

NCERT 10.5.3 10Q

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Question: Show that $a_0, a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below :

- 1) $a_n = (3 + 4n)$
- 2) $a_n = (9 - 5n)$

Also find the sum of the first 15 terms in each case.

Answer:

- (i) We know that, The AP has constant common difference between two consecutive terms.

$$\begin{aligned} \therefore \text{common difference } (d) &= a_{n+1} - a_n \quad (1) \\ &= (3 + 4(n+1)) - (3 + 4n) \quad (2) \\ &= 4 \quad (3) \end{aligned}$$

\therefore Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'

Hence given sequence is in AP.

Parameter	Description	Value
a_n	General term of AP	$(3 + 4n)$
$x(n)$	Discrete signal	$(3 + 4n)u(n)$
a_0	first term	3
d	common difference	4

TABLE (i): Given parameters in 1st AP

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (4)$$

$$= \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)z^{-n} \quad (5)$$

$$nu(n) \xrightarrow{Z} -z.U'(z) \quad (6)$$

$$X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \quad (7)$$

$$\therefore y(n) = x(n) * u(n) \quad (8)$$

$$Y(z) = X(z)U(z) \quad (9)$$

$$= \left[\frac{3}{1 - z^{-1}} + \frac{4z^{-1}}{(1 - z^{-1})^2} \right] \cdot \frac{1}{1 - z^{-1}} \quad (10)$$

Using contour integration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j} \int Y(z)z^{13}dz \quad (11)$$

$$= \frac{1}{2\pi j} \int \frac{3.z^{15}}{(z-1)^2}dz + \frac{1}{2\pi j} \int \frac{4.z^{15}}{(z-1)^3}dz \quad (12)$$

$$\therefore R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (13)$$

$$R_1 = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z-1)^2 \cdot \frac{3.z^{15}}{(z-1)^2} \right) \quad (14)$$

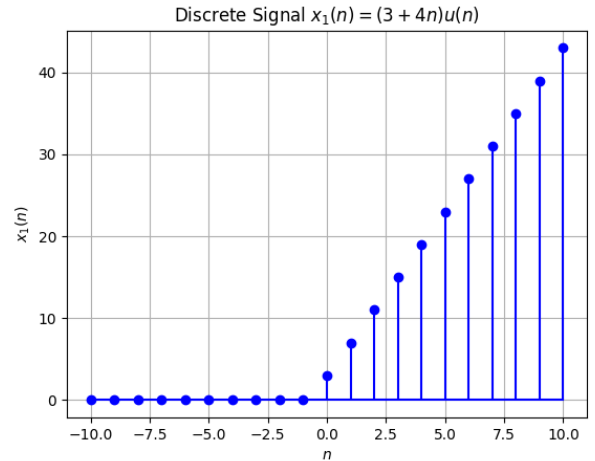
$$= 45 \quad (15)$$

$$R_2 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \cdot \frac{4.z^{15}}{(z-1)^3} \right) \quad (16)$$

$$= 420 \quad (17)$$

$$\Rightarrow y(14) = R_1 + R_2 \quad (18)$$

$$= 465 \quad (19)$$



- (ii) We know that, The AP has constant common difference between two consecutive terms.

$$\begin{aligned} \therefore \text{common difference } (d) &= a_{n+1} - a_n \quad (20) \\ &= (9 - 5(n + 1)) - (9 - 5n) \quad (21) \\ &= -5 \quad (22) \end{aligned}$$

\therefore Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'
Hence given sequence is in AP.

Parameter	Description	Value
a_n	General term of AP	$(9 - 5n)$
$x(n)$	Discrete signal	$(9 - 5n)u(n)$
a_0	first term	9
d	common difference	-5

TABLE (ii): Given parameters in 2st AP

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (23)$$

$$= \sum_{n=-\infty}^{\infty} [9 - 5n].u(n) z^{-n} \quad (24)$$

$$nu(n) \xrightarrow{Z} -z.U'(z) \quad (25)$$

$$X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \quad (26)$$

$$\therefore y(n) = x(n) * u(n) \quad (27)$$

$$Y(z) = X(z)U(z) \quad (28)$$

$$= \left[\frac{9}{1 - z^{-1}} - \frac{5z^{-1}}{(1 - z^{-1})^2} \right] \cdot \frac{1}{1 - z^{-1}} \quad (29)$$

$$y(14) = \frac{1}{2\pi j} \int Y(z)z^{13} dz \quad (30)$$

$$= \frac{1}{2\pi j} \int \frac{9.z^{15}}{(z - 1)^2} dz - \frac{1}{2\pi j} \int \frac{5.z^{15}}{(z - 1)^3} dz \quad (31)$$

$$\therefore R = \frac{1}{(m - 1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z - a)^m f(z)) \quad (32)$$

$$R_1 = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left((z - 1)^2 \cdot \frac{9.z^{15}}{(z - 1)^2} \right) \quad (33)$$

$$= 135 \quad (34)$$

$$R_2 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z - 1)^3 \cdot \frac{5.z^{15}}{(z - 1)^3} \right) \quad (35)$$

$$= 525 \quad (36)$$

$$\Rightarrow y(14) = R_1 - R_2 \quad (37)$$

$$= -390 \quad (38)$$

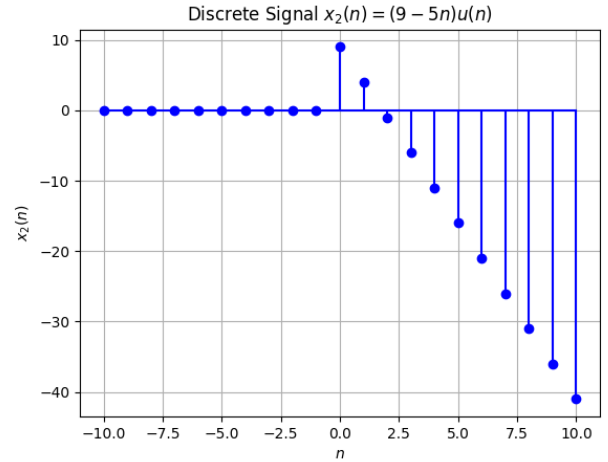


Fig. (ii): $x_2(n) = (9 - 5n)u(n)$

Using contour integration for inverse Z transformation,