GATE 2022 BM.38

EE23BTECH11010 - VENKATESH BANDAWAR*

Question: An input $x(t)$ is applied to a system with a frequency transfer function given by $H(j\omega)$ as shown below. The magnitude and phase response of the transfer function are shown below. If $y(t_d) = 0$ for $x(t) = u(t)$, the time $t_d(> 0)$ is.
(Gate 2022 BM.38)

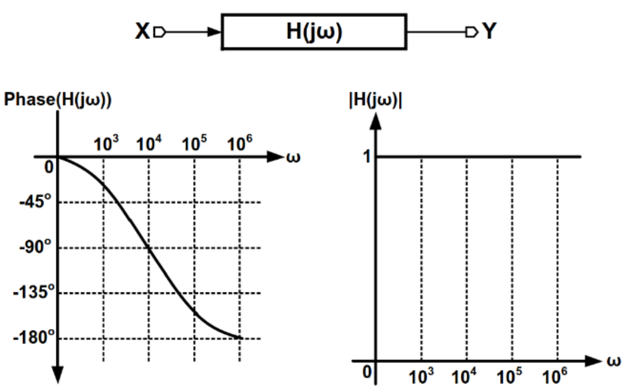

Fig. 1: Graph of y(t)

Solution:

Parameter	Description
$x(t) = u(t)$	Input signal
$y(t)$	Output signal
$X(j\omega)$	Fourier Transform of $x(t)$
$Y(j\omega)$	Fourier Transform of $y(t)$
$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$	Transfer function

TABLE I: Caption

$$\because |H(j\omega| = 1 \tag{1}$$

$$|z| = |\bar{z}| \tag{2}$$

$$\therefore H(j\omega) = \frac{a - j\omega}{a + j\omega} \tag{3}$$

$$\theta_{H(j\omega)} = \tan^{-1}(\frac{-\omega}{a}) - \tan^{-1}(\frac{\omega}{a}) \tag{4}$$

$$= -2\tan^{-1}(\frac{\omega}{a}) \tag{5}$$

At $\omega = 10^4, \theta_{H(j\omega)} = -\frac{\pi}{2}$

$$\implies a = 10^4 \tag{6}$$

$$u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{j\omega} \tag{7}$$

$$Y(j\omega) = \frac{1}{j\omega}\frac{a - j\omega}{a + j\omega} \tag{8}$$

$$= \frac{1}{j\omega} - \frac{2}{a + j\omega} \tag{9}$$

$$\frac{1}{j\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} u(t) \tag{10}$$

$$\frac{1}{a + j\omega} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} e^{-at}u(t) \tag{11}$$

$$y(t) = (1 - 2e^{-at})u(t) \tag{12}$$

$$\because y(t_d) = 0 \tag{13}$$

$$t_d = 100 \ln 2\mu s \tag{14}$$

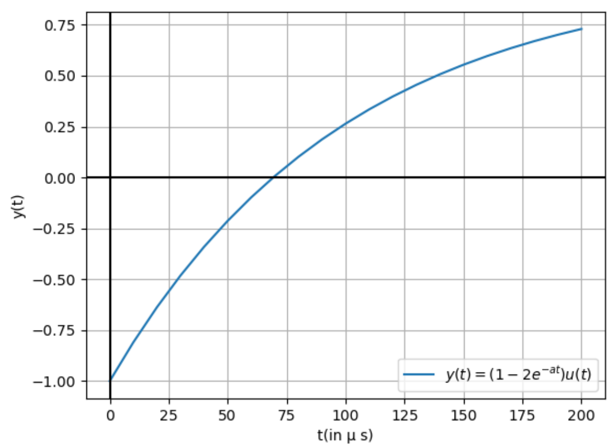

Fig. 2: Caption