NCERT 10.5.3 10Q

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Question: Show that a_1 , a_2 , . . ., a_n , . . . form an AP where an is defined as below :
(i) $a_n = 3 + 4n$
(ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Answer:

(i) Given: $a_n = 3 + 4n$
we know that, The AP has constant common difference between two consecutive terms

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \tag{1}$$
$$= (3 + 4(n + 1)) - (3 + 4n)$$
$$= 4 \tag{2}$$

$\because$ Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
a_n	n^{th} term	$3 + 4n$
a_0	first term	3
d	common differnce	4
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	465

TABLE (i): *Given parameters in* 1^{st} *AP*

Z - Transformation of a_n :

$$\because X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \tag{3}$$
$$x(n) = a_n\, u(n) \tag{4}$$
$$= [3 + 4n]\; u(n) \tag{5}$$

$$\text{Here, } u(n) = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$

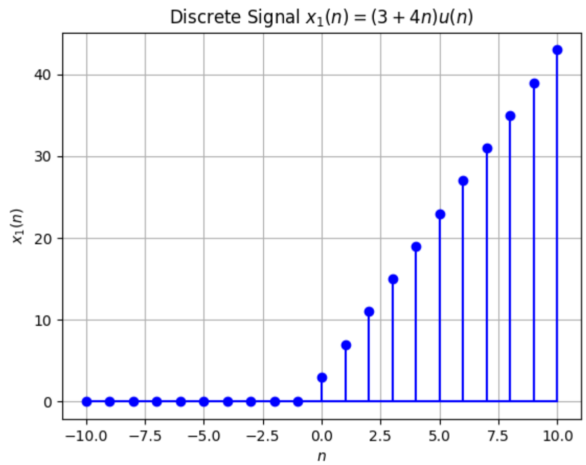

Fig. (i)

$$\implies X(z) = \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)\, z^{-n} \tag{6}$$

$$\implies X(z) = 3\sum_{n=0}^{\infty} 1.z^{-n} + 4\sum_{n=0}^{\infty} 1.n.z^{-n} \tag{7}$$

Since,

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad ; |r| < 1 \tag{8}$$

Differentiating w.r.t. 'r' and multiplying by 'r' we get,

$$\sum_{n=0}^{\infty} n.r^n = \frac{r}{(1-r)^2} \tag{9}$$

Here, $r = z^{-1}$

$$\implies X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2} \tag{10}$$

$\{Where, |z| > 1\}$

(ii) Given: $a_n = 9 - 5n$
we know that, The AP has constant common difference between two consecutive terms

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \quad (11)$$
$$= (9 - 5(n+1)) - (9 - 5n)$$
$$= -5 \quad (12)$$

$\because$ Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
a_n	n^{th} term	$9 - 5n$
a_0	first term	9
d	common differnce	-5
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	-390

TABLE (ii): *Given parameters in* 2^{st} *AP*

Z - Transformation of a_n :

$$\because X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \quad (13)$$
$$x(n) = a_n\, u(n) \quad (14)$$
$$= [9 - 5n]\ u(n) \quad (15)$$

$$\text{Here, } u(n) = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$$

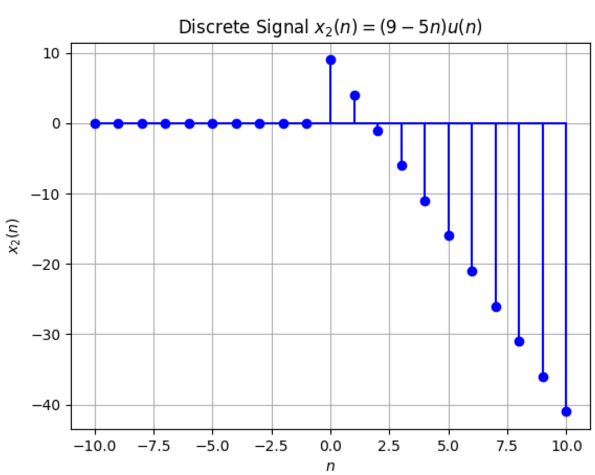

Fig. (ii)

$$\implies X(Z) = \sum_{n=-\infty}^{\infty} [9 - 5n].u(n)\, z^{-n} \quad (16)$$

$$\implies X(z) = 9 \sum_{n=0}^{\infty} 1.z^{-n} - 5 \sum_{n=0}^{\infty} 1.n.z^{-n} \quad (17)$$

Since,

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad ; |r| < 1 \quad (18)$$

Differentiating w.r.t. 'r' and multiplying by 'r' we get,

$$\sum_{n=0}^{\infty} n.r^n = \frac{r}{(1-r)^2} \quad (19)$$

Here, $r = z^{-1}$

$$\implies X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2} \quad (20)$$

$\{Where, |z| > 1\}$