GATE 2021 EE.20

EE23BTECH11010 - VENKATESH BANDAWAR*

I. Laplace Transform

Laplace transform of integrals:
Let the function defined as $y(t) = \int_0^t f(u)du$ for all $t > 0$
Laplace transform of y(t) in t

$$\mathcal{L}(y(t)) = \int_0^\infty e^{-st} y(t)dt \tag{1}$$

$$= \int_0^\infty e^{-st} \int_0^t f(u)dudt \tag{2}$$

$$= \int_0^t f(u)du \left[-\frac{e^{-st}}{s}\right]_0^\infty + \int_0^\infty \frac{e^{-st}}{s} f(t)dt \tag{3}$$

$$= \frac{F(s)}{s} \tag{4}$$

II. RLC Low Pass Filter

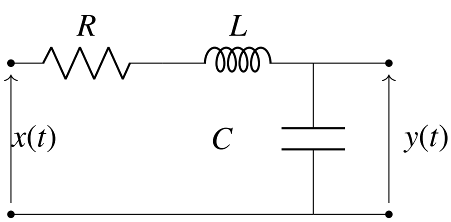

Fig. 1: RLC Low pass filter

$$Y(s) = I(s)Z_C \tag{5}$$

$$= \frac{X(s)}{Z_L + Z_R + Z_C} Z_C \tag{6}$$

$$H(s) = \frac{Y(s)}{X(s)} \tag{7}$$

$$= \frac{Z_C}{Z_L + Z_R + Z_C} \tag{8}$$

$$= \frac{\frac{1}{sC}}{sL + R + \frac{1}{sC}} \tag{9}$$

$$= \frac{1}{s^2LC + sRC + 1} \tag{10}$$

$$\implies H(s) = \omega_0^2 \frac{1}{(s-p_1)(s-p_2)} \tag{11}$$

where,

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{12}$$

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{13}$$

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \tag{14}$$

where

$$\alpha = \frac{R}{2L} \tag{15}$$

Damping Factor is given by,

$$\zeta = \frac{\alpha}{\omega_0} \tag{16}$$

$$= \frac{R}{2}\sqrt{\frac{C}{L}} \tag{17}$$

ζ	**Pole Location**	**Referred to as**	**Condition**
$\zeta > 1$	Different locations on the negative real axis	Overdamped	$R > 2\sqrt{\frac{L}{C}}$
$\zeta = 1$	Coincide on the negative real axis	Critically Damped	$R = 2\sqrt{\frac{L}{C}}$
$\zeta < 1$	Complex Conjugate poles in the left half of s-plane	Underdamped	$R < 2\sqrt{\frac{L}{C}}$

TABLE I: Effect of Damping Coefficient ζ on system behaviour

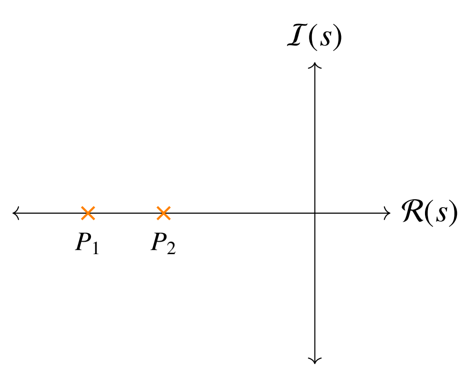

Fig. 2: s-Plane for Overdamped case

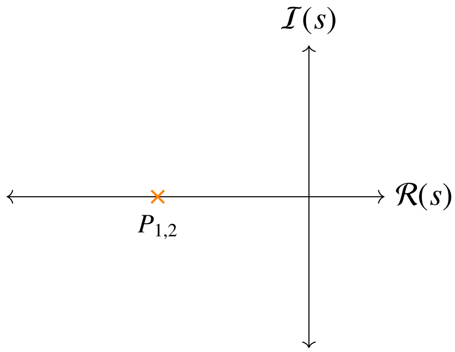

Fig. 3: s-Plane for Critically damped case

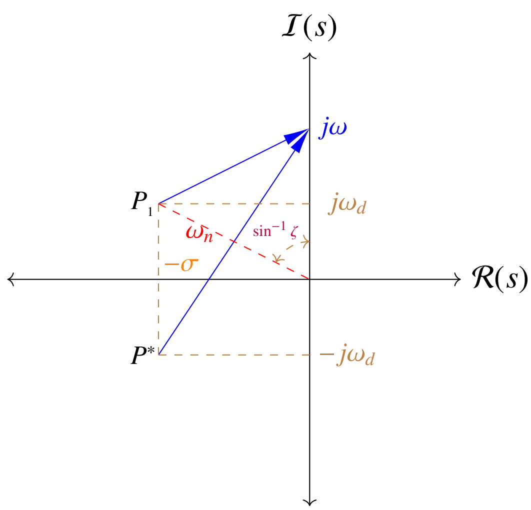

Fig. 4: s-Plane for Under damped case