Linear Algebra-4

Agenda → • Recap
• Loss function
• Perceptron learning algorithm
  ↳ Code
• Circle

Recap → a) Vectors → $\bar{x} \in R^d$ $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$

b) Norm → $||x||$ → $||x||_2 = \sqrt{x_1^2 + x_2^2 + \dots x_d^2}$

$||x||_1 = |x_1| + |x_2| + \dots + |x_d|$

c) Dot product (inner product) → $\vec{x}, \vec{y} \in R^d$

$\vec{x} \cdot \vec{y} = \vec{x}^T y = \sum_{i=1}^{d} x_i y_i$

$= x_1 y_1 + x_2 y_2 + \dots + x_d y_d$

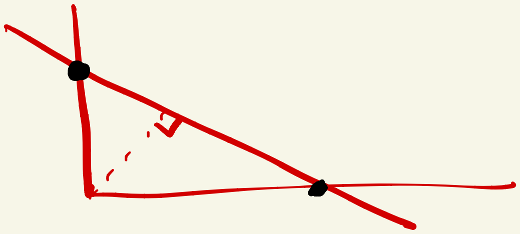
d) Angle b/w 2 vectors $\cos\theta = \dfrac{\vec{x}^T y}{||x|| \, ||y||}$

e) Dist^n b/w a line and origin →

line → $w^T x + w_0 = 0$

origin → $0,0$

$$\boxed{d = \frac{w_0}{||w||}}$$



f) Dist^n b/w any line and any point

line → $\omega^T x + \omega_0 = 0$

point → $\vec{x}_1$

$$\boxed{d = \frac{\omega^T \vec{x}_1 + \omega_0}{\|\omega\|}}$$

$\omega_1 x_1 + \omega_2 x_2 + \omega_0 = 0$

Q How to Calculate dist^n b/w 2 lines ??

Lines intersect → dist^n = 0

dist^n makes sense only if 2 lines are parrallel.

line$_1$ → $\omega_1^T x + \omega_{10} = 0$

line$_2$ → $\omega_2^T x + \omega_{20} = 0$

if they are parallel $\omega_1 = \omega_2$.

⇒ L_1 → $\omega^T x + 5 = 0$

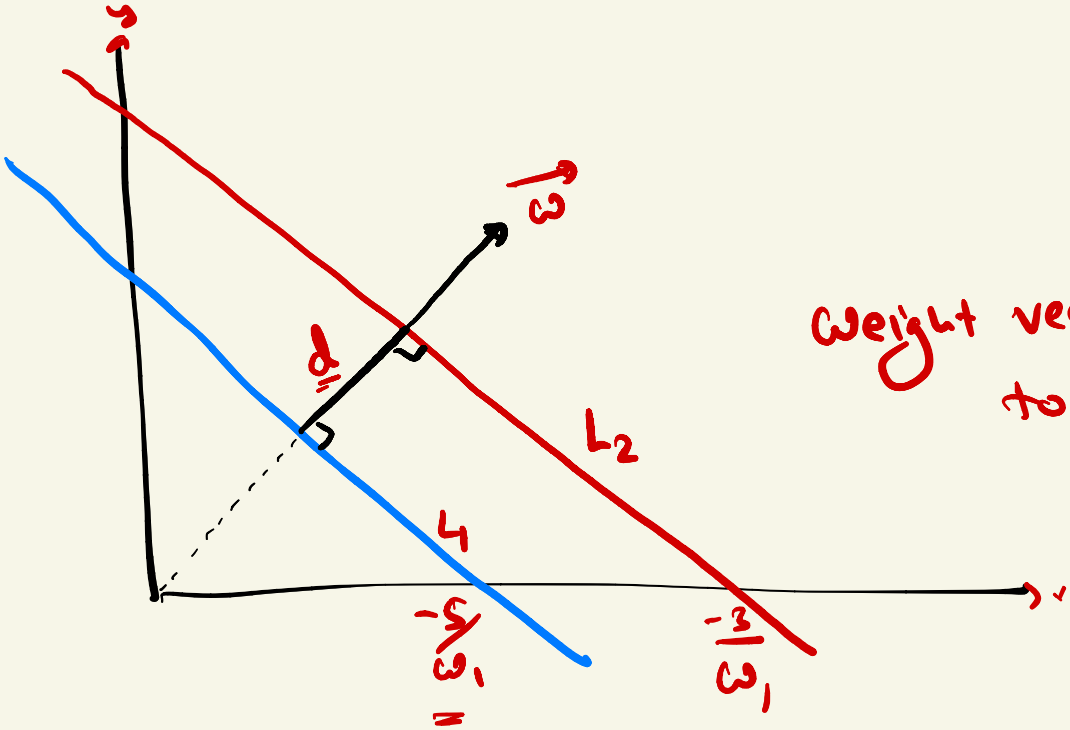
L_2 → $\omega^T x + 3 = 0$

Yes they are parrellell

→ Dist^n b/w L_1 & L_2

$w_1 x_1 + w_2 y + 5 = 0$

$w_1 x_2 + w_2 y + 3 = 0$



weight vector is perpendicular to the decision boundary

$\vec{w}$ $$\underline{w^T x + w_0 = 0}$$

distance of L_1 from origin $\Rightarrow$ $\frac{5}{\|w\|}$ d_1

distance of L_2 from origin $\Rightarrow$ $\frac{3}{\|w\|}$ d_2

$$\text{dist}^n \text{ b/w line} = d_1 - d_2$$

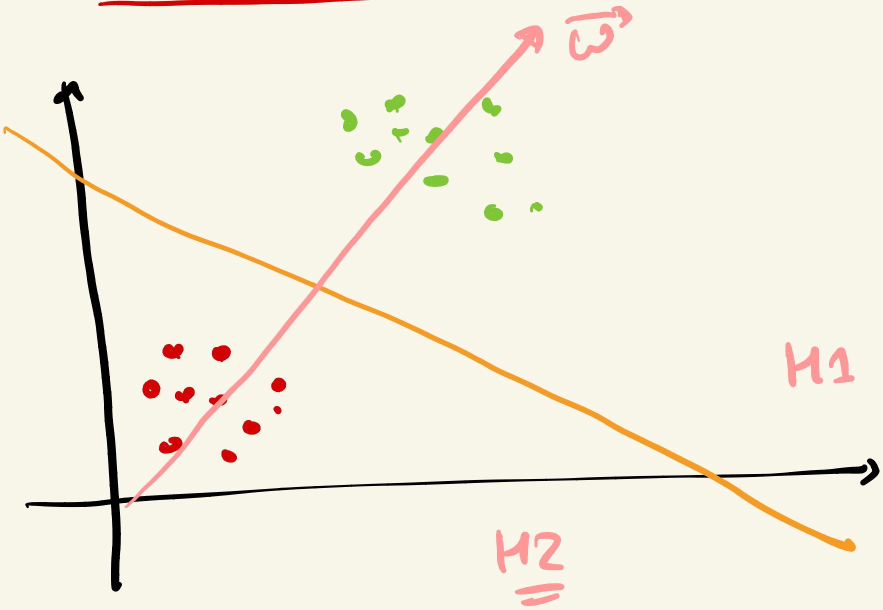
$$= \frac{5-3}{\|w\|}$$

$L_1 \rightarrow w^T x + w_{10} = 0$

$L_2 \rightarrow w^T x + w_{20} = 0$

$$\boxed{\text{dist}^n = \frac{w_{20} - w_{10}}{\|w\|}}$$

⇒ How to find which half space a point belongs to ??



⇒ Given there are 2 Categories

Cat vs dog.

↓

identify the Correct Category

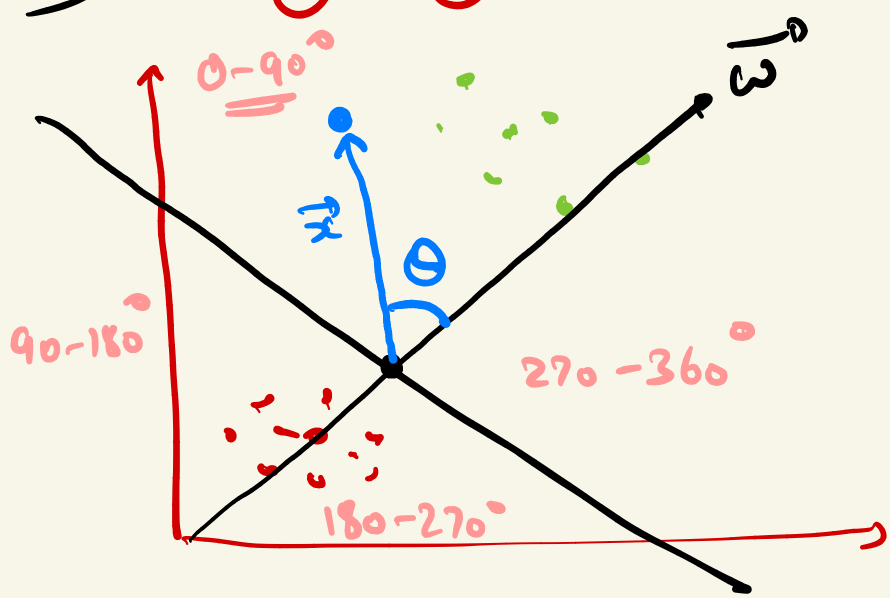
if we know decision boundary $\vec{w}^T x + w_0 = 0$

1st way → Calculate distⁿ from the line.

if d → +ve ⇒ H1

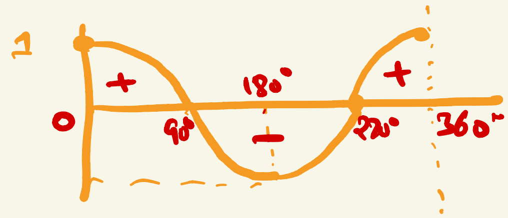
d → -ve ⇒ H2

2nd way Using angles →



if I try to find angle b/w $\vec{w}$ and point vector ($\vec{x}$)

$$\cos\theta = \frac{x^T \omega}{\|x\| \, \|\omega\|}$$

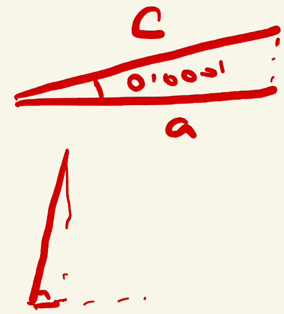


+ve → 0−90°

−ve → 90−180°

+ve → 180−270°

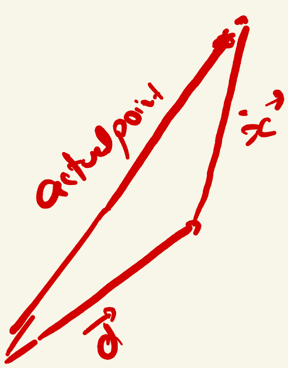
+ve → 270−360°



$$\cos\theta = \frac{x^T \omega}{\|x\| \, \|\omega\|}$$

if +ve → H1

−ve → H2



Actual = $\vec{d} + \vec{x}$

When we shift a Line →

$$\bar{\omega}_1 x_1 + \omega_2 x_2 + \omega_0 = 0$$

⇒ x, y

10, 20

12, 15

18, 20

30, 35

→ 0,0 → 10,20

x	y
0	20
2	5
8	10
20	25

$df[x] = df[x] - 10$

$\Rightarrow \cos\theta \to +ve \quad \to HS\ 1$

$\cos\theta \to -ve \quad \to HS\ 2$

$d \to +ve \quad \Rightarrow HS1$

$d \to -ve \quad \Rightarrow HS2$

e.g. Categorize Cat and dog $\to$

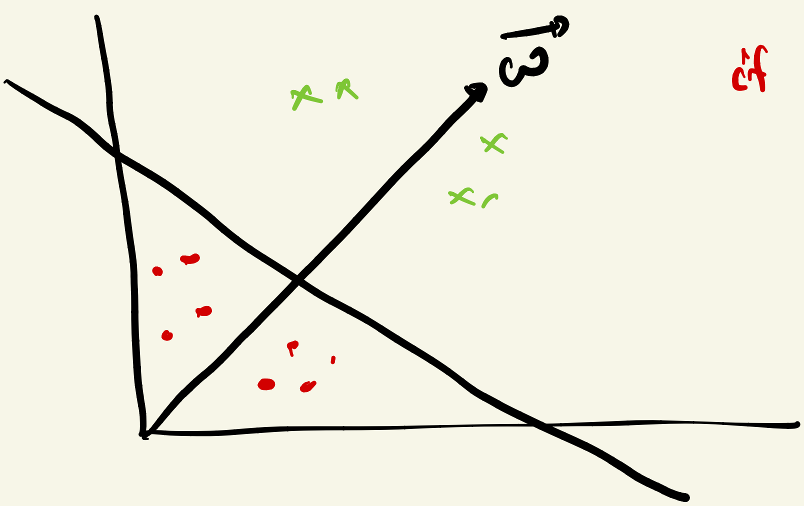
Cat $\to$ HS 1 $\Rightarrow$ +1 | 1

dog $\to$ HS2 $\Rightarrow$ −1 | 0

$\Rightarrow d \to +ve \quad \Rightarrow$ outcome $\to +1$

$d \to -ve \quad \Rightarrow$ outcome $\to -1$

Quiz



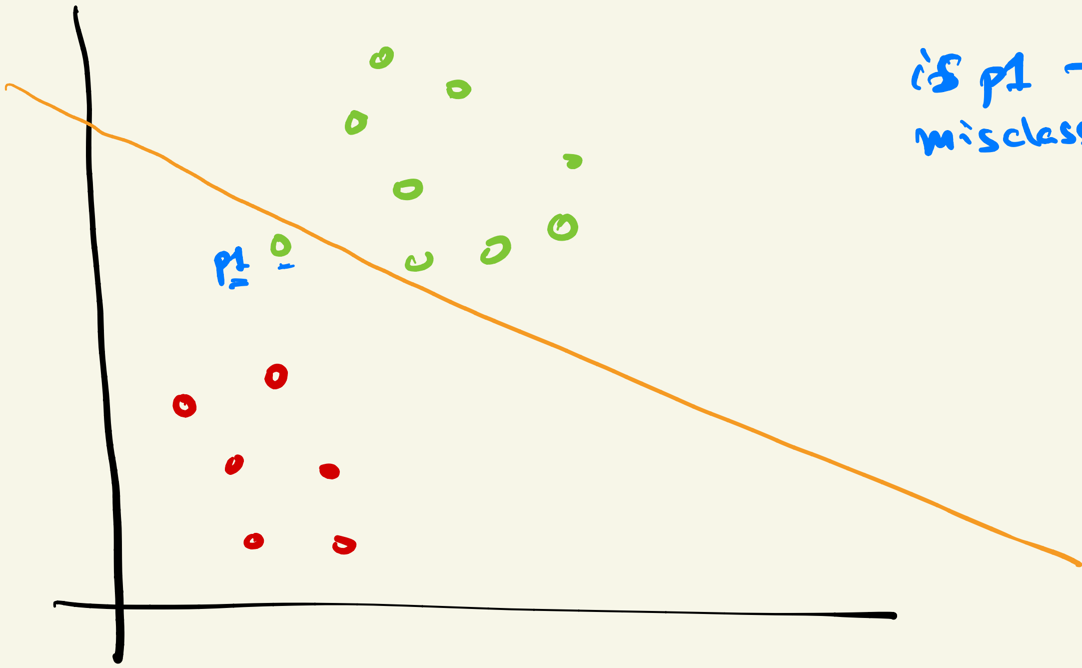
if point lies on the $\vec{\omega}$ direction from decision boundary $\to$ +ve

else −ve

⇒ Put everything together –

Colur → actual Category

is p1 misclassified → $\underline{\underline{\text{Yes}}}$



What is the actual label for P_1 → $+1$

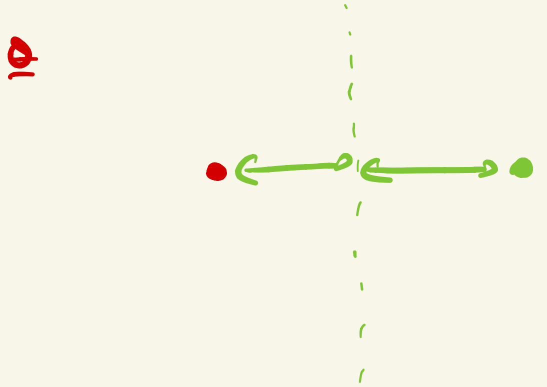
What is the assigned label for P_1 as per decision boundary → $\underline{-1}$

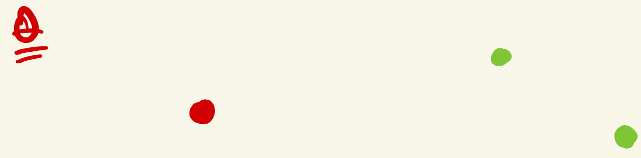
decision boundary → $\omega^T x + \omega_0 = 0$

$$d = \frac{\omega^T \vec{x} + \omega_0}{\|\omega\|} \nearrow \begin{array}{l} d > 0 \rightarrow +1 \\ d < 0 \rightarrow -1 \end{array}$$

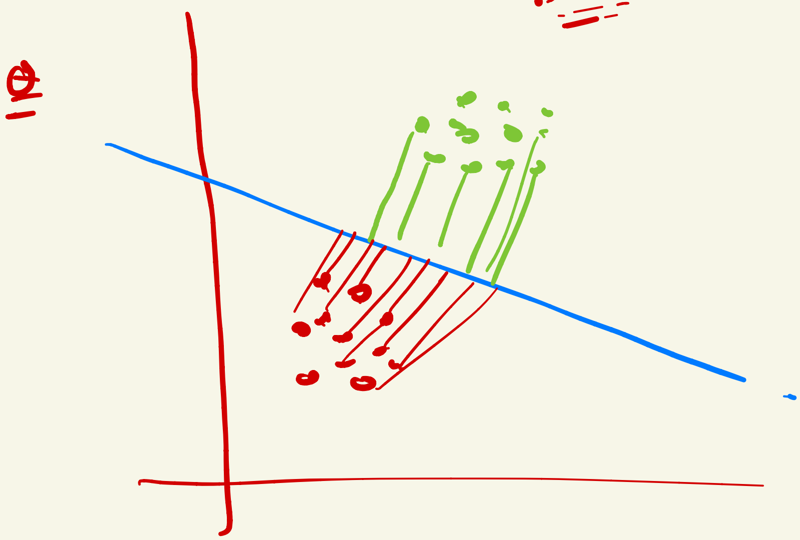
For P_1 if we Calculate distⁿ from Current line → $-ve$
⇒ $\boxed{-1}$

[This Can be improved by shifting to a better line].

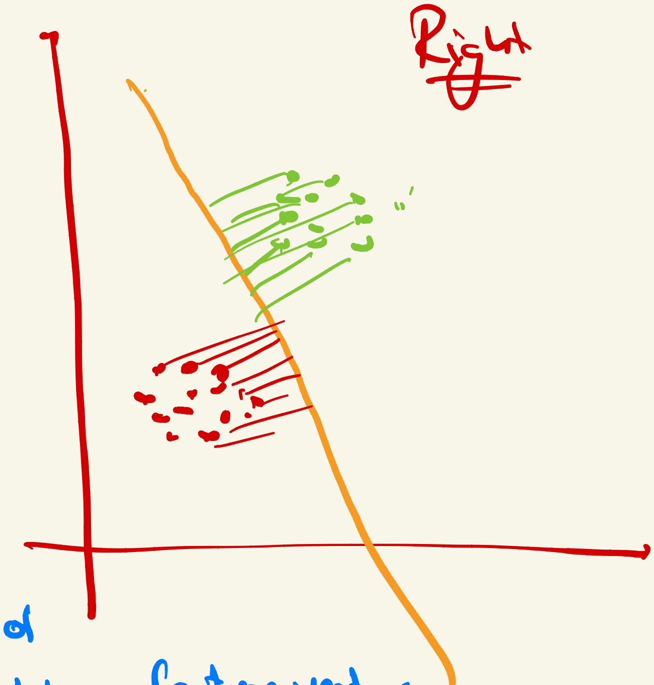




Left

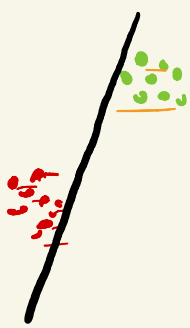


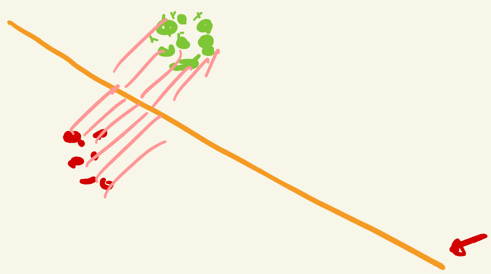
Right



Total sum of

we want to maximize distances b/w feature vector and line.





Which is better 2nd ✓

why distance from line is higher

e.g.

$\Rightarrow$ Cat vs dog

whisker	eyecolor	Height	weight	Label

$$\underbrace{\boxed{X}}_{\text{Dataset}} = \{\underbrace{x_i}_{\text{feature vector}}, \underbrace{y_i}_{\text{actual label}}\}_{i=1}^{n}$$

actual label $\rightarrow +1$, $\rightarrow -1$ [actual].

$\Rightarrow$ equation for Classifier $\boxed{w^Tx + w_0 = 0}$

$$d_{x_1} \rightarrow \frac{w^Tx_1 + w_0}{||w||}$$

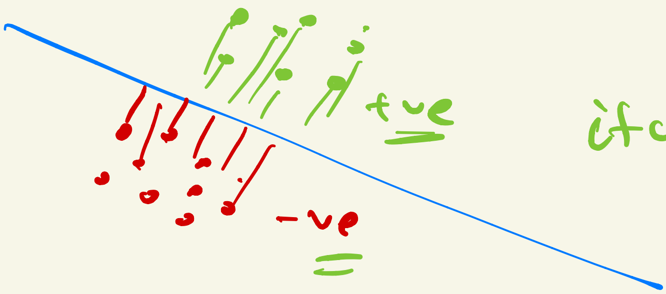
$$d_{x_2} \rightarrow \frac{w^Tx_2 + w_0}{||w||}$$

$\Rightarrow$ To get the maximum overall distance

Sum of all the distance b/w Classifier and feature vector.

Gain function →

$$G(x, \bar{\omega}, \omega_0) = \sum_{i=1}^{n} \frac{\omega^T x_i + \omega_0}{\|\omega\|}$$



if we directly sum they will negate each other

⇒ How to get better Gain function

① Take absolute distance ⇒ $f(d) = |d|$

② Take square of distance ⇒ $f(d) = d^2$

③ multiply distance with actual Label.

$$f(d) = d \cdot y_i$$

① $G(x \quad) = \sum \left| \frac{\omega^T x_i + \omega_0}{\|\omega\|} \right|$

② $G(\quad) = \sum \left(\frac{\omega^T x_i + \omega_0}{\|\omega\|} \right)^2$

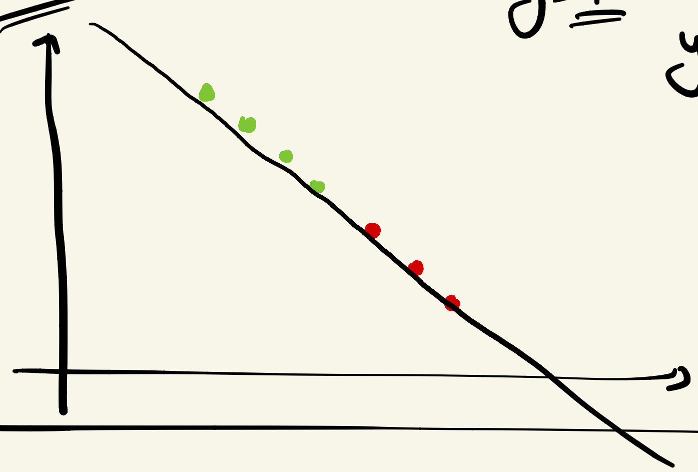
③ $G(\quad) = \sum \left(\frac{\omega^T x_i + \omega_0}{\|\omega\|} \right) \times y_0$

② → need lessly complicated.

① and ③ →

$y=|x|$ → this equation is non differentiable at $x=0$ [which breaks some of the complex ML algorithm]

Practical issue.



$y=|x|$ → you/machine will never be able to get a better classifier

⇒ Gain function ⇒

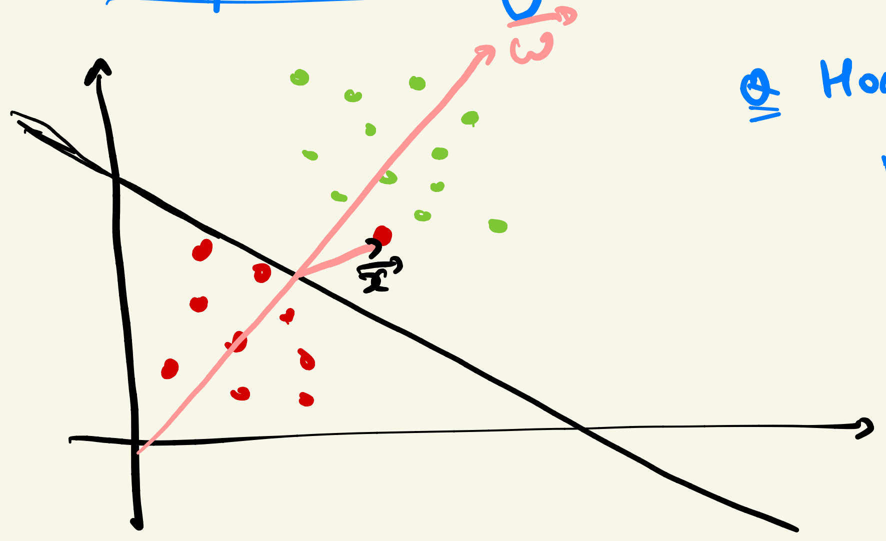
$$G(x,\omega,\omega_0) = \sum_{i=1}^{n} \left(\frac{\omega^T x_i + \omega_0}{\|\omega\|}\right) y_i$$

Best fit classifier → with = $\max \left(\sum_{i=1}^{n} \frac{\omega^T x_i + \omega_0}{\|\omega\|} y_i \right)$

⇒ In most ML application we minimize loss function.

$$\boxed{\mathcal{L}(x,\bar{\omega},\omega_0) = -G(x,\bar{\omega},\omega_0)}$$

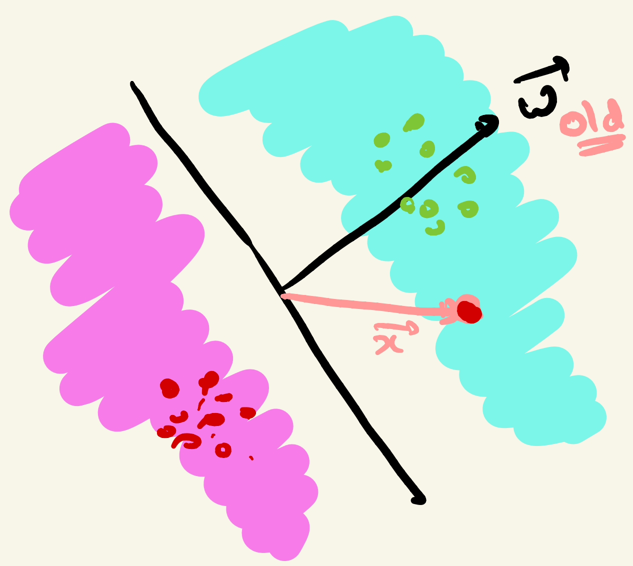
⇒ Perceptron learning algorithm

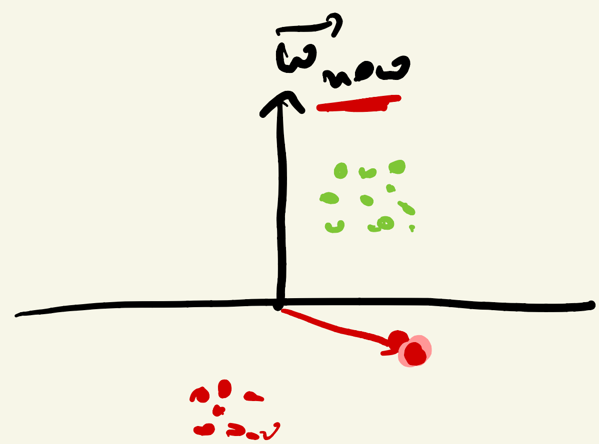


Q How do you identify if a point is mis classified.

$$\underbrace{\text{actual label}}_{y_i} \neq \underbrace{\text{assigned label}}_{\text{Sign}\left(\frac{w^T x_i + w_0}{\|w\|}\right)}$$

```
if  yi == Sign(        )
        print("correct")
Else:
        print("Incorrect")
```

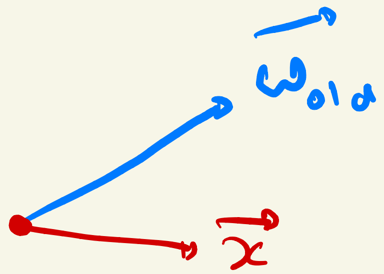




⇒ If HS2 [oppo of weight vector] is incorrectly classified

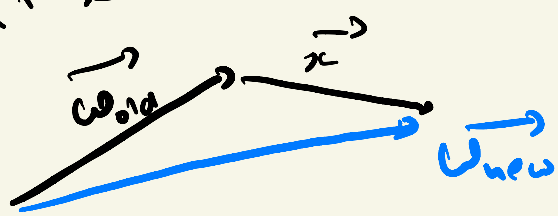
rotate anticlockwise.

$\Rightarrow$ actual label $\rightarrow$ -ve for misclassified point $\Rightarrow$ rotate anticlockwise



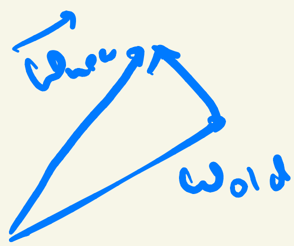
$$\vec{w}_{new} = \vec{w}_{old} - \vec{x}$$

$\Rightarrow \vec{w}_{old} + \vec{x}$



$\vec{w}_{old} + \vec{x} \Rightarrow$ clockwise

$\Rightarrow \vec{w}_{old} - \vec{x}$



$\vec{w}_{old} - \vec{x} \Rightarrow$ anticlockwise

$\Rightarrow$ if $\vec{x}$ is misclassified and actual label for x is -ve $\Rightarrow$ anticlockwise

$$\vec{w}_{new} = \vec{w}_{old} - \vec{x}$$

⇒ If $\vec{x}$ is misclassified and actual label for $\vec{x}$ is +ve

⇒ Clockwise

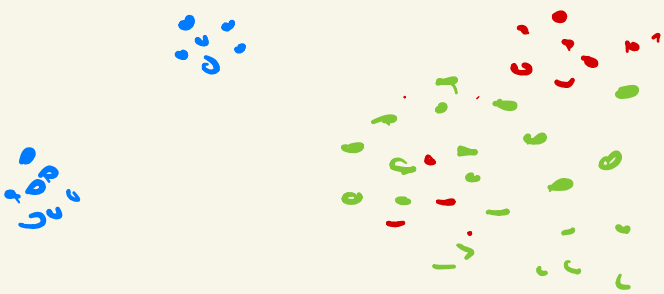
$$\vec{\omega}_{new} = \vec{\omega}_{old} + \vec{x}$$

⇒ if x is misclassified we should rotate $\vec{\omega}$.

$$\boxed{\vec{\omega}_{new} = \vec{\omega}_{old} + [\vec{x} \times y_i]}$$

⇒ • 100 iteration → we can pick the best one.

↳ accuracy →



⇒

Assignment

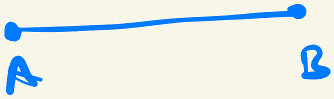
→ If there 1 point → Yes

→ If there 2 points → Yes

→ If there 3 points → Maybe

A B C

Write equation of line b/w A & B.



for all next points just check if equation of line is satisfied

(C, D, E, F,) ..

$\Rightarrow$ 3, 4 2, 0

$y = mx + c$

$y_1 = mx_1 + c$

$y_2 = mx_2 + c$

$$y_1 - y_2 = \boxed{m(x_1 - x_2)}$$

$$\rightarrow m = \frac{y_1 - y_2}{x_1 - x_2}$$

$$y_2 = \left(\frac{y_1 - y_2}{x_1 - x_2}\right) x_2 + c$$

$$y_2 x_1 - \cancel{y_2 x_2} - y_1 x_2 + \cancel{y_2 x_2} = c$$

$$\boxed{c = y_2 x_1 - y_1 x_2}$$

$\Rightarrow \quad \square \quad \underline{3,4} \qquad \frac{y-4}{x-3} = \_\_\_ \quad \_ \quad \_$

pick 1 element $\Rightarrow$ Calculate $\dfrac{y-y_0}{x-x_0}$