NCERT 10.5.3 10Q

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Question: Show that a_0 , a_1 , a_2 , . . ., a_n , . . . form an AP where an is defined as below :

1) $a_n = (3 + 4n)$
2) $a_n = (9 - 5n)$

Also find the sum of the first 15 terms in each case.

Answer:

(i) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \tag{1}$$
$$= (3 + 4(n + 1)) - (3 + 4n) \tag{2}$$
$$= 4 \tag{3}$$

$\because$ Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'
Hence given sequence is in AP.

Parameter	Description	Value
a_n	General term of AP	$(3 + 4n)$
$x(n)$	Discrete signal	$(3 + 4n)u(n)$
a_0	first term	3
d	common differnce	4

TABLE (i): Given parameters in 1^{st} AP

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \tag{4}$$
$$= \sum_{n=-\infty}^{\infty} [3 + 4n].u(n)\, z^{-n} \tag{5}$$
$$nu(n) \overset{Z}{\longleftrightarrow} -z.U'(z) \tag{6}$$
$$X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2}; |z| > 1 \tag{7}$$
$$\because y(n) = x(n) * u(n) \tag{8}$$
$$Y(z) = X(z)U(z) \tag{9}$$
$$= \left[\frac{3}{1 - z^{-1}} + \frac{4z^{-1}}{(1 - z^{-1})^2}\right].\frac{1}{1 - z^{-1}} \tag{10}$$

Using contour integeration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z)z^{13}dz \tag{11}$$
$$= \frac{1}{2\pi j}\int \frac{3.z^{15}}{(z - 1)^2}dz + \frac{1}{2\pi j}\int \frac{4.z^{15}}{(z - 1)^3}dz \tag{12}$$
$$\because R = \frac{1}{(m - 1)!}\lim_{z\to a}\frac{d^{m-1}}{dz^{m-1}}\left((z - a)^m f(z)\right) \tag{13}$$
$$R_1 = \frac{1}{1!}\lim_{z\to 1}\frac{d}{dz}\left((z - 1)^2.\frac{3.z^{15}}{(z - 1)^2}\right) \tag{14}$$
$$= 45 \tag{15}$$
$$R_2 = \frac{1}{2!}\lim_{z\to 1}\frac{d^2}{dz^2}\left((z - 1)^3.\frac{4.z^{15}}{(z - 1)^3}\right) \tag{16}$$
$$= 420 \tag{17}$$
$$\implies y(14) = R_1 + R_2 \tag{18}$$
$$= 465 \tag{19}$$

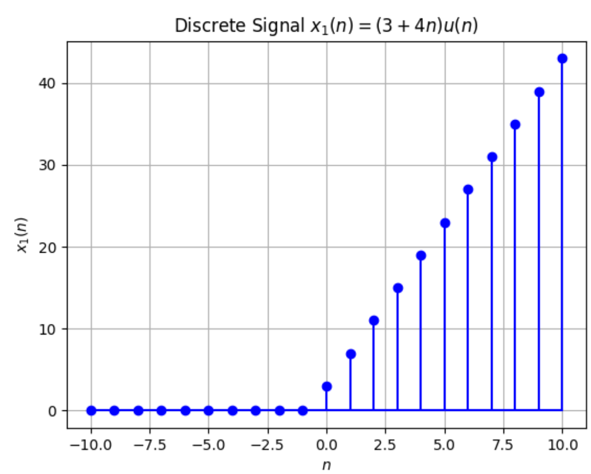

Fig. (i): $x_1(n) = (3 + 4n)u(n)$

(ii) We know that, The AP has constant common difference between two consecutive terms.

$$\therefore \text{common difference } (d) = a_{n+1} - a_n \quad (20)$$
$$= (9 - 5(n+1)) - (9 - 5n) \quad (21)$$
$$= -5 \quad (22)$$

$\because$ Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'
Hence given sequence is in AP.

Parameter	Description	Value
a_n	General term of AP	$(9-5n)$
$x(n)$	Discrete signal	$(9-5n)u(n)$
a_0	first term	9
d	common differnce	-5

TABLE (ii): Given parameters in 2^{st} AP

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \quad (23)$$
$$= \sum_{n=-\infty}^{\infty} [9 - 5n].u(n)\, z^{-n} \quad (24)$$
$$nu(n) \overset{\mathcal{Z}}{\longleftrightarrow} -z.U'(z) \quad (25)$$
$$X(z) = \frac{9}{1-z^{-1}} - \frac{5.z^{-1}}{(1-z^{-1})^2}; |z| > 1 \quad (26)$$
$$\because y(n) = x(n) * u(n) \quad (27)$$
$$Y(z) = X(z)U(z) \quad (28)$$
$$= \left[\frac{9}{1-z^{-1}} - \frac{5z^{-1}}{(1-z^{-1})^2}\right].\frac{1}{1-z^{-1}} \quad (29)$$

Using contour integeration for inverse Z transformation,

$$y(14) = \frac{1}{2\pi j}\int Y(z) z^{13} dz \quad (30)$$
$$= \frac{1}{2\pi j}\int \frac{9.z^{15}}{(z-1)^2} dz - \frac{1}{2\pi j}\int \frac{5.z^{15}}{(z-1)^3} dz \quad (31)$$
$$\because R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \quad (32)$$
$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 . \frac{9.z^{15}}{(z-1)^2}\right) \quad (33)$$
$$= 135 \quad (34)$$
$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 . \frac{5.z^{15}}{(z-1)^3}\right) \quad (35)$$
$$= 525 \quad (36)$$
$$\implies y(14) = R_1 - R_2 \quad (37)$$
$$= -390 \quad (38)$$

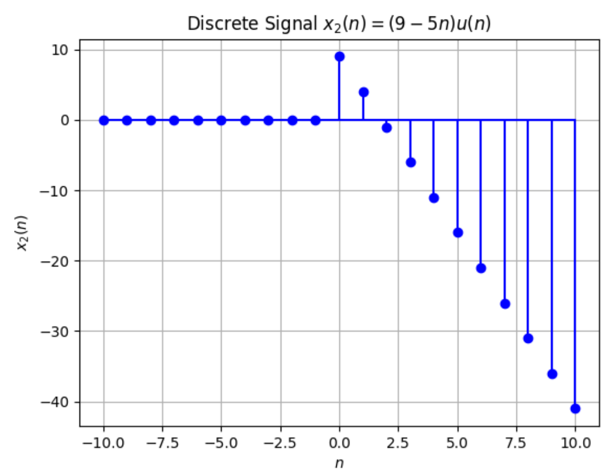

Fig. (ii): $x_2(n) = (9 - 5n)u(n)$