# NCERT 10.5.3 10Q

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**Question:** Show that $a_0$ , $a_1$ , $a_2$ , . . ., $a_n$ , . . . form an AP where an is defined as below :

1) $a_n = (3 + 4n)$
2) $a_n = (9 - 5n)$

Also find the sum of the first 15 terms in each case.

**Solution:**

| Parameter | Description | Value |
|---|---|---|
| $x_i(n)$ | $i^{th}$Discrete signal | $(3 + 4n)u(n)$ |
| | | $(9 - 5n)u(n)$ |
| $x_i(0)$ | First term of $i^{th}$AP | 3 |
| | | 9 |
| $d_i$ | common difference of $i^{th}$AP | 4 |
| | | $-5$ |

TABLE 2: Given parameters

1) From equation (??)

$$X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \tag{1}$$

$$\because y(n) = x(n) * u(n) \tag{2}$$

$$Y(z) = X(z)U(z) \tag{3}$$

$$= \left[\frac{3}{(1 - z^{-1})^2} + \frac{4z^{-1}}{(1 - z^{-1})^3}\right] \tag{4}$$

Using contour integration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{5}$$

$$= \frac{1}{2\pi j}\int \frac{3.z^{15}}{(z - 1)^2}dz + \frac{1}{2\pi j}\int \frac{4.z^{15}}{(z - 1)^3}dz \tag{6}$$

$$\because R = \frac{1}{(m - 1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z - a)^m f(z)\right) \tag{7}$$

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z - 1)^2.\frac{3.z^{15}}{(z - 1)^2}\right) \tag{8}$$

$$= 45 \tag{9}$$

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z - 1)^3.\frac{4.z^{15}}{(z - 1)^3}\right) \tag{10}$$

$$= 420 \tag{11}$$

$$\implies y(14) = R_1 + R_2 \tag{12}$$

$$= 465 \tag{13}$$

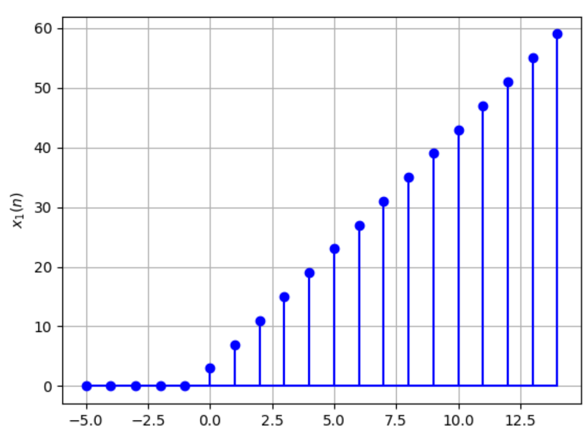


Fig. 1: $x_1(n) = (3 + 4n)u(n)$

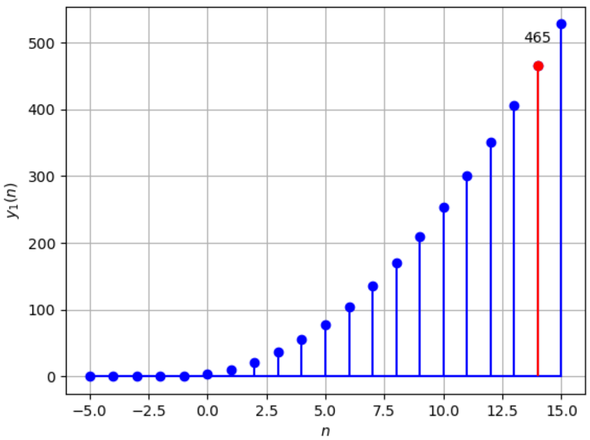


Fig. 1: $x_1(n) = (2n^2 + 5n + 3)u(n)$

2) From equation (??)

$$X(z) = \frac{9}{1-z^{-1}} - \frac{5.z^{-1}}{(1-z^{-1})^2}; |z| > 1 \tag{14}$$

$$\because y(n) = x(n) * u(n) \tag{15}$$

$$Y(z) = X(z)U(z) \tag{16}$$

$$= \left[\frac{9}{(1-z^{-1})^2} - \frac{5z^{-1}}{(1-z^{-1})^3}\right] \tag{17}$$

Using contour integration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{18}$$

$$= \frac{1}{2\pi j}\int \frac{9.z^{15}}{(z-1)^2}dz - \frac{1}{2\pi j}\int \frac{5.z^{15}}{(z-1)^3} \tag{19}$$

$$\because R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}((z-a)^m f(z)) \tag{20}$$

$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2.\frac{9.z^{15}}{(z-1)^2}\right) \tag{21}$$

$$= 135 \tag{22}$$

$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3.\frac{5.z^{15}}{(z-1)^3}\right) \tag{23}$$

$$= 525 \tag{24}$$

$$\implies y(14) = R_1 - R_2 \tag{25}$$

$$= -390 \tag{26}$$

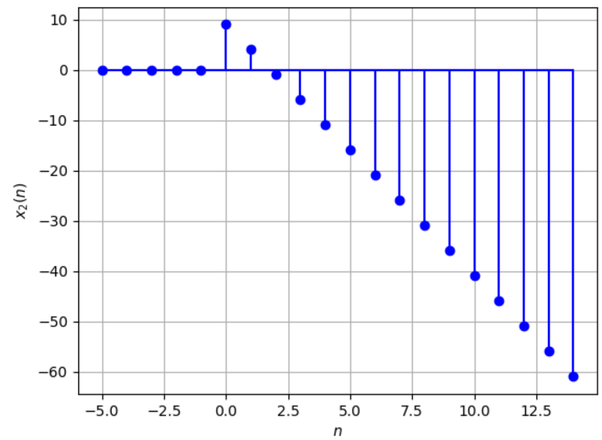


Fig. 2: $x_2(n) = (9 - 5n)u(n)$

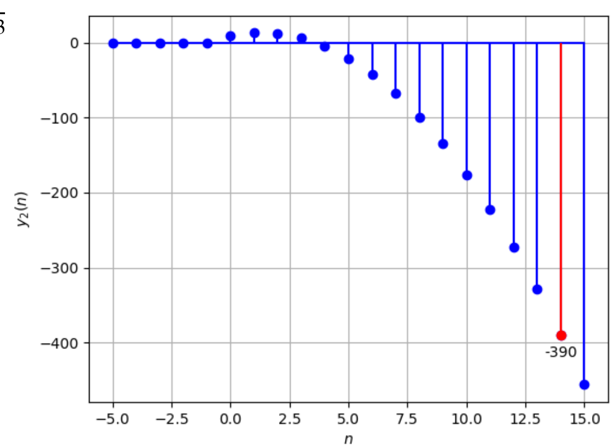


Fig. 2: $x_2(n) = (-5n^2 + 13n + 18)u(n)$