NCERT 11.15. Q10

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Question: For the travelling harmonic wave y(x, t) = 2.0 cos 2π (10t - 0.0080 x + 0.35) where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of
   (a) 4 m,
   (b) 0.5 m,
   (c) $\lambda/2$,
   (d) $3\lambda/4$

Solution:

Method I

K = 2π(0.008)

$\because K = \dfrac{2\pi}{\lambda}$

$\lambda = \dfrac{2\pi}{K}$

$\lambda = \dfrac{2\pi}{2\pi \times 0.008}$

$\lambda = 125$

$\because$ phase difference(in radians) =

$\dfrac{\text{distance between two points}}{\lambda} \times 2\pi$

(a) phase difference = $\dfrac{4}{125} \times 2\pi$

$\qquad = \dfrac{8\pi}{125}$ radians

(b) phase difference = $\dfrac{0.5}{125} \times 2\pi$

$\qquad = \dfrac{\pi}{125}$ radians

(c) phase difference = $\dfrac{125/2}{125} \times 2\pi$

$\qquad = \pi$ radians

(d) phase difference = $\dfrac{3 \times 125/4}{125} \times 2\pi$

$\qquad = 3\pi/2$ radians

Method II

harmonic wave :

y(x, t) = 2.0 cos 2π (10t - 0.0080 x + 0.35)

phase of harmonic wave (at x):

= 2π (10t - 0.0080x + 0.35)

(a) phase of harmonic wave (at x=4) :

= 2π (10t - 0.0080(x+4) + 0.35)

⇒ phase difference :

= 2π (10t - 0.0080(x+4) + 0.35) - 2π (10t - 0.0080x +0.35)

= 2π × 0.0080 × 4

= $\dfrac{8\pi}{125}$ radians

(b) phase of harmonic wave (at x=0.5) :

= 2π (10t - 0.0080(x+0.5) + 0.35)

⇒ phase difference :

= 2π (10t - 0.0080(x+0.5) + 0.35) - 2π (10t - 0.0080x +0.35)

= 2π × 0.0080 × 0.5

= $\dfrac{\pi}{125}$ radians

(c) phase of harmonic wave (at x=$\lambda/2$) :

= 2π (10t - 0.0080(x+$\lambda/2$) + 0.35)

⇒ phase difference :

= 2π (10t - 0.0080(x+$\lambda/2$) + 0.35) - 2π (10t - 0.0080x +0.35)

= 2π × 0.0080 × $\lambda/2$

($\because \lambda = 125$)

= 2π × 0.0080 × 125/2

= π radians

(d) phase of harmonic wave (at x=$3\lambda/4$) :

= 2π (10t - 0.0080(x+$3\lambda/4$) + 0.35)

⇒ phase difference :

= 2π (10t - 0.0080(x+$3\lambda/4$) + 0.35) - 2π (10t - 0.0080x +0.35)

= 2π × 0.0080 × $3\lambda/4$

($\because \lambda = 125$)

= 2π × 0.0080 × 3 × 125/4

= 3π/2 radians