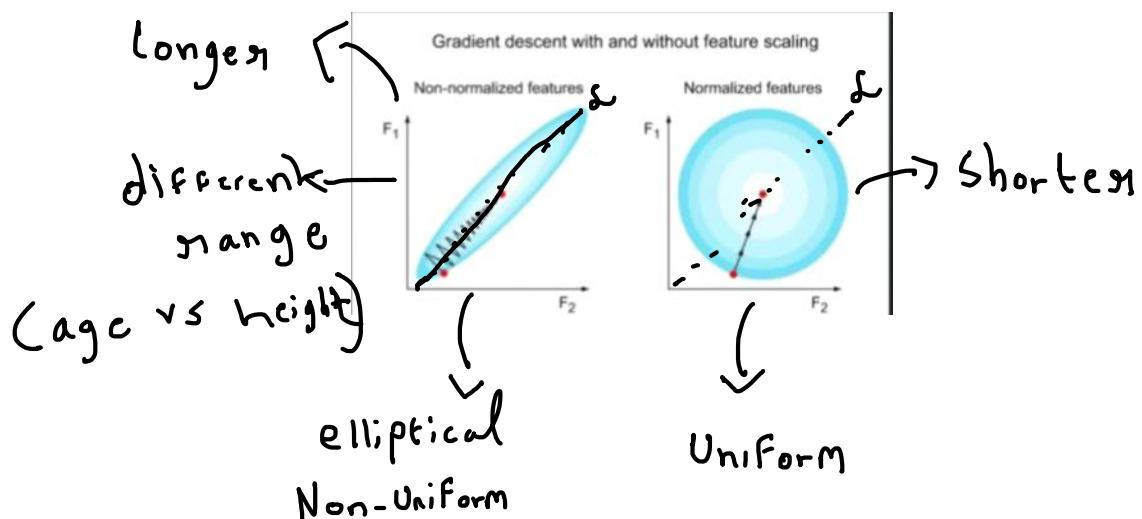


Is feature scaling a must everytime we perform linear regression ?

No!!.

When ?

- i) Interpretability with respect to Feature importance if features are in different scale
- ii) Gradient descent solution is found faster converges Faster}  $\rightarrow$  Why ?



## Evaluation Metrics (in case of doubts)

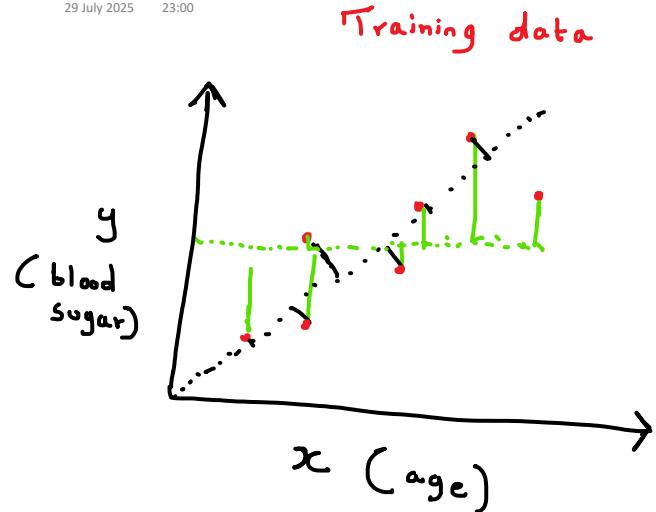
30 July 2025 19:15

y_true	y_pred	Error (y_pred - y_true)	Absolute Error	Squared Error
3.0	2.5	-0.5	0.5	0.25
4.5	5.0	0.5	0.5	0.25
2.0	2.4	0.4	0.4	0.16
5.5	5.1	-0.4	0.4	0.16
6.0	6.0	0.0	0.0	0.00

- Mean Error (ME) =  $0.0 / 5 = 0.0$
- MAE =  $(0.5 + 0.5 + 0.4 + 0.4 + 0.0) / 5 = 0.36$
- MSE =  $(0.25 + 0.25 + 0.16 + 0.16 + 0.0) / 5 = 0.164$

Absolute error is not  
differentiable at some points {  $y = y_{pred}$  }

	x	y	y_pred	mean_of_actual_y	tss_for_datapoint (square of (actual_y-mean_of_actual_y))	rss_for_datapoint (square of (actual_y - pred_y))	
<b>0</b>	1	2	2.8	4.0		4.0	0.64
<b>1</b>	2	4	3.4	4.0		0.0	0.36
<b>2</b>	3	5	4.0	4.0		1.0	1.00
<b>3</b>	4	4	4.6	4.0		0.0	0.36
<b>4</b>	5	5	5.2	4.0		1.0	0.04



$$\left\{ R^2 = 1 - \frac{RSS}{TSS} \right\}$$

Mean =  $\bar{y}$   $\Rightarrow$  target variable mean  
 $TSS = \sum (y_i - \bar{y})^2$   
 (Total Variation)  
 in value of  $y$  (target)

$$RSS = \sum (y_i - \hat{y})^2$$

after line is fit  
 { squared difference between predicted and actual }

$\left\{ 1 - \frac{\text{Model can't explain}}{\text{Total information in data}} \right\}$

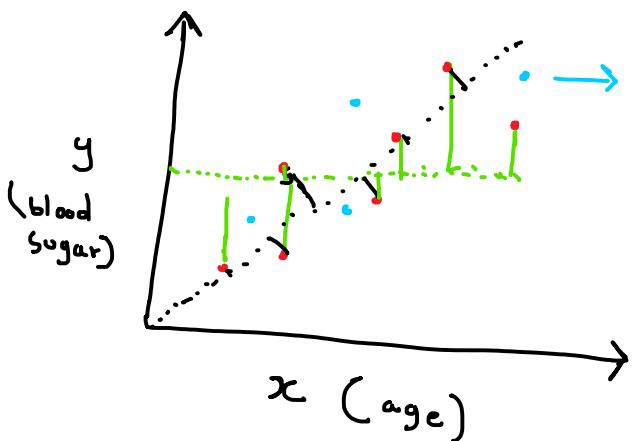
Total error  $\Rightarrow$  how much variation the best fit line (Model) could not explain

$= 1 - \text{Proportion of info not explained by model}$

$= \text{Proportion of info/Variance that}$

model was able to explain

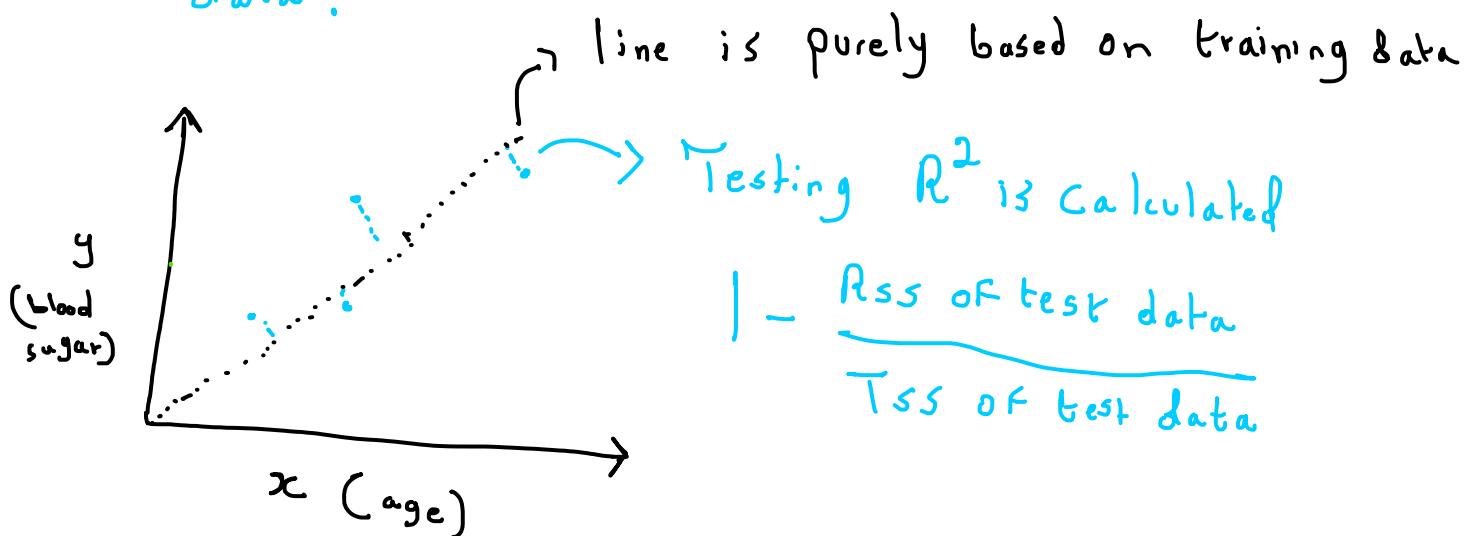
80% to train      20% to test



→ Test data }  $\Rightarrow$  Why do we need?

How will my model perform on unseen data

Let us separate test data from train data?



Test  $R^2$  gives a more realistic evaluation of the model!!

Train  $R^2$  vs Test  $R^2$  → how much variance unseen does it capture

Training data used for?

Testing data used for?

## Examination Analogy

Range of  $R^2$ ?  $-\infty$  to 1

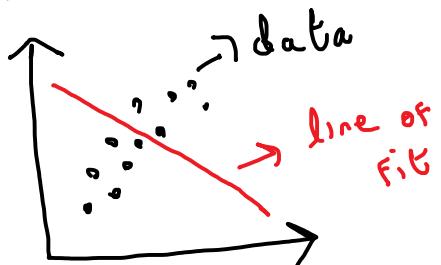
$$R^2 = 1 - \frac{\text{RSS}}{\text{TSS}} > 1$$

Can it be negative? Yes!!

$$\frac{\text{RSS}}{\text{TSS}} > 1 \text{ or } \text{RSS} > \text{TSS}$$

Can it be greater than 1?

No => You cannot capture more information than 1.



is already there

$$\left. \begin{array}{l} 1 \cdot \alpha = 1 - \frac{RSS}{TSS} \\ 1 \cdot \alpha - 1 = - \frac{RSS}{TSS} \\ 0 \cdot \alpha = - \frac{RSS}{TSS} \end{array} \right\} \text{Not Possible !!}$$

What happens to my  $\{ \text{training } R^2 \}$  if I keep adding variables?  $\Rightarrow$  Always stay the same or increase

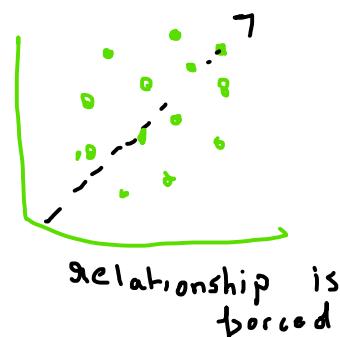
A model wants to make full use of all variables/data provided to it.

How about test  $R^2$ ?

$\hookrightarrow$  Can move in any direction on addition of variables

Sometimes we don't have enough data to have train and test data. I have only 50 datapoints

Solution?  $\Rightarrow$  Adjusted R-squared



Problem with R-Square

What if we add one extra feature to our model?

N features  $\Rightarrow$  R-Square

+  
1 feature or More

$$w_0x_0 + w_1x_1 + w_2x_2 + \dots + w_nx_n + (w_{n+1}x_{n+1})$$

$$+ w_{n+2}x_{n+2} - \dots$$

Train  $R^2 \geq$

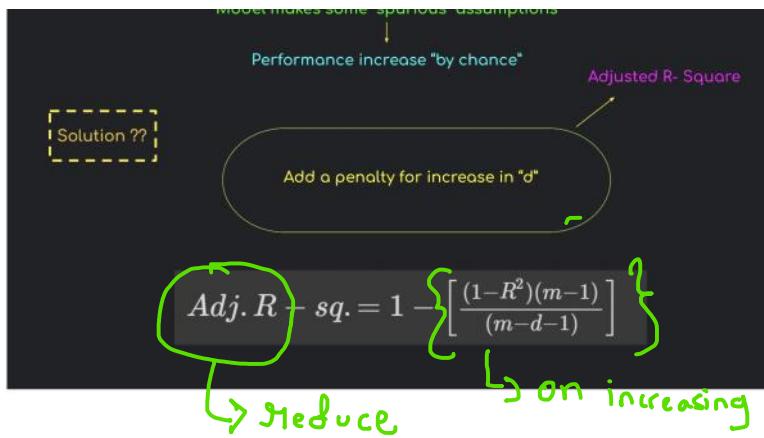
Without testing data, how do I know if the increase is significant?

When will R-square increase (irrelevant features)?

Model makes some "spurious" assumptions  
 $\downarrow$   
Performance increase "by chance"

Adjusted R-Square

$\Rightarrow$  Sample size  
 $m \cdot \text{No. of vars} / \dots$



$\hat{m}$  → Sample size  
 $m$  = No. of rows/observations  
 $d$  = No. of features  
 Adding more features  
 {penalizes  $R^2$ }  
 $\uparrow$  (Adj R<sup>2</sup>)

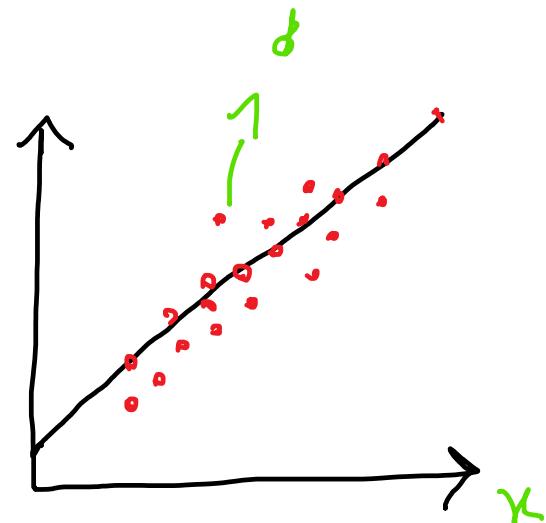
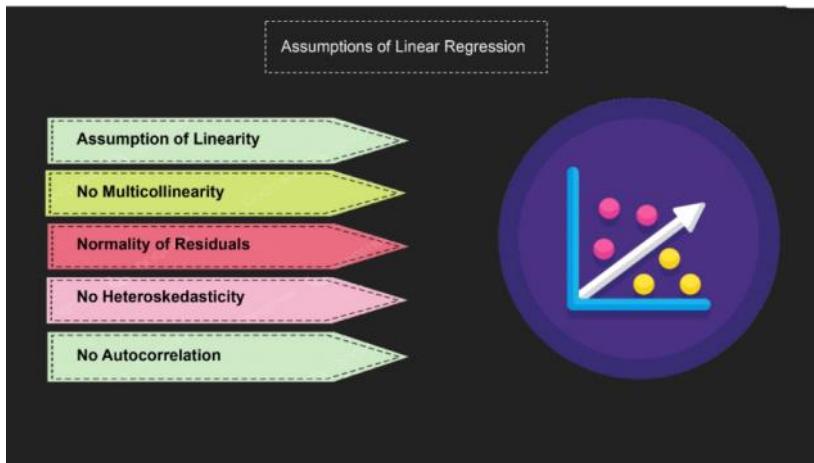
IF I build 5 different models to predict the same target but with different number of variables, how will I pick the best model?

Range of Adj R<sup>2</sup> ⇒ -∞ to 1

- ↳ i) Train v Test R<sup>2</sup> ⇒ Access to enough data
- ii) Adjusted R<sup>2</sup> ⇒ If not

## Assumption of Linearity

30 July 2025 16:02



## Regression Plane

