GATE 2021 EE.20

EE23BTECH11010 - VENKATESH BANDAWAR*

I. Frequency Response

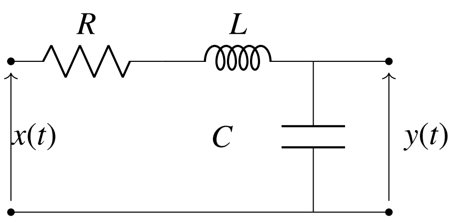

Fig. 1: RLC Low pass filter

The frequency response $y_{ss}(t)$ is defined as the steady state response to a sinusoidal input signal $x(t) = \sin \omega t$. It describes how well the filter can distinguish between different frequencies.

$$y_{ss}(t) = |H(j\omega)| \sin(\omega t + \angle H(j\omega)) \tag{1}$$

1) Overdamped Case

$$H(s) = \omega_0^2 \frac{1}{(s-p_1)(s-p_2)} \tag{2}$$

where,

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \tag{3}$$

$$H(s) = |H(s)| \; e^{j\angle H(s)} \tag{4}$$

$$|H(s)| = \omega_0^2 \frac{1}{|s-p_1|\,|s-p_2|} \tag{5}$$

$$= \omega_0^2 \frac{1}{|j\omega - p_2| \; |j\omega - p_2|} \tag{6}$$

$$= \omega_0^2 \frac{1}{\sqrt{\omega^2 + {p_1}^2}\sqrt{\omega^2 + {p_2}^2}} \tag{7}$$

$$\angle H(s) = -(\angle(s-p_1) + \angle(s-p_2)) \tag{8}$$

$$= \tan^{-1}\frac{\omega}{p_1} + \tan^{-1}\frac{\omega}{p_2} \tag{9}$$

The magnitude of the transfer function expressed on a logarithmic scale:

$$|H_{dB}(\omega)| = 20\log(\omega_0^2) - 20\log\sqrt{\omega^2 + {p_1}^2} - 20\log\sqrt{\omega^2 + {p_2}^2} \tag{10}$$

2) Critically damped case

$$H(s) = \omega_0^2 \frac{1}{(s-p)^2} \tag{11}$$

where,

$$p = \sqrt{\frac{1}{LC}} \tag{12}$$

$$H(s) = |H(s)| \; e^{j\angle H(s)} \tag{13}$$

$$|H(s)| = \omega_0^2 \frac{1}{|s-p|^2} \tag{14}$$

$$|H(j\omega)| = \omega_0^2 \frac{1}{|j\omega - p|^2} \tag{15}$$

$$= \omega_0^2 \frac{1}{\omega^2 + p^2} \tag{16}$$

$$\angle H(s) = -(\angle(s-p) + \angle(s-p)) \tag{17}$$

$$= 2\tan^{-1}\frac{\omega}{p} \tag{18}$$

The magnitude of the transfer function expressed on a logarithmic scale:

$$|H_{dB}(\omega)| = 20\log(\omega_0^2) - 20\log(\omega^2 + p^2) \tag{19}$$

3) Underdamped Case

$$H(s) = \omega_0^2 \frac{1}{(s-p)(s-p^*)} \tag{20}$$

where,

$$p,\ p^* = \omega_n\left(-\zeta \pm j\sqrt{1-\zeta^2}\right) \tag{21}$$

$$= -\sigma \pm j\omega_d \tag{22}$$

$$H(s) = |H(s)| \; e^{j\angle H(s)} \tag{23}$$

$$|H(s)| = \omega_0^2 \frac{1}{\sqrt{({\omega_n}^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \tag{24}$$

$$\angle H(s) = -(\angle(s-p) + \angle(s-p^*)) \tag{25}$$

$$= -\tan^{-1}\frac{\omega + \omega_d}{\sigma} - \tan^{-1}\frac{\omega - \omega_d}{\sigma} \tag{26}$$

The magnitude of the transfer function expressed on a logarithmic scale:

$$|H_{dB}(\omega)| = 20\log(\omega_0^2) - 10\log(({\omega_n}^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2) \tag{27}$$

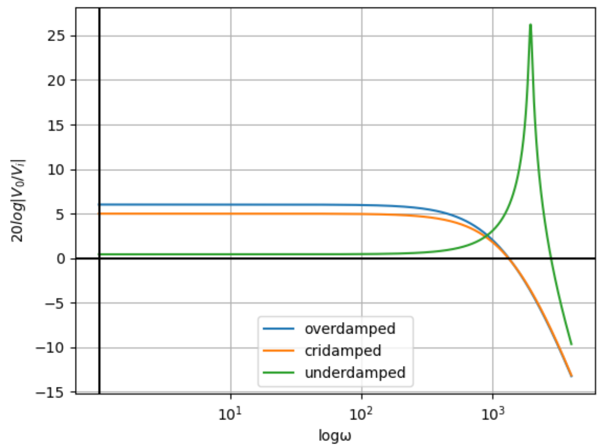

Fig. 2: Frequency response of all cases

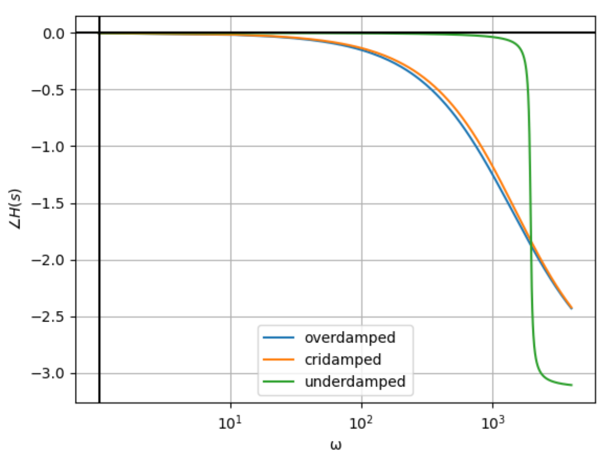

Fig. 3: Plot of $\angle H(s)$