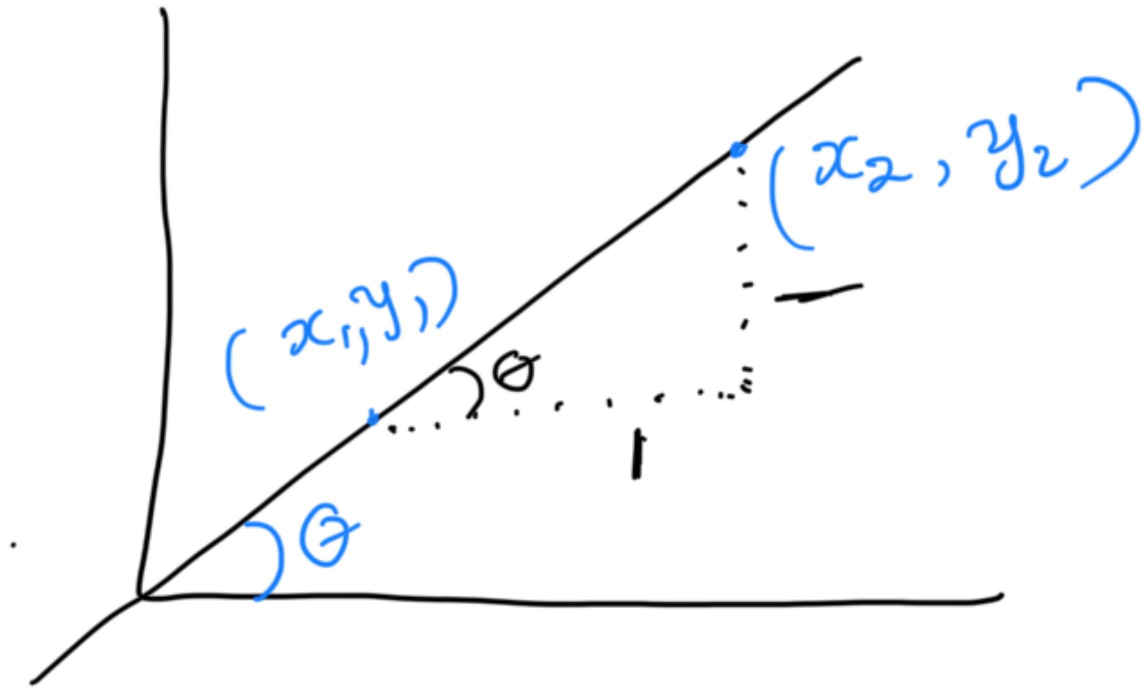
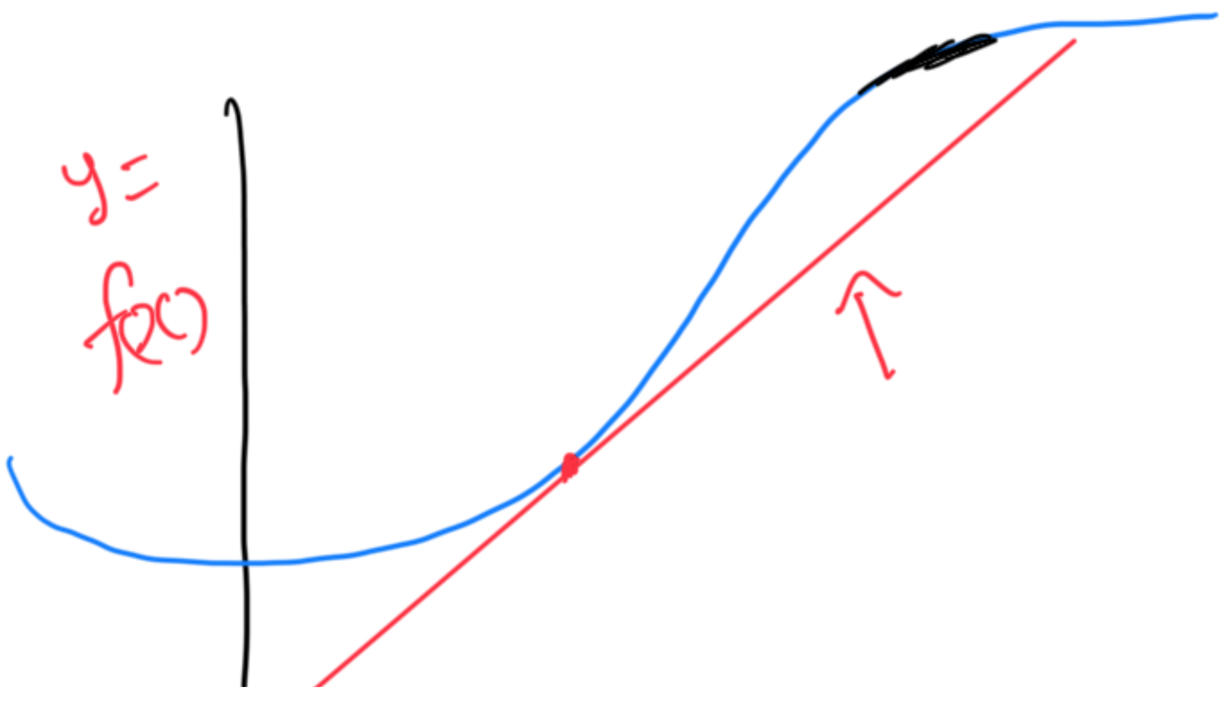
# Optimization - 2

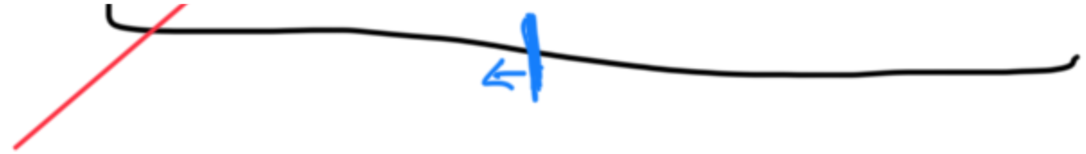



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

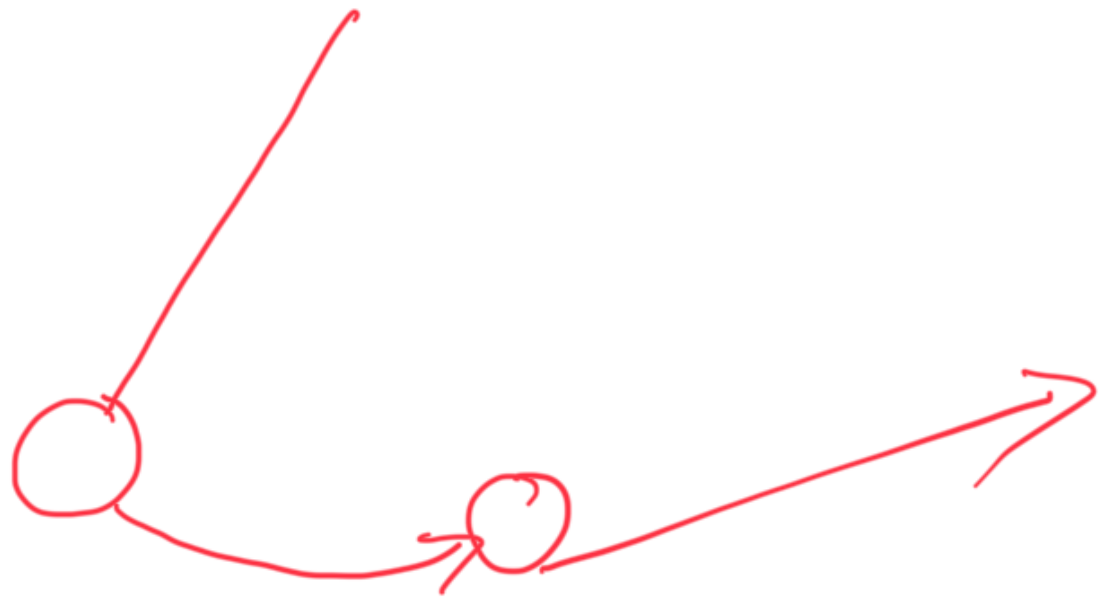
$$= \tan(\theta)$$




a small change in
$x$
$\downarrow$
what change in



$f(x)$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x_2) - f(x_1)}{}$$

$$x_2 - x_1$$

$$= \frac{f(x_1 + \Delta x) - f(x_1)}{(x_1 + \Delta x) - x_1}$$

$$= \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

$$\boxed{\frac{d}{dx} f(x) = f'(x) = \lim_{\Delta x \to 0} \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}}$$

$$f(x) = x$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + (\Delta x)^2 + 2x \cdot \Delta x - \cancel{x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \Delta x + 2x$$

$$= \underline{\underline{2x}}$$

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 6 \end{cases} \qquad f(x) = |x|$$

$0 \quad \text{if} \quad x = 0$

$\boxed{\text{Case 1 : } x > 0}$

$f(x) = x$

$\boxed{f'(x) = 1}$

$\boxed{\text{Case 2} \quad x < 0}$

$f(x) = -x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-(x+\Delta x) - (-x)}{\Delta x}$$

$$= \frac{\cancel{-x} - \cancel{\Delta x} + \cancel{x}}{\cancel{\Delta x}}$$

$$= \boxed{-1} \checkmark$$

$\boxed{\text{Case 3}}$ $x = 0$

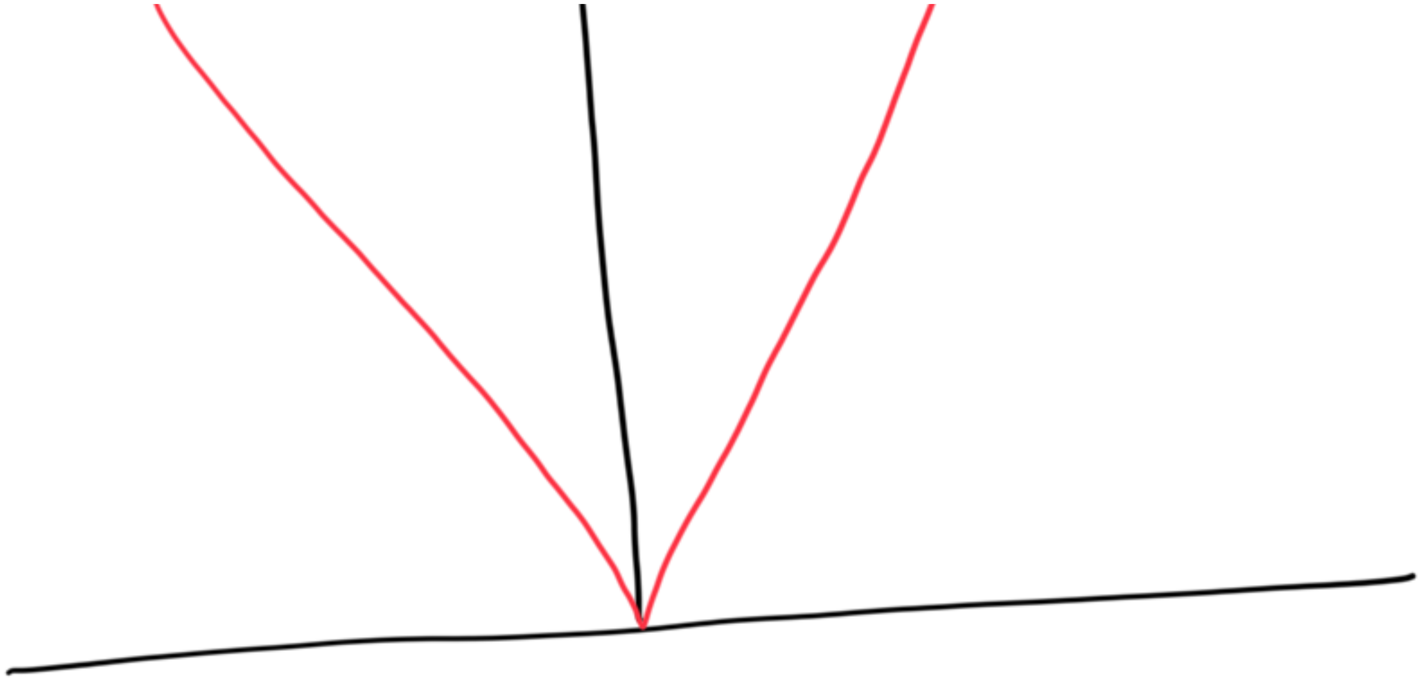
**LHL** $\lim_{x \to 0^-} f'(x) = -1$

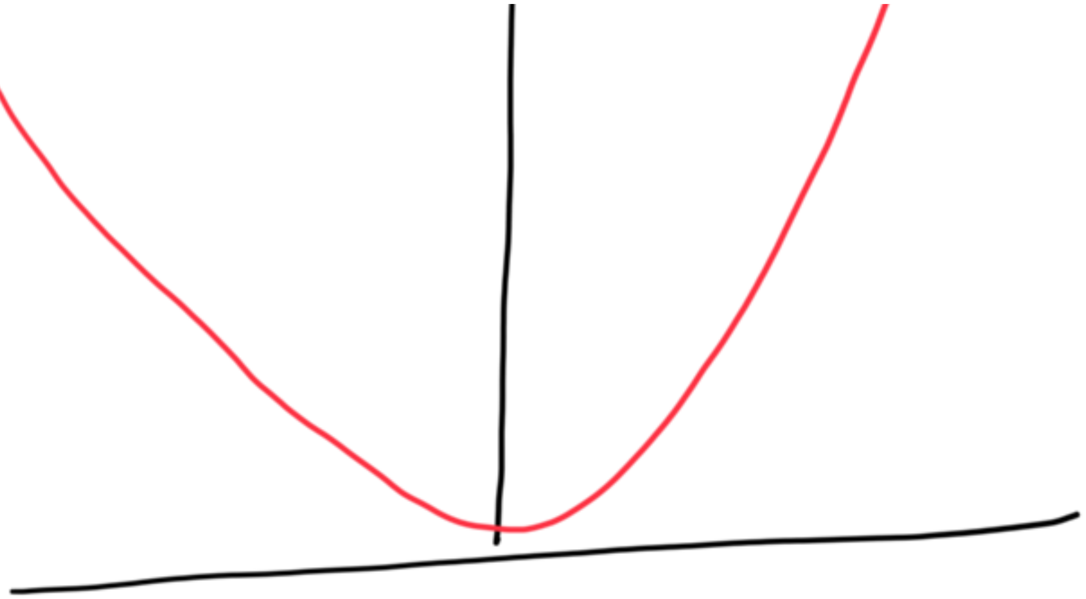
RHL $\lim_{x \to 0^+} f'(x) = 1$

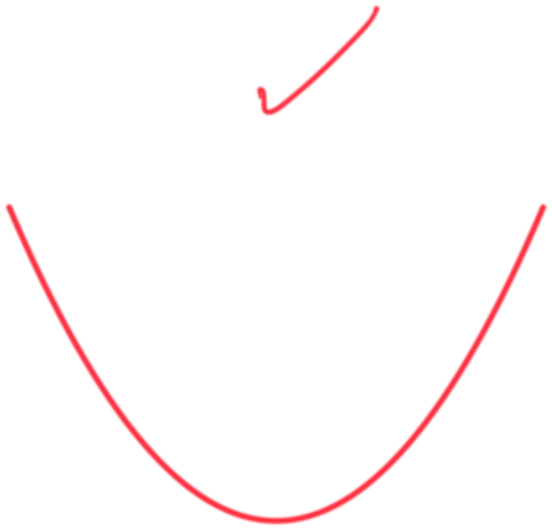
$\boxed{x = 0}$ $f(x) = |x|$ is not differentiable

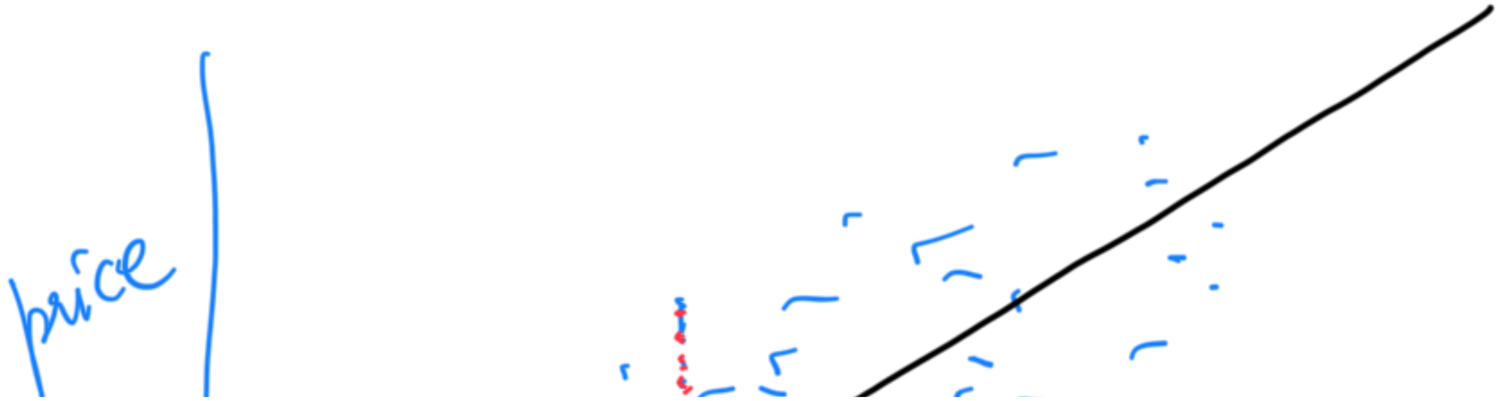
$x^2$

$|x|$



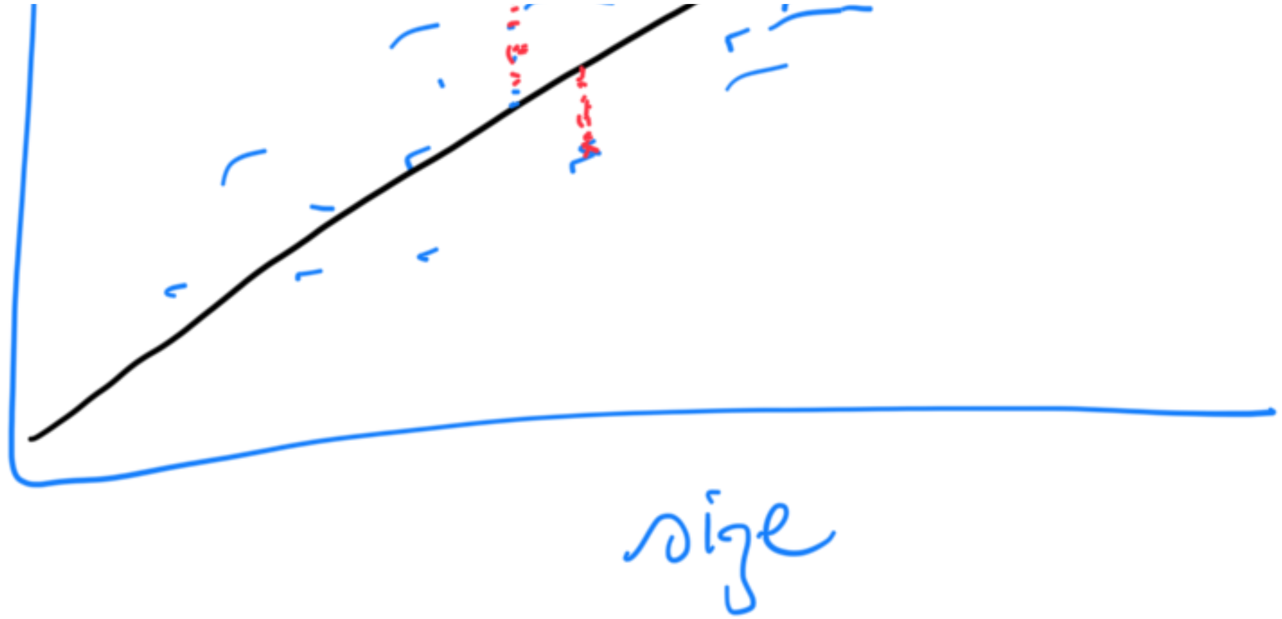






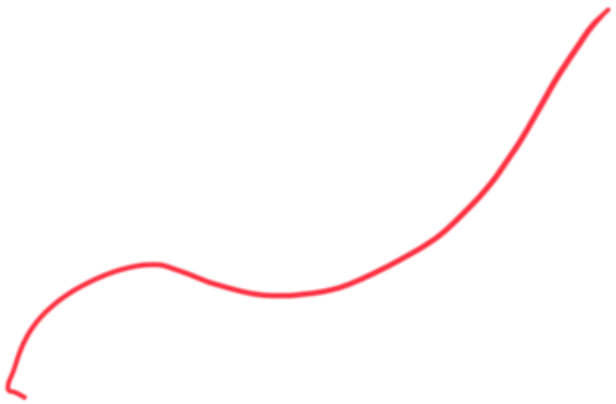

$$\sum_{i=1}^{n} |x|$$

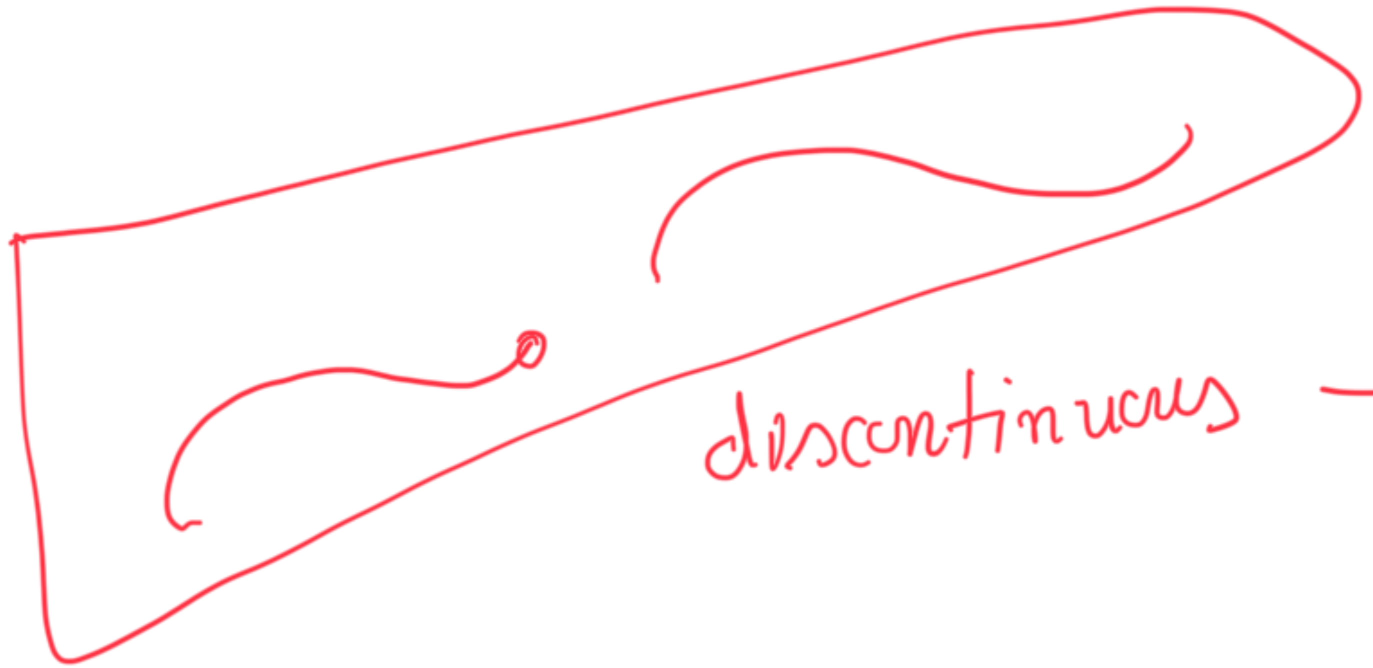
$$(x^2)$$




MAE

MSE





discontinuous $\longrightarrow$ non differentiable.

if $\underbrace{f'(x)}$ is continuous, then $f$ is differentiable.

$d \quad n \qquad n \cdot x^{n-1}$

$$\frac{d}{dx} x = n \cdot x$$

$$\frac{d}{dx} c = \underline{\underline{0}}$$

$$\frac{d}{dx} \log x = 1/x$$

$$\frac{d}{dx} e^{x} = e^{x}$$

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## Rules of differentiation

**Linearity:**

$$h(x) = g(x) + f(x)$$

$$h'(x) = g'(x) + f'(x)$$

$$f(x) = x^{3} + \log(x)$$

$$f'(x) = 3x^2 + 1/x$$

## Product Rule

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = g'(x) \cdot h(x) + h'(x) \cdot g(x)$$

## Chain Rule

$$h(x) = f(g(x)) \rightarrow \qquad e^{g(x)}$$

$$h'(x) = f'(g(x)) \cdot g'(x) \qquad f = e^{g(x)}$$

$$g(x) = -x$$

$$f(x) = \log(x^2) \qquad y = x^2$$

$$\frac{d}{dx}(\log y)$$

$$= \frac{1}{y} \cdot \frac{d}{dx} y$$

$$= \frac{1}{y} \cdot \frac{d}{dx} x^2$$

$$= \frac{1}{x^2} \cdot 2x$$

$$= \frac{2}{x}$$

$$f(x) = e^{-x}$$

$$y = -x$$

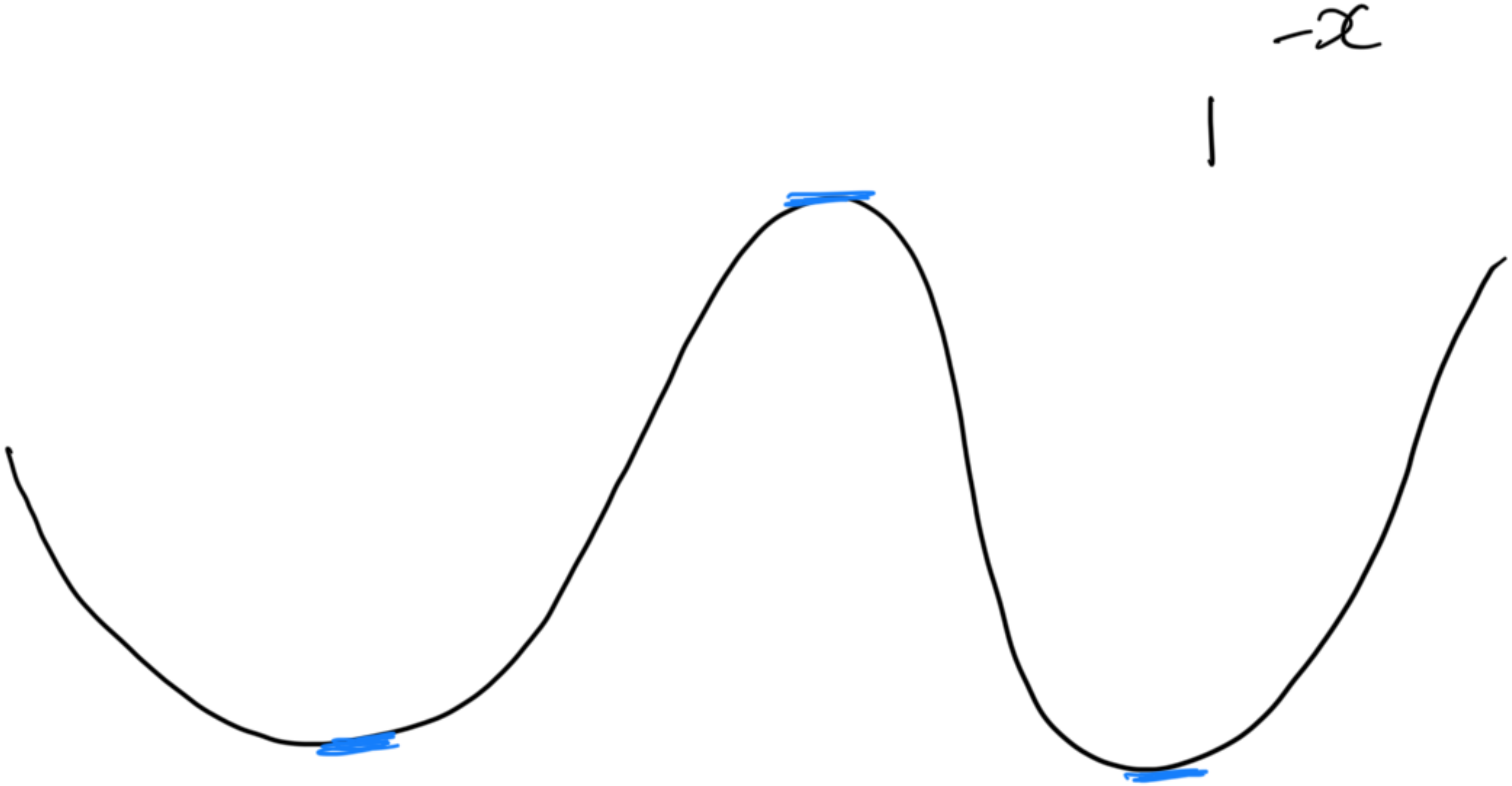
$$\frac{d}{dx} e^{y}$$

$$= e^{y} \cdot \frac{d}{dx} y$$

$$= e^{-x} \cdot \frac{d}{dx}(-x)$$

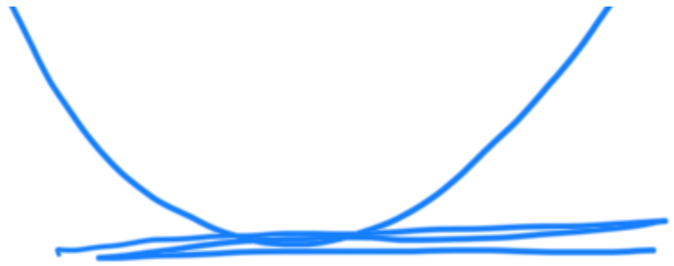
$$= e^{-x} \times -1$$

$$= \boxed{-e^{-x}}$$



minima

$$f'(x) = 0$$



How to distinguish b/w maxima and minima