Practice Problems

Recap →

→ 1 line → Explanation

⇒ Dataset feature Target/Label

dataset

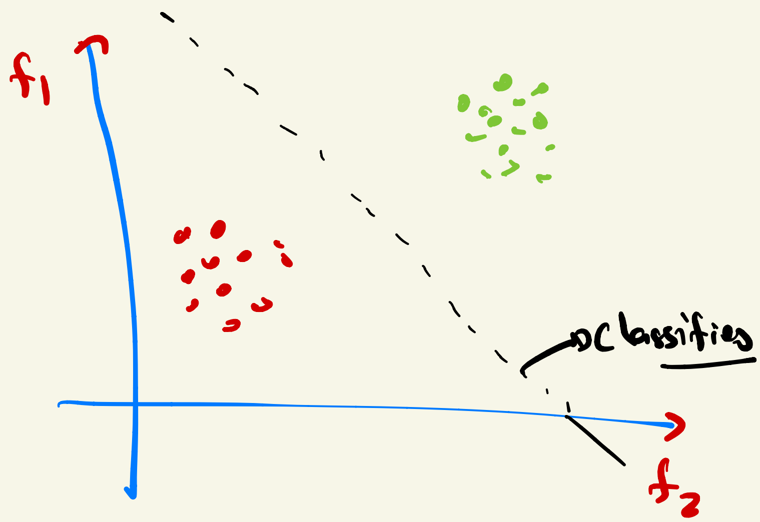
Applicant Income	Age	Gender	Loan Approved

feature

} Label

if there are only 2 feature

⇒



All axis → feature

Color → Target

⇒ easiest Classifier → line

$y = mx + c$

m → Slope, c → y-intercept

[works only in 2D]

$$\boxed{w_1 x_1 + w_2 x_2 + w_0 = 0}$$

Parameters

⇒ Vectors → $\bar{x} = [x_1 \ x_2 \ x_3 \ldots x_d]$

Length of vector → magnitude → $\sqrt{x_1^2 + x_2^2 + \ldots x_d^2}$

$||x||$

L1 norm = $|x_1| + |x_2| + |x_3| + \ldots$

L2 Norm = $||x||_2 = \sqrt{x_1^2 + x_2^2 + \ldots}$

⇒ Vector → $x \to [x_1 \, x_2 \, x_3]$

$y \to [y_1 \, y_2 \, y_3]$

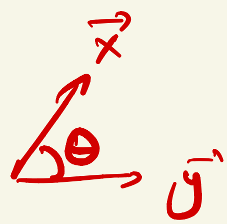
dot product $\quad \underline{x^T y} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} [y_1 \ y_2 \ y_3]$

$\underline{w_1 x_1 + w_2 x_2 + w_0 = 0}$]

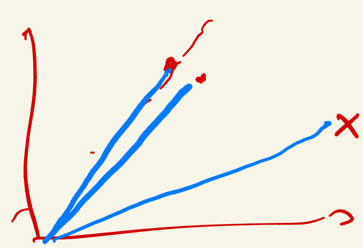
$[w_1, w_2] \qquad [x_1 \, x_2]$

$$\boxed{w^T x + w_0 = 0}$$

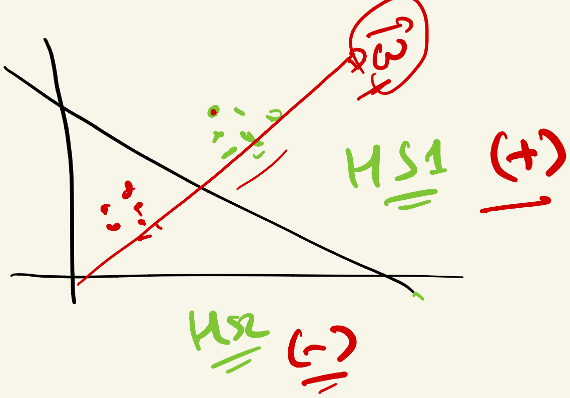
⇒ What if we only care about.



$$\underline{\cos}\theta = \frac{x^T y}{||x|| \, ||y||}$$



⇒ Half Spaces →



$\Rightarrow$ $\vec{\omega}$ are perpendicular to decision boundary.

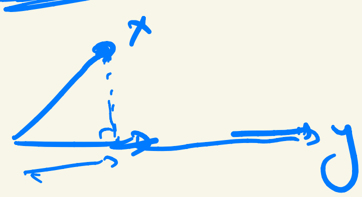
dist^n b/w a point and a line $\Rightarrow$

$$\boxed{d = \frac{\omega^T x + \omega_0}{\|\omega\|}}$$

weight $\rightarrow$ $[\omega_1, \omega_2]$

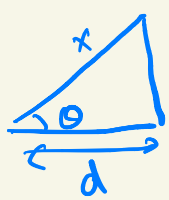
dist^n b/w origin and a line $\rightarrow$

$\Rightarrow$ Projection of x in y direction



projection of x in y direc

$$\frac{x^T y}{\|y\|} = x^T \hat{y}$$

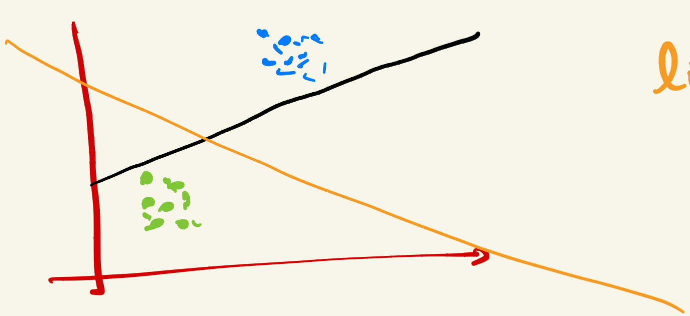


$$d/x = \cos\theta = \frac{x^T y}{\|y\|}$$

$\Rightarrow$ end goal of any ml algorith to identify best classifier.

$$\omega^T x + \omega_0 = 0$$

best classifier weight vector



line with maximum total distance from all the points.

$$d = \frac{w^T x + w_0}{||w||} \quad \begin{cases} \rightarrow +ve \rightarrow +ve \text{ HS} \\ \rightarrow -ve \rightarrow -ve \text{ HS} \end{cases}$$

⇒ • use absolute value → $|d|$

• use square distance

if we convert labels → +1, −1

$$\boxed{\text{Total distance} = \sum \frac{w^T x_i + w_0}{||w||} \times y_i}$$

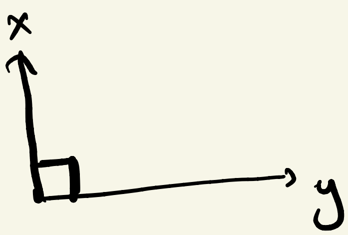
suppose we have two vectors

vector x =[2,1,-3]

vector y=[5,8,6]

- What is the length of the projection of x onto y?

Projection of x on y ⇒ $\frac{x^T y}{||y||}$



$$\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} \begin{bmatrix} 5 & 8 & 6 \end{bmatrix}$$

$2 \times 5 + 1 \times 8 + 6 - 3 = 0$

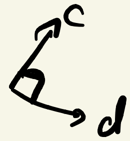
$10 + 8 - 18 = 0$

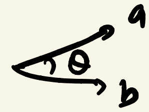
length of projection = $\underline{0}$

given a and b are two unit vectors such that,

c=a+2b
d=5a-4b

these two are perpendicular, then what is the angle between a and b?





$$\cos 90^\circ = 0$$

$$c^T d \Rightarrow c\cdot d = 0$$

$$\cos\theta = \frac{c^T d}{\|c\|\,\|d\|} =$$

$$c^T d \rightarrow 0$$

$$(a+2b)\cdot(5a-4b) = 0$$

$$5\|a\|^2 + 6\, a\cdot b - 8\|b\|^2 = 0$$

$$5 + 6\, a\cdot b - 8 = 0$$

$$a\cdot b = \frac{3}{6} = \frac{1}{2}$$

$a\cdot b$ = dot product

$$\|a\|\,\|b\|\cos\theta = \frac{1}{2} = a\cdot b$$

$$\cos\theta = \frac{1}{2}$$

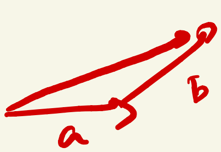
$$\theta = 60^\circ$$

a is a vector

b is a vector

$$a+b \Rightarrow \|a+b\| \equiv \|a\| + \|b\|$$

$$\sqrt{x_1^2 + x_2^2 + \dots}$$



$a = [2, 3, 4]$ $\quad$ $c = a+b = [3, 5, 7]$

$b = [1, 2, 3]$ $\quad$ $\|c\| = \sqrt{9+25+49}$

$\|a\| = \sqrt{2^2+3^2+4^2} = \sqrt{29}$ $\quad$ $= \sqrt{83}$

$\|b\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$(5a+3d)\cdot(6b+4c)$

$30\,\boxed{a\cdot b} + \ldots$

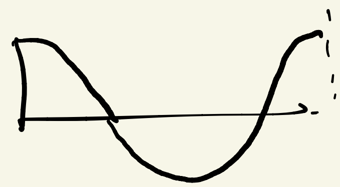
$$\cos\theta = \frac{x^T y}{\|x\|\,\|y\|}$$

$x^T y = \|x\|\,\|y\| \cos\theta$

$x\cdot y = \|x\|\,\|y\| \cos\theta$

$a\cdot b = \|a\|\,\|b\| \cos\theta$

$a\cdot a = \|a\|\,\|a\| \cos\theta_1$ $\quad$ $\theta_1 = 0$ $\quad$ a, a



$\cos 0^\circ = 1$

$a\cdot a = (\|a\|)^2$

$a\cdot b = (\|a\|)(\|b\|) \cos\theta$

$x^T y$

Revisit

$c\cdot d = 0 = \|c\|\,\|d\|\,\cancel{\cos 90^\circ}^{\,0}$

$= 0$

$(a+2b)\cdot(5a-4b) = 0$

$5\, a\cdot a + 10\, a\cdot b - 4\, a\cdot b - 8\, b\cdot b = 0$

$5\|a\|^2 + 6\, a\cdot b - 8\|b\|^2 = 0$

$$a \cdot b = \frac{3}{6} = \frac{1}{2}$$

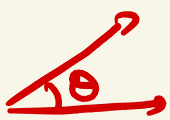
$$a \cdot b = \frac{1}{2}$$

$$\|a\| \, \|b\| \cos\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\boxed{\theta = 60^\circ}$$

Quiz $a = [3, 4]$ $b = [5, 0]$



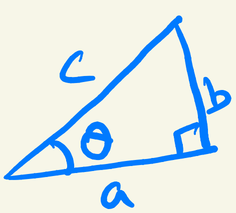
$$\cos\theta = \frac{a \cdot b}{\|a\| \, \|b\|} = \frac{a^T b}{\|a\| \, \|b\|}$$

$$\|a\| = \sqrt{3^2 + 4^2} = 5$$

$$\|b\| = \sqrt{5^2 + 0} = 5$$

$$a^T b = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \begin{bmatrix} 5 & 0 \end{bmatrix} = 3 \times 5 + 4 \times 0 = 15$$

$$\cos\theta = \frac{15}{5 \cdot 5} = \frac{3}{5}$$

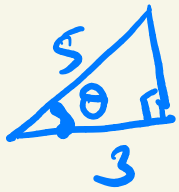


$a/c =$

$$\boxed{\cos\theta = \frac{3}{5}}$$

$$\boxed{\theta = \cos^{-1} \frac{3}{5}}$$

$$\cos 60^\circ = \frac{1}{2}$$



θ L1 $\Rightarrow w_1^T x + w_{01} = 0$ Both lines are parallel

L2 $\Rightarrow w_2^T x + w_{02} = 0$

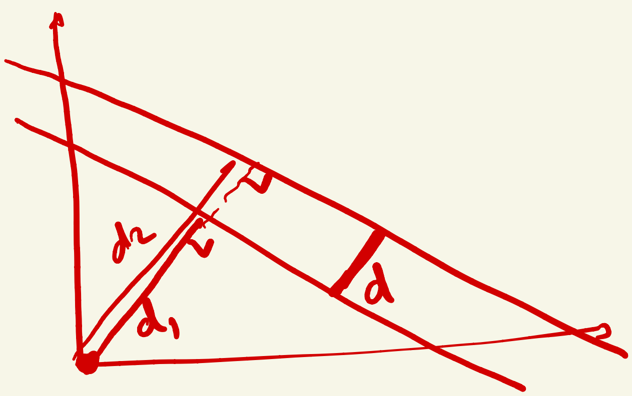
$w_1 = [4,3]^T$

$w_2 = [16,12]^T$] in same direction,

$w_{01} = 3$

$w_{02} = 7$

$$w^T x + w_0 = 0 \Rightarrow \frac{w_0}{\|w\|}$$



$$d_1 = \frac{w_{01}}{\|w_1\|} \qquad d_2 = \frac{w_{02}}{\|w_2\|}$$

$$d = d_2 - d_1$$

$$d_1 = \frac{3}{\sqrt{4^2+3^2}} = 3/5$$

$$d_2 = \frac{7}{\sqrt{16^2+12}} = 7/20$$

$$|d_1 - d_2| = \frac{3}{5} - \frac{7}{20} = \frac{12}{20} - \frac{7}{20} = 5/20 = \boxed{1/4}$$

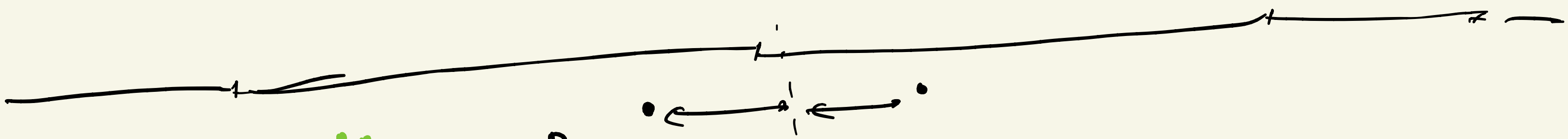
Quiz

$4x - 3y = 5$ $\Rightarrow$ $4x - 3y - 5 = 0$

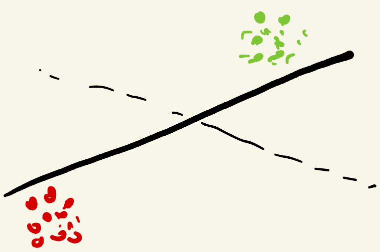
$4x - 3y = 9$ $\quad$ $4x - 3y - 9 = 0$

$$\frac{-5}{\sqrt{4^2+3^2}} \qquad \frac{-9}{\sqrt{4^2+3^2}}$$

$$-1 \qquad -9/5 \qquad = \boxed{4/5}$$



$\Rightarrow$



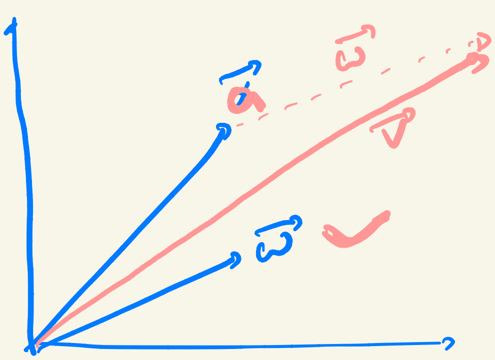
Sum of dist^n $\neq 0$

(work if both side has $\approx$ equal no. of pts)

In the classification context we have two vectors x and w Define v=x+w

In which direction should we move w to reach v ,options are:

- clockwise
- anti-clockwise



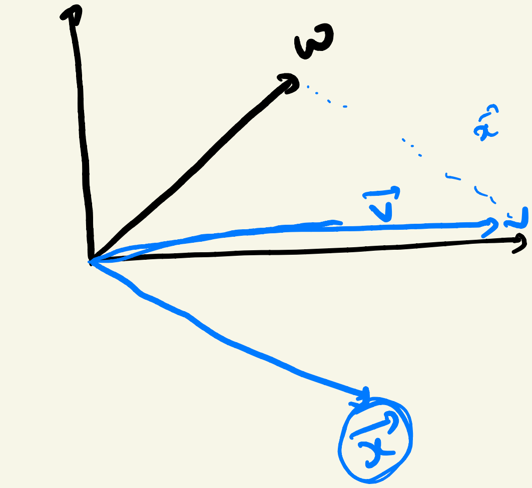
$a \to 3x \quad 5y$

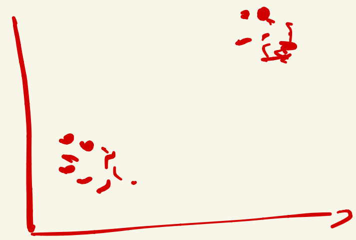
$w \to 4x \quad 7y$

if both $\vec{w}$ & $\vec{x}$ are positive

then $\Rightarrow$ anticlockwise

if we $v = \vec{w} + \vec{x}$



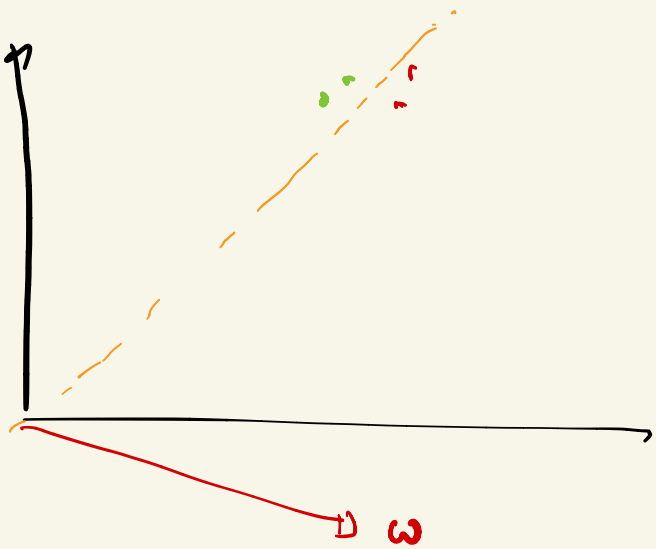


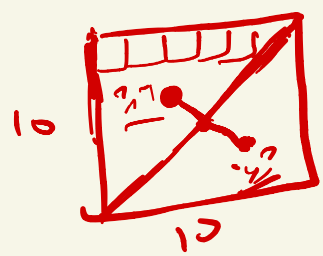
Suppose we have points with verly little data points and the boundry is w with $\omega_0 = 0$

- will this be a good classifier?
- in which direction should w rotate to make it better

$\omega_0 = 0$

ω





$100 \to 100c_2$

$\frac{50c_1}{100c_2} = \frac{1}{101}$

Good vs Bad →

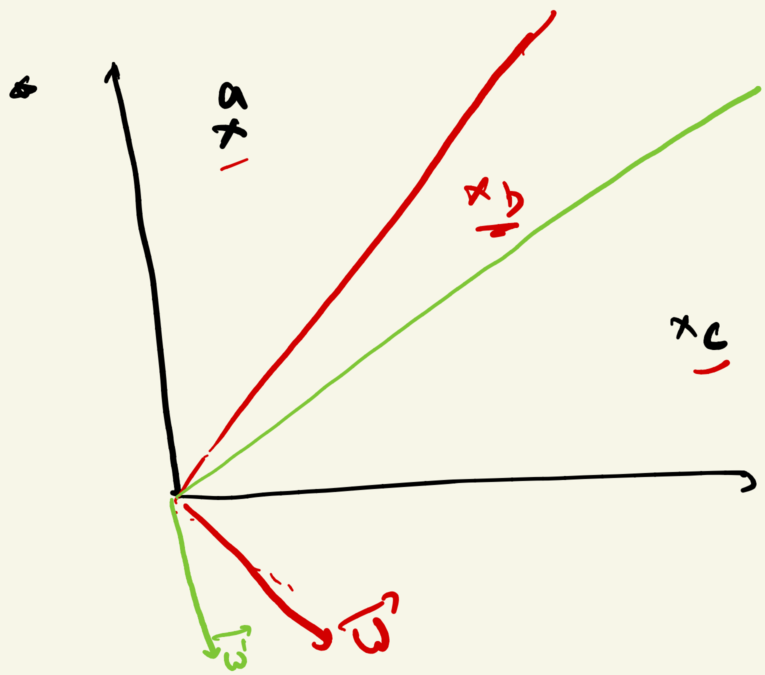
if there is a small error in best fit classifier results should not change

even with a small change everything breaks

⇒ Based on misclassified point → decide how to rotate

-ve →

+ve →

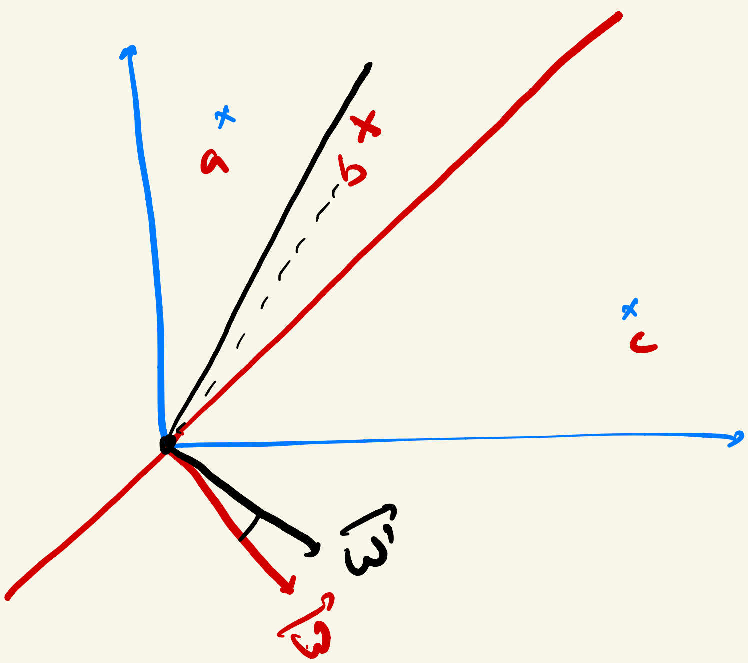


a → -ve

b → actual -ve
but
assign +ve

c → +ve

$\boxed{\text{Clockwise}}$



a → -ve

b → +ve (actual)
-ve (assign)

c → +ve

<u>anticlockwise</u>

<u>little rotate</u>
<u>little shift</u>

⇒ • Either rotate in direction of misclassified
• Shift in that direction

Q $3x-4y=5$ $\quad x=(2,-3)$

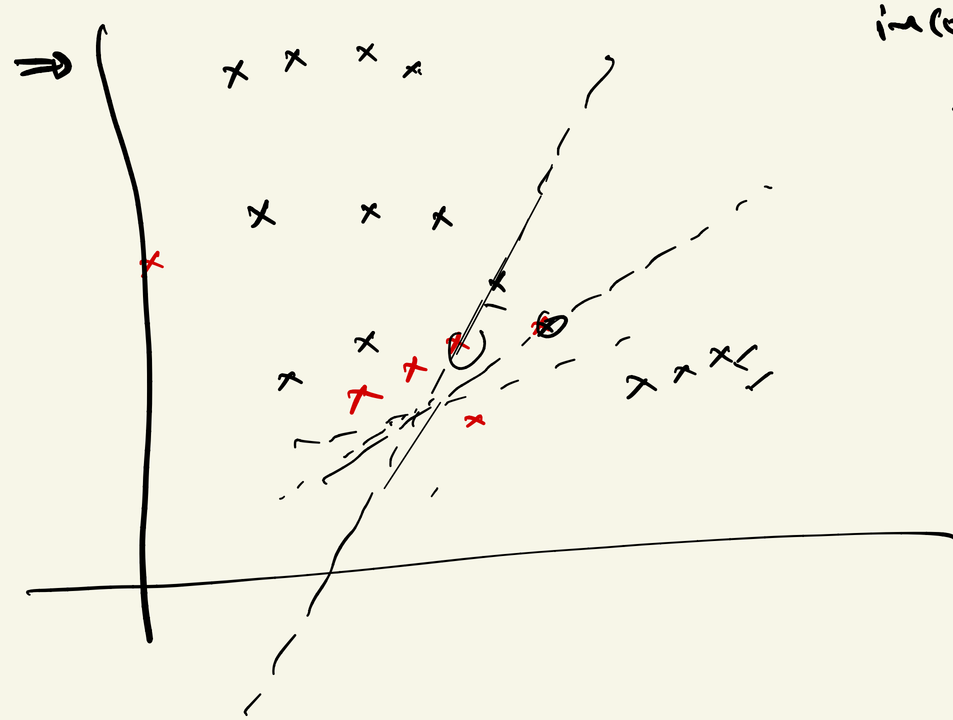
$3x-4y-5=0$

$$\frac{13}{\|w\|} = \frac{13}{\sqrt{3^2+4^2}} = \frac{13}{5}$$

$3\times 2-4(-3)-5=0$

$6+12-5 \Rightarrow 13$

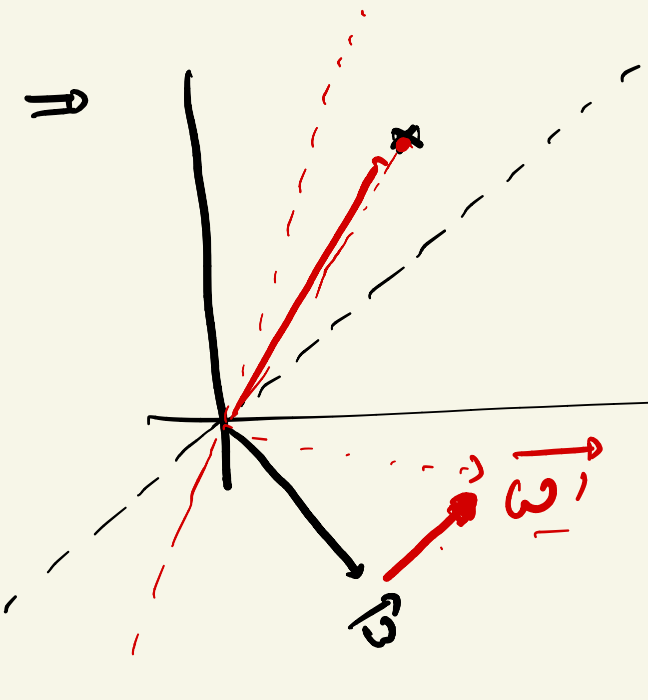
⇒



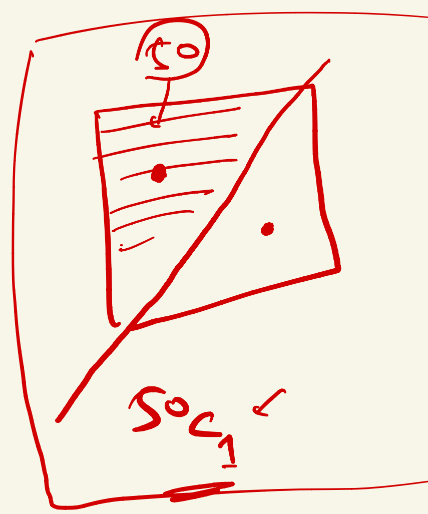
incorrect pt →

rotate + shift =

⇒



anti clockwise



$100c_2$

$$\frac{50c_1}{100c_2}$$