

NCERT 11.9.4 8Q

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Question: Find the sum to n terms of series ,
whose n^{th} term is : $n(n+1)(n+4)$.

Solution

Parameter	Description	Value
$x(n)$	n^{th} term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

From equation (??) to (??),

$$X(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})^2} \{ |z| > 1 \} \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

$$= \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{4z^{-1}}{(1-z^{-1})^3} \quad (3)$$

$$= \frac{1}{4} \left[\frac{z^{-1}(1+11z^{-1}+11z^{-2}+z^{-3})}{(1-z^{-1})^5} \right] + \frac{13}{6} \left[\frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} \right] + \frac{19}{4} \left[\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \right] + \frac{17}{6} \left[\frac{z^{-1}}{(1-z^{-1})^2} \right] \{ |z| > 1 \} \quad (4)$$

where,

$$nu(n) \xleftrightarrow{z} \frac{z^{-1}}{(1-z^{-1})^2} \{ |z| > 1 \} \quad (5)$$

$$n^2u(n) \xleftrightarrow{z} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3} \{ |z| > 1 \} \quad (6)$$

$$n^3u(n) \xleftrightarrow{z} \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} \{ |z| > 1 \} \quad (7)$$

$$n^4u(n) \xleftrightarrow{z} \frac{z^{-1}(1+11z^{-1}+11z^{-2}+z^{-3})}{(1-z^{-1})^5} \{ |z| > 1 \} \quad (8)$$

Taking reverse z transform, using equations (5) to (8)

$$y(n) = \left(\frac{n^4}{4} + \frac{13n^3}{6} + \frac{19n^2}{4} + \frac{17n}{6} \right) u(n) \quad (9)$$

$$= \left(\frac{n^4}{4} + \frac{2n^3}{4} + \frac{10n^3}{6} + \frac{n^2}{4} + \frac{15n^2}{6} + \frac{4n^2}{2} + \frac{5n}{6} + \frac{4n}{2} \right) u(n) \quad (10)$$

$$= \left(\frac{n^4 + 2n^3 + n^4}{4} \right) u(n) + \left(\frac{10n^3 + 15n^2 + 5n}{6} \right) u(n) + \left(\frac{4n^2 + 4n}{2} \right) u(n) \quad (11)$$

$$= \left(\frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} \right) u(n) \quad (12)$$

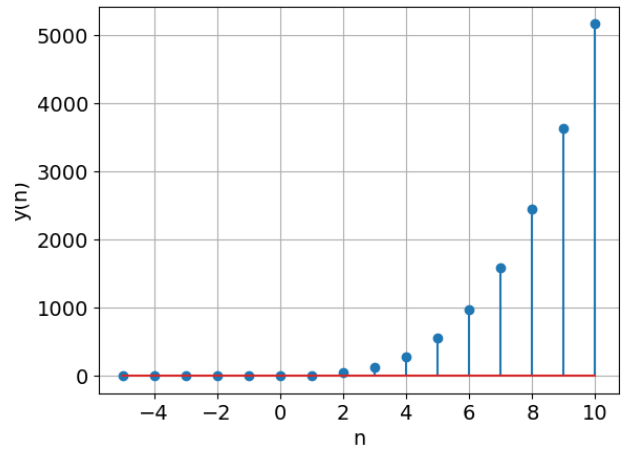


Fig. 0: Sum of n terms of series