

# GATE 2022 IN.53

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**Question:** In a unity-gain feedback control system, the plant  $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$  is controlled by a lag compensator  $C(s) = \frac{s+10}{s+0.1}$ . The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at  $\omega = 3\text{rad/s}$  is \_\_\_\_\_ (in integer) (GATE 2022 IN)

**Solution:**

Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(\frac{s}{0.5}+1)(\frac{s}{100}+1)}$
$C(s)$	Lag Compensator	$\frac{100(\frac{s}{10}+1)}{\frac{s}{0.1}+1}$
$T(s)$	Loop gain	$P(s)C(s)$
$\omega$	Angular Frequency	3rad/s

TABLE I: Given Parameters list

$$|T(s)| = \frac{0.1 \left( \frac{s}{10} + 1 \right)}{s \left( \frac{s}{0.5} + 1 \right) \left( \frac{s}{100} + 1 \right) \left( \frac{s}{0.1} + 1 \right)} \quad (1)$$

Here, 10, 0, 0.5, 100, 0.1 are corner frequencies of loop gain  $L(s)$

Corner Frequency	Description	Change in slope
10	Zero	$20\text{dB/dec}$
0.1	Pole	$-20\text{dB/dec}$
0.5	Pole	$-20\text{dB/dec}$
100	Pole	$-20\text{dB/dec}$

TABLE II: Caption

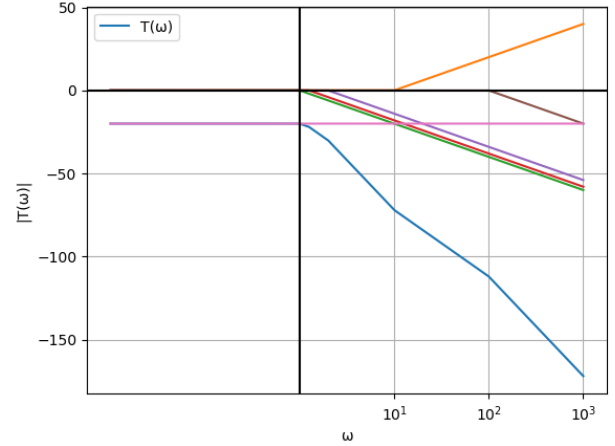


Fig. 1: Pink Line = Bode magnitude plot of loop gain

$$\text{Gain}(K) = \lim_{s \rightarrow 0} sT(s) \quad (2)$$

$$K = 0.1 \quad (3)$$

$$|T(s)| = 20 \log_{10} K \quad (4)$$

$$= -20\text{dB} \quad (5)$$

$$T(\omega) = \begin{cases} -20 \log_{10}(\omega) & \omega < 0.1 \\ -20.0(2 \log_{10}(\omega) - 0.1) & 0.1 \leq \omega < 0.5 \\ -20.0(3 \log_{10} \omega - 0.1 + \log_{10} 0.5) & 0.5 \leq \omega < 10 \\ -20.0(2 \log_{10} \omega + 0.9 + \log_{10} 0.5) & 10 \leq \omega < 100 \\ -20.0(3 \log_{10} \omega - 1.9 + \log_{10} 0.5) & \omega \geq 100 \end{cases}$$

Slope of Bode magnitude plot (at  $\omega = 3$ ) = -60 dB/decade