

NCERT 11.9.4 8Q

EE22BTECH11010 - Venkatesh D Bandawar *

Question: Find the sum to n terms of series ,
whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n-1)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n-1)$	sum to n terms of series or sum to $(n-1)^{th}$ term of series	

TABLE 0: Given parameters

from equation (??) to (??),

$$X(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})^2} \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

$$= \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{4z^{-1}}{(1-z^{-1})^3} \quad (3)$$

Using contour integration for inverse Z transformation,

$$y(n-1) = \frac{1}{2\pi j} \oint_c Y(z)z^{n-2}dz \quad (4)$$

$$= \frac{1}{2\pi j} \oint_c \frac{(z^2+4z+1)}{(z-1)^5} z^n dz + \frac{1}{2\pi j} \oint_c \frac{5(z+1)}{(z-1)^4} z^n dz + \frac{1}{2\pi j} \oint_c \frac{4}{(z-1)^3} z^n dz \quad (5)$$

$$\therefore R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (6)$$

$$R_1 = \frac{1}{4!} \lim_{z \rightarrow 1} \frac{d^4}{dz^4} \left((z-1)^5 \frac{(z^2+4z+1)z^n}{(z-1)^5} \right) \quad (7)$$

$$= \frac{(n+2)(n+1)(n)(n-1)}{4!} + \frac{4(n+1)(n)(n-1)(n-2)}{4!} + \frac{n(n-1)(n-2)(n-3)}{4!} \quad (8)$$

$$R_2 = \frac{1}{3!} \lim_{z \rightarrow 1} \frac{d^3}{dz^3} \left((z-1)^4 \frac{5(z+1)z^n}{(z-1)^4} \right) \quad (9)$$

$$= \frac{5(n+1)(n)(n-1)}{3!} + \frac{5n(n-1)(n-2)}{3!} \quad (10)$$

$$R_3 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left((z-1)^3 \frac{4z^n}{(z-1)^3} \right) \quad (11)$$

$$= \frac{4n(n-1)}{2!} \quad (12)$$

$$\Rightarrow y(n-1) = R_1 + R_2 + R_3 \quad (13)$$

$$= \frac{n^2(n-1)^2}{4} + \frac{5n(n-1)(2n-1)}{6} + \frac{4n(n-1)}{2} \quad (14)$$

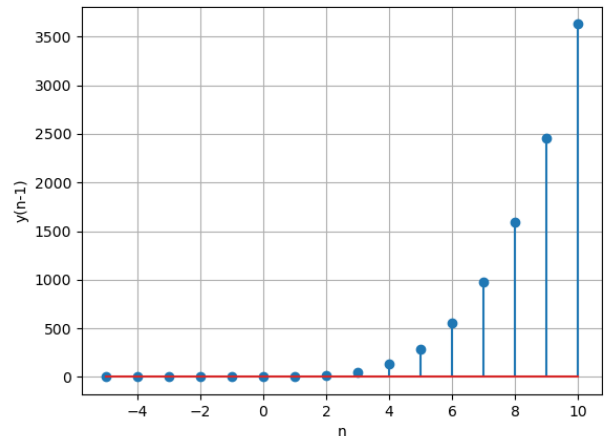


Fig. 0: $y(n) = \frac{n^2(n-1)^2}{4} + \frac{5n(n-1)(2n-1)}{6} + \frac{4n(n-1)}{2}$