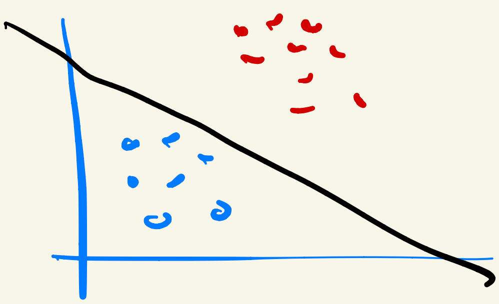
Linear Algebra

Recap →

* ML → Training machine on the basis of Human Intelligence

* New Terms
  - → Features → independent characteristics → size, weight, ..
  - → labels → dependent values | target → Category.
  - → Datapoint → row in the dataset
    - ↳ feature value
    - ↳ label value
  - → Dataset → Collection of data pts
  - → Classifier → mathematical curve that seperate the datapoints into the Category.
    - → line
    - → Curve
    - → Circle
    - → multiple Classifier.
  - → any dimension → 3D → plane
  - → half space → region (of a specific class) defined by classifiers



Framework for ML

→ Collect data

→ Visualise

→ choose the right geometric shape for classifier [shape]

→ Choose a loss function that helps in identifying the best curve.

→ Training

Later classes

Lines → ① $y = mx + c$

Slope (m), y-intercept (c)

② $w_1 x_1 + w_2 x_2 + w_0 = 0$

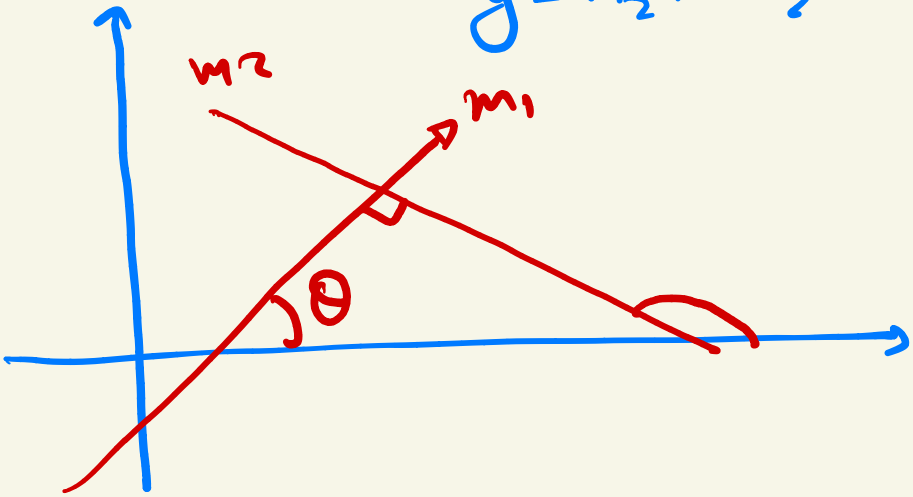
w_1, w_2, w_0 → parameters / weights

Quiz 1 → perpendicular lines → 90°

$y = m_1 x + c_1$

$y = m_2 x + c_2$

$\boxed{m_1 \times m_2 = -1}$



$m_1 = \tan\theta$

Quiz 2

$3x - 2y + 6 = 0$

$y = mx + c$

$3x + 6 = 2y$

$\boxed{\frac{3}{2}} x + 3 = y$

$9x - 6y - 18 = 0$

↓

$9x - 18 = 6y$

$\frac{9}{6} x - 3 = y$

$\boxed{\frac{3}{2}} x - 3 = y$

Q weight → x

height → y

Tall vs Short

$$w_1 x + w_2 y + w_0 = 0$$

$w_1 \to 1$

$w_2 \to 1$

$w_0 \to -98$

$x + y - 98 = 0$

$0.5x + 1.1y - 85 = 0$

$\Rightarrow$ Visually finding best value for w_1, w_2, w_0 $\to$ [mathematical optimization Technique]

It is not always possible to visually get the best outcome.

$\Rightarrow$ $w_1 x + w_2 y + w_0 = 0$ [2 Dimensional data] 2 feature.

2D hyperplane | Line

$\Rightarrow$ $w_1 x + w_2 y + w_3 z + w_0 = 0$

3D hyperplan plane

$\Rightarrow$ $w_1 x + w_2 y + w_3 z + w_4 t + w_0 = 0$

4D hyperplane

$$\boxed{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + \ldots + w_nx_n + w_0 = 0}$$

general equation of n dimensional hyper plane.

Machine learning → finding best values for $w_0, w_1, w_2, w_3 \ldots w_n$

Vectors →

physics → direction + magnitude

python/maths → Collections of numbers stored in a list.

- Representation $\bar{x}$ $\vec{x}$

- n dimensional vector
  - → Column $\begin{bmatrix} 1, \\ 2, \\ 3, \\ 4, \\ 5 \end{bmatrix}$
  - → Row $[1, 2, 3, 4, 5]$

⇒ $\boxed{x \in R^d}$ x belongs to real numbers of d dimension.

$d \to 3$ $\quad$ $x \to$ $[1, 2, 3]$

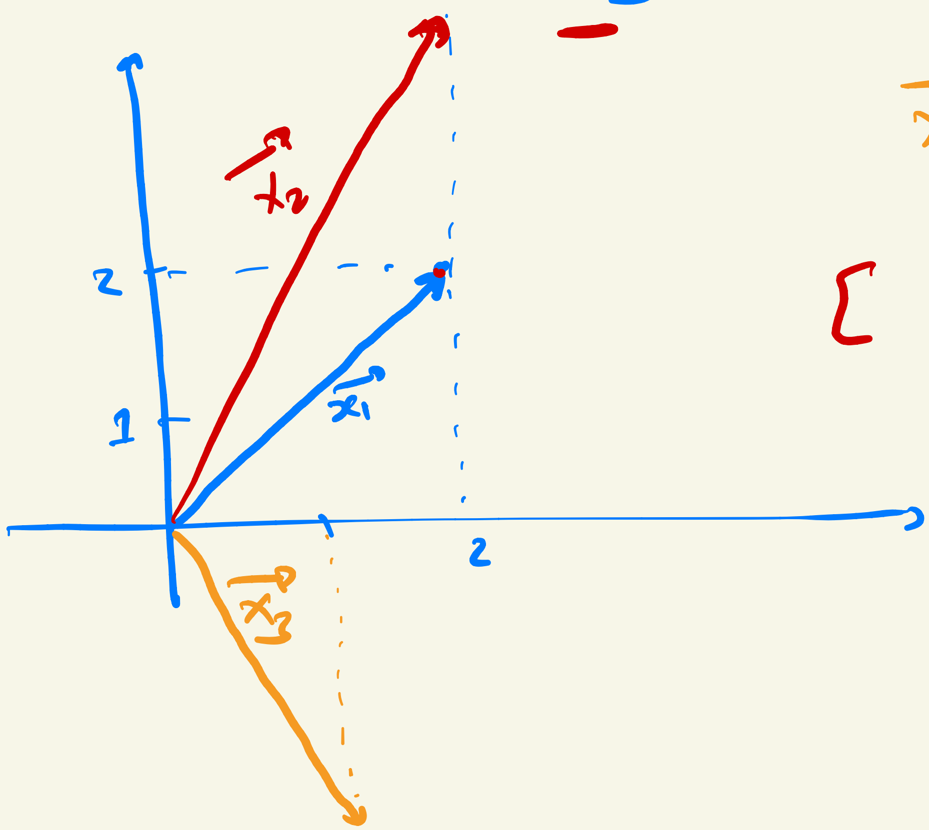
$[2, 10, 12]$

$[6.5, 10.2, 11.9]$

⇒ Draw vectors in 2D →

$\vec{x}_1 = \begin{bmatrix}2\\2\end{bmatrix}$ $\qquad$ $\vec{x}_2 = \begin{bmatrix}2\\4\end{bmatrix}$

$\vec{x}_3 = \begin{bmatrix}1\\-3\end{bmatrix}$



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⇒ Magnitude of Vector →

Length of the vector

⇒ 2D → $\sqrt{x_1^2 + y_1^2}$ $\qquad$ $x_1 \to \sqrt{4+4}$

$\sqrt{8}$

$\Rightarrow$ 3D $\rightarrow$

2D $\rightarrow$ $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ mag $\rightarrow$ $\sqrt{x_1^2 + x_2^2}$

3D $\rightarrow$ $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ mag $\rightarrow$ $\sqrt{x_1^2 + x_2^2 + x_3^2}$

nD $\rightarrow$ $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ mag $\rightarrow$ $\sqrt{x_1^2 + x_2^2 + \dots x_n^2}$

Norm of a Vector $\rightarrow$ (Length of vector)

$\|\bar{x}\|$ $\leftarrow$ norm of vector $\bar{x}$

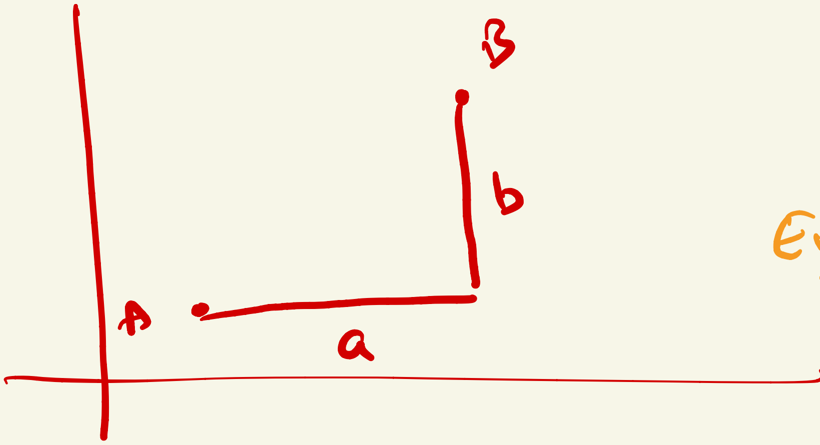
$$\|\bar{x}\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

Eucleadian distance

magnitude of vector

$$\|x\|_1 = |x_1| + |x_2| + |x_3| + \dots |x_n|$$

manhattan distance

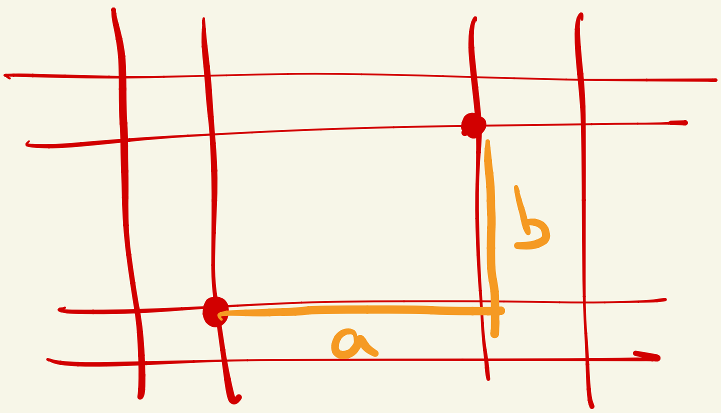


How will a bird go → A → B

fly → shortest distance

Eucleadean distance

$\sqrt{a^2+b^2}$



drive → $a+b$

manhattan distance

Norm of vector →

$\|\bar{x}\|_2$ → L_2 Norm of vector → $\sqrt{x_1^2+x_2^2+\dots}$

$\|\bar{x}\|_1$ → L_1 Norm of vector → $|x_1|+|x_2|+\dots$

default Norm → L_2 Norm

$\|\bar{x}\|$ → L_2 norm

⇒ Matrix Multiplication →

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1\times5+2\times7 & 1\times6+2\times8 \\ 3\times5+4\times7 & 3\times6+4\times8 \end{bmatrix}$$

A B

$m \times p$ $p \times n$

final dimension → $m \times n$

$A_{m\times n}$ $B_{a\times b}$ ⇒ only be done if $n = a$

⇒ Two Column vectors

2×1 2×1

A B

$\vec{x}_1$ $\vec{x}_2$

we can't multiply 2 column vectors

&

what if we convert one of them into row vector

$2\times 1 \longrightarrow 1\times 2$

(Column) (row)

$\vec{x} \longrightarrow \vec{x}^T$ (transpose)

$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$$\underline{\vec{x}^T \vec{y}} \Rightarrow \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1\times 3 + 2\times 4 \end{bmatrix} = \underline{11}$$

$\downarrow$

$\boxed{\text{Dot product}} \qquad \boxed{x^T y}$

if x and y are Column vectors.

2×1	2×1		2×1	2×1
$\Downarrow$				
1×2	2×1		2×1	1×2
$\Downarrow$				$\cancel{\times}$
$\boxed{1\times 1}$			$\underline{2\times 2}$	

$x = \begin{bmatrix}2\\1\end{bmatrix}$ $y = \begin{bmatrix}3\\4\end{bmatrix}$ $x^T y$

$$[2, 1]\begin{bmatrix}3\\4\end{bmatrix} = [2\times3+1\times4] = [10]$$

1×1

Dotproduct

$$x y^T \Rightarrow \begin{bmatrix}2\\1\end{bmatrix}\begin{bmatrix}3 & 4\end{bmatrix} = \begin{bmatrix}2\times3 & 2\times4\\1\times3 & 1\times4\end{bmatrix}$$

2×1 1×2 2×2

Not Dot product

Q → Loan Example

app income	Co app income	Credit hist	Loan Amount	Status

feature

target

x_1 → app income

x_2 → Co app income

x_3 → credit history

x_4 → amount

$$\omega_1 x_1 + \omega_2 x_2 + \omega_3 x_3 + \omega_4 x_4 + \omega_0 = 0$$

$$\boxed{10x_1 + 1x_2 + 100x_3 + 0.01\,x_4 +} \;\leq \text{''}$$

≐ On the basis of this

Can we predict status

$$\vec{\omega} = \begin{bmatrix} 10 \\ 1 \\ 100 \\ 0.01 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

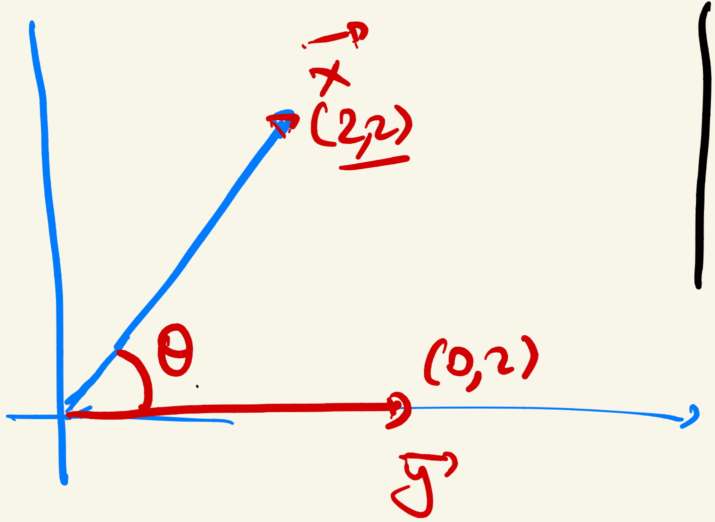
Equation of classifier

$$\boxed{\vec{\omega}^{T}\vec{x} + \omega_0 = 0}$$

entire machine Learning → finding the best value for $\vec{\omega}$.

→ Learning value for $\vec{\omega}$.

⇒ Angles b/w 2 vectors →



$$\boxed{\cos(\theta) = \frac{\bar{x}^T \bar{y}}{\|\bar{x}\| \cdot \|\bar{y}\|}}$$

$\theta \rightarrow 0 \leq \quad < 90$

$270 \leq \quad \leq 360$

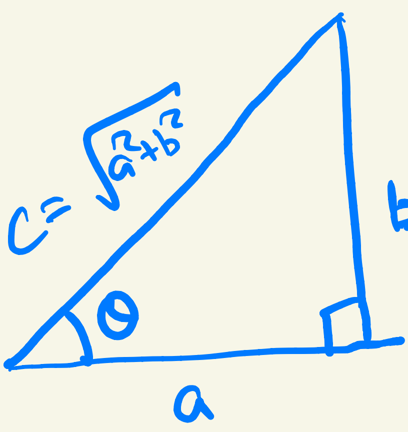
$\cos\theta \rightarrow +ve$

else $-ve$.

$\theta = 0 \rightarrow \cos\theta = 1$

$\theta = 90 \rightarrow \cos\theta = 0$

⇒



$\sin\theta = b/c$

$\cos\theta = a/c$

$\tan\theta = b/a$

$\theta = 0 \rightarrow 1$

$\theta = 90 \rightarrow 0$

$\Rightarrow$ $\omega_1 x_1 + \omega_2 x_2 + \omega_0 = 0$ [Co-ordinate geometry].

$$\boxed{\bar{\omega}^T \bar{x} + \omega_0 = 0}$$

$$\bar{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \qquad \bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

weight vector

feature vector.

$$\boxed{\omega_0 \rightarrow \text{bias}}$$

$\omega_1 x_1 + \omega_2 x_2 + \omega_0 = 0$ $\rightarrow$ equation of line

equation of 2D hyperplane

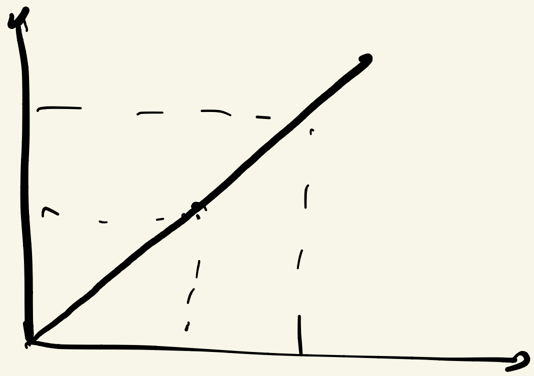
$\boxed{\omega^T x + \omega_0 = 0}$ Vector equation of hyperplane.

$\Rightarrow$ weight vector is perpendicular to hyperplane.

$$\Rightarrow w_1 x_1 + w_2 x_2 + w_0 = 0$$

$w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\Rightarrow$ $x_1 + x_2 - 98 = 0$

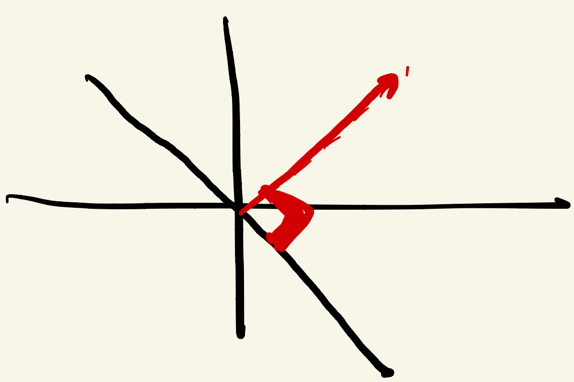
$w_0 = -98$



$\Rightarrow$ $x_1 + x_2 = 0$

1,1

$\underline{w^T x} \rightarrow 0$



$\bar{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\underline{\cos 90^\circ = 0}$ $\cos 0 = 1$

$$\cos\theta = \frac{w^T x}{\|w\| \, \|x\|}$$

$$\boxed{0 = w^T x}$$

$\Rightarrow$

$$\cos\theta = \frac{w^T x}{\|w\| \, \|x\|}$$

angle b/w w & x = $\underline{\underline{90^\circ}}$

line is defined as $\underline{\underline{w^T x + w_0 = 0}}$

thats why if

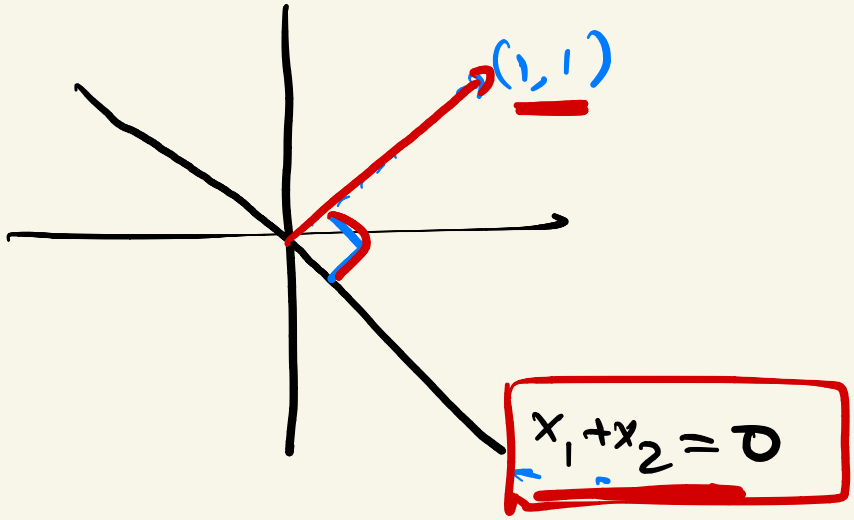
we try to get angle b/w line and $\vec{w}$

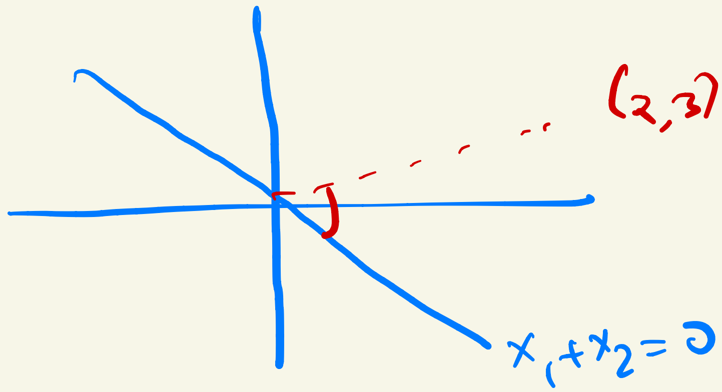
$\underline{90^\circ}$

$w_0 \rightarrow \underline{0}$

$$\cos\theta = \frac{w^T x}{|w| \, |x|} =$$

$\Rightarrow$





Unit Vector $\rightarrow$ Vector with magnitude = 1

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\boxed{\hat{w} = \frac{\vec{w}}{\|w\|}}$$

Unit vector.

$$\vec{w} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \qquad \text{mag} \rightarrow \sqrt{4+4}$$

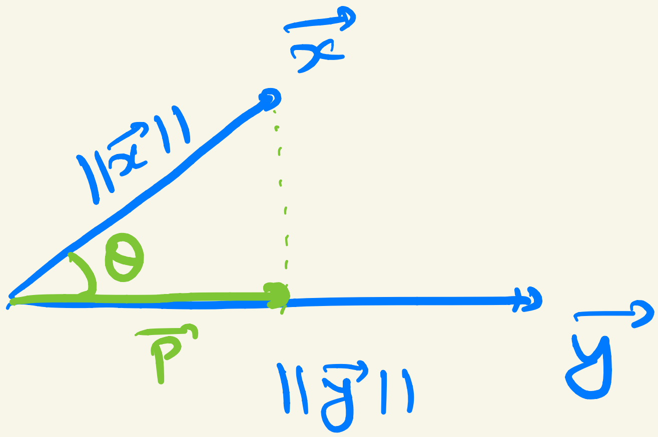
$$\sqrt{8}$$

$$\hat{w} = \begin{bmatrix} 2/\sqrt{8} \\ 2/\sqrt{8} \end{bmatrix}$$

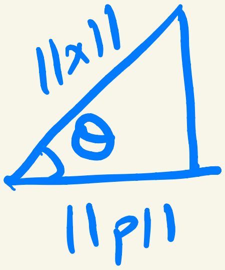
$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \qquad \hat{w} = \begin{bmatrix} \dfrac{w_1}{\sqrt{w_1^2 + w_2^2 + w_3^2}} \\ \dfrac{w_2}{\sqrt{}} \\ \dfrac{w_3}{\sqrt{}} \end{bmatrix}$$

$$\|\hat{w}\| \rightarrow 1$$

$\Rightarrow$ Vector Projection $\rightarrow$



$\vec{p}$ projection of $\vec{x}$ in $\vec{y}$ vector direction.



$$\boxed{\cos\theta = \frac{||p||}{||x||}}$$

$$\cos\theta = \frac{x^T y}{||x|| \, ||y||}$$

$\theta_1 =$ angle b/w $\vec{p}$ & $\vec{y}$ $= 0$

$$\cos\theta_1 = \frac{\vec{p}^{\,T} y}{||p|| \, ||y||} = 1$$

$$\boxed{||p|| = \frac{\vec{p}^{\,T} y}{||y||}}$$

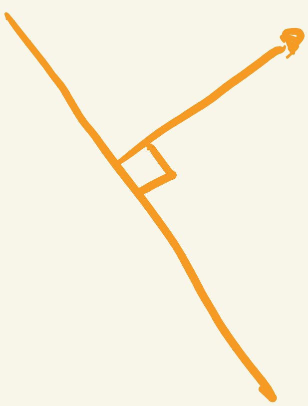
$\Rightarrow$ $y = mx + c$ $\rightarrow$

$\hookrightarrow$ $w_1 x_1 + w_2 x_2 + w_0 = 0$ [easier to extrapolate in n dimension]

$w^T x + w_0 = 0$

$\omega \rightarrow \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \qquad x \rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\boxed{\omega^T x + \omega_0 = 0}$$



→ $\vec{\omega}$ 90° to hyperplane

→

$\cos\theta =$

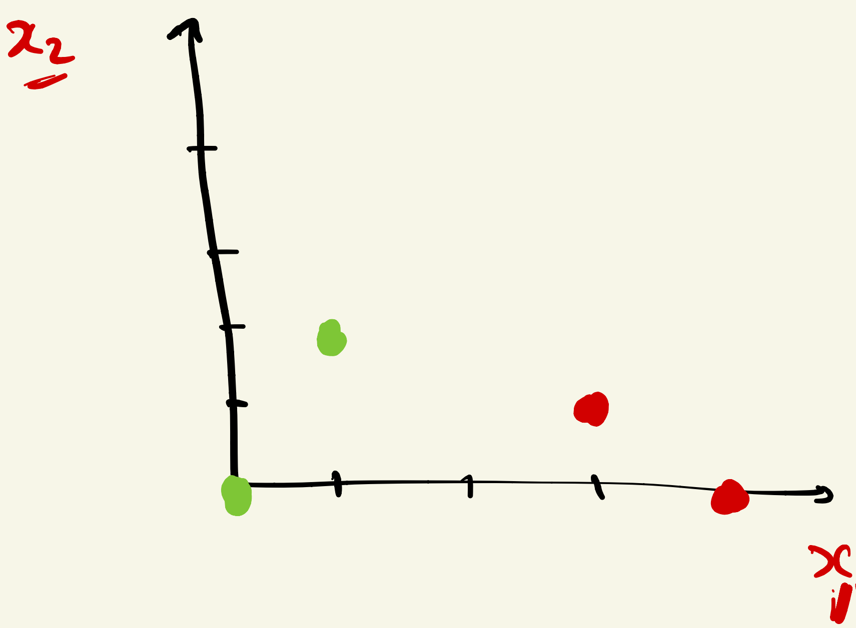
⇒ Assignment → 5

⇒ 1, 2 → 0 G

3, 1 → 1

0, 0 → 0 G

4, 0 → 1



- All Questions → 1st
  - → answers →
  - →

Silly mistakes

- SQL
  PA
  CLT / Normal Distⁿ

- Python
- DAV
  - PA

- Pick the weakest segment. → → Attempt
  - 10 Ques | 1 hr ⇒ Sorting during revision
  -