

GATE 2021 EE.20

EE23BTECH11010 - VENKATESH BANDAWAR*

I. FREQUENCY RESPONSE

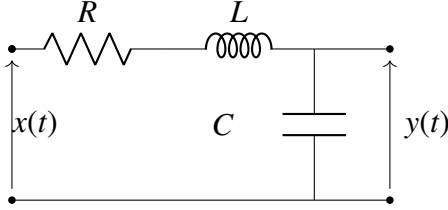


Fig. 1: RLC Low pass filter

The frequency response $y_{ss}(t)$ is defined as the steady state response to a sinusoidal input signal $x(t) = \sin \omega t$. It describes how well the filter can distinguish between different frequencies.

$$y_{ss}(t) = |H(j\omega)| \sin(\omega t + \angle H(j\omega)) \quad (1)$$

1) Overdamped Case

$$H(s) = \omega_0^2 \frac{1}{(s - p_1)(s - p_2)} \quad (2)$$

where,

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad (3)$$

$$H(s) = |H(s)| e^{j\angle H(s)} \quad (4)$$

$$|H(s)| = \omega_0^2 \frac{1}{|s - p_1| |s - p_2|} \quad (5)$$

$$= \omega_0^2 \frac{1}{|j\omega - p_1| |j\omega - p_2|} \quad (6)$$

$$= \omega_0^2 \frac{1}{\sqrt{\omega^2 + p_1^2} \sqrt{\omega^2 + p_2^2}} \quad (7)$$

$$\angle H(s) = -(\angle(s - p_1) + \angle(s - p_2)) \quad (8)$$

$$= \tan^{-1} \frac{\omega}{p_1} + \tan^{-1} \frac{\omega}{p_2} \quad (9)$$

The magnitude of the transfer function expressed on a logarithmic scale:

$$|H_{dB}(\omega)| = 20 \log(\omega_0^2) - 20 \log \sqrt{\omega^2 + p_1^2} - 20 \log \sqrt{\omega^2 + p_2^2} \quad (10)$$

2) Critically damped case

$$H(s) = \omega_0^2 \frac{1}{(s - p)^2} \quad (11)$$

where,

$$p = \sqrt{\frac{1}{LC}} \quad (12)$$

$$H(s) = |H(s)| e^{j\angle H(s)} \quad (13)$$

$$|H(s)| = \omega_0^2 \frac{1}{|s - p|^2} \quad (14)$$

$$|H(j\omega)| = \omega_0^2 \frac{1}{|j\omega - p|^2} \quad (15)$$

$$= \omega_0^2 \frac{1}{\omega^2 + p^2} \quad (16)$$

$$\angle H(s) = -(\angle(s - p) + \angle(s - p)) \quad (17)$$

$$= 2 \tan^{-1} \frac{\omega}{p} \quad (18)$$

The magnitude of the transfer function expressed on a logarithmic scale:

$$|H_{dB}(\omega)| = 20 \log(\omega_0^2) - 20 \log(\omega^2 + p^2) \quad (19)$$

3) Underdamped Case

$$H(s) = \omega_0^2 \frac{1}{(s - p)(s - p^*)} \quad (20)$$

where,

$$p, p^* = \omega_n (-\zeta \pm j \sqrt{1 - \zeta^2}) \quad (21)$$

$$= -\sigma \pm j\omega_d \quad (22)$$

$$H(s) = |H(s)| e^{j\angle H(s)} \quad (23)$$

$$|H(s)| = \omega_0^2 \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \quad (24)$$

$$\angle H(s) = -(\angle(s - p) + \angle(s - p^*)) \quad (25)$$

$$= -\tan^{-1} \frac{\omega + \omega_d}{\sigma} - \tan^{-1} \frac{\omega - \omega_d}{\sigma} \quad (26)$$

The magnitude of the transfer function expressed on a logarithmic scale:

$$|H_{dB}(\omega)| = 20 \log(\omega_0^2) - 10 \log((\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2) \quad (27)$$

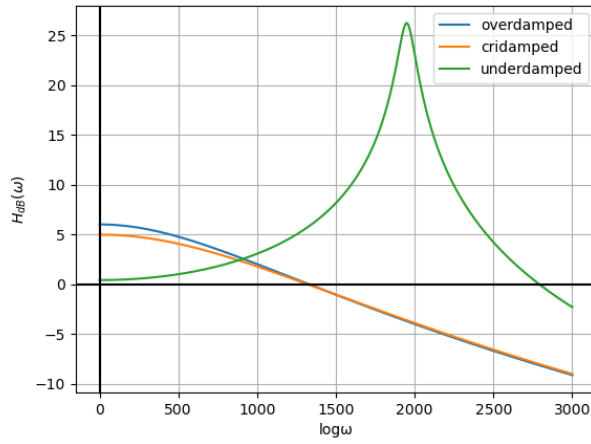


Fig. 2: Frequency response of all cases

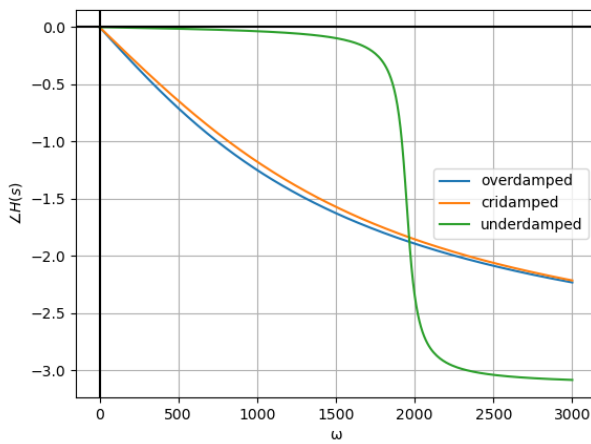


Fig. 3: Plot of $\angle H(s)$