NCERT 10.5.3 10Q

EE22BTECH11010 - Venkatesh D Bandawar *

Question: Show that a_0 , a_1 , a_2 , . . ., a_n , . . . form an AP where an is defined as below :

1) $a_n = (3 + 4n)$
2) $a_n = (9 - 5n)$

Also find the sum of the first 15 terms in each case.

Solution:

Parameter	Description	Value
		$(3 + 4n)u(n)$
$x_i(n)$	i^{th} Discrete signal	$(9 - 5n)u(n)$
		3
$x_i(0)$	First term of ith AP	9
		4
d_i	common difference of ith AP	-5

TABLE 2: Given parameters

1) From equation (**??**)

$$X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2} ; |z| > 1 \quad (1)$$

$$\because y(n) = x(n) * u(n) \quad (2)$$

$$Y(z) = X(z)U(z) \quad (3)$$

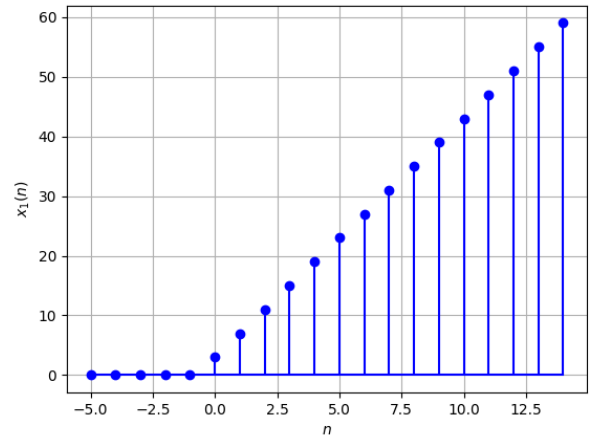$$= \left[\frac{3}{(1 - z^{-1})^2} + \frac{4z^{-1}}{(1 - z^{-1})^3} \right] \quad (4)$$

Using contour integration for inverse Z trans-

formation,

$$y(14) = \frac{1}{2\pi j} \int Y(z)z^{13} dz \quad (5)$$

$$= \frac{1}{2\pi j} \int \frac{3.z^{15}}{(z - 1)^2} dz + \frac{1}{2\pi j} \int \frac{4.z^{15}}{(z - 1)^3} dz \quad (6)$$

$$\because R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (7)$$

$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z - 1)^2 . \frac{3.z^{15}}{(z - 1)^2} \right) \quad (8)$$

$$= 45 \quad (9)$$

$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 . \frac{4.z^{15}}{(z - 1)^3} \right) \quad (10)$$

$$= 420 \quad (11)$$

$$\implies y(14) = R_1 + R_2 \quad (12)$$

$$= 465 \quad (13)$$

Fig. 1: $x_1(n) = (3 + 4n)u(n)$

2) From equation (**??**)

$$X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \quad (14)$$

$$\because y(n) = x(n) * u(n) \quad (15)$$

$$Y(z) = X(z)U(z) \quad (16)$$

$$= \left[\frac{9}{(1 - z^{-1})^2} - \frac{5z^{-1}}{(1 - z^{-1})^3} \right] \quad (17)$$

Using contour integration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j} \int Y(z)z^{13}dz \quad (18)$$

$$= \frac{1}{2\pi j} \int \frac{9.z^{15}}{(z - 1)^2}dz - \frac{1}{2\pi j} \int \frac{5.z^{15}}{(z - 1)^3}dz \quad (19)$$

$$\because R = \frac{1}{(m - 1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (20)$$

$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z - 1)^2 . \frac{9.z^{15}}{(z - 1)^2} \right) \quad (21)$$

$$= 135 \quad (22)$$

$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z - 1)^3 . \frac{5.z^{15}}{(z - 1)^3} \right) \quad (23)$$

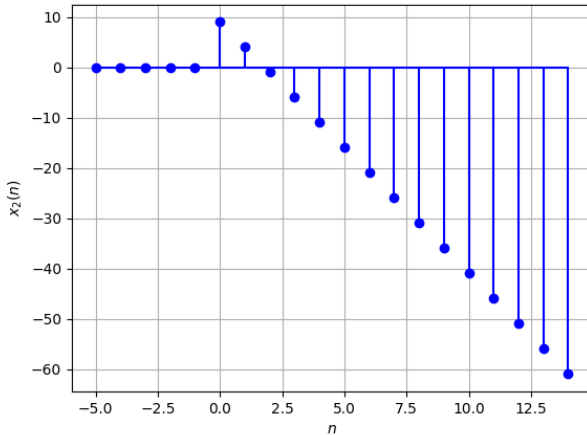$$= 525 \quad (24)$$

$$\implies y(14) = R_1 - R_2 \quad (25)$$

$$= -390 \quad (26)$$

Fig. 2: $x_2(n) = (9 - 5n)u(n)$