

GATE 2022 BM.38

EE23BTECH11010 - VENKATESH BANDAWAR*

Question: An input $x(t)$ is applied to a system with a frequency transfer function given by $H(j\omega)$ as shown below. The magnitude and phase response of the transfer function are shown below. If $y(t_d) = 0$ for $x(t) = u(t)$, the time $t_d(> 0)$ is.
(Gate 2022 BM.38)

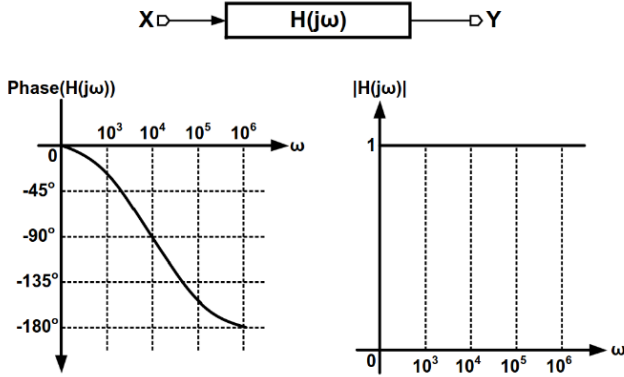


Fig. 1: Graph of $y(t)$

Solution:

Parameter	Description
$x(t) = u(t)$	Input signal
$y(t)$	Output signal
$X(j\omega)$	Fourier Transform of $x(t)$
$Y(j\omega)$	Fourier Transform of $y(t)$
$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$	Transfer function

TABLE I: Input Parameters Table

$$\because |H(f)| = 1 \quad (1)$$

$$|z| = |\bar{z}| \quad (2)$$

$$\therefore H(f) = \frac{a - j2\pi f}{a + j2\pi f} \quad (3)$$

$$\angle H(f) = \tan^{-1}\left(\frac{-2\pi f}{a}\right) - \tan^{-1}\left(\frac{2\pi f}{a}\right) \quad (4)$$

$$= -2 \tan^{-1}\left(\frac{2\pi f}{a}\right) \quad (5)$$

$$\text{At } 2\pi f = 10^4, \angle H(f) = -\frac{\pi}{2}$$

$$\Rightarrow a = 10^4 \quad (6)$$

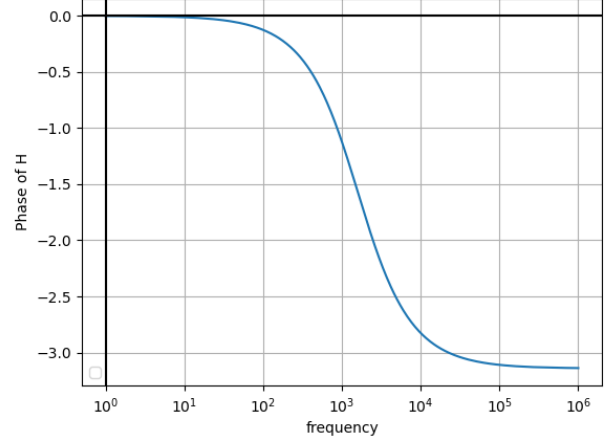


Fig. 2: Phase of $H(f)$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{j2\pi f} \quad (7)$$

$$Y(f) = \frac{1}{j2\pi f} \frac{a - j2\pi f}{a + j2\pi f} \quad (8)$$

$$= \frac{1}{j2\pi f} - \frac{2}{a + j2\pi f} \quad (9)$$

$$\frac{1}{j2\pi f} \xleftrightarrow{\mathcal{F}^{-1}} u(t) \quad (10)$$

$$\frac{1}{a + j2\pi f} \xleftrightarrow{\mathcal{F}^{-1}} e^{-at}u(t) \quad (11)$$

$$y(t) = (1 - 2e^{-at})u(t) \quad (12)$$

$$\because y(t_d) = 0 \quad (13)$$

$$t_d = 100 \ln 2 \mu s \quad (14)$$

