GATE: CE - 30.2023

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Question: In the differential equation $\frac{dy}{dx} + \alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is (rounded off to three decimal places). (GATE CE 2023)

Solution:

Parameter	**Value**
x	0.0
	1.0
y	1.0
	0.8

TABLE I: Given parameters

let, t=x

$$\frac{dy}{dt} + \alpha ty = 0 \tag{1}$$

Taking fourier transform,

where, $\mathcal{F}\{\frac{dy}{dt}\} = 2\pi f jY(f)$

$\mathcal{F}\{a \cdot t \cdot y(t)\} = a \cdot j\frac{d}{df}Y(f)$

$$\frac{2\pi f}{\alpha}Y(f) + \frac{d}{df}Y(f) = 0 \tag{2}$$

I.F.= $e^{\int \frac{2\pi}{\alpha} f df} = e^{\frac{\pi}{\alpha} f^2}$

$$e^{\frac{\pi}{\alpha} f^2}Y(f) = K \tag{3}$$

$$Y(f) = Ke^{-\frac{\pi}{\alpha} f^2} \tag{4}$$

Taking inverse fourier transform, since,

$$\mathcal{F}^{-1}\{e^{-af^2}\} = \sqrt{\frac{\pi}{a}} \cdot e^{-\frac{\pi^2}{a}t^2}$$

$$y(t) = K\sqrt{\alpha}e^{-\pi\alpha t^2} \tag{5}$$

$$\frac{y(0)}{y(1)} = \frac{1}{e^{-\pi\alpha}} \tag{6}$$

$$\ln\frac{5}{4} = \pi\alpha \tag{7}$$

$$\alpha = \frac{0.223}{\pi} \tag{8}$$