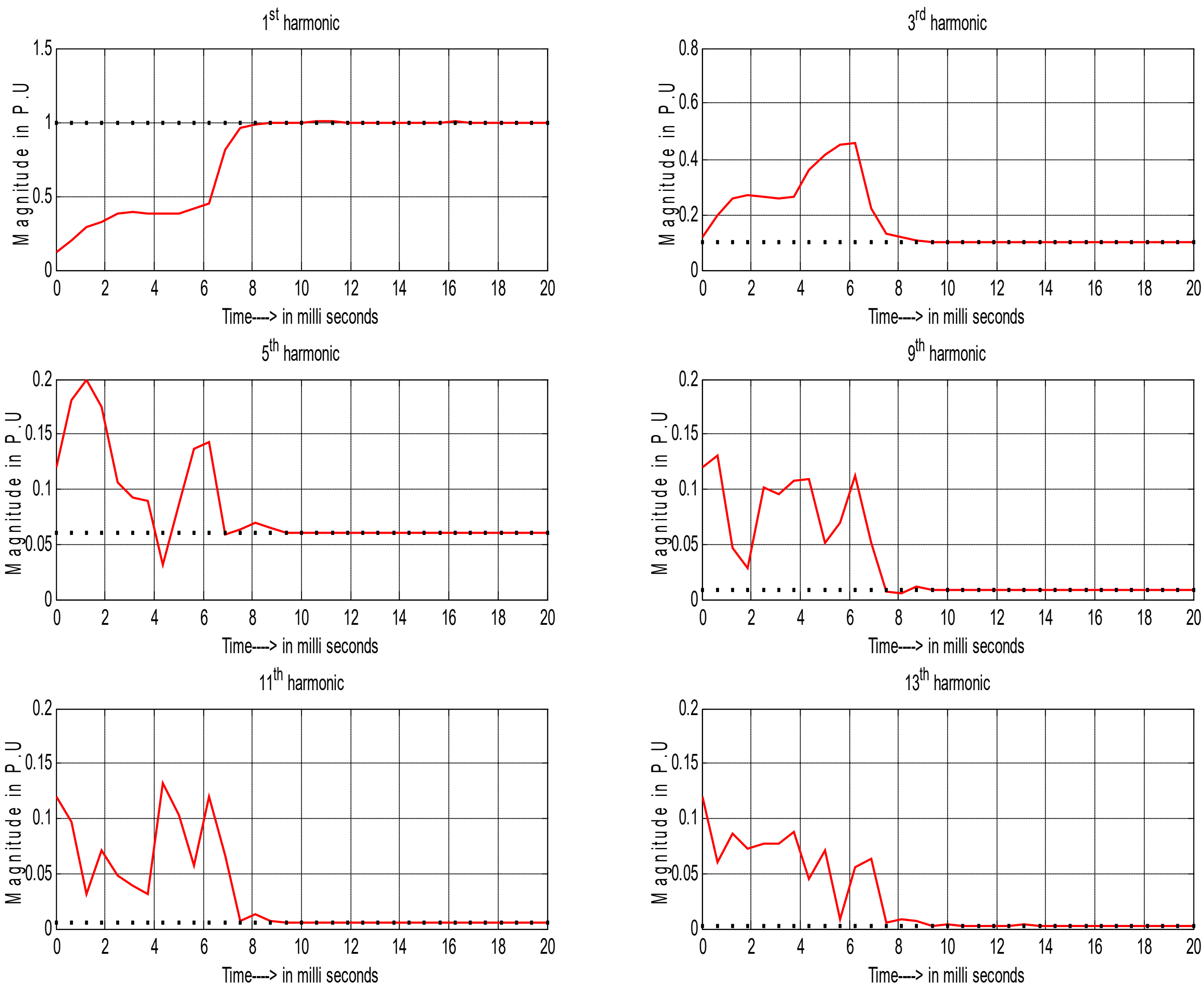


Figure (9): Estimation of Harmonics by the Kalman filter when Model-2 is used

5. Conclusions

In this report, in the introduction, I have mentioned the general algorithms; FFT & DFT, for harmonic analysis. At the same time I highlighted the downsides of those and reason for choosing Kalman filtering approach. Later I derived two types of state space representation of system and described the parameters chosen for the Kalman filter. In the results section I described the performance of the Kalman filter in different scenarios.

Two models are discussed in this, in order to show the flexibility of Kalman filtering scheme. There are many applications, where the results of FFT algorithm are as accurate as Kalman filter model. However, there are many other applications where Kalman filter becomes superior to other algorithms. However, the state equations, measurement equations and covariance matrices need to be correctly defined.

For Model-1, usage of cyclic gains makes it optimal and it is suitable to implement on MCUs. The usage of cyclic gains eliminates the on-line computation of gains. So, the MCU has to bother about the update and prediction of states. It can successfully complete the computation before next sample from ADC module of MCU arrives, when deployed in real-time environment.

The implementation of Kalman filter is suitable for real-time because we can observe from the plots that it is able to react to sudden changes in less than 10ms. It is also evident that it requires very less time from the start time to reach steady state. Moreover, the linear Kalman filter can mitigate the noise, if present in the measurements, to some extent successfully.

6. References

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