

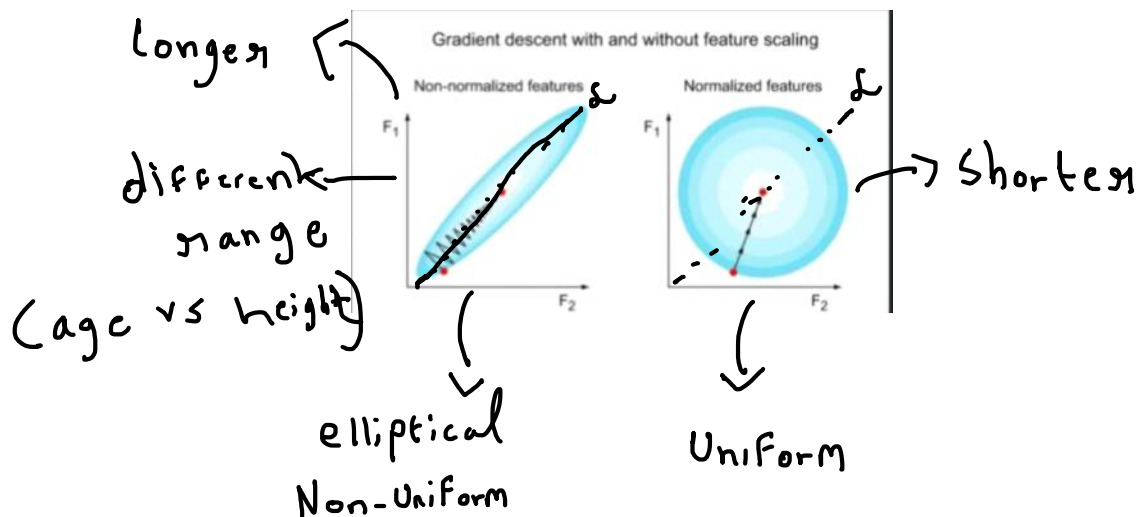
Is feature scaling a MUST everytime we perform linear regression ?

No!!

When ?

i) Interpretability with respect to Feature importance ^{if features are in different scale}

ii) Gradient descent {converges faster} ^{solution is found faster} → why?



Evaluation Metrics (in case of doubts)

30 July 2025 19:15

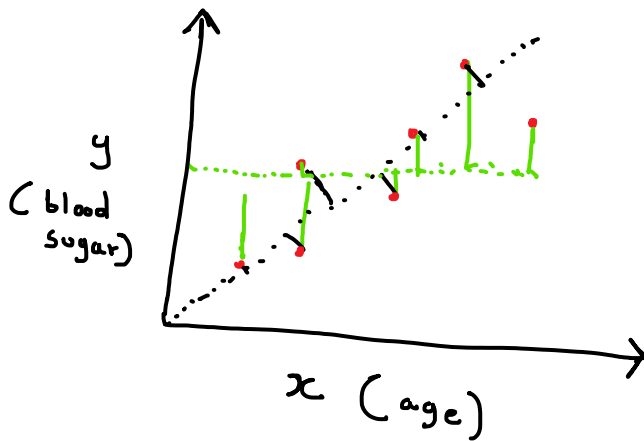
y_true	y_pred	Error (y_pred - y_true)	Absolute Error	Squared Error
3.0	2.5	-0.5	0.5	0.25
4.5	5.0	0.5	0.5	0.25
2.0	2.4	0.4	0.4	0.16
5.5	5.1	-0.4	0.4	0.16
6.0	6.0	0.0	0.0	0.00

- Mean Error (ME) = $0.0 / 5 = 0.0$
- MAE = $(0.5 + 0.5 + 0.4 + 0.4 + 0.0) / 5 = 0.36$
- MSE = $(0.25 + 0.25 + 0.16 + 0.16 + 0.0) / 5 = 0.164$

Absolute error is not
differentiable at some points $\{ y = y_{pred} \}$

	x	y	y_pred	mean_of_actual_y	tss_for_datapoint (square of (actual_y-mean_of_actual_y))	rss_for_datapoint (square of (actual_y - pred_y))
0	1	2	2.8	4.0	4.0	0.64
1	2	4	3.4	4.0	0.0	0.36
2	3	5	4.0	4.0	1.0	1.00
3	4	4	4.6	4.0	0.0	0.36
4	5	5	5.2	4.0	1.0	0.04

Training data



Mean = \bar{y} \Rightarrow target variable mean

$$TSS = \sum (y_i - \bar{y})^2$$

Total Variation
in Value of
 y (target)
Amount of info

$$RSS = \sum (y_i - \hat{y})^2$$

after
line
is
fit

Squared difference
between predicted
and actual

$$\left\{ R^2 = 1 - \frac{RSS}{TSS} \right\}$$

\Downarrow

$\left\{ 1 - \frac{\text{Model can't explain}}{\text{Total information in data}} \right\}$

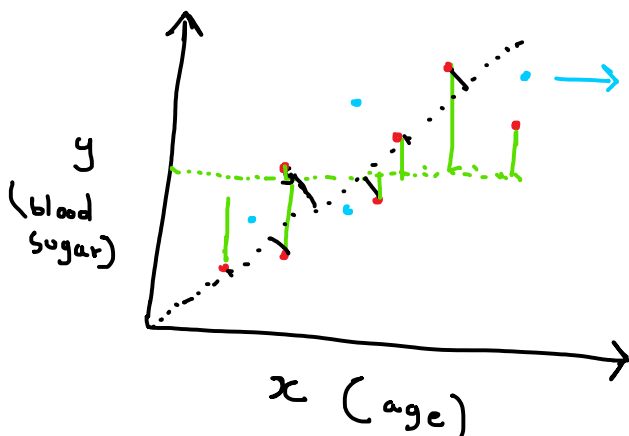
\Downarrow

Total error \Rightarrow how much Variation
the best fit line (Model)
could not explain

$= 1 - \text{Proportion of info not explained by model}$

$= \text{Proportion of info/Variance that}$
model was able to explain

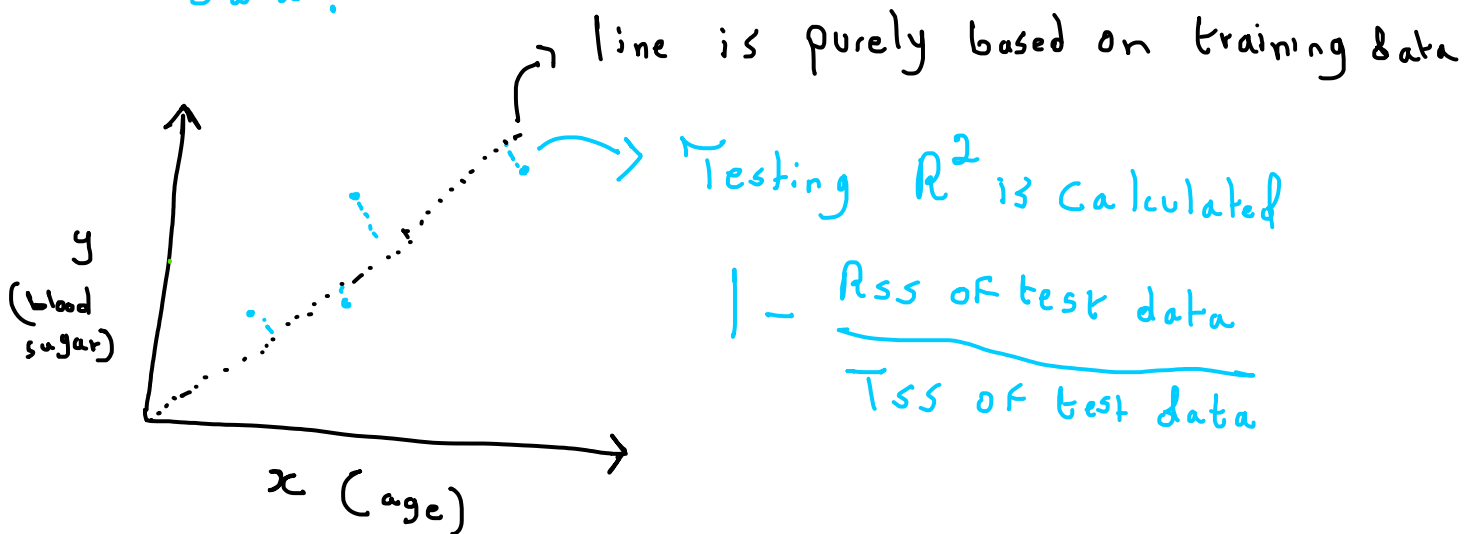
80% to train 20% to test



Test data \Rightarrow Why do
we need?

How will my model
perform on unseen data

Let us separate test data from train data?



Test R^2 gives a more realistic evaluation of the model!!

Train R^2 vs Test R^2 → how much variance unseen
 Training data used for?
 Testing data used for?

Examination Analogy

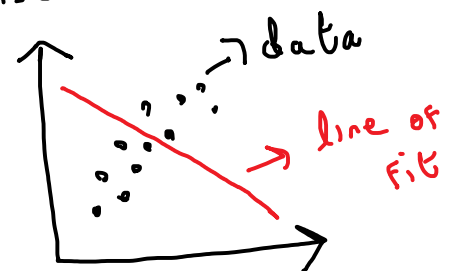
Range of R^2 ? $-\infty$ to 1

$$R^2 = 1 - \frac{\text{RSS}}{\text{TSS}} > 1$$

Can it be negative? Yes!!

$$\frac{\text{RSS}}{\text{TSS}} > 1 \text{ or } \text{RSS} > \text{TSS}$$

Can it be greater than 1?



No ⇒ You cannot capture more information than what

is already there

$$\left\{ \begin{array}{l} 1.2 = 1 - \frac{RSS}{TSS} \\ 1.2 - 1 = -\frac{RSS}{TSS} \\ 0.2 = -\frac{RSS}{TSS} \end{array} \right. \quad \begin{array}{l} \text{NOT} \\ \text{Possible!!} \\ \frac{RSS}{TSS} = -0.2 \end{array}$$

What happens to my $\{ \text{training } R^2 \}$ if I keep adding variables? \Rightarrow Always stay the same or increase

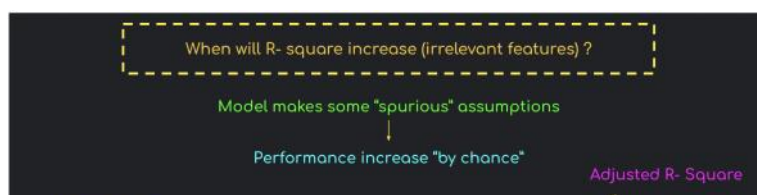
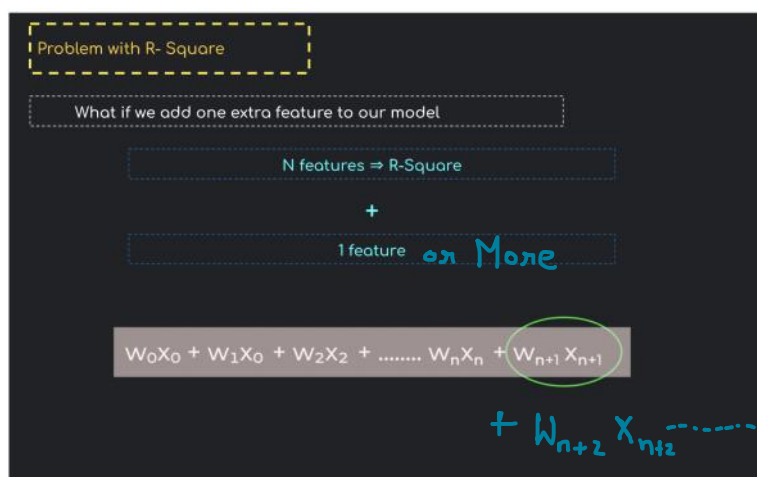
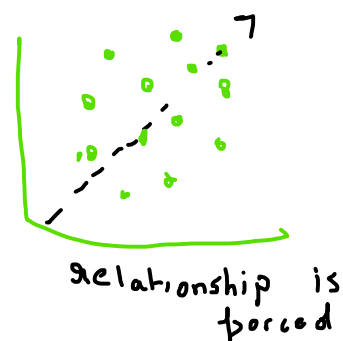
A model wants to make full use of all Variables/data provided to it.

How about test R^2 ?

\hookrightarrow Can move in any direction on addition of variables

Sometimes we don't have enough data to have Train and test data. I have only 50 datapoints

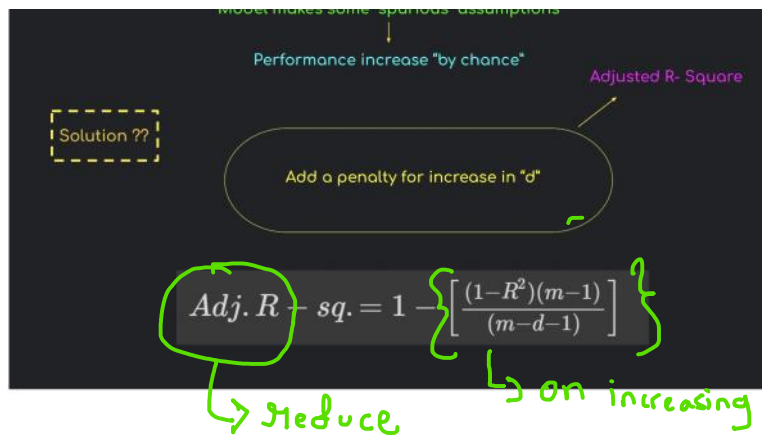
Solution? \Rightarrow Adjusted R-squared



$\text{Train } R^2 \geq$

Without testing data, how do I know if the increase is significant?

\Rightarrow Sample size



Sample size
 $M = \text{No. of rows/observations}$

$d = \text{No. of features}$

Adding more features
 {penalizes R^2 }
 (Adj R^2)

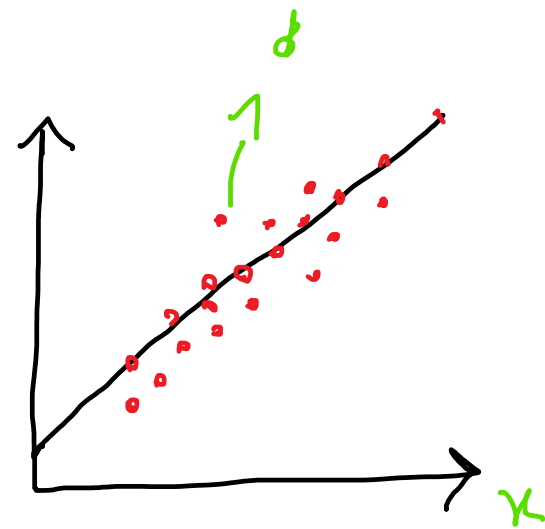
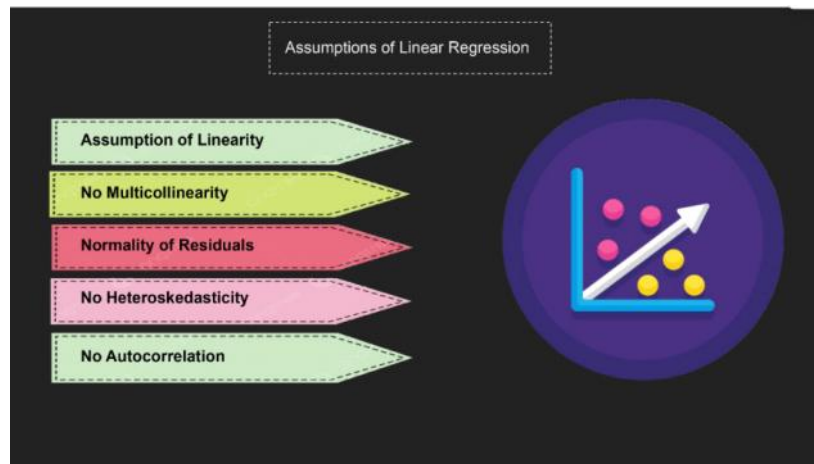
If I build 5 different models to predict the same target but with different number of variables, how will I pick the best model?

Range of Adj $R^2 \Rightarrow -\infty$ to 1

- i) Train v Test $R^2 \Rightarrow$ Access to enough data
- ii) Adjusted $R^2 \Rightarrow$ If not

Assumption of Linearity

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Regression Plane

