

# Online Computation and Competitive Analysis

---

Allan Borodin

*University of Toronto*

Ran El-Yaniv

*Technion – Israel Institute of Technology*



**CAMBRIDGE**  
UNIVERSITY PRESS

# Contents

---

|  |                  |
|--|------------------|
| <i>Preface</i>   | <i>page xiii</i> |
| <b>1 Introduction to Competitive Analysis: The List Accessing Problem</b>                    | <b>1</b>         |
| 1.1 Basic Ideas and Terminology  | 1                |
| 1.2 The List Accessing Problem   | 4                |
| 1.3 The Sleator–Tarjan Result  | 6                |
| 1.4 The Potential Function Method  | 9                |
| 1.5 Some Lower Bounds  | 10               |
| 1.6 The List Factoring Technique   | 13               |
| 1.7 Historical Notes and Open Questions  | 19               |
| <b>2 Introduction to Randomized Algorithms: The List Accessing Problem</b>                   | <b>23</b>        |
| 2.1 The Competitive Ratio of Randomized Algorithms   | 23               |
| 2.2 Algorithm BIT  | 24               |
| 2.3 Algorithm RMTF: Barely Random Versus Random  | 26               |
| 2.4 List Factoring–Phase Partitioning Revisited  | 27               |
| 2.5 COMB: An $\frac{8}{5}$ -Competitive Algorithm  | 29               |
| 2.6 Historical Notes and Open Questions  | 29               |
| <b>3 Paging: Deterministic Algorithms</b>  | <b>32</b>        |
| 3.1 Some Paging Algorithms   | 33               |
| 3.2 The $(h, k)$ -Paging Problem   | 34               |
| 3.3 List Accessing Algorithms as Paging Algorithms   | 35               |
| 3.4 LFD – An Optimal Offline Paging Algorithm  | 35               |
| 3.5 Marking and Conservative Algorithms and the Competitiveness of LRU, CLOCK, FIFO, and FWF | 36               |
| 3.6 LIFO and LFU Are Not Competitive   | 39               |
| 3.7 The Full Access Cost Model   | 40               |

|          |  |            |
|----------|--|------------|
| 3.8      | Theory Versus Practice   | 41         |
| 3.9      | Historical Notes and Open Questions                                | 42         |
| <b>4</b> | <b>Paging: Randomized Algorithms</b>                               | <b>44</b>  |
| 4.1      | Randomized Competitive Analysis                                    | 44         |
| 4.2      | The Competitiveness of RANDOM                                      | 46         |
| 4.3      | The MARK Algorithm   | 49         |
| 4.4      | A Lower Bound for Randomized Paging Algorithms                     | 51         |
| 4.5      | Historical Notes and Open Questions                                | 52         |
| <b>5</b> | <b>Alternative Paging Models: Beyond Pure Competitive Analysis</b> | <b>54</b>  |
| 5.1      | The Access Graph Model   | 54         |
| 5.2      | Dynamic Access Graphs and Experimental Studies                     | 65         |
| 5.3      | Distributional Paging Models                                       | 68         |
| 5.4      | Historical Notes and Open Questions                                | 75         |
| <b>6</b> | <b>Game Theoretic Foundations</b>                                  | <b>78</b>  |
| 6.1      | Games in Extensive and Strategic Forms                             | 78         |
| 6.2      | Randomized Strategies: Mixed, Behavioral, and General              | 83         |
| 6.3      | Equivalence Theorems for Linear Games and Games of Perfect Recall  | 89         |
| 6.4      | An Application to Paging and Competitive Analysis                  | 93         |
| 6.5      | Historical Notes and Open Questions                                | 95         |
| <b>7</b> | <b>Request–Answer Games</b>  | <b>98</b>  |
| 7.1      | Request–Answer Games   | 98         |
| 7.2      | Randomized Adversaries   | 102        |
| 7.3      | Relating the Adversaries   | 104        |
| 7.4      | Historical Notes and Open Questions                                | 107        |
| <b>8</b> | <b>Competitive Analysis and Zero-Sum Games</b>                     | <b>109</b> |
| 8.1      | Two-Person Zero-Sum Games  | 109        |
| 8.2      | On Generalizations of the Minimax Theorem for Infinite Games       | 114        |
| 8.3      | Yao’s Principle: A Technique for Obtaining Lower Bounds            | 115        |
| 8.4      | Paging Revisited   | 120        |
| 8.5      | Historical Notes   | 122        |
| <b>9</b> | <b>Metrical Task Systems</b>                                       | <b>123</b> |
| 9.1      | Formulation of (Metrical) Task Systems                             | 123        |
| 9.2      | An $8(N - 1)$ -Competitive Traversal Algorithm                     | 127        |
| 9.3      | A $2N - 1$ Lower Bound   | 128        |

|           |   |            |
|-----------|---|------------|
| 9.4       | An Optimal Work Function MTS Algorithm  | 131        |
| 9.5       | A Randomized Algorithm for a Uniform MTS  | 134        |
| 9.6       | A Randomized Polylogarithmic Competitive Algorithm for Any MTS                              | 135        |
| 9.7       | Historical Notes and Open Questions   | 146        |
| <b>10</b> | <b>The <math>k</math>-Server Problem</b>  | <b>150</b> |
| 10.1      | The Formulation of the Model  | 150        |
| 10.2      | Some Basic Aspects of the $k$ -Server Problem   | 151        |
| 10.3      | A Deterministic Lower Bound   | 153        |
| 10.4      | $k$ -Servers on a Line and a Tree   | 155        |
| 10.5      | An Efficient 3-Competitive 2-Server Algorithm for Euclidean Spaces                          | 159        |
| 10.6      | Balancing Algorithms  | 161        |
| 10.7      | The $k$ -Server Work Function Algorithm   | 164        |
| 10.8      | On Generalizations of the $k$ -Server Conjecture That Fail                                  | 175        |
| 10.9      | Historical Notes and Open Questions   | 178        |
| <b>11</b> | <b>Randomized <math>k</math>-Server Algorithms</b>  | <b>182</b> |
| 11.1      | Oblivious Adversaries and Two Randomized $k$ -Server Algorithms for the Circle              | 182        |
| 11.2      | A Lower Bound Against an Adaptive-Online Adversary  | 185        |
| 11.3      | The Cat and Rat Game and Applications to Randomized $k$ -Server Algorithms                  | 186        |
| 11.4      | The Harmonic Random Walk  | 191        |
| 11.5      | The HARMONIC $k$ -Server Algorithm on an Arbitrary Metric Space                             | 192        |
| 11.6      | The Resistive Approach  | 196        |
| 11.7      | Historical Notes and Open Questions   | 199        |
| <b>12</b> | <b>Load Balancing</b>   | <b>201</b> |
| 12.1      | Defining the Problem  | 201        |
| 12.2      | Online Algorithms for Load Balancing of Permanent Jobs                                      | 204        |
| 12.3      | Formulating the Machine Assignment Problem as a Generalized Virtual Circuit Routing Problem | 210        |
| 12.4      | Load Balancing of Temporary Jobs  | 213        |
| 12.5      | Bin Packing   | 218        |
| 12.6      | Historical Notes and Open Questions   | 222        |
| <b>13</b> | <b>Call Admission and Circuit Routing</b>   | <b>226</b> |
| 13.1      | Specifying the Problem  | 226        |

|           |  |            |
|-----------|--|------------|
| 13.2      | Throughput Maximization for Permanent Calls in Networks with Large Edge Capacities | 227        |
| 13.3      | Throughput Maximization for Limited Duration Calls                                 | 232        |
| 13.4      | Experimental Results   | 234        |
| 13.5      | Call Admission for Particular Networks: The Disjoint Paths Problem                 | 237        |
| 13.6      | The Disjoint Paths Problem: A Lower Bound for a Difficult Network                  | 245        |
| 13.7      | Routing on Optical Networks  | 250        |
| 13.8      | Path Coloring for Particular Networks  | 253        |
| 13.9      | A Lower Bound for Path Coloring on the Brick Wall Graph                            | 259        |
| 13.10     | Historical Notes and Open Problems   | 260        |
| <b>14</b> | <b>Search, Trading, and Portfolio Selection</b>                                    | <b>264</b> |
| 14.1      | Online Search and One-Way Trading  | 264        |
| 14.2      | Online Portfolio Selection   | 273        |
| 14.3      | Two-Way Trading: Statistical Adversaries and “Money Making” Algorithms             | 277        |
| 14.4      | Two-Way Trading and the Fixed Fluctuation Model                                    | 281        |
| 14.5      | Weighted Portfolio Selection Algorithms  | 290        |
| 14.6      | Historical Notes and Open Questions  | 307        |
| <b>15</b> | <b>On Decision Theories and the Competitive Ratio</b>                              | <b>312</b> |
| 15.1      | Certainty, Risk, and Strict Uncertainty  | 312        |
| 15.2      | Decision Making Under Strict Uncertainty   | 315        |
| 15.3      | The Competitive Ratio Axioms   | 321        |
| 15.4      | Characterization of the Competitive Ratio  | 325        |
| 15.5      | Characterizations of the Classical Criteria for Strict Uncertainty                 | 333        |
| 15.6      | An Example – The Leasing Problem   | 335        |
| 15.7      | Decision Making Under Risk   | 339        |
| 15.8      | Bayesian Approaches for Decision Making Under Uncertainty                          | 346        |
| 15.9      | Historical Notes and Open Questions  | 348        |
| <b>A</b>  | <b>Glossary</b>  | <b>355</b> |
| <b>B</b>  | <b>Stochastic Analyses for List Accessing Algorithms</b>                           | <b>357</b> |
| <b>C</b>  | <b>The Harmonic Random Walk and Its Connection to Electrical Networks</b>          | <b>361</b> |

|          |  |     |
|----------|--|-----|
| <b>D</b> | <b>Proof of Lemmas 5.4 and 5.5 in Theorem 5.11: FAR Is a Uniformly Optimal Online Paging Algorithm</b> | 364 |
| D.1      | Proof of Lemma 5.4: <i>Type 1</i> Reps and the Construction of $T'$                                    | 364 |
| D.2      | Proof of Lemma 5.5: <i>Type 2</i> Reps and the Construction of $H$                                     | 365 |
| <b>E</b> | <b>Some Tools from Renewal Theory</b>  | 369 |
| E.1      | Renewal Processes  | 369 |
| E.2      | Wald's Equation  | 370 |
| E.3      | The Elementary Renewal Theorem   | 373 |
| <b>F</b> | <b>Proof of Theorem 13.14: Disjoint Paths in an Array</b>  | 375 |
| F.1      | Short Distance Calls   | 375 |
| F.2      | Long Distance Calls  | 376 |
| <b>G</b> | <b>Some Tools from the Theory of Types</b>   | 379 |
| <b>H</b> | <b>Two Technical Lemmas</b>  | 382 |
|          | <i>Bibliography</i>  | 389 |
|          | <i>Index</i>   | 403 |