GATE: CE - 30.2023

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Question: In the differential equation $\frac{dy}{dx} + \alpha xy = 0$, α is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is (rounded off to three decimal places). (GATE CE 2023)

Solution:

Parameter	Value
x	0.0
	1.0
y	1.0
	0.8

TABLE I: Given parameters

$$\frac{dy}{dx} + \alpha xy = 0 \tag{1}$$

Taking laplace transform,

$$sY(s) - y(0) - \alpha\frac{dY(s)}{ds} = 0 \tag{2}$$

$$\frac{dY(s)}{ds} - \frac{s}{\alpha}Y(s) = -\frac{Y(0)}{\alpha} \tag{3}$$

$$\tag{4}$$

I.F.$= e^{\int -\frac{s}{\alpha}ds} = e^{-\frac{s^2}{2\alpha}}$

$$e^{-\frac{s^2}{2\alpha}}Y(s) = \frac{y(0)}{\alpha}\int_{-\infty}^{\infty} e^{-\frac{s^2}{2\alpha}}ds \tag{5}$$

$$= -\frac{y(0)}{\alpha}\sqrt{2\pi\alpha} \tag{6}$$

$$Y(s) = -\sqrt{\frac{2\pi}{\alpha}}y(0)e^{\frac{s^2}{2\alpha}} \tag{7}$$

$$Y'(s) = -\sqrt{\frac{2\pi}{\alpha}}y(0)e^{\frac{s^2}{2\alpha}}\frac{s}{\alpha} \tag{8}$$