NCERT 10.5.3 10Q

EE22BTECH11010 - Venkatesh D Bandawar *

Question: Show that a_0 , a_1 , a_2 , . . ., a_n , . . . form an AP where an is defined as below :

1) $a_n = (3 + 4n)$
2) $a_n = (9 - 5n)$

Also find the sum of the first 15 terms in each case.

Solution:

Parameter	Description	Value
$x_i(n)$	i^{th}Discrete signal	$(3+4n)u(n)$
		$(9-5n)u(n)$
$x_i(0)$	First term of i^{th}AP	3
		9
d_i	common difference of i^{th}AP	4
		-5

TABLE 2: Given parameters

1)

$$X(z) = \frac{3}{1-z^{-1}} + \frac{4.z^{-1}}{(1-z^{-1})^2}; |z| > 1 \tag{1}$$

$$\because y(n) = x(n) * u(n) \tag{2}$$

$$Y(z) = X(z)U(z) \tag{3}$$

$$= \left[\frac{3}{1-z^{-1}} + \frac{4z^{-1}}{(1-z^{-1})^2}\right].\frac{1}{1-z^{-1}} \tag{4}$$

Using contour integration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{5}$$

$$= \frac{1}{2\pi j}\int \frac{3.z^{15}}{(z-1)^2}dz + \frac{1}{2\pi j}\int \frac{4.z^{15}}{(z-1)^3}dz \tag{6}$$

$$\because R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{7}$$

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2.\frac{3.z^{15}}{(z-1)^2}\right) \tag{8}$$

$$= 45 \tag{9}$$

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3.\frac{4.z^{15}}{(z-1)^3}\right) \tag{10}$$

$$= 420 \tag{11}$$

$$\implies y(14) = R_1 + R_2 \tag{12}$$

$$= 465 \tag{13}$$

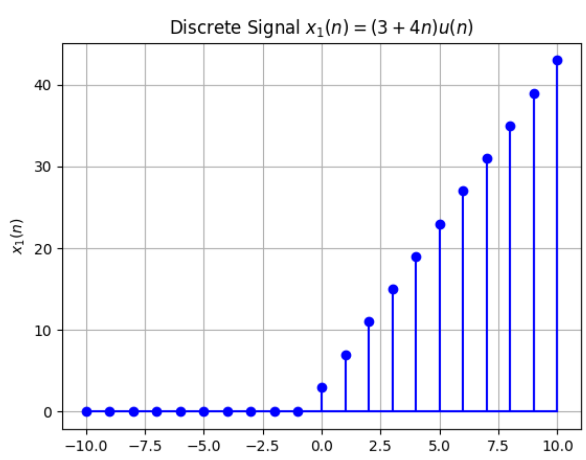

Fig. 1: $x_1(n) = (3 + 4n)u(n)$

2)

$$X(z) = \frac{9}{1-z^{-1}} - \frac{5.z^{-1}}{(1-z^{-1})^2}; |z| > 1 \tag{14}$$

$$\because y(n) = x(n) * u(n) \tag{15}$$

$$Y(z) = X(z)U(z) \tag{16}$$

$$= \left[\frac{9}{1-z^{-1}} - \frac{5z^{-1}}{(1-z^{-1})^2}\right] . \frac{1}{1-z^{-1}} \tag{17}$$

Using contour integration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{18}$$

$$= \frac{1}{2\pi j}\int \frac{9.z^{15}}{(z-1)^2}dz - \frac{1}{2\pi j}\int \frac{5.z^{15}}{(z-1)^3}dz \tag{19}$$

$$\because R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{20}$$

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2 . \frac{9.z^{15}}{(z-1)^2}\right) \tag{21}$$

$$= 135 \tag{22}$$

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3 . \frac{5.z^{15}}{(z-1)^3}\right) \tag{23}$$

$$= 525 \tag{24}$$

$$\implies y(14) = R_1 - R_2 \tag{25}$$

$$= -390 \tag{26}$$

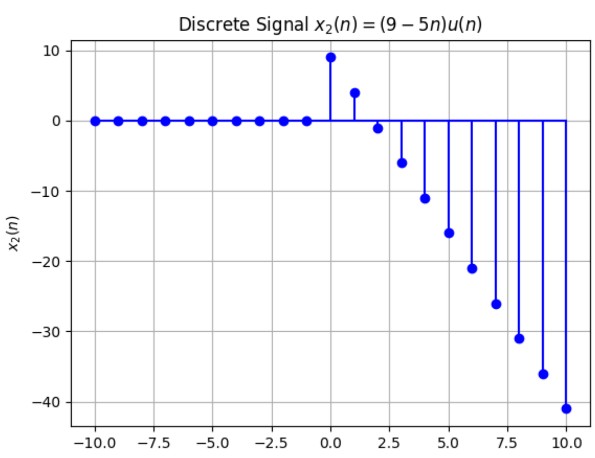

Fig. 2: $x_2(n) = (9 - 5n)u(n)$