GATE 2022 IN.53

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Question: In a unity-gain feedback control system, the plant $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$ is controlled by a lag compensator $C(s) = \frac{s+10}{s+0.1}$ The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at $\omega = 3$rad/s is $_____$ (in integer) (GATE 2022 IN)

Solution:

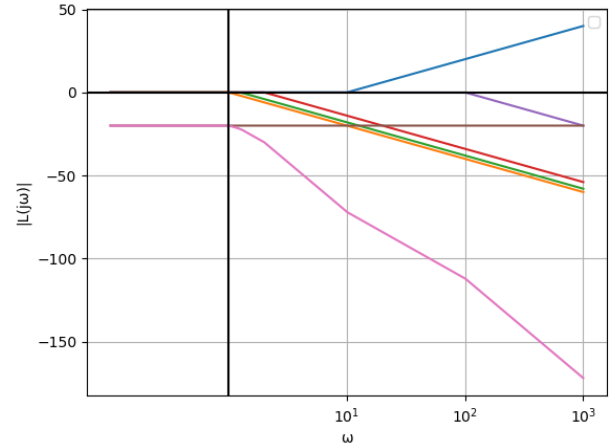| Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(2s+1)(0.01s+1)}$
$C(s)$	Lag Compensator	$\frac{s+10}{s+0.1}$
$L(s)$	Loop gain=$P(s)C(s)$	$\frac{0.1\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{0.5}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{0.1}+1\right)}$
ω	Angular Frequency	3rad/s

TABLE I: Given Parameters list

Fig. 1: Pink Line = Bode magnitude plot of loop gain

$$Gain(K) = \lim_{s \to 0} L(s) \tag{1}$$

Excluding s and $\frac{1}{s}$,

$$K = \lim_{s \to 0} \frac{0.1\left(\frac{s}{10}+1\right)}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{0.1}+1\right)} \tag{2}$$

$$= 0.1 \tag{3}$$

$$|L(s)| = 20\log_{10} K \tag{4}$$

$$= -20dB/decade \tag{5}$$

Here, $10, 0, 0.5, 100, 0.1$ are corner frequencies of loop gain L(s)

In a Bode magnitude plot,

Zero of L(s) = 10

$$|L(s)| = 20\log_{10}\omega \tag{6}$$

Poles of L(s) = 0, 0.1, 0.5, 100

$$|L(s)| = -20\log_{10}\omega \tag{7}$$

Slope of Bode magnitude plot (at $\omega = 3$) = -60 dB/decade