Linear Algebra - 3

Recap → * Vectors → • direction + magnitude
• Collection of points (Row, coloumn)

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix} \qquad \vec{x} = [x_1 \ x_2 \ x_3 \ldots]$$

$$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad \vec{y}^T = [y_1, y_2, y_3]$$

* magnitude → $||\vec{x}|| = \sqrt{x_1^2 + x_2^2 + \ldots}$

Norm → $||\vec{x}||_1 = |x_1| + |x_2| + |x_3| + \ldots$ [L1 Norm, Manhattan]

$||\vec{x}||_2 = \sqrt{x_1^2 + x_2^2 + \ldots}$ [L2 Norm, Eucledean]

$$\Rightarrow \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

as 3×1, 3×1 Cannot be directly multiplied

$$\vec{x}^T \Rightarrow [x_1, x_2, x_3]$$

Dot product $\boxed{\vec{x}^T \vec{y}}$ $\quad x_1 y_1 + x_2 y_2 + x_3 y_3 + \ldots$

↳ final outcome → 1 value.

$\Rightarrow \omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 + \dots \omega_n x_n + \omega_0 = 0$

$$\boxed{\vec{\omega}^{T} \vec{x} + \omega_0 = 0}$$

Vector Representation.

↳ helps in Computation & optimization.

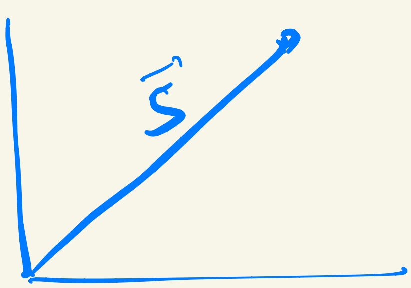
numpy has extremely powerful vector dot product.

↳ Angles b/w 2 vectors →

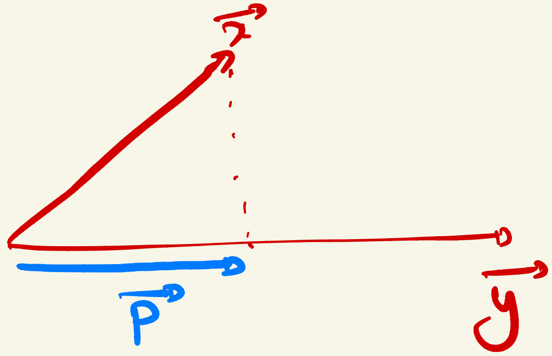
$$\cos\theta = \frac{x^T y}{\|\bar{x}\| \, \|\bar{y}\|}$$

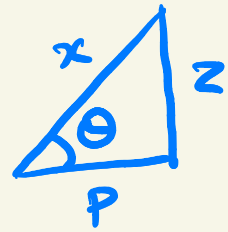
→ check Closeness b/w $\vec{\omega}$ & actual data using the angle.

⇒ Unit Vector → $\hat{y} = \dfrac{\vec{y}}{\|y\|}$ $\quad \|\hat{y}\| = 1$



⇒ Projection of $\vec{x}$ in direction $\vec{y}$.





$$\cos\theta = \frac{P}{x} = \frac{\|\vec{p}\|}{\|\vec{x}\|}$$

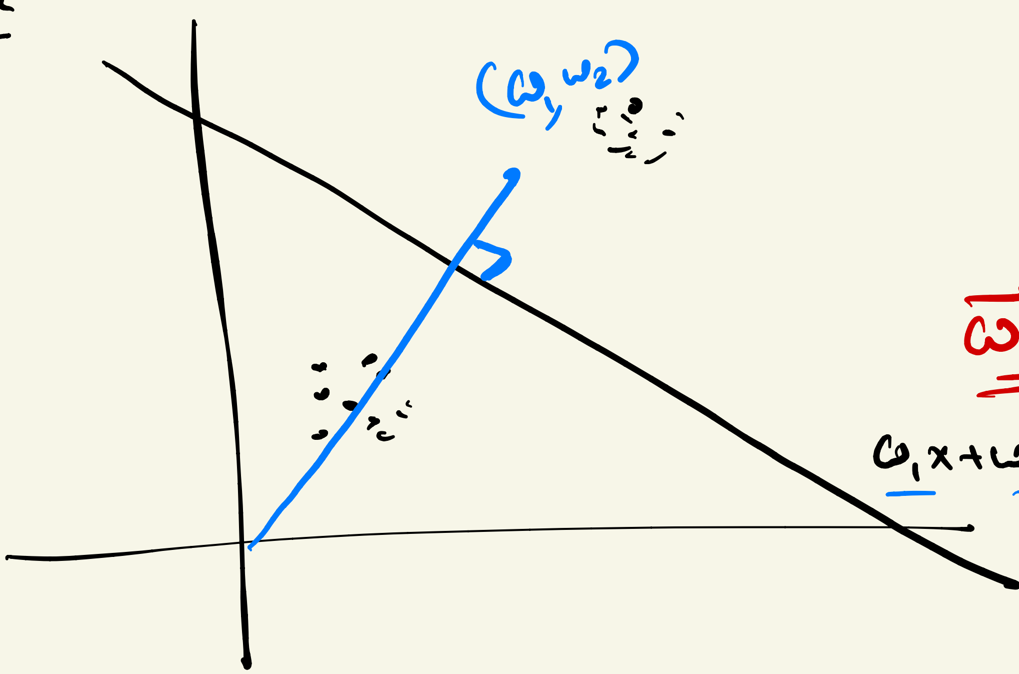
$$\frac{x^T y}{\|x\|\,\|y\|} = \frac{\|\vec{p}\|}{\|\vec{x}\|}$$

$$\|\vec{p}\| = \frac{x^T y}{\|y\|} = x^T \hat{y}$$

$\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ divide by 2

$$\boxed{\|\vec{p}\| = \vec{x}^T \hat{y}}$$

Quiz



$\vec{w}^T x + w_0 = 0$

$w_1 x + w_2 y + w_0 = 0$

Why should I care if $\vec{w}$ is perpendicular to decision boundary?

$\Rightarrow$ $\vec{w}$ is perpendicular to line $\vec{w}^T x + w_0 = 0$

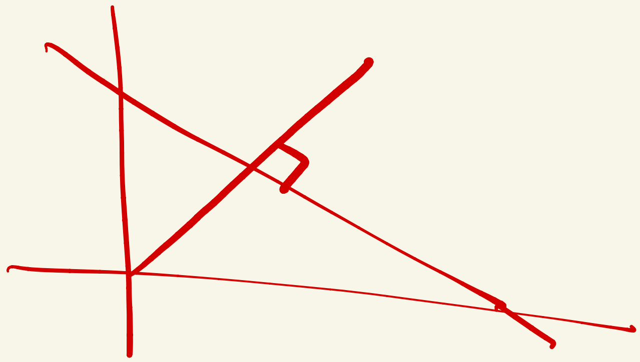
$w_1 x_1 + w_2 x_2 + \dots + w_0 = 0$

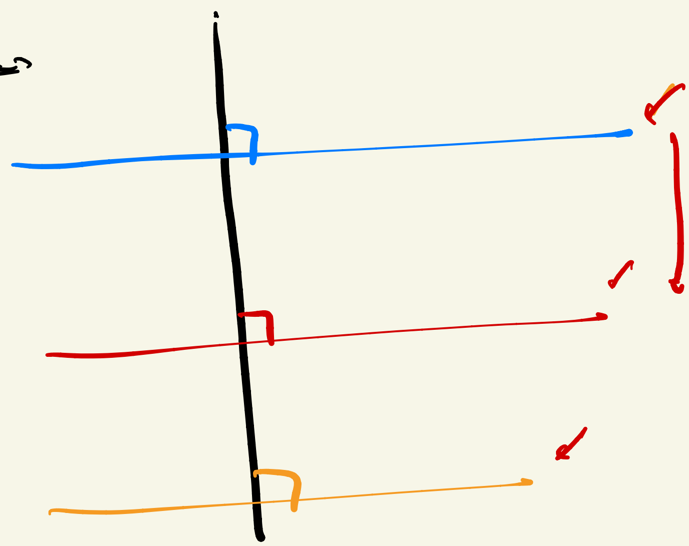
$\Downarrow$

$w^T x + w_0 = 0$

$\Downarrow$

$\vec{w}$ | when we will use optimization, loss function,





$\Rightarrow$ Shifting 2D Lines

Prove that $\vec{w}$ is perpendicular to $w^T x + w_0 = 0$

$\Rightarrow$ it $\rightarrow$

x	y	x'	y
10	20	15	20
20	30	25	30
30	40	35	40
40	50	45	50

dist^n in X direction from origin.

$$\Rightarrow \omega_1 x_1 + \omega_2 y_2 + \omega_0 = 0$$

$$\omega_2 y_2 = -\omega_1 x_1 - \omega_0$$

$$y_2 = \frac{-\omega_1}{\omega_2} x_1 - \frac{\omega_0}{\omega_2}$$

$$y_2 = \frac{-\omega_1}{\omega_2}(x_1 - 50) - \frac{\omega_0}{\omega_2}$$

$$y_2 = \frac{-\omega_1}{\omega_2} x_1 + \boxed{\frac{50\omega_1}{\omega_2} - \frac{\omega_0}{\omega_2}}$$

When I say move 50 units to right.

$$w_1(x_1+50) + w_2 x_2 + w_0 = 0$$

$$w_1(x_1-50) + w_2 x_2 + w_0 = 0$$

---

x	y
10	20
20	30
30	40
40	50

$$y - x = 10$$

$$-1x + 1y - 10 = 0$$ original line

$$\boxed{-1(x+5) + 1y - 5 = 0}$$ original equat

x'	y
15	20
25	30
35	40
45	50

new line

$$\boxed{-1x' + 1y - 5 = 0}$$

new line

$$\boxed{-1(x'-5) + 1y - 10 = 0}$$

Shifting right by a units

$$w_1(x-a) + w_2 y + w_0 = 0$$

Left by a unit

$$w_1(x+a) + w_2 y + w_0 = 0$$

Shifting up by a unit

$$w_1(x) + w_2(y-a) + w_0 = 0$$

Q Shift line by a unit right & b unit up.

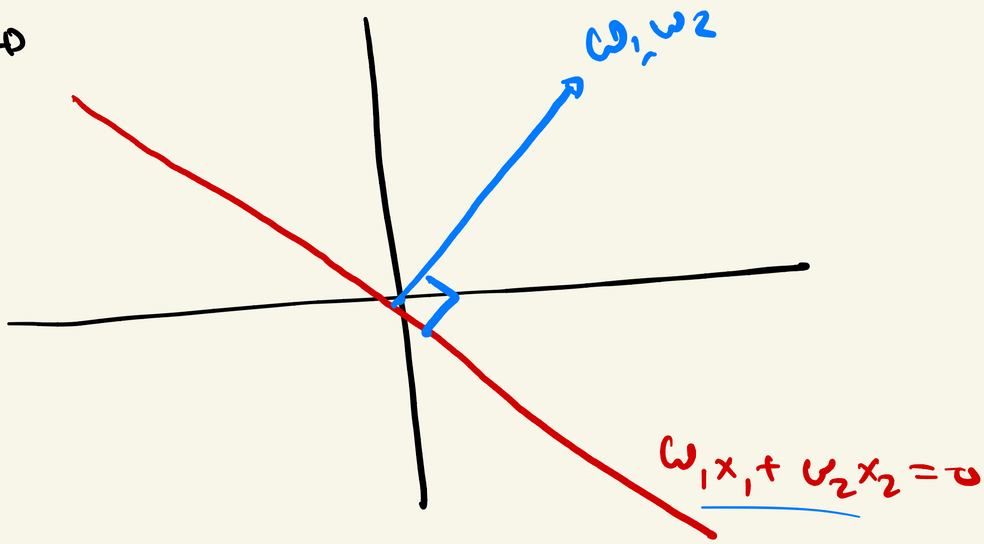
$$w_1(x_1-a) + w_2(x_2-b) + w_0 = 0$$

$$\underline{w_1x_1 + w_2x_2} \quad \underbrace{-w_1a - w_2b + w_0}_{w'} = 0$$

⇒ if you only move the Line → only intercepts chang

if you chang weights → the slope change.

⇒



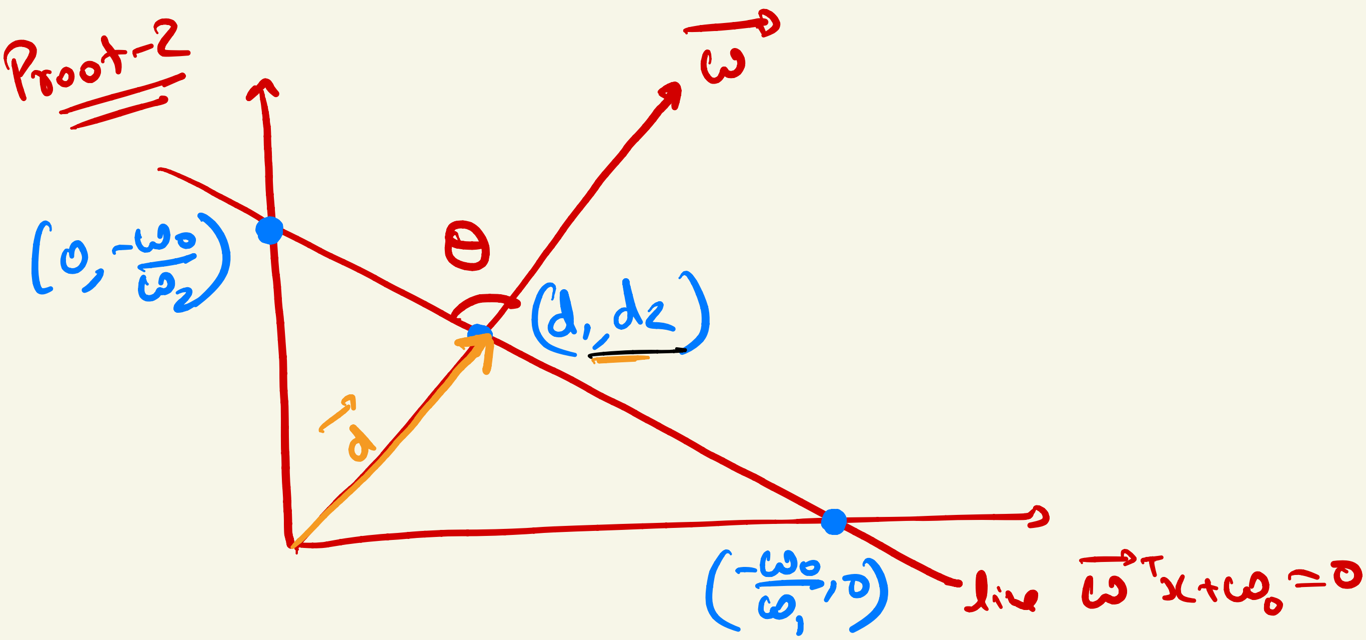
$$\underline{w_1 x_1 + w_2 x_2 + w_0 = 0}$$

because shifting doesnot chay the slope

$\underline{w^T x + w_0 = 0}$ is always perpendicular to

$$\vec{w} = [w_1, w_2].$$

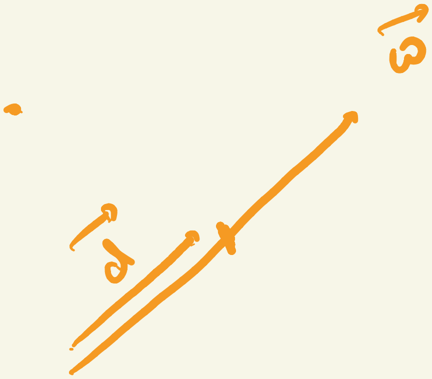
Proof-2



$$\underline{w_1 \cancel{x} + w_2 y + w_0 = 0}$$

if weight vector cuts line vector at $\underline{d_1, d_2}$.

$$\boxed{w_1 d_1 + w_2 d_2 + w_0 = 0}$$



$$\vec{d} = K \frac{\vec{\omega}}{\|\omega\|}$$

$$\|\vec{d}\| = \sqrt{d_1^2 + d_2^2}$$

$$\vec{d} = k \frac{\vec{\omega}}{\|\omega\|} \qquad d_1 = \frac{k\omega_1}{\|\omega\|} \qquad d_2 = k \frac{\omega_2}{\|\omega\|}$$

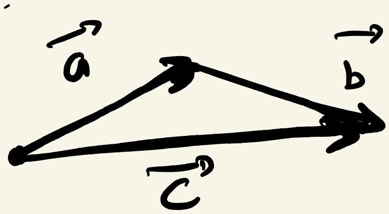
$$\vec{d} = [d_1, d_2] \qquad \vec{d} = \frac{k}{\|\omega\|}\left[\omega_1, \ \omega_2\right]$$

$$\omega_1 d_1 + \omega_2 d_2 + \omega_0 = 0$$

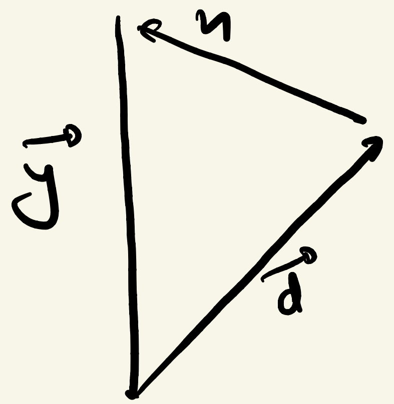
$$K \frac{\omega_1^2}{\|\omega\|} + K \frac{\omega_2^2}{\|\omega\|} + \omega_0 = 0$$

$$\boxed{K = -\frac{(\omega_0)(\|\omega\|)}{\omega_1^2 + \omega_2^2}}$$

$\Rightarrow$ Re...



$$\boxed{\vec{c} = \vec{a} + \vec{b}}$$



$\vec{d}$,

$$\vec{y} = \left[0, -\frac{w_0}{w_2}\right]$$

$$\vec{y} = \vec{d} + \vec{n}$$

$$\boxed{\vec{n} = \vec{y} - \vec{d}}$$

$$\vec{y} = \begin{bmatrix} 0 \\ -\frac{w_0}{w_2} \end{bmatrix} \qquad \vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} k\frac{w_1}{\|w\|} \\ k\frac{w_2}{\|w\|} \end{bmatrix}$$

$$=$$

$$\vec{n} = \vec{y} - \vec{d} = \begin{bmatrix} 0 - k\frac{w_1}{\|w\|} \\ -\frac{w_0}{w_2} - k\frac{w_2}{\|w\|} \end{bmatrix}$$

$$\cos\theta = \frac{\vec{d}^T \vec{n}}{(\qquad)}$$

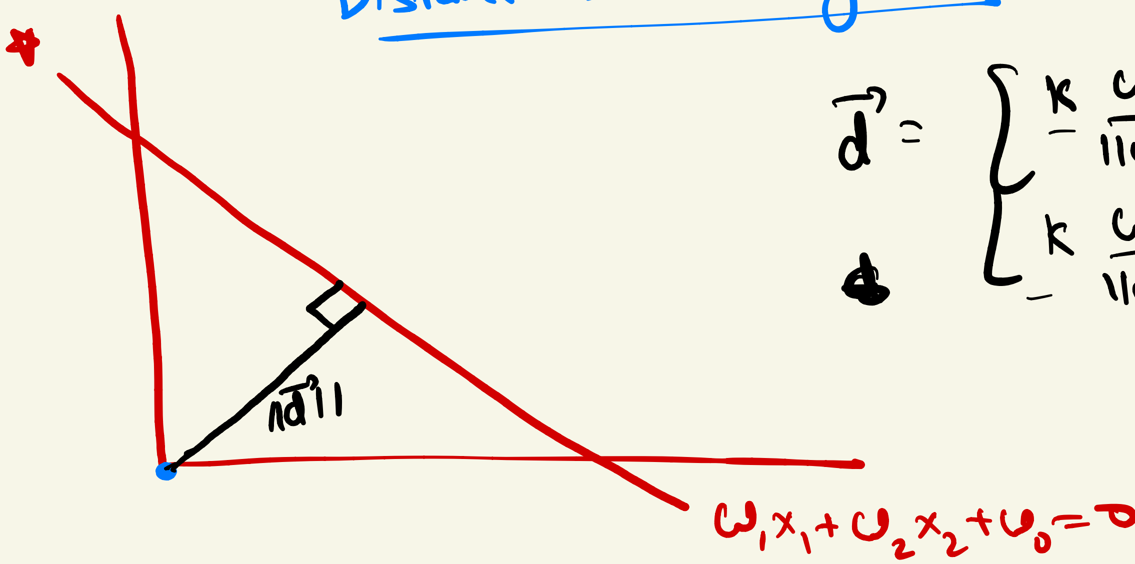
$$\begin{bmatrix} k\dfrac{\omega_1}{\|\omega\|} & k\dfrac{\omega_2}{\|\omega\|} \end{bmatrix} \begin{bmatrix} -\dfrac{k\omega_1}{\|\omega\|} \\ -\dfrac{\omega_0}{\omega_2} - \dfrac{k\omega_2}{\|\omega\|} \end{bmatrix}$$

$$-\frac{k^2\omega_1^2}{\|\omega\|^2} + \left(\frac{-\omega_0 k}{\|\omega\|} - \frac{k^2\omega_2^2}{\|\omega\|} \right)$$

$$\cos\theta = 0 \Rightarrow \boxed{\theta = 90^\circ}$$

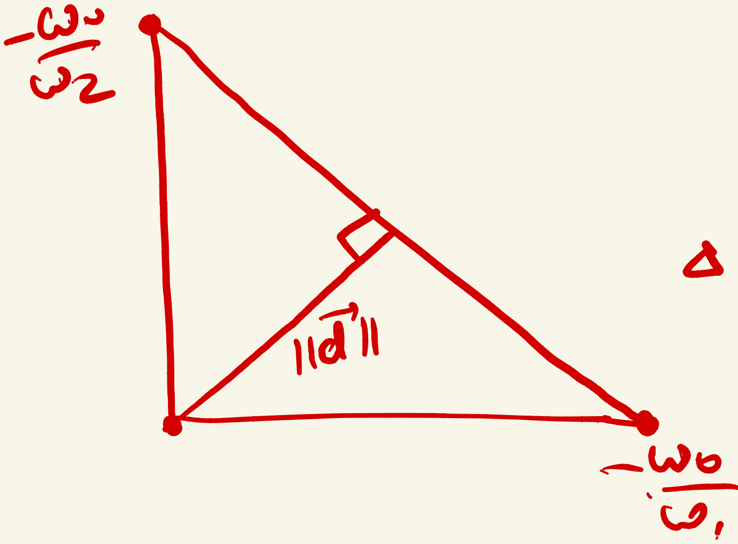
$\Rightarrow$ $\boxed{\vec{\omega} \text{ is orthogonal to decision boundary}}$

Distance from origin



$$\vec{d} = \begin{bmatrix} k\dfrac{\omega_1}{\|\omega\|} \\ k\dfrac{\omega_2}{\|\omega\|} \end{bmatrix}$$

$$\vec{d} = \sqrt{k^2\left(\frac{\omega_1^2 + \omega_2^2}{\|\omega\|}\right)} = \frac{\omega_0}{\|\omega\|}$$



$w_1 x + w_0 = 0$

$$\Delta = \left(\frac{1}{2}\right)\left(\frac{-w_0}{w_2}\right)\left(\frac{-w_0}{w_1}\right)$$

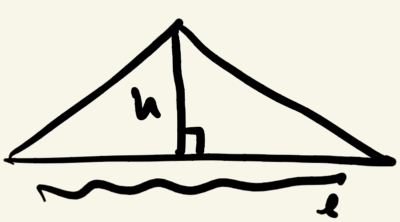
$$\frac{1}{2}\|\vec{d}\| \quad c = \frac{1}{2}\left(-\frac{w_0}{w_2}\right)\left(\frac{w_0}{w_1}\right)$$

$$\cancel{\frac{1}{2}}\|\vec{d}\|\left(\sqrt{\frac{w_0^2}{w_1^2}+\frac{w_0^2}{w_2^2}}\right) = \cancel{\frac{1}{2}}\left(\frac{w_0^2}{w_1 w_2}\right)$$

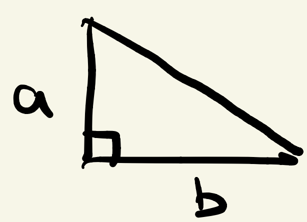
$$\|\vec{d}\| \frac{\cancel{w_0}}{\cancel{w_1 w_2}}\left(w_1^2+w_2^2\right)^{1/2} = \frac{w_0^{\cancel{2}}}{\cancel{w_1 w_2}}$$

$$\|\vec{d}\| = \frac{w_0}{\sqrt{w_1^2+w_2^2}} = \frac{w_0}{\|w\|}$$

Area of any triang = $\frac{1}{2}$ × length × height.



$\frac{1}{2} lh$

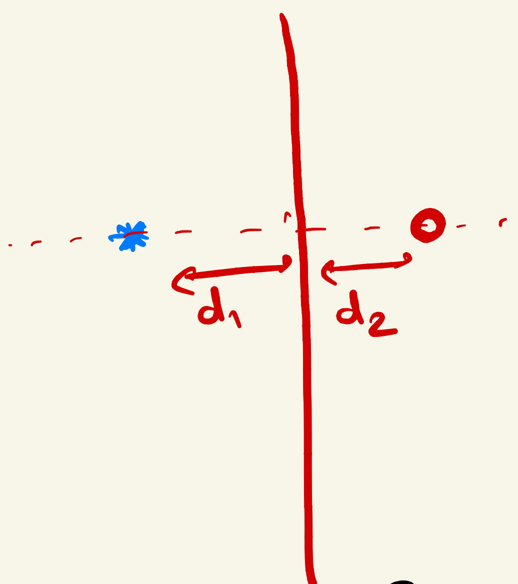


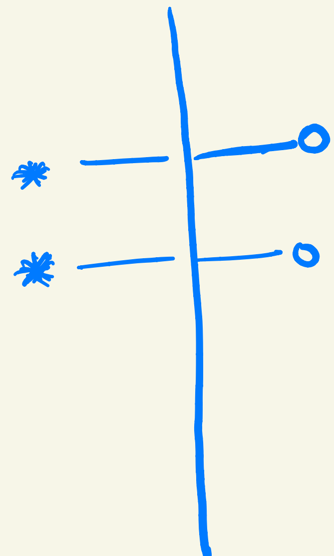
$\frac{1}{2} a \cdot b$

$$\boxed{\|\vec{d}\| = \frac{-w_0}{\|\vec{w}\|}}$$

distance b/w origin and a line

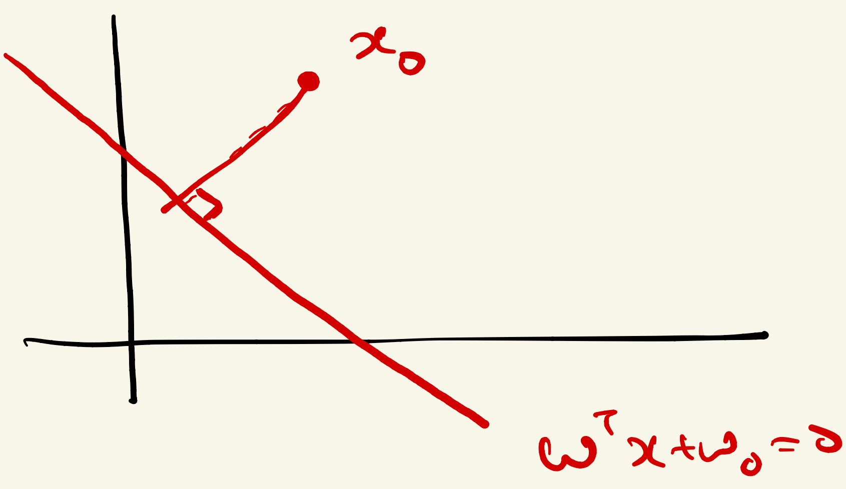
$$w_1 x_1 + w_2 x_2 + w_0 = 0 \implies \|\vec{d}\| = \frac{w_0}{\sqrt{w_1^2 + w_2^2}}$$

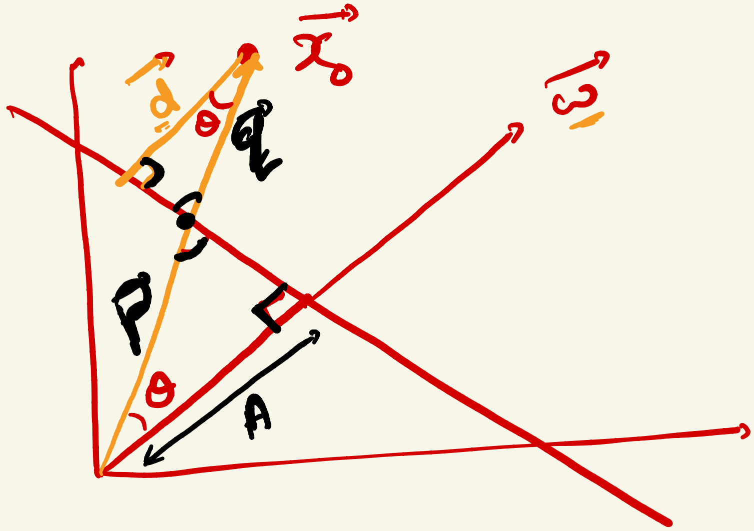




Basic identification of best-fit line v/s done on the basis of distⁿ from actual pt.

⇒ if we can get distⁿ of a point from line →

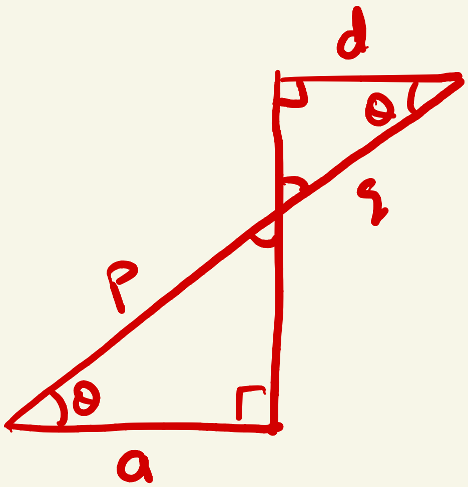




$$\|\vec{x}_0\| = p + q$$

$$\to \frac{A}{p} = \cos\theta = \frac{d}{q}$$

$$A = \frac{-\omega_0}{\|\omega\|}$$



$$\cos\theta = \frac{d}{q}$$

$$\cos\theta = \frac{a}{p}$$

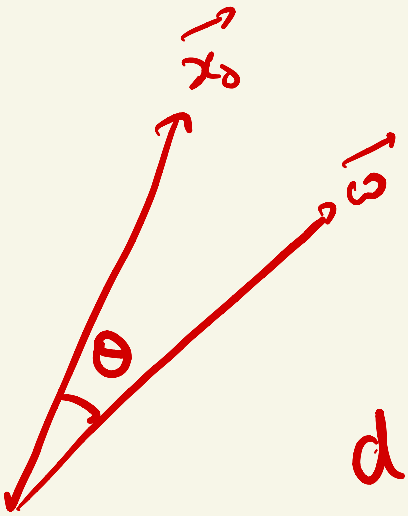
$$a = \frac{-\omega_0}{\|\omega\|}$$

$$\|\vec{x}\| = p + q$$

$$\|\vec{x}_0\| = \frac{a}{\cos\theta} + \frac{d}{\cos\theta}$$

$$d = \|\vec{x}_0\| \cos\theta - a$$

$$= \|\vec{x}_0\| \cos\theta + \frac{w_0}{\|w\|}$$



$$\cos\theta = \frac{w^T x}{\|x\|\,\|w\|}$$

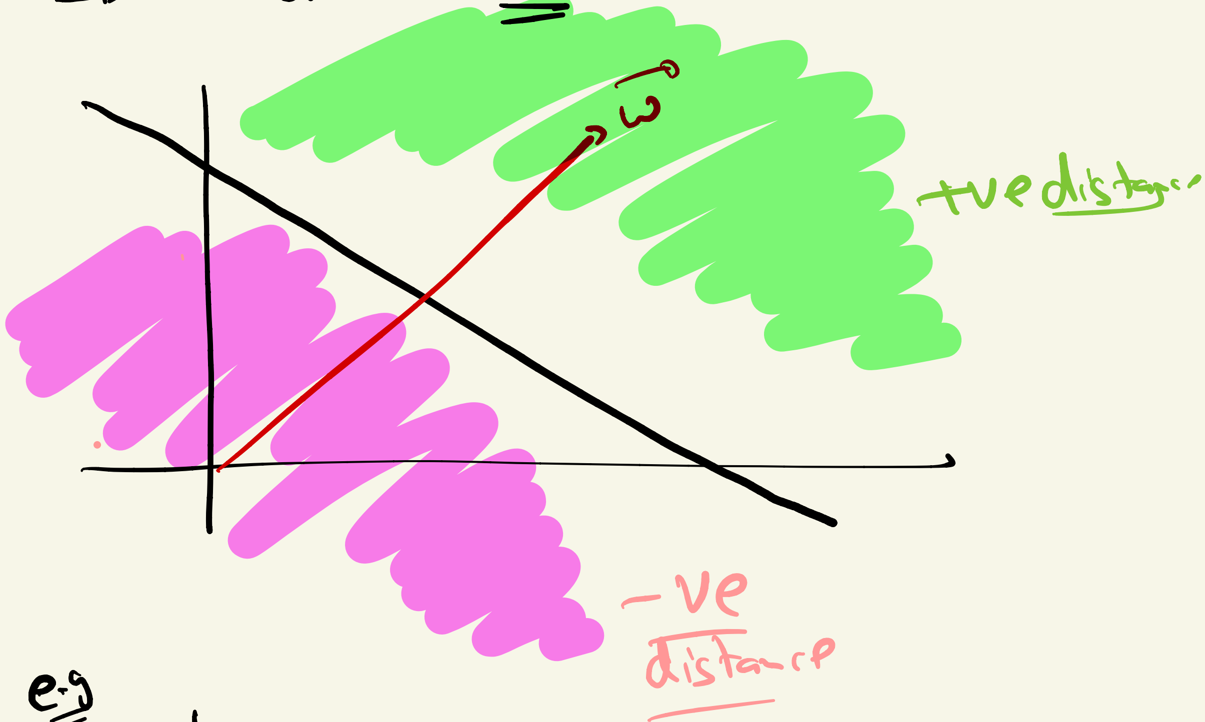
$$d = \frac{w^T x}{\|w\|} + \frac{w_0}{\|w\|}$$

$$\boxed{d = \frac{w^T x_0 + w_0}{\|w\|}}$$

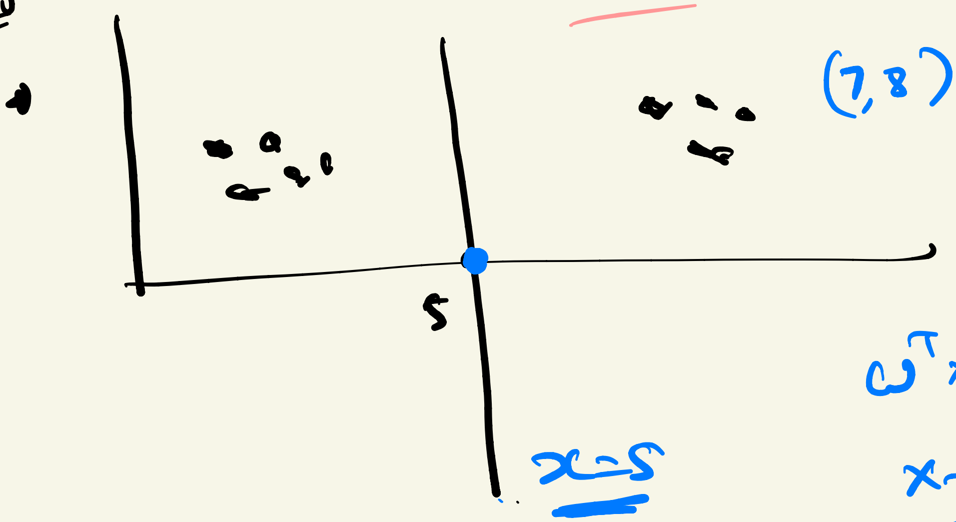
distance between point $\vec{x}_0$ and line $w^T x + w_0 = 0$

$$\boxed{d = \frac{w^T x_0 + w_0}{\|w\|}}$$

$\Rightarrow$ if $d \rightarrow +ve$.



e.g



$w^T x + w_0 =$

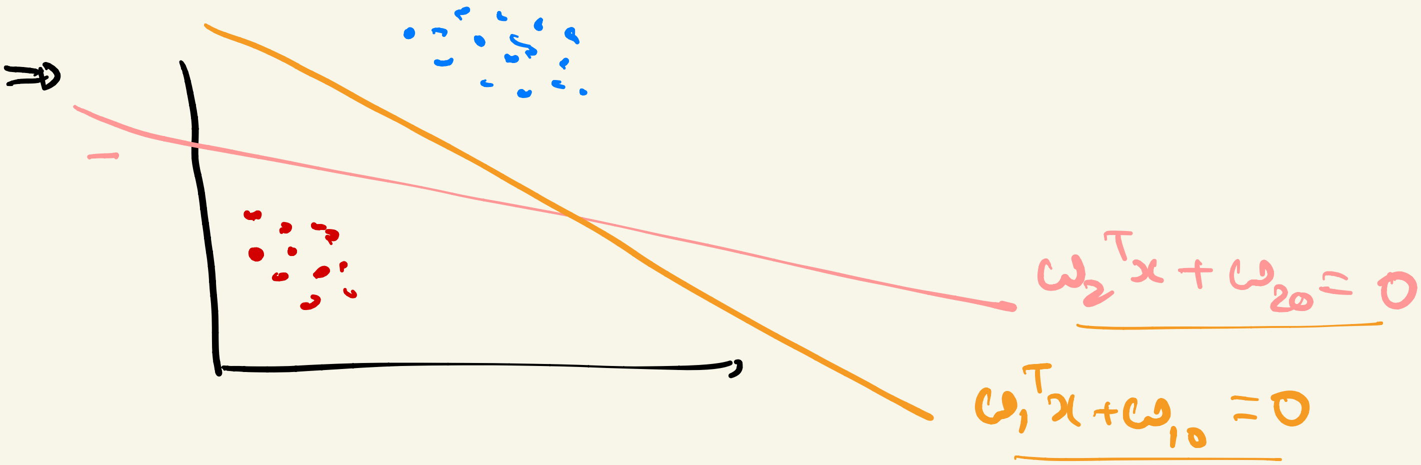
$x - 5 = 0$

$1, 0 \qquad \|w\| \Rightarrow 1$

$(7,8) \qquad 2 \rightarrow +ve$

$(3,4) \qquad -2 \rightarrow -ve$

$$\boxed{\vec{d} = \frac{\omega^T x + \omega_0}{\|\omega\|}}$$



Just Calculate $\sum \vec{d}_{x_i}$

- Why / what are doing
- final formula (understand what it means)
- Proof

Conclude →

1) $\vec{\omega}$ is orthogonal to decision boundary

2) distance of a point from a line $\Rightarrow \dfrac{\omega^T x + \omega_0}{\|\omega\|}$

$\Rightarrow$ Assignment dot product $\longrightarrow$ $\underline{\underline{1 \times 1}}$

$m \times 1$ $\quad$ $m \times 1$ $\quad$ $x^T y = \underline{\underline{1 \times m}} \quad \underline{\underline{m \times 1}}$

x $\quad$ y

$\underline{\underline{1 \times 1}}$

$x \in R^n \longrightarrow$ n dimension

$\in R^1$

(2 times d)

$R^{\boxed{2d}} \longrightarrow \underline{2d}$ $\underline{\underline{\text{dimensions}}}$

$R^2 \longrightarrow$ 2 dim

$R^d \longrightarrow$ d dim

$R^1 \longrightarrow$ 1 dim