NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series , whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}}{\left(1-z^{-1}\right)^2} \{|z|>1\} \tag{1}$$

$$n^2u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3} \{|z|>1\} \tag{2}$$

$$n^3u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4} \{|z|>1\} \tag{3}$$

$$n^4u(n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{z^{-1}\left(1+11z^{-1}+11z^{-2}+z^{-3}\right)}{\left(1-z^{-1}\right)^5} \{|z|>1\} \tag{4}$$

From equation (**??**) to (**??**),

$$X(z) = \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4} + \frac{5z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^3} + \frac{4z^{-1}}{\left(1-z^{-1}\right)^2} \{|z|>1\} \tag{5}$$

$$Y(z) = X(z)U(z) \tag{6}$$

$$= \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^5} + \frac{5z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^4} + \frac{4z^{-1}}{\left(1-z^{-1}\right)^3} \tag{7}$$

$$= \frac{1}{4}\left[\frac{z^{-1}\left(1+11z^{-1}+11z^{-2}+z^{-3}\right)}{\left(1-z^{-1}\right)^5}\right] + \frac{13}{6}\left[\frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}\right] + \frac{19}{4}\left[\frac{z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}\right] + \frac{17}{6}\left[\frac{z^{-1}}{\left(1-z^{-1}\right)^2}\right] \{|z|>1\} \tag{8}$$

Taking reverse z transform, using equations (1) to (4)

$$y(n) = \left(\frac{n^4}{4} + \frac{13n^3}{6} + \frac{19n^2}{4} + \frac{17n}{6}\right)u(n) \tag{9}$$

$$= \left(\frac{n^4}{4} + \frac{2n^3}{4} + \frac{10n^3}{6} + \frac{n^2}{4} + \frac{15n^2}{6} + \frac{4n^2}{2} + \frac{5n}{6} + \frac{4n}{2}\right)u(n) \tag{10}$$

$$= \left(\frac{n^4+2n^3+n^4}{4}\right)u(n) + \left(\frac{10n^3+15n^2+5n}{6}\right)u(n) + \left(\frac{4n^2+4n}{2}\right)u(n) \tag{11}$$

$$= \left(\frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}\right)u(n) \tag{12}$$

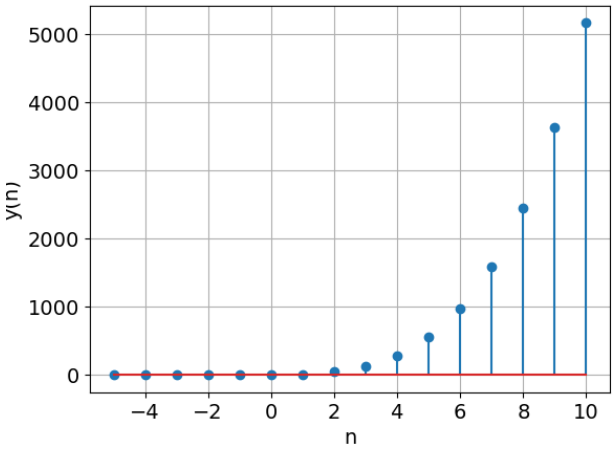

Fig. 0: Sum of n terms of series