## THE ACTINIDE (5f) GROUP

### 6.1. Ions and compounds of the actinide group

ALTHOUGH Bohr had suggested as early as 1923 that another paramagnetic group might occur at the upper end of the periodic table, with electrons in the 5f shell, it was not until much later that the point at which such a series started became clear. In a review of the evidence Seaborg (1949) postulated that the 5f series starts with actinium as the analogue of lanthanum, thorium that of cerium, etc., the closed shells of electrons corresponding to a radon core. Later Dawson (1952), on the basis of magnetic data, suggested that at the start of the series the magnetic electrons occupied 6d states rather than 5f, but it is now accepted that this interpretation is incorrect (see, for example, Bleaney (1955)). The number of unpaired electron spins is shown in Table 6.1 for the ions where the evidence is most complete. These electrons occupy the 5f states, and at 7 electrons (Am<sup>2+</sup>, Cm<sup>3+</sup>) we reach the half-filled shell; presumably the shell would be complete with 14 electrons at the tripositive ion of element 103. Measurements on the heavier ions are, of course, very difficult because of the high level of radioactivity, and magnetic resonance measurements have been confined to the lighter members of the group. Since most of the isotopes are a-particle emitters, radiation damage is heavy, causing magnetic resonance spectra to deteriorate rapidly in a single crystal, or even to be unobservable immediately after crystal growth. Ionic radii and some other data for the first half of the actinide group are given in Table 6.2.

In the solid state the tripositive ions have been studied mainly in the compounds  $LaCl_3$  and  $La(C_2H_5SO_4)_3$ ,  $9H_2O$ , which have been extensively used in work on the lanthanide group. The site symmetry is  $C_{3h}$  (see the discussion in the previous chapter) and analysis of the results is similar to that for the 4f group. Uranium ions have been incorporated into  $CaF_2$ , and its analogues, where charge compensation results in magnetic resonance spectra that show tetragonal or trigonal symmetry as well as cubic symmetry.

The actinide ions in higher valency states have been studied in compounds that are, in the main, different from those of the 4f group. The two sets of compounds of greatest importance have regular octahedral symmetry at the actinide ion site. In the gas phase the molecules UF<sub>6</sub>, NpF<sub>6</sub>, and PuF<sub>6</sub> have a structure in which the metal

#### TABLE 6.1

Number of unpaired electron spins for the principal ions of the actinide group. In addition the complex ions (UO<sub>2</sub>)<sup>2+</sup>, (NpO<sub>2</sub>)<sup>2+</sup>, (PuO<sub>2</sub>)<sup>2+</sup> have 0, 1, and 2 unpaired electron spins respectively. For these ions, as for those in the table, the unpaired electrons occupy the 5f states

|               | 0                 | 1         | 2                | 3             | 4         | 5                | 6    | 7    |
|---------------|-------------------|-----------|------------------|---------------|-----------|------------------|------|------|
| Thorium       | Th4+              |           |                  |               |           |                  |      |      |
| Protoactinium | Pa <sup>5+</sup>  | $Pa^{4+}$ |                  |               |           |                  |      |      |
| Uranium       | $\mathbf{U}^{6+}$ |           | $U^{4+}$         | $\Omega_{s+}$ | $U^{2+}$  |                  |      |      |
| Neptunium     |                   | $Np^{6+}$ |                  | $Np^{4+}$     | $Np^{3+}$ |                  |      |      |
| Plutonium     |                   | •         | Pu <sup>6+</sup> | •             | Pu4+      | Pu <sup>s+</sup> |      |      |
| Americium     |                   |           |                  |               |           |                  | Am³+ | Am2+ |
| Curium        |                   |           |                  |               |           |                  |      | Cm3+ |

atom is at the centre of a regular octahedral arrangement of six fluorine atoms, the metal to fluorine distances being 0·1996, 0·1981, and 0·1971 nm respectively. A magnetic resonance experiment shows that cubic symmetry is preserved in the solid state for NpF<sub>6</sub> (Hutchison and Weinstock 1960). Some quadripositive ions have been studied in the diluted state in the compounds  $Cs_2ZrCl_6$  and  $[(CH_3)_4N]_2PtCl_6$ , in each

Table 6.2

Some data for ions in the first half of the actinide series

| Tripositive ions |                                    |                         | Quadripositive ions                           |                                    |                         |   |
|------------------|------------------------------------|-------------------------|---|------------------------------------|-------------------------|---|
| $\boldsymbol{z}$ | Ion                                | Ionic<br>radius<br>(nm) | Spin-orbit<br>coupling<br>(cm <sup>-1</sup> ) | Ion                                | Ionic<br>radius<br>(nm) | Spin-orbit<br>coupling<br>(cm <sup>-1</sup> ) |
| 90               | Th <sup>3+</sup> , $5f^1$          | 0.108                   |   | Th4+, 5f0                          | 0.095                   |   |
| 91               | Pa <sup>3+</sup> , 5f <sup>2</sup> |                         |   | $Pa^{4+}$ , $5f^1$                 | 0.091                   | 1490  ASJ                                     |
| 92               | U <sup>3+</sup> , 5f <sup>3</sup>  |                         | 1666 CW<br>1659 GCCN                          | $U^{4+}$ , $5f^2$                  | 0.089                   | 1800 JSSW                                     |
| 93               | Np <sup>8+</sup> , 5f <sup>4</sup> | 0.102                   | 2070 CW<br>1969 GCCN                          | Np4+, 5f3                          | 0.088                   |   |
| 94               | Pu <sup>8+</sup> , 5f <sup>5</sup> | 0.101                   | 2292 CW<br>2260 GCCN                          | Pu <sup>4+</sup> , 5f <sup>4</sup> | 0.086                   |   |
| 95               | Am <sup>3+</sup> , 5f <sup>6</sup> | 0.100                   | 2548 CW<br>2605 C                             | Am <sup>4+</sup> , 5f <sup>5</sup> | 0.085                   |   |
| 96               | Cm <sup>8+</sup> , 5f <sup>7</sup> |                         | 2948 CW<br>2842 GCCN                          |                                    |                         |   |

The ionic radii are from Evans (1964); the other references are

ASJ Axe, Stapleton, and Jeffries (1961).

CW Carnall and Wybourne (1964), Wybourne (1964).

GCCN Gruber, Cochran, Conway, and Wong (1966).

JSSW Johnston, Satten, Schreiber, and Wong (1966).

C Conway (1964).

of which the metal ion is at the centre of a regular octahedron of chlorine ions.

The actinide series differs also from the lanthanide series in the formation of complex ions such as the uranyl ion (UO2)2+ and its analogues (NpO<sub>2</sub>)<sup>2+</sup> and (PuO<sub>2</sub>)<sup>2+</sup>. These ions are linear, of the form O-U-O, with a rather short distance for the U-O bond which provides evidence for a covalent type of bonding. Uranyl sodium acetate, (UO<sub>2</sub>)Na(C<sub>2</sub>H<sub>3</sub>O<sub>2</sub>)<sub>3</sub> is cubic, with four (UO<sub>2</sub>) groups per unit cell whose axes lie along the threefold axes of the cube; paramagnetic resonance has been observed in the plutonyl salt (Hutchison and Lewis 1954). A salt that has been used more extensively is uranyl rubidium nitrate, (UO2)Rb(NO3)3. This salt is hexagonal and all the (UO2) groups lie parallel to the c-axis, surrounded by three nitrate groups in the equatorial plane, making the c-axis a threefold axis for each (UO<sub>0</sub>) group. Paramagnetic resonance has been observed in the neptunyl and plutonyl ions present in dilute form in uranyl rubidium nitrate (Bleaney, Llewellyn, Pryce, and Hall 1954a, b) and the properties of these ions have been the subject of a detailed theoretical study by Eisenstein and Prvce (1955, 1956).

#### 6.2. Tripositive actinide ions

Optical studies have been made of a number of the tripositive ions of the actinide group, mostly incorporated in crystals of  $LaCl_3$ . An analysis of these results together with data obtained on ions in solution shows that the behaviour is similar to that of the 4f group except that the spin-orbit coupling is considerably larger, as can be seen from a comparison of Tables 6.2 and 5.3. Two sets of values are given for the tripositive ions, which illustrate the spread in the experimental values.

The electrostatic parameters such as  $F_2$  are rather smaller than in the 4f group, as can be seen from Tables 6.18, 6.19 in Wybourne (1964), so that departures from LS-coupling are much more pronounced. For example, Conway (1964) finds that in the ground multiplet  $5f^6$ ,  $^7F$  of  $Am^{3+}$  the level J=1 lies  $2720~\rm cm^{-1}$  above J=0, and J=6 lies  $12350~\rm cm^{-1}$  above J=0. Thus these two splittings are in the ratio  $2:9\cdot 1$  instead of the ratio 2:42 that is appropriate to pure LS-coupling, a departure that is far greater than those given in Table 5.2 for the corresponding  $4f^6$  ions. It follows that intermediate coupling is an important factor in interpreting any resonance results; however no analysis of the crystal field will be attempted here and the resonance results given below summarize mainly the spin Hamiltonian parameters.

5f3, U3+ in LaCla

The resonance results for this substance have been fitted to the usual spin Hamiltonian for effective spin  $S = \frac{1}{2}$  with axial symmetry; they are summarized in Table 6.3.

The g-values are remarkably close to those  $(g_{\parallel}=4\cdot00,\,g_{\perp}=1\cdot76)$  measured for the corresponding ion  $4f^3$ ,  $\mathrm{Nd}^{3+}$  in  $\mathrm{LaCl_3}$ , a result which in itself strongly suggests that the resonance is due to a doublet split off by the crystal field from a ground state characterized fairly accurately

Table 6.3

Spin Hamiltonian parameters for U<sup>3+</sup> in LaCl<sub>3</sub>

| $ g_{\parallel} =4$  | $\cdot 153(5),  g_{\perp}  = 1 \cdot 520(2)$   | Hutchison, Llewellyn, Wong, and Dorain (1956)                                       |
|--|--|---|
| $egin{array}{c}  A_{\parallel}  \  A_{\perp}  \  P_{\parallel}  \end{array}$ | Isotope 233 ( $I=\frac{5}{2}$ 378·6(12)×10 <sup>-4</sup> cm 123·6(10)×10 <sup>-4</sup> cm 9·9(10)×10 <sup>-4</sup> cm Bleaney, Hutchison, Llev and Pope (1956) | $58.5(5) \times 10^{-4} \text{ cm}^{-1}$<br>$5.5(5) \times 10^{-4} \text{ cm}^{-1}$ |

as  $5f^3$ ,  ${}^4I_2$ . The ratio  $(A_{\parallel}/A_{\perp})/(g_{\parallel}/g_{\perp})$  is 1·12, which is also very close to that for Nd3+ in LaCl3, and suggests that crystal field strength and spin-orbit coupling are roughly in the same ratio as for Nd3+. The nuclear spins of  $\frac{5}{2}$  for isotope 233 and  $\frac{7}{2}$  for isotope 235 are clearly established from the hyperfine structure, the ratio of the nuclear moments being  $\mu_I^{235}/\mu_I^{233}=0.66$ . The quadrupole interaction measured by observation of the 'forbidden' hyperfine lines that appear when the external magnetic field is not along the symmetry axis (cf. § 3.9). The values of  $P_{\parallel}$  are very much larger than in the case of Nd<sup>3+</sup>, and the correction for the pseudo-quadrupolar interaction  $\Delta P_{\parallel}$  which was quite important for Nd3+ (§ 4.8) should be quite small for U3+. Thus the ratio of the quadrupole moments,  $Q^{235}/Q^{233} = 1.17(20)$  should be quite accurate. As in the case of Nd3+, the lattice may make a substantial contribution to the electric field gradient at the nucleus, since the anti-shielding factor  $\gamma_{\infty}$  is likely to be larger for U<sup>3+</sup> as well as  $\langle r^{-3} \rangle$ , which would increase the gradient due to the f-electrons.

# $5f^7$ , $Cm^{3+}$ in $LaCl_3$ , and $La(C_2H_5SO_4)_3$ , $9H_2O$

This ion is expected to be in an  ${}^8S_{\frac{7}{2}}$  state, like Gd<sup>3+</sup>, where crystal field effects produce splittings only through higher-order effects. Abraham, Judd, and Wickman (1963) have shown that some spectra attributed earlier to Cm<sup>3+</sup> must be due to Gd<sup>3+</sup>; they find that only a

single anisotropic resonance line is visible, with the following g-values:

$$\begin{array}{lll} {\rm Cm^{3+}\ in\ La(C_2H_5SO_4)_3,\ 9H_2O} & |g_{\parallel}| = 1\cdot 925(2) & |g_{\perp}| = 7\cdot 73(2), \\ {\rm Cm^{3+}\ in\ LaCl_3} & |g_{\parallel}| = 1\cdot 925(2) & |g_{\perp}| = 7\cdot 67(2). \\ \end{array}$$

Since  $g_{\perp}$  is very close to  $4g_{\parallel}$ , this must correspond to the transition  $|+\frac{1}{2}\rangle\leftrightarrow|-\frac{1}{2}\rangle$  of  $S=\frac{7}{2}$  split by an axial term  $D\{S_z^2-\frac{1}{3}S(S+1)\}$  (or  $B_2^0\{3S_z^2-S(S+1)\}$ ) which is large compared with the frequency at which resonance is observed ( $\sim 0.3$  cm<sup>-1</sup>). The value of  $g_{\parallel}$  should be equal to the g-value of the free ion, and its rather low value is due to a fair degree of intermediate coupling, being in reasonable agreement with that calculated from the wave-functions

$$0.8884 | {}^{8}S\rangle + 0.4197 | {}^{6}P\rangle - 0.0909 | {}^{6}D\rangle + \dots$$

suggested by Carnall and Wybourne (1964) (see also Gruber, Cochran, Conway, and Nicol (1966)). The larger degree of intermediate coupling means that we should also expect the crystal field splittings to be much larger for Cm³+ than for Gd³+, in agreement with experiment. A proportionate increase in line-width for the other transitions due to crystal imperfections would also be expected, and the damage due to the high radioactivity would readily make such lines too broad to be observed.

### 6.3. Actinide ions in CaF<sub>2</sub>

The electron spin resonance and optical spectra of several actinide ions have been studied in CaF<sub>2</sub> and isomorphic crystals, partly in a search for suitable laser crystals.

$$5f^4$$
,  $U^{2+}$ ;  $5f^3$ ,  $U^{3+}$  and  $5f^2$ ,  $U^{4+}$  in  $CaF_2$ 

Magnetic resonance was observed by Bleaney, Llewellyn, and Jones (1956) due to uranium in CaF<sub>2</sub> and SrF<sub>2</sub>; the spectra showed axial symmetry about a fourfold axis with the following g-values:

|                  | Uran               | nium                          | Neodymium                      |               |  |
|------------------|--------------------|-------------------------------|--------------------------------|---------------|--|
|                  | $ g_{\parallel} $  | $ g_{\perp} $                 | $ g_{\parallel} $              | $ g_{\perp} $ |  |
| $\mathbf{CaF_2}$ | 3.501              | 1.866                         | $4 \cdot 412$                  | 1.301         |  |
| $SrF_2$          | 3.433              | 1.971                         | 4.289                          | 1.505         |  |
|                  | (Errors: $\pm 0$ · | 008 in $g_{\parallel}, \pm 0$ | $0.002 \text{ in } g_{\perp}.$ |               |  |

The similarity of the results suggests that the ions are  $U^{3+}$ ,  $Nd^{3+}$ . A fluorine hyperfine structure was observed in the  $U^{3+}$  spectrum, but not in the  $Nd^{3+}$  spectrum, suggesting that the 5f electrons take part in an appreciable amount of covalent bonding.

Title et al. (1962) have reported resonance lines whose angular variation can be fitted to that of a  $\Gamma_8$  quartet in cubic symmetry, which would be expected as the ground state for  $5f^3$  in the cubic field of CaF<sub>2</sub>; however the observed g-values have not been interpreted, and do not seem to correspond to those expected from simple crystal field theory using the calculations of Lea, Leask, and Wolf (1962), though this is perhaps not surprising.

A number of asymmetrical lines typical of non-Kramers doublets with  $g_{\perp} \leqslant 0.1$  and axial symmetry about a threefold axis have also been observed in CaF<sub>2</sub> doped with uranium and, for comparison, with praseodymium. The values of  $g_{\parallel}$  are

- (a)  $g_{\parallel} = 5.65(1)$  McLaughlan (1966) Pr<sup>3+</sup>,  $4f^2$ ,  $^3H_4$ .
- (b)  $g_{\parallel} = 5.83(1)$  McLaughlan (1966) Pr<sup>3+</sup>,  $4f^2$ ,  $^3H_4$ .
- (c)  $g_{\parallel} = 5.66(2)$  McLaughlan (1966)  $U^{4+}$ ,  $5f^2$ ,  ${}^3H_4$ .
- (d)  $g_{\parallel} = 4.02(1)$  McLaughlan (1966)  $U^{4+}$ ,  $5f^2$ ,  ${}^3H_4$ .
- (e)  $g_{\parallel} = 3.238(5)$  Yariv (1962); Title et al. (1962).

The first three lines are so similar in the value of  $g_{\parallel}$  that they are assigned to  $f^2$  ions, where the  $^3H_4$  manifold is split by the trigonal field to give as ground state the non-Kramers doublet (cf. eqn (5.67))

$$\alpha \mid \pm 4 \rangle \pm \beta \mid \pm 1 \rangle + \gamma \mid \mp 2 \rangle.$$

For this doublet the maximum possible value of  $g_{\parallel}$  is  $8\langle J \parallel \Lambda \parallel J \rangle$  (corresponding to  $\alpha=1$ ,  $\beta=\gamma=0$ ); this maximum is 6·4 for  $f^2$ ,  $^3H_4$  but only 4·8 for  $f^4$ ,  $^5H_4$ , so that the lines cannot be due to  $5f^4$ ,  $U^{2+}$  ions. The last value (e), also ascribed to  $5f^2$ ,  $U^{4+}$  ions by the authors quoted, is assigned to  $5f^4$ ,  $U^{2+}$  by Hargreaves (1967).

In the absence of local charge compensation, an  $f^2$ ,  ${}^3H_4$  ion in the purely cubic field of  $\operatorname{CaF_2}$  may have as ground state (see Lea, Leask and Wolf (1962)) a  $\Gamma_5$  triplet with an isotropic  $\tilde{g}=2$  and an effective spin  $\tilde{\mathbf{S}}=1$ . Such an isotropic line has been observed by McLaughlan (1967), and the line (d) above is attributed to a  $|+\tilde{1}\rangle \leftrightarrow |-\tilde{1}\rangle$  transition for such a triplet split by a trigonal distortion, the  $|\tilde{0}\rangle$  level lying higher by some 7 cm<sup>-1</sup> (McDonald 1969; Wetsel and Donoho 1965).

$$5f^7$$
, Am<sup>2+</sup> and Cm<sup>3+</sup> in CaF<sub>2</sub>

The magnetic resonance spectra of these ions at sites of cubic symmetry in  $CaF_2$  have been observed by Edelstein and Easley (1968). At low field strengths a single resonance line with an isotropic g-value of  $(-)4\cdot492$  is observed for <sup>244</sup>Cm<sup>3+</sup>, but at higher field strengths (frequencies of 35 GHz) some anisotropy is found. Similar results are

#### TABLE 6.4

Magnetic resonance results for the Γ<sub>6</sub> doublet of the <sup>8</sup>S<sub>\frac{7}{2}</sub> state split by a cubic field in CaF<sub>2</sub> (Edelstein and Easley 1968). Similar but slightly smaller values of (—)4·484(2) and (—)4·475(2) have been obtained for <sup>244</sup>Cm<sup>3+</sup> in ThO<sub>2</sub> and CeO<sub>2</sub> respectively (Abraham, Finch, and Clark 1968)

| Ion                             | $g(\Gamma_6)$     | I        | A  (cm <sup>-1</sup> ) |
|---------------------------------|-------------------|----------|------------------------|
| <sup>244</sup> Cm <sup>3+</sup> | $(-)4\cdot492(2)$ | 0        | 0                      |
| <sup>241</sup> Am <sup>2+</sup> | (-)4.490(2)       | <u>5</u> | 0.01837(2)             |
| $^{243}Am^{2+}$                 | $(-)4\cdot490(2)$ | 5 2      | 0.01821(2)             |

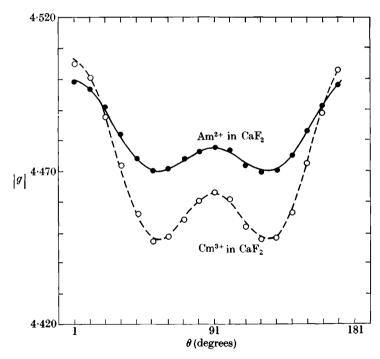


Fig. 6.1. The apparent g-factor of the  $\Gamma_6$  doublet of two 5f' ions on cubic sites in CaF<sub>2</sub>, as measured at 35 GHz. The magnetic field is rotated in an  $\langle 011 \rangle$  plane, and  $\theta=0$  corresponds to a  $\langle 100 \rangle$  axis. The anisotropy is due to admixture of the excited  $\Gamma_8$  states by the Zeeman interaction (Edelstein and Easley 1968).

$$- \bigoplus - \operatorname{Am}^{2+}, W(\Gamma_8) - W(\Gamma_6) = 18.6 \text{ cm}^{-1};$$

$$- \bigcirc - \bigcirc - \operatorname{Cm}^{3+}, W(\Gamma_8) - W(\Gamma_6) = 13.4 \text{ cm}^{-1}.$$

$$\Gamma_7$$
(doublet), if  $b_6 = -0 \cdot 1 \text{cm}^{-1}$ 
(58.4)  $\frac{b_6}{-1} = -0 \cdot 1 \text{cm}^{-1}$ 

$$\begin{array}{ccc} \Gamma_7(\text{doublet}), \text{if} & (49\cdot6) \overline{\Gamma_7(\text{doublet})}, \text{if} \\ b_6 = -0 \cdot 1 \text{cm}^{-1} & b_6 = 0 \end{array}$$

$$(35.7)$$
  $\frac{-}{\Gamma_7}$   $\frac{-}{\Gamma_7}$  doublet), if  $\mathbf{b_6} = 0$ 

$$18.6(5) \frac{\Gamma_8 \text{ (quartet)}}{\Gamma_8 \text{ (quartet)}}$$

Fig. 6.2. Splitting of the ground  $J=\frac{7}{2}$  manifold of two  $5f^7$  ions on sites of cubic symmetry in  $CaF_2$ . The energy of the  $\Gamma_8$  quartet level is deduced from the anisotropy shown in Fig. 6.1. Two calculated energies are given for the  $\Gamma_7$  doublet, using two sets of values for the splitting parameters  $b_4=60B_4$ ,  $b_6=1260B_6$ , either of which gives the correct energy for the  $\Gamma_8$  level. These sets are (Edelstein and Easley 1968):

$$b_4 \; (\mathrm{cm^{-1}}) \quad b_6 \; (\mathrm{cm^{-1}}) \quad W(\Gamma_7) (\mathrm{cm^{-1}})$$

$$Cm^{3+} \begin{cases} -1.115 & 0 & 35.7 \\ -1.433 & -0.1 & 45.1 \end{cases}$$

$$Am^{2+} \begin{cases} -1.549 & 0 & 49.6 \\ -1.849 & -0.1 & 58.4 \end{cases}$$

obtained for Am<sup>2+</sup>, with a hyperfine structure due to the odd isotopes 241, 243, each of which has  $I=\frac{5}{2}$ . The resonance is attributed to the  $\Gamma_6$  doublet of an  ${}^8S_{\frac{7}{2}}$  state with a large cubic field splitting; the effective g-value (see Table 22) is  $(-\frac{7}{3})$  times the true g-value for  $S=\frac{7}{2}$  at low fields, while at higher fields the anisotropy arises from admixture of the excited  $\Gamma_6$  quartet by the Zeeman interaction. The energy of the  $\Gamma_6$  level in zero field can be deduced from this anisotropy.

The results are listed in Table 6.4; the anisotropy is shown in Fig. 6.1 and the energy levels in Fig. 6.2. For the  $S = \frac{7}{2}$  configuration the

value of  $g_J$  is deduced to be 1.9261(10) for Cm<sup>3+</sup> and 1.9258(10) for Am<sup>2+</sup>; the latter is slightly smaller than the value  $g_J = 1.93788(7)$  measured by Armstrong and Marrus (1966) for the free atom Am, which has the configuration  $5f^77s^2$ ,  $^8S_{\frac{7}{2}}$ , but both are in good agreement with those for Cm<sup>3+</sup> in  $C_{3h}$  symmetry (see § 6.2). The cubic field splittings (see Fig. 6.2) are enormously greater than those listed in Table 5.19 for the corresponding  $4f^7$  ions.

### 6.4. Actinide ions in octahedral symmetry

We consider first the quadripositive ions. Optical measurements on  $\mathrm{Pa^{4+}}$ ,  $5f^1$  in  $\mathrm{Cs_2ZrCl_6}$  (Axe, reported by Raubenheimer, Boesman, and Stapleton (1965)) yields the value  $\zeta=1490~\mathrm{cm^{-1}}$  for the spin-orbit parameter. This is in line with values for the tripositive ions (Table 6.2) and for  $\mathrm{U^{4+}}$ ,  $5f^2$  (see below). On the other hand, the fourth degree term in the crystal potential is much larger than that for tripositive actinide or lanthanide ions. For  $\mathrm{Pa^{4+}}$  the above authors give  $A_4\langle r^4\rangle=+888~\mathrm{cm^{-1}}$ ,  $A_6\langle r^6\rangle=+42~\mathrm{cm^{-1}}$ , and similar values are found for  $\mathrm{U^{4+}}$  (see below).

Magnetic resonance has been observed for Pa<sup>4+</sup> in Cs<sub>2</sub>ZrCl<sub>6</sub> by Axe, Stapleton, and Jeffries (1961) who also measured the hyperfine structure of <sup>231</sup>Pa  $(I=\frac{3}{2})$  by Endor. In an octahedral field the ground manifold of  $5f^1$ ,  $^2F_{\frac{5}{2}}$  is split into a doublet  $(\Gamma_7)$  and a quartet  $(\Gamma_8)$ , the former lying lower (from optical measurements) by about 1900 cm<sup>-1</sup>. The magnetic resonance spectrum of the doublet was fitted to the usual spin Hamiltonian

$$\mathcal{H} = g\beta \mathbf{H} \cdot \mathbf{S} + A\mathbf{I} \cdot \mathbf{S} - g^{(I)}\beta \mathbf{H} \cdot \mathbf{I}$$
 (6.1)

with the isotropic values  $|g|=1\cdot1423(14), \ |A|=1578\cdot6(14) \ \mathrm{MHz}$   $|g^{(I)}|=7\cdot74\times10^{-4}.$  After applying an 8 per cent correction for the pseudo-nuclear Zeeman effect, a value of 1·96 nuclear magnetons is obtained for the nuclear moment. The experiments show that g has the opposite sign to  $g^{(I)}$ , and hence also to  $g_I$ ; since the nuclear moment is expected to be positive this gives a negative sign for g (and also for g). For a pure  $g=\frac{5}{2}$  manifold split by a small cubic field the g-value for the ground doublet. In general, one can write the ground doublet as

$$\cos \alpha |J = \frac{5}{2}, \Gamma_7\rangle + \sin \alpha |J = \frac{7}{2}, \Gamma_7\rangle$$
 (6.2)

for which

$$g = -\frac{10}{7}\cos^2\alpha + \frac{16}{7\sqrt{3}}\cos\alpha\sin\alpha + \frac{24}{7}\sin^2\alpha,$$
 (6.3)

showing that g can be positive or negative according to the value of  $\alpha$ . The hyperfine constant is

$$A = 2g_{\rm n}\beta\beta_{\rm n}\langle r^{-3}\rangle \left\{ -\frac{16}{7}\cos^2\alpha - \frac{8}{7\sqrt{3}}\cos\alpha\sin\alpha + \frac{16}{7}\sin^2\alpha - \kappa \left(\frac{5}{21}\cos^2\alpha + \frac{16}{7\sqrt{3}}\cos\alpha\sin\alpha + \frac{3}{7}\sin^2\alpha\right) \right\}$$
(6.3a)

where the term in  $\kappa$  allows for the possibility of a contribution from core polarization. An alternative approach is outlined below, in the theory for NpF<sub>6</sub>.

Measurements of the susceptibility of  $[(CH_3)_4N]_2UCl_6$ , where the U<sup>4+</sup> ion is in an octahedral environment, show that it has a small temperature-independent susceptibility up to 300°K (Hutchison and Candela 1957). This is consistent with a ground term  $5f^2$ ,  $^3H_4$ , which in an octahedral field of the expected sign gives a  $\Gamma_1$  singlet as the lowest level; the small constant susceptibility shows that the crystal field splittings are large compared with room temperature (200 cm<sup>-1</sup>). Analysis of the optical spectrum (see, for example, Satten, Schreiber, and Wong (1965)) gives values of  $A_4\langle r^4\rangle\approx 900$  cm<sup>-1</sup>,  $A_6\langle r^6\rangle\approx 56$  cm<sup>-1</sup>,  $\zeta=1796$  cm<sup>-1</sup>; the first excited state (the  $\Gamma_4$  triplet of  $^3H_4$ ) lies at about 900 cm<sup>-1</sup>.

The magnetic resonance spectrum of NpF<sub>6</sub> at dilutions of 2 and 4 per cent in UF<sub>6</sub> has been observed at 4°K by Hutchison and Weinstock (1960). The results are fitted to the spin Hamiltonian (6.1); g is found to be isotropic with the value g = -0.604(3), the sign being determined using circularly polarized radiation (cf. § 3.2). The hyperfine structure, also isotropic, is due to the isotope <sup>237</sup>Np ( $I = \frac{5}{2}$ ), and its value is |A| = 0.0665(5) cm<sup>-1</sup>. These results are consistent with a structure in which the neptunium ion is surrounded by a regular octahedron of fluorine ions, with a single electron in the 5f shell. This is similar to that of Pa<sup>4+</sup> in Cs<sub>2</sub>ZrCl<sub>6</sub> (see above, but the much smaller g-value shows that a weak crystal field approach is an even poorer approximation in this case).

The theory of NpF<sub>6</sub> has been considered in some detail by Eisenstein and Pryce (1960). In a cubic field the sevenfold orbital degeneracy of an f-state is lifted, giving a singlet ( $a_2$  or  $\Gamma_2$ ), and two triplets ( $t_1$  or  $\Gamma_4$ , and  $t_2$  or  $\Gamma_5$ ); the wave-functions are listed in Table 6.5. In an octahedral

TABLE 6.5

Splitting of the orbital states of an f-electron in a cubic field (Eisenstein and Pryce 1960). The wave-functions to the right are in terms of  $|l_z\rangle$ , and are real linear combinations of those given in Fig. 7.5. (A common factor of  $\sqrt{7}$  has been omitted from the functions given in Cartesian coordinates as compared with Eisenstein and Pryce). If the orbital splitting is related to the spin operator form  $B_4(O_4^0+5O_4^0)+B_6(O_6^0-21O_6^0)$ , then  $V=10b_4+84b_6$ ;  $V'=18b_4+28b_6$ , and for a single f-electron  $B_4=b_4/60=2A_4\langle r^4\rangle/(11\times45)$ ,  $B_6=b_6/180=-4A_6\langle r^6\rangle/(11\times13\times27)$ 

| <br>Symmetry       | Energy | Wave-functions  |
|--------------------|--------|---|
|                    |        | $\begin{cases}  \delta_2\rangle \equiv \frac{1}{2}x(5x^2 - 3r^2) \equiv \frac{\sqrt{(3)}}{4}\{ +1\rangle -  -1\rangle\} - \frac{\sqrt{(5)}}{4}\{ +3\rangle -  -3\rangle\} \\  \delta_3\rangle \equiv \frac{1}{2}y(5y^2 - 3r^2) \equiv -i\frac{\sqrt{(3)}}{4}\{ +1\rangle +  -1\rangle\} - i\frac{\sqrt{(5)}}{4}\{ +3\rangle +  -3\rangle\} \\  \delta_1\rangle \equiv \frac{1}{2}z(5z^2 - 3r^2) \equiv  0\rangle \end{cases}$   |
| $t_1$ $(\Gamma_4)$ | V'     | $\left\{  \delta_3 angle \equiv rac{1}{2}y(5y^2-3r^2) \equiv -\mathrm{i}rac{\sqrt{(3)}}{4}\{ +1 angle +  -1 angle\} - \mathrm{i}rac{\sqrt{(5)}}{4}\{ +3 angle +  -3 angle\}  ight.$  |
|                    |        |   |
|                    |        | $\begin{cases}  \epsilon_2\rangle \equiv \frac{1}{2}\sqrt{(15)}x(y^2-z^2) \equiv \frac{\sqrt{(5)}}{4}\{ +1\rangle- -1\rangle\} + \frac{\sqrt{(3)}}{4}\{ +3\rangle- -3\rangle\} \\  \epsilon_3\rangle \equiv \frac{1}{2}\sqrt{(15)}y(z^2-x^2) \equiv i\frac{\sqrt{(5)}}{4}\{ +1\rangle+ -1\rangle\} - i\frac{\sqrt{(3)}}{4}\{ +3\rangle+ -3\rangle\} \\  \epsilon_1\rangle \equiv \frac{1}{2}\sqrt{(15)}z(x^3-y^2) \equiv \frac{1}{\sqrt{(2)}}\{ +2\rangle+ -2\rangle\} \end{cases}$ |
| $t_2$ $(\Gamma_5)$ | v      | $\left\{  \epsilon_3\rangle \equiv \frac{1}{2}\sqrt{(15)}y(z^2-x^2) \equiv i\frac{\sqrt{(5)}}{4}\{ +1\rangle+ -1\rangle\} - i\frac{\sqrt{(3)}}{4}\{ +3\rangle+ -3\rangle\} \right.$   |
|                    |        | $ \epsilon_1 angle \equiv rac{1}{2}\sqrt{(15)}z(x^2-y^2) \equiv rac{1}{\sqrt{(2)}}\{ +2 angle +  -2 angle\}$  |
| $a_2$ $(\Gamma_2)$ | 0      | $ \beta\rangle \equiv \sqrt{(15)xyz} \qquad \equiv \frac{1}{\sqrt{(2)}}\{ +2\rangle -  -2\rangle\}$   |

field the  $a_2$  state is lowest, since it has zero density along each fourfold axis and so avoids the negatively charged ligand ions as much as possible; the  $t_2$  states lie next, and the  $t_1$  states highest. In crystal field theory this is equivalent to assuming that the fourth degree terms in the octahedral potential outweigh the sixth degree terms. In terms of bonding, the  $a_2$  orbital is non-bonding, the  $t_1$  orbitals can form  $\sigma$ -bonds, and the  $t_2$  orbitals  $\pi$ -bonds. If we assume that the magnetic electron occupies an anti-bonding orbital, and that  $\sigma$ -bonding is stronger than

Table 6.6

Basis functions of f<sup>1</sup> including spin in a cubic field (Eisenstein and Pryce 1960)

|                | <del>-</del>   | $m_J$                          |
|----------------|--|--------------------------------|
| $\Gamma_{7}$   | $ \vec{A}\rangle = i \vec{\beta}\rangle$<br>$ \vec{B}\rangle = (\frac{1}{2})^{\frac{1}{2}}[- \vec{\epsilon}_1\rangle +  \vec{\epsilon}_2\rangle - i \vec{\epsilon}_3\rangle]$  | }+3                            |
|                | $ A\rangle = i  \beta\rangle   B\rangle = (\frac{1}{3})^{\frac{1}{2}}[ \epsilon_{1}\rangle +  \tilde{\epsilon}_{2}\rangle + i  \tilde{\epsilon}_{3}\rangle]$   | $\left\{-\frac{3}{2}\right\}$  |
| $\Gamma_{8}$   | $ C\rangle = (\frac{1}{6})^{\frac{1}{2}}[-2 \epsilon_1\rangle -  \epsilon_2\rangle + i \epsilon_3\rangle]$ $ \overline{D}\rangle = -(\frac{1}{2})^{\frac{1}{2}}[ \delta_2\rangle + i \delta_3\rangle]$   | $\left.\right\} + \frac{3}{2}$ |
|                | $ \begin{aligned}  C'  &= (\frac{1}{2})^{\frac{1}{2}} [ \bar{\epsilon}_2\rangle - \mathrm{i} \bar{\epsilon}_3\rangle] \\  D'\rangle &= (\frac{1}{2})^{\frac{1}{2}} [2 \delta_1\rangle -  \bar{\delta}_2\rangle - \mathrm{i} \bar{\delta}_3\rangle] \\  \bar{C}'\rangle &= (1)^{\frac{1}{2}} [1 + 1 + 1 + 1 + 1] \end{aligned} $  | }+±                            |
|                | $\begin{aligned}  \overline{C}'\rangle &= -(\frac{1}{2})^{\frac{1}{2}}[ \epsilon_{2}\rangle + i  \epsilon_{3}\rangle] \\  \overline{D}'\rangle &= (\frac{1}{6})^{\frac{1}{2}}[2  \overline{\delta}_{1}\rangle +  \delta_{2}\rangle - i  \delta_{3}\rangle] \\  C\rangle &= (\frac{1}{6})^{\frac{1}{2}}[-2  \epsilon_{1}\rangle +  \tilde{\epsilon}_{2}\rangle + i  \tilde{\epsilon}_{3}\rangle] \end{aligned}$ | }-1                            |
| г              | $ D angle=(rac{1}{2})^{rac{1}{2}}[ ar{\delta}_2 angle-\mathrm{i} ar{\delta}_3 angle]$  | }-3                            |
| Γ <sub>6</sub> | $ \begin{aligned}  E\rangle &= (\frac{1}{3})^{\frac{1}{2}} [ \delta_1\rangle +  \tilde{\delta}_2\rangle + \mathrm{i}  \tilde{\delta}_3\rangle] \\  \vec{E}\rangle &= (\frac{1}{3})^{\frac{1}{2}} [- \tilde{\delta}_1\rangle +  \delta_2\rangle - \mathrm{i}  \delta_3\rangle] \end{aligned} $  | +½<br>-½                       |

The orbital functions are given in Table 6.5; the notation is that  $\beta$  is an orbital function combined with 'up' spin  $(m_8 = +\frac{1}{2})$  and  $\bar{\beta}$  is an orbital function combined with 'down' spin  $(m_8 = -\frac{1}{2})$ . The values in the column headed  $m_J$  are the values of  $l_z + s_z$ .

 $\pi$ -bonding, this gives the same order of levels as assumed above, the anti-bonding  $\sigma$ -orbitals  $t_1$  lying highest, and the non-bonding orbital  $a_2$  lowest.

When the electron spin is introduced the product wave-functions can be classified as  $\Gamma_6$  (doublet),  $\Gamma_7$  (doublet), and  $\Gamma_6$  (quartet), where  $\Gamma_6$  occurs only once, but  $\Gamma_7$  and  $\Gamma_6$  occur twice, as given in Table 6.6. From the energy matrices given in Table 6.7 it is clear that the ground state will be a  $\Gamma_7$  doublet, which can be written as

$$|+\rangle = \cos \theta |A\rangle - \sin \theta |B\rangle |-\rangle = \cos \theta |\bar{A}\rangle - \sin \theta |\bar{B}\rangle$$
(6.4)

#### TABLE 6.7

Energy matrices for the  $\Gamma_7$  (doublets),  $\Gamma_8$  (quartets) and  $\Gamma_6$  (doublet) states of NpF<sub>6</sub>. The quantities V, V' are the energies of the  $t_2$  and  $t_1$  levels respectively (see Table 6.5);  $\sqrt{k'}$ ,  $\sqrt{k}$  are factors by which the amplitude of the 5f wave-functions are reduced in the  $t_1$  and  $t_2$  orbitals respectively.  $t_1$  orbitals are  $\sigma$ -bonding,  $t_2$  orbitals  $\pi$ -bonding and the remaining  $a_2$  orbital is non-bonding (Eisenstein and Pryce 1960)

whose g-value is

$$g = 2\cos^2\theta - 4(k/3)^{\frac{1}{2}}\sin 2\theta - \frac{2}{3}(1-k)\sin^2\theta \tag{6.5}$$

if an orbital reduction factor  $k^{\frac{1}{2}}$  is assumed to allow for any  $\pi$ -bonding in the  $t_2$  orbitals (note that the nomenclature used by Eisenstein and Pryce differs from that normally used for bonding effects with d-orbitals). Equation (6.5) with k=1 is equivalent to eqn (6.3), as can be verified by noting that in the limit of a weak crystal field the two  $\Gamma_7$  states belonging to  $J=\frac{5}{2}$  and  $J=\frac{7}{2}$  have the form

$$J = \frac{7}{2}, \Gamma_7; \qquad \sqrt{\left(\frac{4}{7}\right)} |A\rangle + \sqrt{\left(\frac{3}{7}\right)} |B\rangle$$

$$J = \frac{5}{2}, \Gamma_7; \qquad \sqrt{\left(\frac{3}{7}\right)} |A\rangle - \sqrt{\left(\frac{4}{7}\right)} |B\rangle$$

$$(6.6)$$

and

$$\cos \theta = \sqrt{\left(\frac{3}{7}\right)} \cos \alpha + \sqrt{\left(\frac{4}{7}\right)} \sin \alpha$$

$$\sin \theta = \sqrt{\left(\frac{4}{7}\right)} \cos \alpha - \sqrt{\left(\frac{3}{7}\right)} \sin \alpha.$$
(6.6a)

In their analysis for NpF<sub>6</sub> Eisenstein and Pryce attempted to fit the g-value, the optical spectrum, and the temperature-independent term in the magnetic susceptibility. With k=1 (and also no orbital reduction factor for the  $t_1$  orbitals), they adopted the values

$$\zeta = 2405 \text{ cm}^{-1}, \qquad V = 5442 \text{ cm}^{-1}, \qquad V' = 22220 \text{ cm}^{-1},$$

which give a temperature-independent susceptibility term of  $158 \times 10^{-6}$  per mol, in good agreement with the value of  $165 \times 10^{-6}$  obtained by Hutchison, Tsang, and Weinstock (1962) for NpF<sub>6</sub> in UF<sub>6</sub>, extrapolated

to infinite dilution. These parameters are considerably larger than those for the single f-electron of Pa<sup>4+</sup> in Cs<sub>2</sub>ZrCl<sub>5</sub> of

$$\zeta = 1490 \text{ cm}^{-1}, \qquad V = 1496 \text{ cm}^{-1}, \qquad V' = 3656 \text{ cm}^{-1}$$

(see Raubenheimer, Boesman, and Stapleton 1965). The corresponding crystal field parameters are

$$\begin{array}{ll} {\rm Pa^{4+}\; in\; Cs_2 Zr Cl_6} & A_4 \langle r^4 \rangle = 888\; {\rm cm^{-1}}, & A_6 \langle r^6 \rangle = 41.9\; {\rm cm^{-1}} \\ {\rm Np^{6+}\; in\; UF_6} & A_4 \langle r^4 \rangle = 5700\; {\rm cm^{-1}}, & A_6 \langle r^6 \rangle = 320\; {\rm cm^{-1}}, \end{array}$$

which showed a marked rise with increasing valency. Both parameters are positive, as would be expected for an octahedral configuration.

The magnetic hyperfine constant is

$$A = -2g_{\rm n}\beta\beta_{\rm n}\langle r^{-3}\rangle\left\{\frac{4}{\sqrt{3}}\sin 2\theta + \frac{1}{3}\kappa(1+2\cos 2\theta)\right\}$$
 (6.7)

and it can be verified by using eqns (6.6a) that this is identical with eqn (6.3a). The value of  $2\theta$  required to fit the experimental g-value of NpF<sub>6</sub> is 63°, for which

$$A = -2g_{\rm n}\beta\beta_{\rm n}\langle r^{-3}\rangle(2\cdot06 + 0\cdot63\kappa). \tag{6.7a}$$

### 6.5. Neptunyl and plutonyl ions

As mentioned in § 6.1, ions of the type (UO<sub>2</sub>)<sup>2+</sup> are linear. Uranyl salts have only a temperature-independent susceptibility, suggesting that there are no unpaired electrons. The free uranium atom has a closed radon core, and six valence electrons, two of which have been lost in (UO<sub>2</sub>)<sup>2+</sup>. Eisenstein and Pryce (1955) postulate that the remaining electrons are used to form covalent bonds with the oxygen atoms. In a simple model which uses only  $\sigma$ -bonding, the six valence electrons of each oxygen are assumed to be in non-bonding orbitals formed from the 2s, 2p states on the oxygens, which are thus filled. For the central uranium ion, bonding orbitals of  $\sigma$ -type can be formed from 5f, 6d, 7s(and perhaps 7p) using the substates with  $m_i = 0$ ; taking the O—U—O axis as the z-axis, this means the f,  $\delta_1$  orbital in Table 6.5, whose wavefunction  $z(5z^2-3r^2)$  is strongly directed along the z-axis, and similarly the  $(3z^2-r^2)$  orbital belonging to the d-states. The two orbitals forming a bond with the inwardly directed  $sp_z$  orbital on each oxygen can accommodate just four electrons, which are assumed to be those originally associated with the uranium. The situation is therefore different from any considered hitherto for compounds of ions containing f-electrons, in that we have strong bonding involving f-orbitals. In many respects it resembles a linear molecule (cf. Elliott 1953).

We now consider such 'actinyl' groups in the hexagonal compound (UO<sub>2</sub>)Rb(NO<sub>3</sub>)<sub>3</sub>, where the axes of all the groups are parallel to the unique crystal axis.

$$({\rm UO_2})^{2+} \ in \ ({\rm UO_2}) {\rm Rb} \, ({\rm NO_3})_3$$

A strong nuclear electric quadrupole interaction with axial symmetry would be expected in this compound; this has been verified by measurements of anisotropy in the α-particle emission from <sup>233</sup>U, <sup>235</sup>U at temperatures below 1°K by Dabbs, Roberts, and Parker (1958).

$$(\mathrm{NpO_2})^{2+}$$
 in  $(\mathrm{UO_2})\mathrm{Rb}(\mathrm{NO_3})_3$ 

The paramagnetic resonance spectrum of this compound has been measured by Bleaney, Llewellyn, Pryce, and Hall (1954b). As would be expected from the foregoing discussion, it shows strong axial symmetry, and the results have been fitted to a spin Hamiltonian with effective spin  $S = \frac{1}{2}$  of the usual form with axial symmetry. The measured parameters are (see Pryce 1959)

$$\begin{split} g_{\parallel} &= -3.405(8), & |g_{\perp}| &= 0.205(6); \\ A_{\parallel} &= -0.16547(5) \text{ cm}^{-1}, & |A_{\perp}| &= 0.01782(3) \text{ cm}^{-1}; \\ P_{\parallel} &= +0.03015(5) \text{ cm}^{-1}. \end{split}$$

The hyperfine parameters are for the isotope <sup>237</sup>Np, whose nuclear spin is  $I=\frac{5}{2}$ . The actual signs are not determined experimentally, except for the result that  $A_{\parallel}$ ,  $P_{\parallel}$  are of opposite sign. Pryce (1959) has pointed out that theoretically  $g_{\parallel}$  is negative (see below), and that  $A_{\parallel}$  should be negative provided that the nuclear magnetic moment is (as expected) positive. A negative value of  $A_{\parallel}$  and a positive value of  $P_{\parallel}$  have been confirmed by Hanauer, Dabbs, Roberts, and Parker (1961) from the temperature dependence of the anisotropy in the emission of  $\alpha$ -particles in a nuclear alignment experiment.

The concentration of negative charge along the axis of the O—U—O complex means that any additional electron such as the extra electron in the neptunyl ion will be strongly repelled, and in the ground state it would occupy an orbital whose charge distribution is concentrated in the equatorial plane. The 5f orbitals will therefore have very different energy, according to their charge distribution. The states for which  $l_z=\pm 3$  will be lowest, followed by the  $l_z=\pm 2$ ,  $\pm 1$  and the antibonding  $l_z=0$  states in that order. The excited states are estimated by Eisenstein and Pryce to lie at  $20000-30000~\rm cm^{-1}$ , as also are the 6d

states. These splittings are large compared with the spin-orbit coupling, of order 2000 cm<sup>-1</sup>, so that in a first approximation we retain only the diagonal term  $\zeta l_z s_z$ . Since  $\zeta$  is positive for a single electron, the four states with  $l_z=\pm 3$ ,  $s_z=\pm \frac{1}{2}$  are split into two doublets, the lower doublet being that for which the axial components of orbit and spin are anti-parallel. These two states can be denoted by  $j_z=l_z+s_z=|\pm \frac{5}{2}\rangle$ , the other two states  $|j_z=\pm \frac{7}{2}\rangle$  lying higher in energy by  $3\zeta$ . Use of the Zeeman operator  $l_z+2s_z$  shows that in this approximation the ground doublet would have  $g_{\parallel}=4$ ,  $g_{\perp}=0$ . Such a doublet would not show a magnetic resonance line, though the similarity of the g-values to the experimental results suggests that the basic theory is correct.

We now allow for the effect of the crystal field due to other constituents, mainly the three nitrate ions symmetrically disposed in the equatorial plane. These produce a crystal field of threefold symmetry about the z-axis, of whose components  $V_6^6$  is of vital importance because it admixes states with  $l_z=\pm 3$ . The ground doublet is then no longer exactly characterized by  $j_z=\pm \frac{5}{2}$ , but contains an admixture of the  $j_z=\pm \frac{7}{2}$  states, making a resonance transition allowed. In addition off-diagonal elements of the spin-orbit coupling will admix some of the  $l_z=\pm 2$ ,  $j_z=\pm \frac{5}{2}$  states, but since the spin-orbit coupling is rather weak relative to the splitting due to the axial charge concentration, their amplitude is rather small. These two perturbations can be taken into account by writing the wave-functions of the ground doublet in terms of  $|l_z,s_z\rangle$  as

$$|+\rangle = (1+p^2+q^2)^{-\frac{1}{2}}(|-3, +\frac{1}{2}\rangle + p |+3, +\frac{1}{2}\rangle - q |-2, -\frac{1}{2}\rangle) |-\rangle = (1+p^2+q^2)^{-\frac{1}{2}}(|+3, -\frac{1}{2}\rangle + p |-3, -\frac{1}{2}\rangle - q |+2, +\frac{1}{2}\rangle)$$
(6.8)

where q is about 0.1. Use of the Zeeman operator  $l_z+2s_z$  then gives

$$\begin{split} g_{\parallel} &= -(4-8p^2+6q^2)(1+p^2+q^2)^{-1} \\ g_{\perp} &= |p\{4-2\sqrt{(6)q}\}(1+p^2+q^2)^{-1}|. \end{split} \tag{6.9}$$

If q is fixed (and its effect is anyway rather small), the values of  $g_{\parallel}$  and  $g_{\perp}$  are related, depending only on the parameter p, and it is not possible to fit the experimental values of  $g_{\parallel}$  and  $g_{\perp}$  simultaneously. If  $g_{\perp}$  is fitted, the value of  $g_{\parallel}$  given by these equations is very close to -4; since most of the contribution to  $g_{\parallel}$  comes from the orbit, this suggests that an orbital reduction factor k should be introduced to allow for some covalent bonding. The g-values then become

$$\begin{split} g_{\parallel} &= -\{k(6-6p^2+4q^2)-2(2+2p^2-2q^2)\}(1+p^2+q^2)^{-1} \\ g_{\perp} &= |p\{4-2\sqrt{(6)kq}\}(1+p^2+q^2)^{-1}|. \end{split} \tag{6.10}$$

The experimental g-values can then be fitted using k = 0.9, p = 0.056.

In eqns (6.8) we have chosen as the  $|+\rangle$  state that which gives a 1:1 correspondence between the matrix elements of the effective spin and of the true angular momentum 1+s. This makes the sign of  $g_{\parallel}$  negative because it is dominated by the contribution from the orbit, which is opposed to the smaller spin contribution.

The theoretical expressions for the hyperfine constants given by Eisenstein and Pryce (1955) are

$$\begin{split} A_{\parallel} &= -2g_{\rm n}\beta\beta_{\rm n}\langle r^{-3}\rangle \\ &\qquad \qquad \{\frac{2.0}{3} + \frac{2}{3}\sqrt{(6)}q - \frac{1.6}{3}p^2 + 4q^2 + \kappa(1+p^2-q^2)\}(1+p^2+q^2)^{-1} \\ A_{\perp} &= |2g_{\rm n}\beta\beta_{\rm n}\langle r^{-3}\rangle p\{\frac{2}{3} - \frac{5}{3}\sqrt{(6)}q - 2\kappa\}(1+p^2+q^2)^{-1}| \end{split} \tag{6.11}$$

(here the sign of  $A_{\parallel}$  has been reversed from the original, like that of  $g_{\parallel}$ ; see Pryce (1959)). With the value q=0.1 assumed above, and the value p=0.056 required to fit the g-values, these equations can be solved for  $\kappa$  and  $g_{\rm n}\langle r^{-3}\rangle$  using the measured hyperfine constants. To fit the anisotropy, a value of about -3.1 is required for  $\kappa$ , which then gives

$$g_{\rm n}\langle r^{-3}\rangle = +13.7.$$

(An alternative solution,  $\kappa = +120$ , which leads to  $g_n \langle r^{-3} \rangle = 0.4$  seems quite inadmissible.) Even if we take a value of  $g_n = +2$ , which corresponds to a magnetic moment close to the Schmidt limit of 4.8 n.m. for  $I = \frac{5}{2}$ , this result seems to give rather a large value for  $\langle r^{-3} \rangle$ , especially if some bonding occurs, which would tend to reduce  $\langle r^{-3} \rangle$ .

This result may be compared with that for NpF<sub>6</sub>, where a similar value of  $g_n\langle r^{-3}\rangle$  might be expected. If the measured value of A is there taken to be negative, eqn (6.7a) yields the result

$$g_{\rm n}\langle r^{-3}\rangle = 21/(2\cdot06+0\cdot63\kappa),$$

which would be equal to 13.7 if we take  $\kappa = -0.75$ . Thus to reconcile this result with that for the neptunyl ion we again need a negative value of  $\kappa$ , though not so large.

In the 3d and 4f groups the observed values of  $\kappa$  are positive but we shall not hazard a prediction of its sign for the 5f group. The theory given above for the magnetic hyperfine constants of the neptunyl ion should be accepted with reserve, since Eisenstein and Pryce (1956) have pointed out that the crystal field may contain a  $V_3^3$  term, which could admix s-states directly to the 5f,  $l_z=\pm 3$  states. This would affect the hyperfine structure in a way that cannot be represented by the addition of a  $\kappa$ -term, and the formulae (6.11) would no longer be correct.

The theory of Eisenstein and Pryce (1955) gives a nuclear electric quadrupole interaction

$$P_{\parallel} = -\frac{e^2 Q}{I(2I-1)} \{ (\frac{4}{5}\alpha - \frac{1}{2}) \langle r^{-3} \rangle_{5f} + \frac{6}{7}\alpha' \langle r^{-3} \rangle_{6d} \}, \tag{6.12}$$

where  $\alpha$  and  $\alpha'$  are the probabilities that the bonding electrons are in the neptunyl 5f and 6d states rather than on the oxygen atoms. Since  $\langle r^{-3}\rangle_{6d}$  is expected to be considerably larger than  $\langle r^{-3}\rangle_{5f}$ , this leads to a negative nuclear quadrupole moment for a positive value of  $P_{\parallel}$ , the field gradient at the nucleus being due mainly to the  $\sigma$ -bonding electrons. If there were an appreciable amount of  $\pi$ -bonding, this would give a contribution to  $P_{\parallel}$  of opposite sign. Shielding corrections, and lattice contributions have been omitted from (6.12).

$$(PuO_2)^{2+}$$
 in  $(UO_2)Rb(NO_3)_3$ 

Paramagnetic resonance has been observed for this ion by Bleaney, Llewellyn, Pryce, and Hall (1954a). The line shape is asymmetrical, as expected for resonance due to a non-Kramers ion; the spin Hamiltonian parameters for a non-Kramers doublet with an effective spin of  $\frac{1}{2}$  are found to be

$$\begin{split} |g_{\parallel}| &= 5 \cdot 32(2) \\ \text{Isotope } 239(I = \frac{1}{2}) & |A_{\parallel}| = 0 \cdot 0862(5) \text{ cm}^{-1} \\ \text{Isotope } 241(I = \frac{5}{2}) & |A_{\parallel}| = 0 \cdot 0609(4) \text{ cm}^{-1}. \end{split}$$

The theory is discussed by Eisenstein and Pryce (1956). Exchange interaction between the two electrons outside the (UO<sub>2</sub>)<sup>2+</sup> type core is assumed to be strong, so that they will have parallel spin and hence must occupy different orbital states. There are then two possibilities: (a) they occupy the states  $l_z = +3$  and  $l_z = -3$ , giving a resultant  $L_z = 0$  and  $S_z = \pm 1$ ; (b) they occupy the states  $l_z = \pm 3$ ,  $l_z = \pm 2$ with parallel spin, giving four possible states  $L_z = \pm 5$ ,  $S_z = \pm 1$ , which are then split by spin-orbit coupling into two doublets, that where the spin and orbital components are anti-parallel having the lower energy. Because of the Coulomb repulsion between the two electrons, the ground state is (b), where the electrons occupy orbitals with different values of  $l_z$ . The values of  $L_z+2S_z$  for the two states of the ground doublet are then  $\pm (5-2)$ , so that we have  $g_{\parallel} = 6$ ,  $g_{\perp} = 0$ . In the next approximation, the crystal field and spin-orbit coupling will admix some other states into the ground state wave-functions, but the amplitude of the admixtures is expected to be small and does not seriously change the value of  $g_{\parallel}$ , while  $g_{\perp}$  remains zero. To explain the

observed value of  $g_{\parallel}$  it seems necessary to introduce a reduction factor k for the orbital moment, which then gives

$$|g_{\parallel}| = 2(5k-2), \qquad g_{\perp} = 0.$$
 (6.13)

The measured value of  $g_{\parallel}=5.32$  can then be fitted with a value of k of about 0.93, which is not inconsistent with that needed to fit the corresponding neptunyl compound. Magnetic resonance has been observed by Hutchison and Lewis (1954) in another plutonyl compound, plutonyl sodium acetate; they find  $g_{\parallel}=5.92$ , which would require a value of k=0.99.

The magnetic hyperfine interaction arises from the 5f electrons, together with a contribution of s-like character due to core polarization and the fact that the  $V_3^3$  term in the crystal field can directly admix s-states into the  $l_z=\pm 3$  states. The two mechanisms cannot be differentiated in the present case, so that both can be combined in a single term, giving

$$A_{\parallel} = 2g_{\rm n}\beta\beta_{\rm n}\langle r^{-3}\rangle(\frac{32}{3} + 2\kappa) \tag{6.14}$$

with, of course,  $A_{\perp}=0$ . Using the value of -0.200(4) n.m. (Faust, Marrus, and Nierenberg 1965) for the nuclear moment of <sup>239</sup>Pu, a value of about 6 a.u. is found for  $\langle r^{-3} \rangle$  if  $\kappa=0$ , and a rather smaller value if  $\kappa$  is positive. A strong nuclear electric quadrupole interaction may be expected for isotopes with  $I>\frac{1}{2}$ , but cannot be observed in an electron spin resonance experiment. An f-electron with  $l_z=\pm 2$  has

$$3l_z^2 - l(l+1) = 0,$$

so the extra electron in the plutonyl ion gives no contribution to the nuclear electric quadrupole interaction, and eqn (6.12) should apply also to the plutonyl ion.