GATE 2022 IN.53

EE23BTECH11010 - VENKATESH BANDAWAR*

Question: In a unity-gain feedback control system, the plant $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$ is controlled by a lag compensator $C(s) = \frac{s+10}{s+0.1}$ The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at $\omega = 3$rad/s is _____ (in integer) (GATE 2022 IN)

Solution:

Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(\frac{s}{0.5}+1)(\frac{s}{100}+1)}$
$C(s)$	Lag Compensator	$\frac{100(\frac{s}{10}+1)}{\frac{s}{0.1}+1}$
$L(s)$	Loop gain	$P(s)C(s)$
ω	Angular Frequency	3rad/s

TABLE I: Given Parameters list

$$|L(s)| = \frac{0.1\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{0.5}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{0.1}+1\right)} \tag{1}$$

Here, $10, 0, 0.5, 100, 0.1$ are corner frequencies of loop gain L(s)

Corner Frequency	Description	Equation
10	Zero	$\lvert L(\omega)\rvert = 20.0(log_{10}(w) - 1.0)$
0	Pole	$\lvert L(\omega)\rvert = -20.0log_{10}(w)$
0.1	Pole	$\lvert L(\omega)\rvert = -20.0(log_{10}(w) - 0.1)$
0.5	Pole	$\lvert L(\omega)\rvert = -20.0(log_{10}(w) + log_{10}(0.5))$
100	Pole	$\lvert L(\omega)\rvert = -20.0(log_{10}(w) - 2.0)$

TABLE II: Caption

$$Gain(K) = \lim_{s\to 0} L(s) \tag{2}$$

Excluding s and $\frac{1}{s}$, From equation (1)

$$K = 0.1 \tag{3}$$

$$|L(s)| = 20\log_{10} K \tag{4}$$
$$= -20dB \tag{5}$$

$$H[\omega] = \begin{cases} 0 & \omega < 0 \\ -20log_{10}(w) & 0 \le \omega < 0.1 \\ -20.0\,(2log_{10}(w) - 0.1) & 0.1 \le \omega < 0.5 \\ -20.0\,(3log_{10} - 0.1 + log_1 00.5) & 0.5 \le \omega < 10 \\ -20.0\,(2log_{10} + 0.9 + log_1 00.5) & 10 \le \omega < 100 \\ -20.0\,(3log_{10} - 1.9 + log_1 00.5) & \omega \ge 100 \end{cases}$$

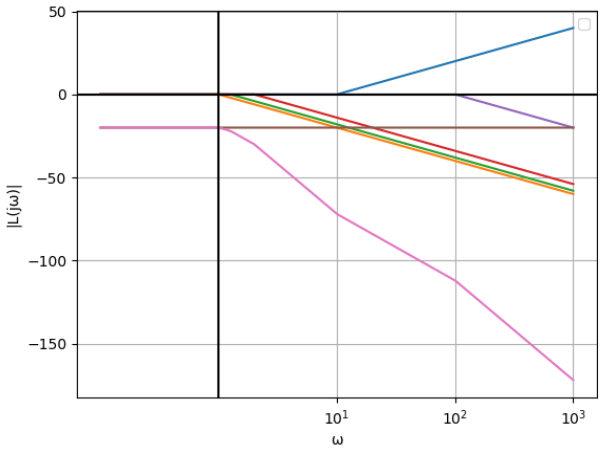

Fig. 1: Pink Line = Bode magnitude plot of loop gain

Slope of Bode magnitude plot (at $\omega = 3$) $= -60$ dB/decade