NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series , whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

From equation (**??**) to (**??**),

$$X(z) = \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4} + \frac{5z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})} \tag{1}$$

$$Y(z) = X(z)U(z) \tag{2}$$

$$= \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^5} + \frac{5z^{-1}\left(z^{-1}+1\right)}{(1-z^{-1})^4} + \frac{4z^{-1}}{(1-z^{-1})} \tag{3}$$

$$= \frac{1}{4}\left[\frac{z^{-1}\left(1+11z^{-1}+11z^{-2}+z^{-3}\right)}{(1-z^{-1})^5}\right] + \frac{13}{6}\left[\frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4}\right] + \frac{19}{4}\left[\frac{z^{-1}\left(1+z^{-1}\right)}{(1-z^{-1})^3}\right] + \frac{17}{6}\left[\frac{z^{-1}}{(1-z^{-1})^2}\right]\{|z|>1\} \tag{4}$$

where,

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1-z^{-1})^2}\{|z|>1\} \tag{5}$$

$$n^2u(n) \xleftrightarrow{Z} \frac{z^{-1}\left(1+z^{-1}\right)}{(1-z^{-1})^3}\{|z|>1\} \tag{6}$$

$$n^3u(n) \xleftrightarrow{Z} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{(1-z^{-1})^4}\{|z|>1\} \tag{7}$$

$$n^4u(n) \xleftrightarrow{Z} \frac{z^{-1}\left(1+11z^{-1}+11z^{-2}+z^{-3}\right)}{(1-z^{-1})^5}\{|z|>1\} \tag{8}$$

Taking reverse z transform, using equations (5) to (8)

$$y(n) = \left(\frac{n^4}{4} + \frac{13n^3}{6} + \frac{19n^2}{4} + \frac{17n}{6}\right)u(n) \tag{9}$$

$$= \left(\frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}\right)u(n) \tag{10}$$

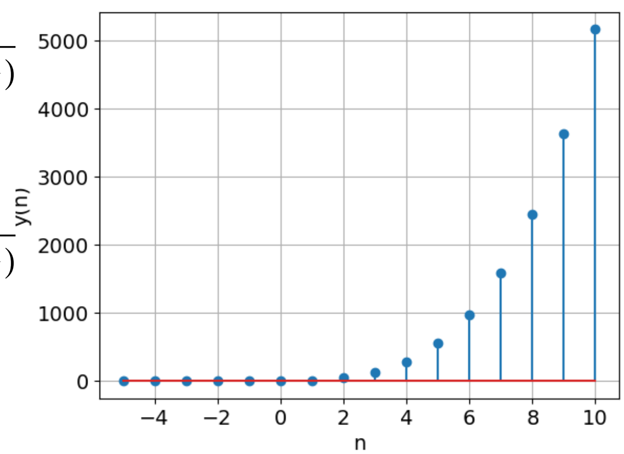

Fig. 0: Sum of n terms of series