Problem 8.2

$$X = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \qquad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

$(b^*, w^*) = ?$ & margin = ?

$\therefore$ ① — $(-1)\left(0(w_1) + 0(w_2) + b\right) \geq 1$

② $(-1)\left(0(w_1) + (-1)w_2 + b\right) \geq 1$

③ $(+1)\left(-2(w_1) + (0)(w_2) + b\right) \geq 1$

① $-b \geq 1$

② $w_2 - b \geq 1$

③ $-2w_1 + b \geq 1$

Add ① & ③ $\Rightarrow$ $-2w_1 \geq 2 \Rightarrow -w_1 \geq 1 \Rightarrow \boxed{w_1 \leq -1}$

from ①, ② $\Rightarrow$ $\cancel{w_2 \geq 1+b}$ $-b \geq 1 - w_2$

$\therefore 1 = 1 - w_2$

$\therefore w_2 = 0. \Rightarrow \boxed{w_2 \geq 0.}$

$\therefore$ Now, we have $\boxed{w_2 \geq 0}$, $\boxed{w_1 \leq -1}$ & $\boxed{-b \geq 1}$

Taking the minimum's, $w_2 = 0$, $w_1 = -1$ & $-b = 1 \Rightarrow b = -1$

$$\boxed{\begin{aligned} &\therefore b^* = -1 \\ &\& \ w^* = [-1, 0] \end{aligned}}$$

Margin $= \dfrac{1}{\|w^*\|} = \dfrac{1}{\sqrt{w_1^2 + w_2^2}} = \dfrac{1}{\sqrt{1+0}} = 1 \therefore \boxed{\text{margin} = 1}$

Problem 8.4

$$X = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 2 & 0 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

Dual problem:- $\min_{\alpha \in \mathbb{R}} \frac{1}{2}\alpha^T Q_D \alpha - 1_N^T \alpha$

subject to, $\xrightarrow{G} -A_D \alpha \geq 0_{N+2}$

$Q_D = X_s X_s^T$ $\quad X_s = \begin{bmatrix} - y_1 x_1^T - \\ - y_2 x_2^T - \\ \vdots \\ - y_N x_N^T - \end{bmatrix}$

$A_D = \begin{bmatrix} y^T \\ -y^T \\ I_{N \times N} \end{bmatrix}$

$P = -1_N = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

Now putting the values to get Q_D, A_D, P, G

$\therefore Q_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix}$ $\quad G = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad A_D = \begin{bmatrix} -1 & -1 & +1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$P = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

Using 'CVXOPT' library's QPsolver,

$$\alpha^* = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

- Attached in the HW submission one more .ipynb file for these calculations done to get α^* with CVXOPT's QPsolver.