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NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series, whose n^{th} term is : n(n+1)(n+4).

Solution

Parameter	Description	Value
x(n)	<i>n</i> th term of series	n(n+1)(n+4)u(n)
y(n)	sum of n terms of series	

TABLE 0: Given parameters

From equation (??) to (??),

$$X(z) = \frac{z^{-1} \left(1 + 4z^{-1} + z^{-2}\right)}{\left(1 - z^{-1}\right)^4} + \frac{5z^{-1} \left(z^{-1} + 1\right)}{\left(1 - z^{-1}\right)^3} + \frac{4z^{-1}}{\left(1 - z^{-1}\right)^2}$$

$$Y(z) = X(z)U(z) \qquad (2)$$

$$= \frac{z^{-1} \left(1 + 4z^{-1} + z^{-2}\right)}{\left(1 - z^{-1}\right)^5} + \frac{5z^{-1} \left(z^{-1} + 1\right)}{\left(1 - z^{-1}\right)^4} + \frac{4z^{-1}}{\left(1 - z^{-1}\right)} \qquad 4000$$

$$= \frac{z^{-1} \left(1 + 11z^{-1} + 11z^{-2} + z^{-3}\right)}{4\left(1 - z^{-1}\right)^5} + \frac{13z^{-1} \left(1 + 4z^{-1} + z^{-2}\right)}{6\left(1 - z^{-1}\right)^4} \qquad 2000$$

$$+ \frac{19z^{-1} \left(1 + z^{-1}\right)}{4\left(1 - z^{-1}\right)^3} + \frac{17z^{-1}}{6\left(1 - z^{-1}\right)^2} \qquad (4)$$

Taking reverse z transform,

Fig. 0: Sum of n terms of series

$$y(n) = \left(\frac{n^4}{4} + \frac{13n^3}{6} + \frac{19n^2}{4} + \frac{17n}{6}\right)u(n)$$

$$= \left(\frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}\right)u(n)$$
(5)