NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series , whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

from equation (**??**) to (**??**),

$$X(z) = \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4} + \frac{5z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})^2} \tag{1}$$

$$Y(z) = X(z)U(z) \tag{2}$$

$$= \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^5} + \frac{5z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4} + \frac{4z^{-1}}{(1-z^{-1})} \tag{3}$$

Using contour integration for inverse Z transformation,

$$y(n) = \frac{1}{2\pi j}\oint_c Y(z)z^{n-1}dz \tag{4}$$

$$= \frac{1}{2\pi j}\oint_c \frac{\left(z^2+4z+1\right)}{(z-1)^5}z^{n+1}dz + \frac{1}{2\pi j}\oint_c \frac{5\,(z+1)}{(z-1)^4}z^{n+1}dz + \frac{1}{2\pi j}\oint_c \frac{4}{(z-1)^3}z^{n+1}dz \tag{5}$$

$$\because R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z-a)^m f(z)\right) \tag{6}$$

$$R_1 = \frac{1}{4!}\lim_{z\to 1}\frac{d^4}{dz^4}\left((z-1)^5\frac{\left(z^2+4z+1\right)z^{n+1}}{(z-1)^5}\right) \tag{7}$$

$$= \frac{(n+3)(n+2)(n+1)(n)}{4!} + \frac{4(n+2)(n+1)(n)(n-1)}{4!} + \frac{(n+1)(n)(n-1)(n-2)}{4!} \tag{8}$$

$$R_2 = \frac{1}{3!}\lim_{z\to 1}\frac{d^3}{dz^3}\left((z-1)^4\frac{5\,(z+1)\,z^{n+1}}{(z-1)^4}\right) \tag{9}$$

$$= \frac{5(n+2)(n+1)(n)}{3!} + \frac{5(n+1)(n)(n-1)}{3!} \tag{10}$$

$$R_3 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3\frac{4z^{n+1}}{(z-1)^3}\right) \tag{11}$$

$$= \frac{4n(n+1)}{2!} \tag{12}$$

$$\implies y(n) = R_1 + R_2 + R_3 \tag{13}$$

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} \tag{14}$$

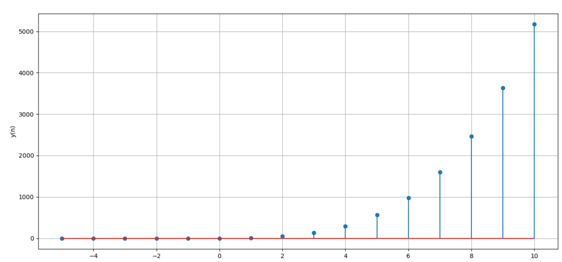

Fig. 0: sum of n terms of series