GATE: CE - 30.2023

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Question: In the differential equation $\frac{dy}{dx} + \alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is (rounded off to three decimal places). (GATE CE 2023)

Solution:

Parameter	**Value**
x	0.0
	1.0
y	1.0
	0.8

TABLE I: Given parameters

Let, $t = x$

$$\frac{dy}{dt} + \alpha ty = 0 \tag{1}$$

$$\int \frac{dy}{y} = -\int \alpha t dt \tag{2}$$

$$\ln(|y|) = -\frac{\alpha t^2}{2} + c \tag{3}$$

$$y(t) = e^c \cdot e^{-\frac{\alpha t^2}{2}} \tag{4}$$

Taking Fourier Transform:
where,

$$e^{-at^2} \overset{\mathcal{F}}{\longleftrightarrow} \sqrt{\frac{\pi}{a}} e^{-\frac{4\pi^2 f^2}{2\alpha}} \tag{5}$$

From equation (5):

$$Y(f) = \sqrt{\frac{2\pi}{\alpha}} e^c \cdot e^{-\frac{4\pi^2 f^2}{2\alpha}} \tag{6}$$

Taking Fourier Transform:
where,

$$\frac{dy}{dt} \overset{\mathcal{F}}{\longleftrightarrow} j2\pi f Y(f) \tag{7}$$

$$a \cdot t \cdot y(t) \overset{\mathcal{F}}{\longleftrightarrow} a \cdot \frac{j}{2\pi} \frac{d}{df} Y(f) \tag{8}$$

From equation (7) and (8):

$$\frac{4\pi^2 f}{\alpha} Y(f) + \frac{d}{df} Y(f) = 0 \tag{9}$$

$$Y(f) = K e^{-\frac{4\pi^2 f^2}{2\alpha}} \tag{10}$$

where,

$$K = \sqrt{\frac{2\pi}{\alpha}} e^c \tag{11}$$

Substituting x and y values:

$$c = \ln(1) = 0 \tag{12}$$

$$\alpha = -2\ln(0.8) = 0.446 \tag{13}$$

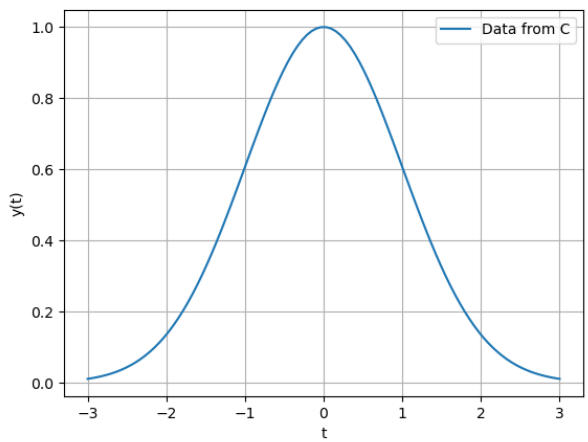

Fig. 1: Graph of y(t)