NCERT 10.5.3 10Q

EE22BTECH11010 - Venkatesh D Bandawar *

Question: Show that $x(0)$, $x(1)$, $x(2)$, . . ., $x(n)$, . . . form an AP where an is defined as below :
(i) $x(n) = (3 + 4n)u(n)$
(ii) $x(n) = (9 - 5n)u(n)$
Also find the sum of the first 15 terms in each case.

Answer:

(i) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = x(n+1) - x(n) \tag{1}$$

$$= (3 + 4(n+1))\, u(n+1) - (3 + 4n)\, u(n)$$

$$= 4 \tag{2}$$

$\because$ Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
$x(n)$	Discrete signal	$(3 + 4n).u(n)$
$x(0)$	first term	3
d	common diffrence	4
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	465

TABLE (i): *Given parameters in 1^{st} AP*

Z - Transformation of $x(n)$:

$$\implies X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{3}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)\, z^{-n} \tag{4}$$

$$\implies X(z) = 3 \sum_{n=0}^{\infty} 1.z^{-n} + 4 \sum_{n=0}^{\infty} 1.n.z^{-n} \tag{5}$$

$$\implies X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2} \tag{6}$$

$\{Where, |z| > 1\}$

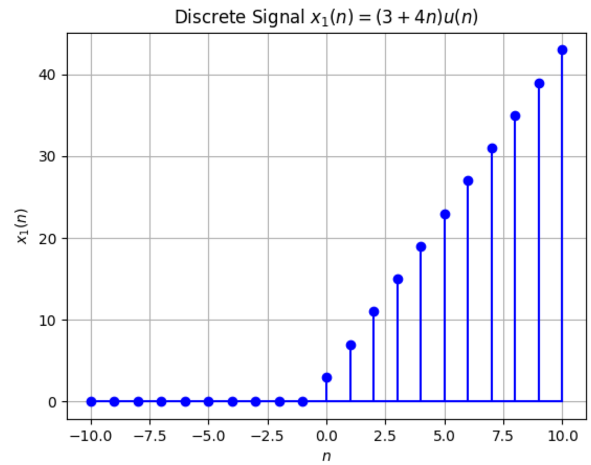

Fig. (i)

(ii) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = x(n+1) - x(n) \tag{7}$$

$$= (9 - 5(n+1))\, u(n+1) - (9 - 5n)\, u(n)$$

$$= -5 \tag{8}$$

$\because$ Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
$x(n)$	Discrete signal	$(9 - 5n).u(n)$
$x(0)$	first term	9
d	common diffrence	-5
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	-390

TABLE (ii): *Given parameters in 2^{st} AP*

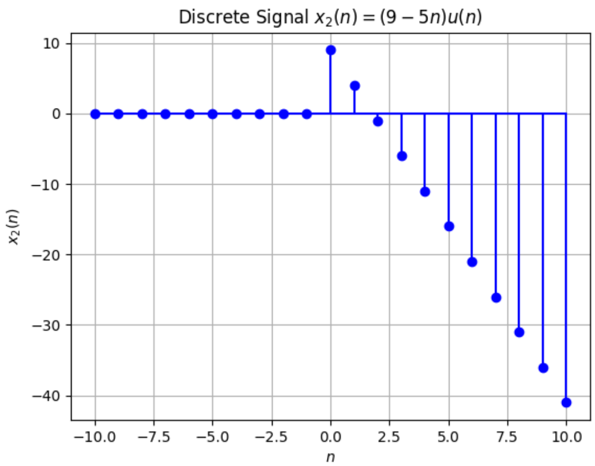

Fig. (ii)

Z - Transformation of $x(n)$:

$$\implies X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{9}$$

$$\implies X(Z) = \sum_{n=-\infty}^{\infty} [9 - 5n].u(n)\, z^{-n} \tag{10}$$

$$\implies X(z) = 9 \sum_{n=0}^{\infty} 1.z^{-n} - 5 \sum_{n=0}^{\infty} 1.n.z^{-n} \tag{11}$$

$$\implies X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2} \tag{12}$$

$$\{Where, |z| > 1\}$$