NCERT 10.5.3 10Q

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Question: Show that $x(0)$, $x(1)$, $x(2)$, . . ., $x(n)$, . . . form an AP where an is defined as below :
(i) $x(n) = (3 + 4n)u(n)$
(ii) $x(n) = (9 - 5n)u(n)$
Also find the sum of the first 15 terms in each case.

Answer:

(i) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = x(n+1) - x(n) \tag{1}$$

$$= (3 + 4(n+1)) u(n+1) - (3 + 4n) u(n)$$

$$= 4 \tag{2}$$

$\because$ Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'

Hence given sequence is in AP.

Parameter	Description	Value
$x(n)$	Discrete signal	$(3 + 4n).u(n)$
$x(0)$	first term	3
d	common differnce	4

TABLE (i): *Given parameters in* 1^{st} *AP*

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{3}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)\, z^{-n} \tag{4}$$

$$nu(n) \xleftrightarrow{Z} -z.U'(z) \tag{5}$$

$$\implies X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \tag{6}$$

$$\because y(n) = x(n) * u(n) \tag{7}$$

$$y(n) \xleftrightarrow{Z} Y(z)$$

$$\implies Y(z) = \left[\frac{3}{1 - z^{-1}} + \frac{4z^{-1}}{(1 - z^{-1})^2}\right] . \frac{1}{1 - z^{-1}} \tag{8}$$

Using contour integeration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j} \int Y(z) z^{13} dz \tag{9}$$

$$y(14) = \frac{1}{2\pi j} \int \frac{3.z^{15}}{(z-1)^2} dz + \frac{1}{2\pi j} \int \frac{4.z^{15}}{(z-1)^3} dz \tag{10}$$

$$\implies R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \tag{11}$$

$$\implies R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 . \frac{3.z^{15}}{(z-1)^2} \right) \tag{12}$$

$$R_1 = 45 \tag{13}$$

$$\implies R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 . \frac{4.z^{15}}{(z-1)^3} \right) \tag{14}$$

$$R_2 = 420 \tag{15}$$

$$\implies y(14) = R_1 + R_2$$

$$= 465 \tag{16}$$

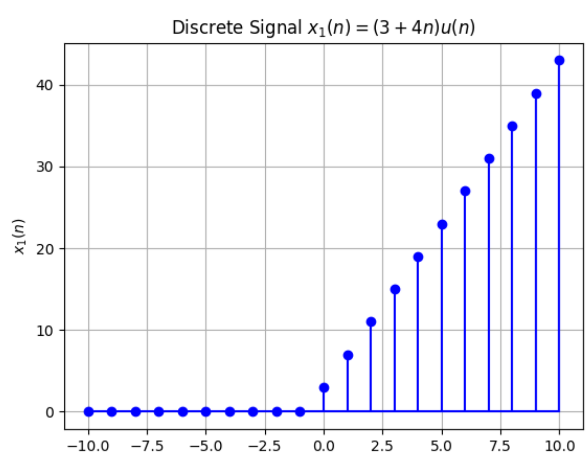

Fig. (i)

(ii) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = x(n+1) - x(n) \tag{17}$$

$$= (9 - 5(n+1))\,u\,(n+1) - (9 - 5n)\,u\,(n)$$

$$= -5 \tag{18}$$

$\because$ Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'

Hence given sequence is in AP.

Parameter	Description	Value
$x(n)$	Discrete signal	$(9-5n).u(n)$
$x(0)$	first term	9
d	common differnce	-5

TABLE (ii): *Given parameters in* 2^{st} *AP*

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{19}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} [9-5n].u(n)\,z^{-n} \tag{20}$$

$$nu(n) \xleftrightarrow{Z} -z.U'(z) \tag{21}$$

$$\implies X(z) = \frac{9}{1-z^{-1}} - \frac{5.z^{-1}}{(1-z^{-1})^2}; |z| > 1 \tag{22}$$

$$\because y(n) = x(n) * u(n) \tag{23}$$

$$y(n) \xleftrightarrow{Z} Y(z)$$

$$\implies Y(z) = \left[\frac{9}{1-z^{-1}} - \frac{5z^{-1}}{(1-z^{-1})^2}\right].\frac{1}{1-z^{-1}} \tag{24}$$

Using contour integeration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{25}$$

$$y(14) = \frac{1}{2\pi j}\int \frac{9.z^{15}}{(z-1)^2}dz - \frac{1}{2\pi j}\int \frac{5.z^{15}}{(z-1)^3}dz \tag{26}$$

$$\implies R = \frac{1}{(m-1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}((z-a)^m f(z)) \tag{27}$$

$$\implies R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z-1)^2.\frac{9.z^{15}}{(z-1)^2}\right) \tag{28}$$

$$R_1 = 135 \tag{29}$$

$$\implies R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z-1)^3.\frac{5.z^{15}}{(z-1)^3}\right) \tag{30}$$

$$R_2 = 525 \tag{31}$$

$$\implies y(14) = R_1 - R_2$$

$$= -390 \tag{32}$$

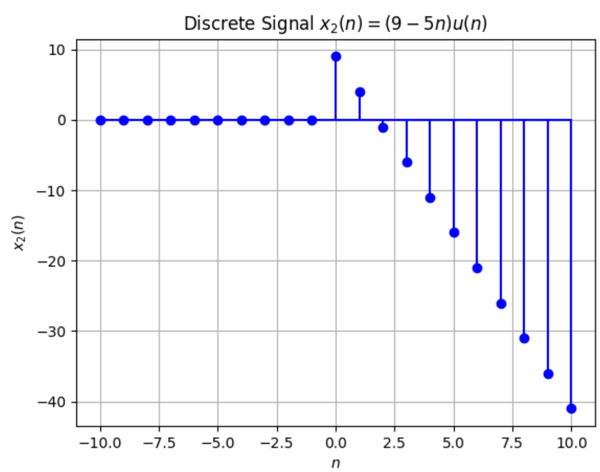

Fig. (ii)