NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series , whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

From equation (**??**) to (**??**),

$$X(z) = \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4} + \frac{5z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^3} + \frac{4z^{-1}}{\left(1-z^{-1}\right)^2} \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

$$= \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^5} + \frac{5z^{-1}\left(z^{-1}+1\right)}{\left(1-z^{-1}\right)^4} + \frac{4z^{-1}}{\left(1-z^{-1}\right)^3} \quad (3)$$

$$= \frac{1}{4}\left[\frac{z^{-1}\left(1+11z^{-1}+11z^{-2}+z^{-3}\right)}{\left(1-z^{-1}\right)^5}\right] + \frac{13}{6}\left[\frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}\right] + \frac{19}{4}\left[\frac{z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}\right] + \frac{17}{6}\left[\frac{z^{-1}}{\left(1-z^{-1}\right)^2}\right]\{|z|>1\} \quad (4)$$

where,

$$nu(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}}{\left(1-z^{-1}\right)^2}\{|z|>1\} \quad (5)$$

$$n^2u(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}\{|z|>1\} \quad (6)$$

$$n^3u(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}\{|z|>1\} \quad (7)$$

$$n^4u(n) \xleftrightarrow{\mathcal{Z}} \frac{z^{-1}\left(1+11z^{-1}+11z^{-2}+z^{-3}\right)}{\left(1-z^{-1}\right)^5}\{|z|>1\} \quad (8)$$

Taking reverse z transform, using equations (5) to (8)

$$y(n) = \left(\frac{n^4}{4}+\frac{13n^3}{6}+\frac{19n^2}{4}+\frac{17n}{6}\right)u(n) \quad (9)$$

$$= \left(\frac{n^4}{4}+\frac{2n^3}{4}+\frac{10n^3}{6}+\frac{n^2}{4}+\frac{15n^2}{6}+\frac{4n^2}{2}+\frac{5n}{6}+\frac{4n}{2}\right)u(n) \quad (10)$$

$$= \left(\frac{n^4+2n^3+n^4}{4}\right)u(n)+\left(\frac{10n^3+15n^2+5n}{6}\right)u(n) + \left(\frac{4n^2+4n}{2}\right)u(n) \quad (11)$$

$$= \left(\frac{n^2(n+1)^2}{4}+\frac{5n(n+1)(2n+1)}{6}+\frac{4n(n+1)}{2}\right)u(n) \quad (12)$$

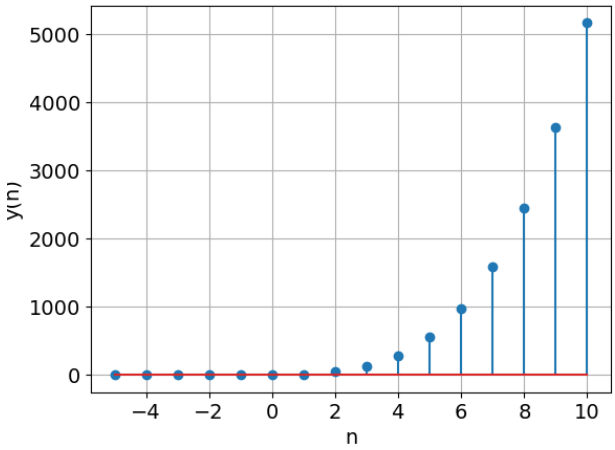

Fig. 0: Sum of n terms of series