

NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series , Taking reverse z transform,
whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

$$y(n) = \left(\frac{n^4}{4} + \frac{13n^3}{6} + \frac{19n^2}{4} + \frac{17n}{6} \right) u(n) \quad (9)$$

$$= \left(\frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} \right) u(n) \quad (10)$$

From equation (??) to (??),

$$X(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})} \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

$$= \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{4z^{-1}}{(1-z^{-1})} \quad (3)$$

$$= \frac{z^{-1}(1+11z^{-1}+11z^{-2}+z^{-3})}{4(1-z^{-1})^5} + \frac{13z^{-1}(1+4z^{-1}+z^{-2})}{6(1-z^{-1})^4}$$

$$+ \frac{19z^{-1}(1+z^{-1})}{4(1-z^{-1})^3} + \frac{17z^{-1}}{6(1-z^{-1})^2} \{ |z| > 1 \} \quad (4)$$

where,

$$nu(n) \xleftrightarrow{z} \frac{z^{-1}}{(1-z^{-1})^2} \{ |z| > 1 \} \quad (5)$$

$$n^2u(n) \xleftrightarrow{z} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \{ |z| > 1 \} \quad (6)$$

$$n^3u(n) \xleftrightarrow{z} \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} \{ |z| > 1 \} \quad (7)$$

$$n^4u(n) \xleftrightarrow{z} \frac{z^{-1}(1+11z^{-1}+11z^{-2}+z^{-3})}{(1-z^{-1})^5} \{ |z| > 1 \} \quad (8)$$

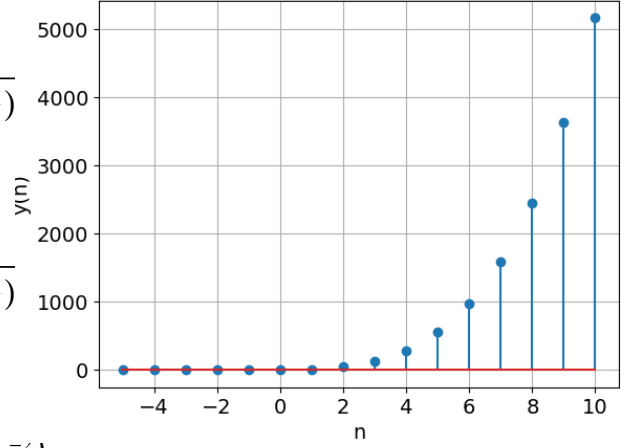


Fig. 0: Sum of n terms of series