NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series , whose n^{th} term is : $n(n + 1)(n + 4)$.
Solution

Parameter	Description	Value
$x(n)$	n^{th} term of discrete signal	$n(n + 1)(n + 4)u(n)$

TABLE 0: Given parameters

from equation (**??**) to (**??**),

$$X(Z) = \frac{z^{-1}\left(1 + 4z^{-1} + z^{-2}\right)}{(1 - z^{-1})^4} + \frac{5z^{-1}\left(z^{-1} + 1\right)}{(1 - z^{-1})^3} + \frac{4z^{-1}}{(1 - z^{-1})^2} \tag{1}$$

$$Y(z) = X(z)U(z) \tag{2}$$

$$= \frac{z^{-1}\left(1 + 4z^{-1} + z^{-2}\right)}{(1 - z^{-1})^5} + \frac{5z^{-1}\left(z^{-1} + 1\right)}{(1 - z^{-1})^4} + \frac{4z^{-1}}{(1 - z^{-1})^3} \tag{3}$$

Using contour integration for inverse Z transformation,

$$y(n - 1) = \frac{1}{2\pi j} \oint_c Y(z)z^{n-2}dz \tag{4}$$

$$= \frac{1}{2\pi j} \oint_c \frac{\left(z^2 + 4z + 1\right)}{(z - 1)^5}z^n dz + \frac{1}{2\pi j} \oint_c \frac{5\left(z + 1\right)}{(z - 1)^4}z^n dz + \frac{1}{2\pi j} \oint_c \frac{4}{(z - 1)^3}z^n dz \tag{5}$$

$$\because R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z - a)^m f(z)\right) \tag{6}$$

$$R_1 = \frac{1}{4!} \lim_{z \to 1} \frac{d^4}{dz^4}\left((z - 1)^5 \frac{\left(z^2 + 4z + 1\right)z^n}{(z - 1)^5}\right) \tag{7}$$

$$= \frac{(n + 2)(n + 1)(n)(n - 1)}{4!} + \frac{4(n + 1)(n)(n - 1)(n - 2)}{4!} + \frac{n(n - 1)(n - 2)(n - 3)}{4!} \tag{8}$$

$$R_2 = \frac{1}{3!} \lim_{z \to 1} \frac{d^3}{dz^3}\left((z - 1)^4 \frac{5\left(z + 1\right)z^n}{(z - 1)^4}\right) \tag{9}$$

$$= \frac{5(n + 1)(n)(n - 1)}{3!} + \frac{5n(n - 1)(n - 2)}{3!} \tag{10}$$

$$R_3 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2}\left((z - 1)^3 \frac{4z^n}{(z - 1)^3}\right) \tag{11}$$

$$= \frac{4n(n - 1)}{2!} \tag{12}$$

$$\implies y(n) = R_1 + R_2 + R_3 \tag{13}$$

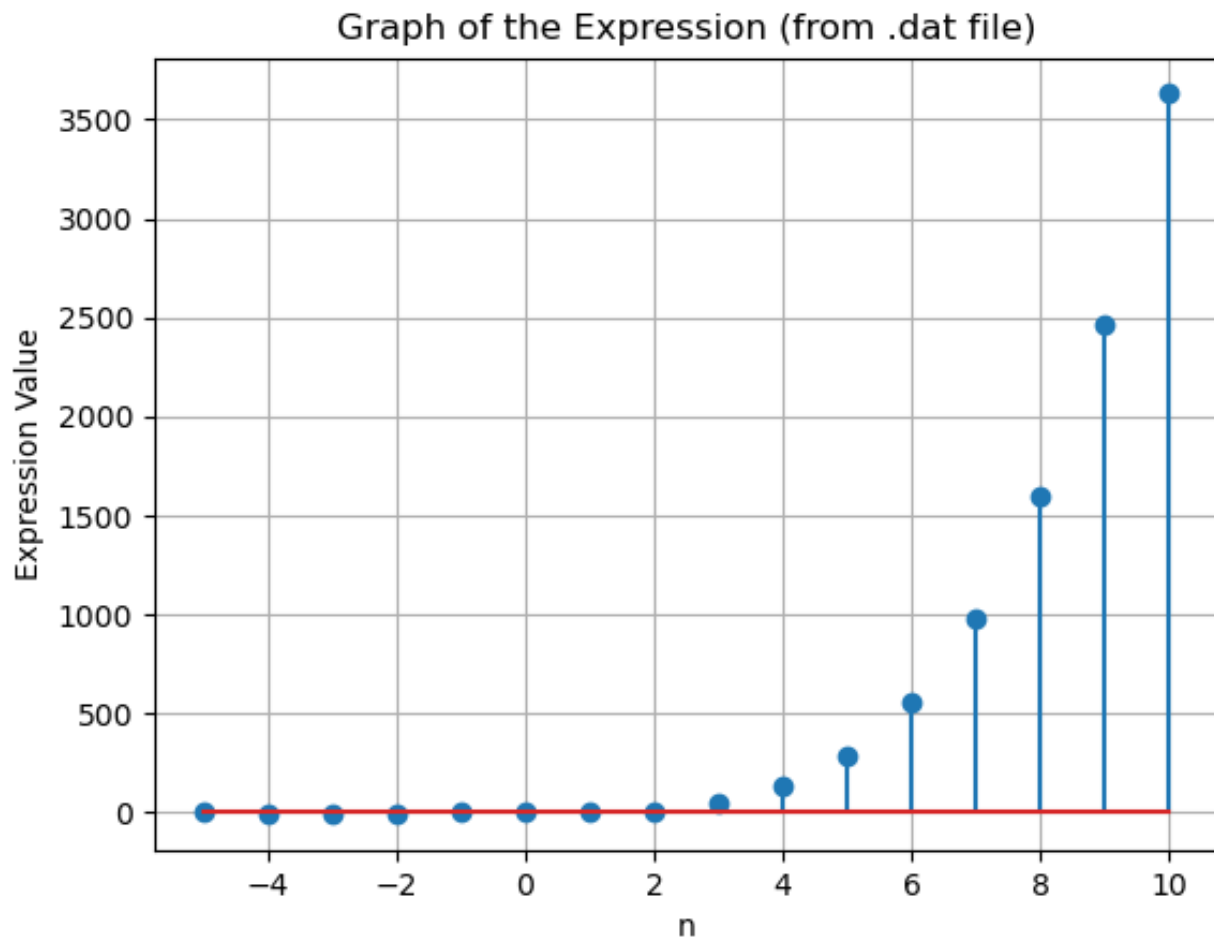$$= \frac{n^2(n - 1)^2}{4} + \frac{5n(n - 1)(2n - 1)}{6} + \frac{4n(n - 1)}{2} \tag{14}$$

Fig. 0: $y(n) = \frac{n^2(n-1)^2}{4} + \frac{5n(n-1)(2n-1)}{6} + \frac{4n(n-1)}{2}$