GATE: CE - 30.2023

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Question: In the differential equation $\frac{dy}{dx} + \alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is (rounded off to three decimal places). (GATE CE 2023)

Solution:

Parameter	Value
x	0.0
	1.0
y	1.0
	0.8

TABLE I: Given parameters

$$\frac{dy}{dx} + \alpha xy = 0 \tag{1}$$

$$\int \frac{dy}{y} = -\int \alpha x dx \tag{2}$$

$$\ln(|y|) = -\frac{\alpha x^2}{2} + c \tag{3}$$

Substituting x and y values,

$$c = \ln(1) = 0 \tag{4}$$

$$\alpha = -2\ln(0.8) = 0.446 \tag{5}$$