GATE 2022 IN.53

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Question: In a unity-gain feedback control system, the plant $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$ is controlled by a lag compensator $C(s) = \frac{s+10}{s+0.1}$ The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at $\omega = 3$rad/s is _____ (in integer) (GATE 2022 IN)

Solution:

Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(\frac{s}{0.5}+1)(\frac{s}{100}+1)}$
$C(s)$	Lag Compensator	$\frac{100(\frac{s}{10}+1)}{\frac{s}{0.1}+1}$
$L(s)$	Loop gain=$P(s)C(s)$	$\frac{0.1(\frac{s}{10}+1)}{s(\frac{s}{0.5}+1)(\frac{s}{100}+1)(\frac{s}{0.1}+1)}$
ω	Angular Frequency	3rad/s

TABLE I: Given Parameters list

$$Gain(K) = \lim_{s\to 0} L(s) \tag{1}$$

Excluding s and $\frac{1}{s}$, From Table Table I

$$K = 0.1 \tag{2}$$

$$|L(s)| = 20\log_{10} K \tag{3}$$

$$= -20dB \tag{4}$$

Here, $10, 0, 0.5, 100, 0.1$ are corner frequencies of loop gain L(s)

Corner Frequency	Description	Equation
10	Zero	$\|L(\omega)\| = 20.0 * (log_{10}(w) - 1.0)$
0	Pole	$\|L(\omega)\| = -20.0 * log_{10}(w)$
0.1	Pole	$\|L(\omega)\| = -20.0 * (log_{10}(w) - 0.1)$
0.5	Pole	$\|L(\omega)\| = -20.0 * (log_{10}(w) + log_{10}(0.5))$
100	Pole	$\|L(\omega)\| = -20.0 * (log_{10}(w) - 2.0)$

TABLE II: Caption

Slope of Bode magnitude plot (at $\omega = 3$) $= -60$ dB/decade

Parameter	Range	Equation
$\|L(\omega)\|$	$\omega < 0$	0
	$0 < \omega < 0.1$	$-20.0 * log_{10}(w)$
	$0.1 < \omega < 0.5$	$-20.0 * (2log_{10}(w) - 0.1)$
	$0.5 < \omega < 10$	$-20.0 * (3log_{10} - 0.1 + log_1 00.5)$
	$10 < \omega < 100$	$-20.0 * (2log_{10} + 0.9 + log_1 00.5)$
	$\omega > 100$	$-20.0 * (3log_{10} - 1.9 + log_1 00.5)$

TABLE III: Caption

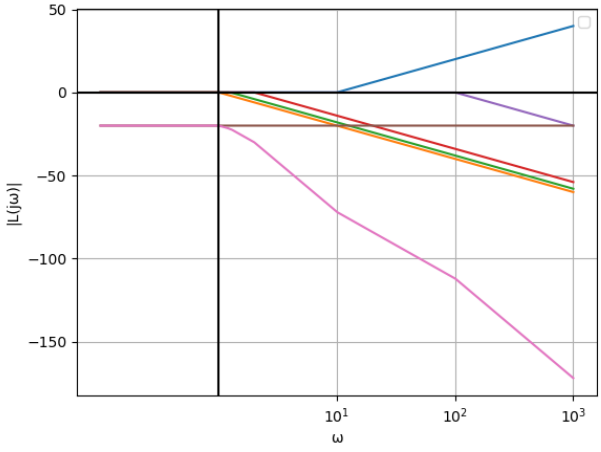

Fig. 1: Pink Line = Bode magnitude plot of loop gain