NCERT 10.5.3 10Q

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Question: Show that a_0 , a_1 , a_2 , . . ., a_n , . . . form an AP where an is defined as below :

1) $a_n = (3 + 4n)$
2) $a_n = (9 - 5n)$

Also find the sum of the first 15 terms in each case.

Answer:

(i) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \tag{1}$$
$$= (3 + 4(n + 1)) u(n + 1) - (3 + 4n)u(n)$$
$$= 4 \tag{2}$$

$\because$ Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'
Hence given sequence is in AP.

Parameter	Description	Value
a_n	General term of AP	$(3 + 4n)$
$x(n)$	Discrete signal	$(3 + 4n)u(n)$
a_0	first term	3
d	common differnce	4

TABLE (i): *Given parameters in* 1^{st} *AP*

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{3}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)\, z^{-n} \tag{4}$$

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} -z.U'(z) \tag{5}$$

$$\implies X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \tag{6}$$

$$\because y(n) = x(n) * u(n) \tag{7}$$

$$y(n) \overset{\mathcal{Z}}{\longleftrightarrow} Y(z)$$

$$\implies Y(z) = \left[\frac{3}{1 - z^{-1}} + \frac{4z^{-1}}{(1 - z^{-1})^2}\right].\frac{1}{1 - z^{-1}} \tag{8}$$

Using contour integeration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{9}$$

$$y(14) = \frac{1}{2\pi j}\int \frac{3.z^{15}}{(z - 1)^2}dz + \frac{1}{2\pi j}\int \frac{4.z^{15}}{(z - 1)^3}dz \tag{10}$$

$$\because R = \frac{1}{(m - 1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z - a)^m f(z)\right) \tag{11}$$

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z - 1)^2.\frac{3.z^{15}}{(z - 1)^2}\right) \tag{12}$$

$$R_1 = 45 \tag{13}$$

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z - 1)^3.\frac{4.z^{15}}{(z - 1)^3}\right) \tag{14}$$

$$R_2 = 420 \tag{15}$$

$$\implies y(14) = R_1 + R_2$$
$$= 465 \tag{16}$$

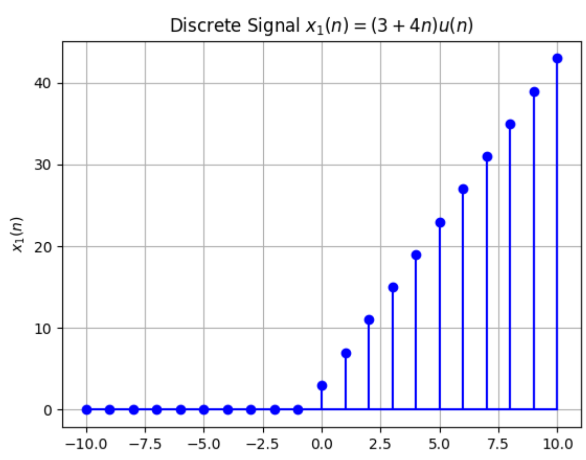

Fig. (i): $x_1(n) = (3 + 4n)u(n)$

(ii) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \tag{17}$$
$$= (9 - 5(n+1))\, u(n+1) - (9 - 5n)\, u(n)$$
$$= -5 \tag{18}$$

$\because$ Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'

Hence given sequence is in AP.

Parameter	Description	Value
a_n	General term of AP	$(9-5n)$
$x(n)$	Discrete signal	$(9-5n)u(n)$
a_0	first term	9
d	common differnce	-5

TABLE (ii): *Given parameters in 2^{st} AP*

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{19}$$
$$\implies X(z) = \sum_{n=-\infty}^{\infty} [9 - 5n].u(n)\, z^{-n} \tag{20}$$
$$nu(n) \xleftrightarrow{Z} -z.U'(z) \tag{21}$$
$$\implies X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \tag{22}$$
$$\because y(n) = x(n) * u(n) \tag{23}$$
$$y(n) \xleftrightarrow{Z} Y(z)$$
$$\implies Y(z) = \left[\frac{9}{1 - z^{-1}} - \frac{5z^{-1}}{(1 - z^{-1})^2}\right].\frac{1}{1 - z^{-1}} \tag{24}$$

Using contour integeration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j} \int Y(z) z^{13} dz \tag{25}$$
$$y(14) = \frac{1}{2\pi j} \int \frac{9.z^{15}}{(z-1)^2} dz - \frac{1}{2\pi j} \int \frac{5.z^{15}}{(z-1)^3} dz \tag{26}$$
$$\because R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \tag{27}$$
$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 . \frac{9.z^{15}}{(z-1)^2} \right) \tag{28}$$
$$R_1 = 135 \tag{29}$$
$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 . \frac{5.z^{15}}{(z-1)^3} \right) \tag{30}$$
$$R_2 = 525 \tag{31}$$
$$\implies y(14) = R_1 - R_2$$
$$= -390 \tag{32}$$

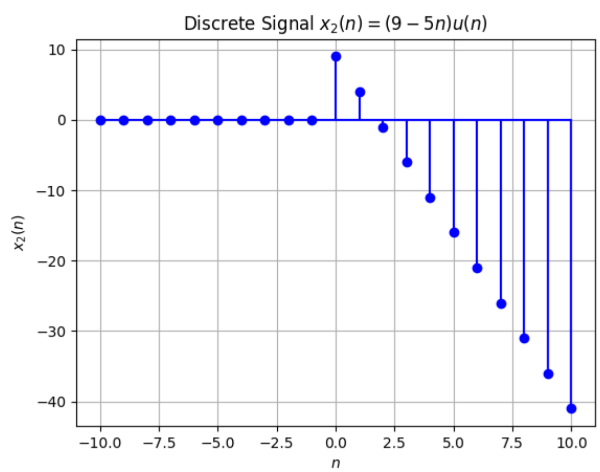

Fig. (ii): $x_2(n) = (9 - 5n)u(n)$