① Exercise 1.3 a)

1.3 a) ~~show~~ $y(t) * w^T(t) * x(t) < 0$

Hint: $x(t)$ is misclassified by $w(t)$

∴ since $x(t)$ is misclassified, that means.

$$\text{sign}(w^T(t) \cdot x(t)) = -1 \quad \& \quad y(t) = +1$$

and $\text{sign}(w^T(t) \cdot x(t)) = +1 \quad \& \quad y(t) = -1$

for $\text{sign}(w^T(t) \cdot x(t))$ to be -1,

we can refer $\text{sign}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$

∴ $\text{sign}(x) = -1$ ∴ $x < 0$.

∴ $\text{sign}(w^T(t) \cdot x(t)) = -1$

which implies, $\boxed{w^T(t) \cdot x(t) < 0.}$

∴ As we already have $y(t) = +1$ for $\text{sign}(w^T(t) \cdot x(t)) = -1$

Then $y(t) * w^T(t) \cdot x(t) = (+1) * (\text{sum number less than } 0)$

$\Rightarrow (+1) * (m < 0)$ where m = some number $< zero$.

∴ $(+1) * (m < 0) \Rightarrow$ output < 0.

$\boxed{\therefore \text{Hence proved, } y(t)\, w^T(t)\, x(t) < 0}$ same for the other case

Exercise 1.3 b)

(b) show that $y(t)\, w^T(t+1)\, x(t) > y(t)\, w^T(t)\, x(t)$

Hint : Use(1.3).

$\therefore$ It is said that with $w(t+1)$ our $x(t)$ will be correctly classified.

$\therefore$ $y(t) = +1$ then ~~sign~~ $\text{sign}\left(w^T(t+1)\, x(t)\right) = +1$

as it $(x(t))$ is correctly classified.

$\therefore$ for $\text{sign}\left(w^T(t+1)\, x(t)\right)$ to be $= +1$

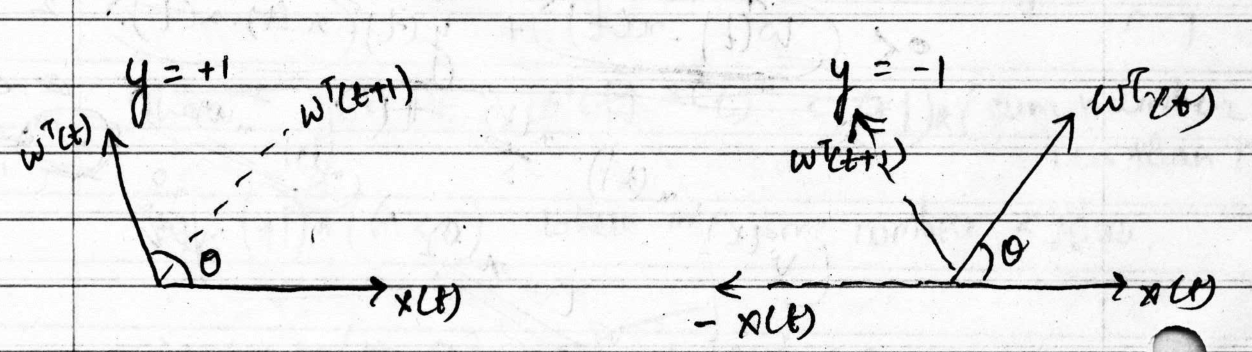
$w^T(t+1)\, x(t) > 0$. from $\text{sign}(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$

$\therefore$ As $y(t) = +1$ $\therefore$ $\boxed{y(t) * w^T(t+1) * x(t) > 0}$

As we have proved in 1.3(a) $y(t)\, w^T(t)\, x(t) < 0$

already which means,

$$\boxed{y(t) * w^T(t+1) * x(t) > y(t)\, w^T(t)\, x(t)}$$



Exercise
Problem 1.6

a) Recommending a book to a user in an online bookstore :-

→ Supervised learning → As we have to have labelled purchase history of user to recommend the book.
→ we know what user has purchased in past & hence for every feature set 'x' we have 'y' as output whether "purchased" or not.

∴ Training data set :-
book author, no. of pages, type of book like its genre, ebook or hardcopy, graphical content or not, etc.

b) Playing a tic-tac-toe :- (Reinforcement learning)

- We do not have output of a player's win|loss given few steps.
→ The model has to learn whether the performed set of steps lead to a winning output or losing one, hence reinforcement learning.

Training data set :- positions of values for 'x' or '0'
i.e first_box, second_box, third_box, Fourth_box, five_box, sixth_box, seventh_box,

eigth_box, ninth_box, haveWon = {0, 1} = 0 for winning and '1' for losing.

each box will have {'x', 'o', winningStep} where winning step = boolean i.e this step performed wining step or losing one.

c] Categorizing movies into different types:-

· It can be supervised or Unsupervised based on the data we have,

∴ Supervised → If we have attributes in dataset for which we have 'movie type' as output attribute for each input vector.

Unsupervised if, if do not have labelled output data for movie category then this becomes clustering problem which will be Unsupervised then.

dataset:-

→ movie_id, isAdult, isHorror, is Action, isRomantic, isDrama, isComedy, budget, actor, director, actress, length_of_movie, rating, language, runtime etc.

d) Learning to play music :-

Reinforcement Learning → As we have to learn by checking if output is proper music adhering to music notes or how.

Dataset :- Bandwidth, frequency, bass, contrast, audio attributes like Chroma, Tonnetz, roll off etc.

e) Credit limit : Deciding the maximum allowed debt for each bank customer :-

→ Supervised :- We have to have history of the customers the bank has & based on that only we can predict the future customer's debt.

dataset :- Age, Income, members in family, education, number of creditcards, creditscores, no. of loans, asset value etc.

Exercise 1.9

$\mu = 0.9$: We have to bound $p(\nu \leq 0.1)$.

$\therefore$ By Hoeffding inequality,

$$p\left[|\nu - \mu| > \varepsilon\right] \leq 2e^{-2\varepsilon^2 N} \quad \text{for any } \epsilon > 0$$

$\therefore$ $|\nu - \mu| > \varepsilon$

$\therefore$ Squaring 2 sides, $(\nu - \mu)^2 > (\varepsilon)^2$

$$(0.1 - 0.9)^2 > (\varepsilon)^2$$

$$\Rightarrow (0.8)^2 > (\varepsilon)^2$$

So we can say for $\leq 2e^{-2\varepsilon^2 N}$, $\varepsilon = 0.8$.

$$\Rightarrow p\left[|\nu - \mu| > \varepsilon\right] \leq 2e^{-2(0.8)^2(10)}$$

$$\leq (2)\, e^{-2(6.4)}$$

$$\leq 5.52 \times 10^{-6}$$

$$\Rightarrow p(\nu \leq 0.1) < p\left[|\nu - \mu| > \varepsilon\right] \leq 5.52 \times 10^{-6}$$

$$\Rightarrow 9.1 \times 10^{-9} \leq p\left[|\nu - \mu| > \varepsilon\right] \leq 5.52 \times 10^{-6}$$

Now, we have given this is, $x_2 = ax_1 + b$

where a = slope & b = intercept (x_2)

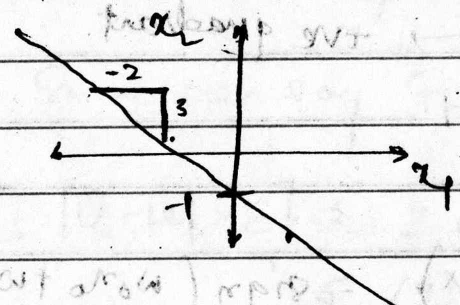
$\therefore x_2 = -w_1/w_2 \, x_1 + (-w_0 x_0 / w_2)$

Ans, part a)

$$\therefore \boxed{\begin{aligned} a &= -w_1/w_2 \\ b &= -w_0/w_2 \text{ as } x_0 = 1 \end{aligned}}$$

part (b) $\therefore$ 1) $w = [1, 2, 3]^T$

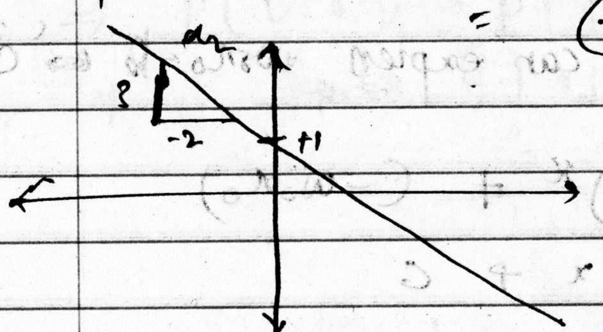
$\therefore$ equation $\rightarrow x_2 = (-2/3)\,x_1 + (-1)$



2) $w = [-1, -2, -3]^T$

$\therefore$ equation $\rightarrow x_2 = (-(-2)/-3)\,x_1 + (-(-1))$

$= (-2/3)\,x_1 + (+1)$



Problem 1.11 :-

a] Supermarket : $\therefore$ matrix $\Rightarrow$

$$\begin{array}{c|cc} & \multicolumn{2}{c}{f} \\ h & +1 & -1 \\ \hline +1 & 0 & 1 \\ -1 & 10 & 0 \end{array}$$

$$\therefore E_{in}(g) = \frac{1}{N} \sum_{n=1}^{N} e(h(x_n), f(x_n))$$

$$\therefore E_{in}(g) = \frac{1}{N} \sum_{n=1}^{N} [y_n \neq g(x_n)]$$

$$\boxed{E_{in}(g) = \frac{1}{N} \left[\left(\sum_{y_n=+1} (g(x_n) \neq +1) \right) \times 10 + \left(\sum_{y_n=-1} (g(x_n) \neq -1) \right) \times 1 \right]}$$

b] For CIA : matrix $\Rightarrow$

$$\begin{array}{c|cc} & \multicolumn{2}{c}{f} \\ h & +1 & -1 \\ \hline +1 & 0 & 1000 \\ -1 & 1 & 0 \end{array}$$

$$\therefore E_{in}(g) = \frac{1}{N} \sum_{n=1}^{N} [y_{(n)} \neq g(x_n)]$$

$$\boxed{E_{in}(g) = \frac{1}{N} \left[\left(\sum_{y_{(n)}=+1} (g(x_n) \neq +1) \right) \times 1 + \left(\sum_{y_{(n)}=-1} (g(x_n) \neq -1) \right) \times 1000 \right]}$$