

# GATE: CE - 30.2023

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**Question:** In the differential equation  $\frac{dy}{dx} + \alpha xy = 0$ ,  $\alpha$  is a positive constant. If  $y = 1.0$  at  $x = 0.0$ , and  $y = 0.8$  at  $x = 1.0$ , the value of  $\alpha$  is (rounded off to three decimal places). (GATE CE 2023)

**Solution:**

Parameter	Value
$x$	0.0
	1.0
$y$	1.0
	0.8

TABLE I: Given parameters

Let,  $t = x$

$$\frac{dy}{dt} + \alpha ty = 0 \quad (1)$$

$$\int \frac{dy}{y} = - \int \alpha t dt \quad (2)$$

$$\ln(|y|) = -\frac{\alpha t^2}{2} + c \quad (3)$$

$$y(t) = e^c \cdot e^{-\frac{\alpha t^2}{2}} \quad (4)$$

Taking Fourier Transform:

where,

$$\frac{dy}{dt} \xleftrightarrow{\mathcal{F}} j2\pi f Y(f) \quad (5)$$

$$\alpha ty(t) \xleftrightarrow{\mathcal{F}} \alpha \frac{j}{2\pi} \frac{d}{df} Y(f) \quad (6)$$

From equation (5) and (6):

$$\frac{4\pi^2 f}{\alpha} Y(f) + \frac{d}{df} Y(f) = 0 \quad (7)$$

$$Y(f) = K e^{-\frac{4\pi^2 f^2}{2\alpha}} \quad (8)$$

Substituting  $x$  and  $y$  values:

$$c = \ln(1) = 0 \quad (9)$$

$$\alpha = -2 \ln(0.8) = 0.446 \quad (10)$$

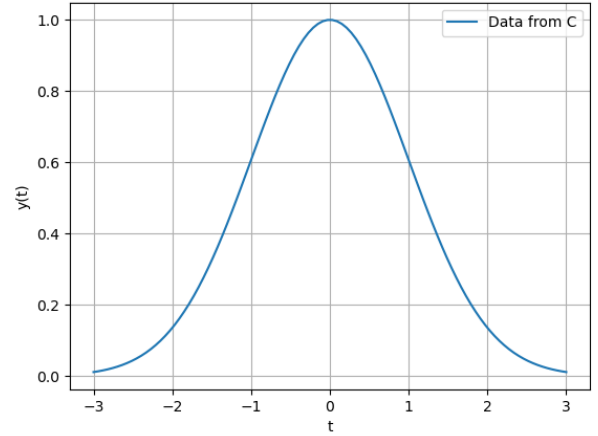


Fig. 1: Graph of  $y(t)$