

GATE: CH - 34.2022

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Question: A process described by the transfer function

$$G_p(s) = \frac{(10s + 1)}{(5s + 1)}$$

is forced by a unit step input at time $t = 0$. The output value immediately after the unit step input (at $t = 0^+$) is ? (Gate 2022 CH 34)

Solution:

Parameters	Description
$X(s)$	Laplace transform of $x(t)$
$Y(s)$	Laplace transform of $y(t)$
$G_p(s) = \frac{Y(s)}{X(s)}$	Transfer function
$x(t) = u(t)$	unit step function

TABLE I: Given parameters

$$G_p(s) = \frac{Y(s)}{X(s)} = \frac{(10s + 1)}{(5s + 1)} \quad (1)$$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \quad (2)$$

From equation (2):

$$Y(s) = \frac{(10s + 1)}{s(5s + 1)} \quad (3)$$

$$= \frac{1}{s} + \frac{5}{5s + 1} \quad (4)$$

Taking inverse laplace transformation,

$$\frac{1}{s} \xleftrightarrow{\mathcal{L}^{-1}} u(t) \quad (5)$$

$$\frac{1}{s - c} \xleftrightarrow{\mathcal{L}^{-1}} e^{ct} u(t) \quad (6)$$

$$y(t) = \left(1 + e^{-\frac{t}{5}}\right) u(t) \quad (7)$$

$$y(0^+) = 2 \quad (8)$$

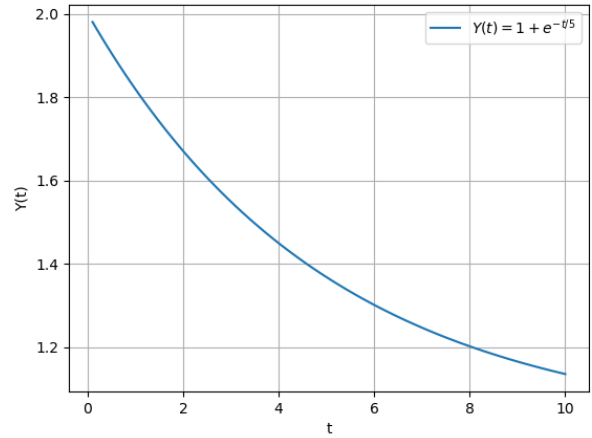


Fig. 1: Graph of $y(t)$