# GATE 2022 BM.38

EE23BTECH11010 - VENKATESH BANDAWAR*

**Question:** An input $x(t)$ is applied to a system with a frequency transfer function given by $H(j\omega)$ as shown below. The magnitude and phase response of the transfer function are shown below. If $y(t_d) = 0$ for $x(t) = u(t)$, the time $t_d(> 0)$ is.
(Gate 2022 BM.38)

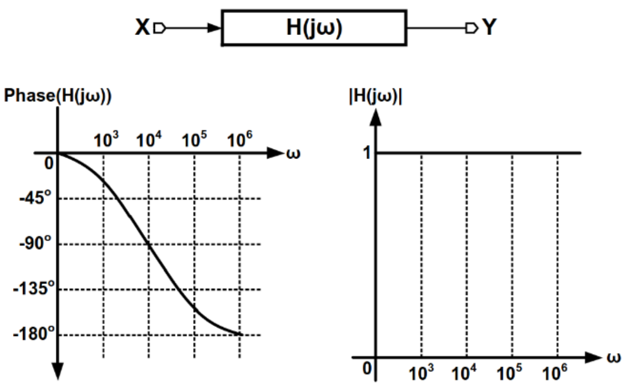



Fig. 1: Graph of y(t)

**Solution:**

| Parameter | Description |
|---|---|
| $x(t) = u(t)$ | Input signal |
| $y(t)$ | Output signal |
| $X(j\omega)$ | Fourier Transform of $x(t)$ |
| $Y(j\omega)$ | Fourier Transform of $y(t)$ |
| $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$ | Transfer function |

TABLE I: Input Parameters Table

from graph 2

$$\angle H(j\omega) = -2\tan^{-1}\left(\frac{\omega}{a}\right) \tag{1}$$

At $\omega = 10^4$, $\angle H(j\omega) = -\frac{\pi}{2}$

$$\implies a = 10^4 \tag{2}$$

$$\angle H(j\omega) = \tan^{-1}\left(\frac{-\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) \tag{3}$$

$$H(j\omega) = \frac{e^{j\tan^{-1}\left(\frac{-\omega}{a}\right)}}{e^{j\tan^{-1}\left(\frac{\omega}{a}\right)}} \tag{4}$$

$$= \frac{a - j\omega}{a + j\omega} \tag{5}$$

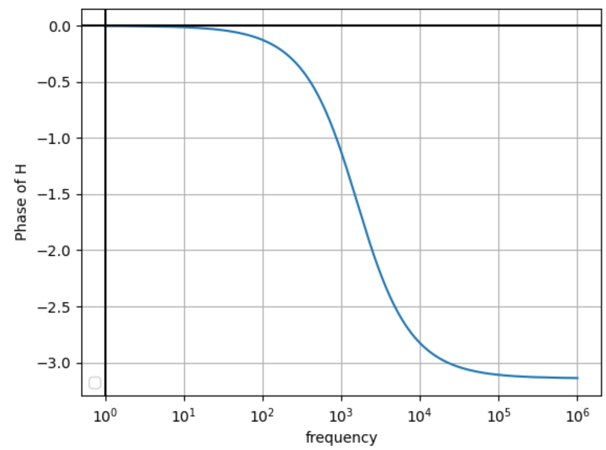



Fig. 2: Phase of H(f)

Substitute $j\omega = s$

$$u(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \tag{6}$$

$$Y(s) = \frac{1}{s}\frac{a-s}{a+s} \tag{7}$$

$$= \frac{1}{s} - \frac{2}{a+s} \tag{8}$$

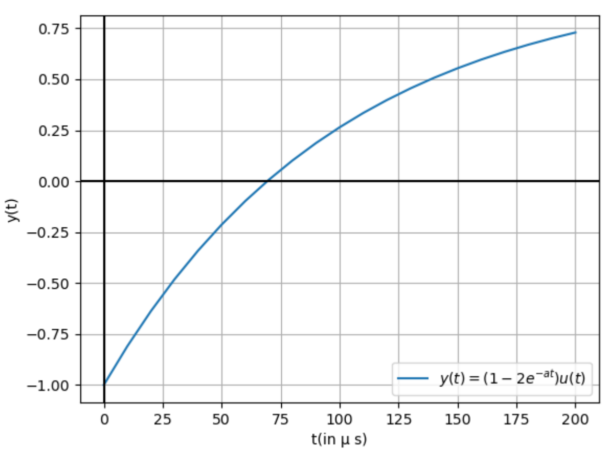
$$\frac{1}{s} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} u(t) \tag{9}$$

$$\frac{1}{a+s} \overset{\mathcal{L}^{-1}}{\longleftrightarrow} e^{-at}u(t) \tag{10}$$

$$y(t) = (1 - 2e^{-at})u(t) \tag{11}$$

$$\because y(t_d) = 0 \tag{12}$$

$$t_d = 100\ln 2\mu s \tag{13}$$


0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
y(t)
0
25
50
75
100
125
150
175
200
t(in μ s)
$y(t) = (1 - 2e^{-at})u(t)$