GATE 2022 IN.53

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Question: In a unity-gain feedback control system, the plant $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$ is controlled by a lag compensator $C(s) = \frac{s+10}{s+0.1}$ The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at $\omega = 3$rad/s is _____ (in integer) (GATE 2022 IN)

Solution:

Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(\frac{s}{0.5}+1)(\frac{s}{100}+1)}$
$C(s)$	Lag Compensator	$\frac{100(\frac{s}{10}+1)}{\frac{s}{0.1}+1}$
$T(s)$	Loop gain	$P(s)C(s)$
ω	Angular Frequency	3rad/s

TABLE I: Given Parameters list

$$|T(s)| = \frac{0.1\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{0.5}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{0.1}+1\right)} \tag{1}$$

Here, $10, 0, 0.5, 100, 0.1$ are corner frequencies of loop gain L(s)

Corner Frequency	**Description**	**Equation**
10	Zero	$\|T(s)\| = 20.0(log_{10}(w) - 1.0)$
0	Pole	$\|T(s)\| = -20.0log_{10}(w)$
0.1	Pole	$\|T(s)\| = -20.0(log_{10}(w) - 0.1)$
0.5	Pole	$\|T(s)\| = -20.0(log_{10}(w) + log_{10}(0.5))$
100	Pole	$\|T(s)\| = -20.0(log_{10}(w) - 2.0)$

TABLE II: Caption

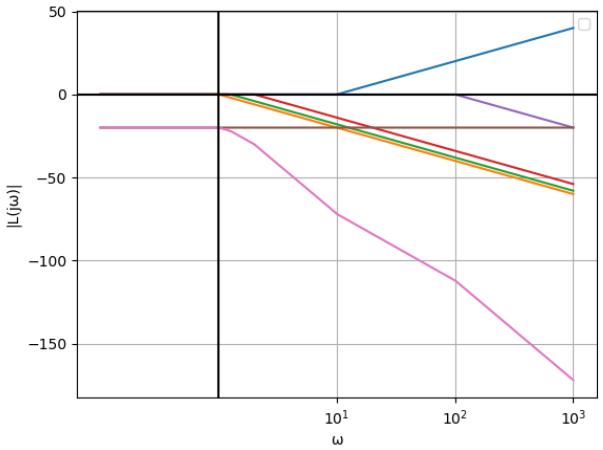

Fig. 1: Pink Line = Bode magnitude plot of loop gain

$$Gain(K) = \lim_{s\to 0} sT(s) \tag{2}$$

$$K = 0.1 \tag{3}$$

$$|T(s)| = 20\log_{10} K \tag{4}$$

$$= -20dB \tag{5}$$

$$T(s) = \begin{cases} -20log_{10}(w) & \omega < 0.1 \\ -20.0\,(2log_{10}(w) - 0.1) & 0.1 \le \omega < 0.5 \\ -20.0\,(3log_{10}\omega - 0.1 + log_{10}0.5) & 0.5 \le \omega < 10 \\ -20.0\,(2log_{10}\omega + 0.9 + log_{10}0.5) & 10 \le \omega < 100 \\ -20.0\,(3log_{10}\omega - 1.9 + log_{10}0.5) & \omega \ge 100 \end{cases}$$

Slope of Bode magnitude plot (at $\omega = 3$) $= -60$ dB/decade