NCERT 10.5.3 10Q

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Question: Show that a_1 , a_2 , . . ., a_n , . . . form an AP where an is defined as below :
(i) $a_n = 3 + 4n$
(ii) $a_n = 9 - 5n$
Also find the sum of the first 15 terms in each case.

Answer:

(i) Given: $a_n = 3 + 4n$
we know that, The AP has constant common difference between two consecutive terms

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \quad (1)$$
$$= (3 + 4(n + 1)) - (3 + 4n)$$
$$= 4 \quad (2)$$

$\because$ Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
a_n	n^{th} term	$3 + 4n$
a_0	first term	3
d	common differnce	4
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	465

TABLE (i): *Given parameters in* 1^{st} *AP*

Z - Transformation of a_n :

$$\implies X(z) = \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)\, z^{-n} \quad (3)$$

$$\implies X(z) = 3\sum_{n=0}^{\infty} 1.z^{-n} + 4\sum_{n=0}^{\infty} 1.n.z^{-n} \quad (4)$$

$$\implies X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

$\{Where, |z| > 1\}$

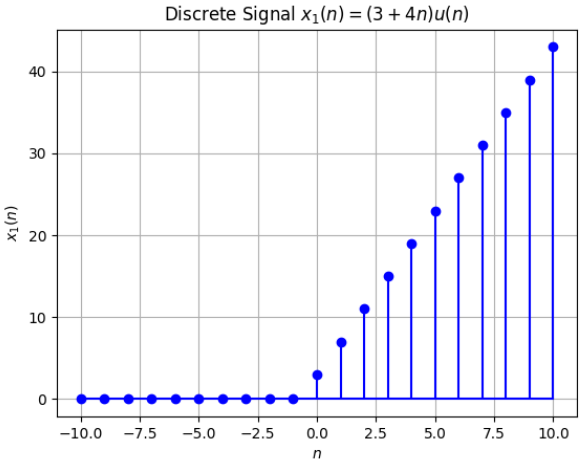

Fig. (i)

(ii) Given: $a_n = 9 - 5n$
we know that, The AP has constant common difference between two consecutive terms

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \quad (6)$$
$$= (9 - 5(n + 1)) - (9 - 5n)$$
$$= -5 \quad (7)$$

$\because$ Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
a_n	n^{th} term	$9 - 5n$
a_0	first term	9
d	common differnce	-5
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	-390

TABLE (ii): *Given parameters in* 2^{st} *AP*

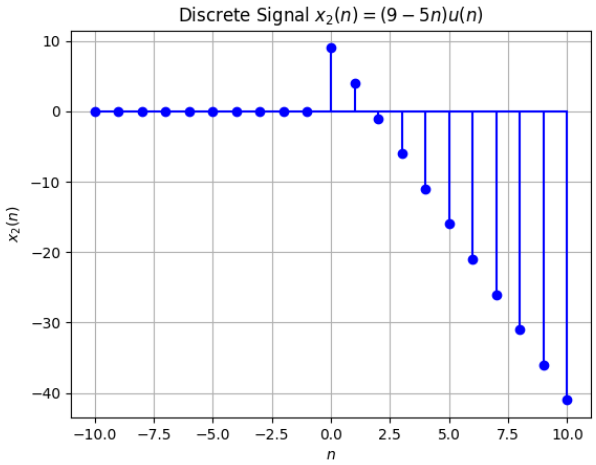

Fig. (ii)

Z - Transformation of a_n :

$$\implies X(Z) = \sum_{n=-\infty}^{\infty} [9 - 5n].u(n)\, z^{-n} \tag{8}$$

$$\implies X(z) = 9 \sum_{n=0}^{\infty} 1.z^{-n} - 5 \sum_{n=0}^{\infty} 1.n.z^{-n} \tag{9}$$

$$\implies X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2} \tag{10}$$

$\{Where, |z| > 1\}$