

GATE 2022 IN.53

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Question: In a unity-gain feedback control system, the plant $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$ is controlled by a lag compensator $C(s) = \frac{s+10}{s+0.1}$. The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at $\omega = 3\text{rad/s}$ is _____ (in integer) (GATE 2022 IN)

Solution:

Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(2s+1)(0.01s+1)}$
$C(s)$	Lag Compensator	$\frac{s+10}{s+0.1}$
$L(s)$	Loop gain	$P(s)C(s)$
ω	Angular Frequency	3rad/s

TABLE I: Given Parameters list

$$|L(s)| = \frac{0.1 \left(\frac{s}{10} + 1 \right)}{s \left(\frac{s}{0.5} + 1 \right) \left(\frac{s}{100} + 1 \right) \left(\frac{s}{0.1} + 1 \right)} \quad (1)$$

Here, 10, 0, 0.5, 100, 0.1 are corner frequencies of loop gain $L(s)$

Corner Frequency	Description	Equation
10	Zero	$ L(\omega) = 20.0(\log_{10}(\omega) - 1.0)$
0	Pole	$ L(\omega) = -20.0\log_{10}(\omega)$
0.1	Pole	$ L(\omega) = -20.0(\log_{10}(\omega) - 0.1)$
0.5	Pole	$ L(\omega) = -20.0(\log_{10}(\omega) + \log_{10}(0.5))$
100	Pole	$ L(\omega) = -20.0(\log_{10}(\omega) - 2.0)$

TABLE II: Caption

$$\text{Gain}(K) = \lim_{s \rightarrow 0} sL(s) \quad (2)$$

$$K = 0.1 \quad (3)$$

$$|L(s)| = 20 \log_{10} K \quad (4)$$

$$= -20\text{dB} \quad (5)$$

$$H[\omega] = \begin{cases} 0 & \omega < 0 \\ -20\log_{10}(\omega) & 0 \leq \omega < 0.1 \\ -20.0(2\log_{10}(\omega) - 0.1) & 0.1 \leq \omega < 0.5 \\ -20.0(3\log_{10} - 0.1 + \log_{10}0.5) & 0.5 \leq \omega < 10 \\ -20.0(2\log_{10} + 0.9 + \log_{10}0.5) & 10 \leq \omega < 100 \\ -20.0(3\log_{10} - 1.9 + \log_{10}0.5) & \omega \geq 100 \end{cases}$$

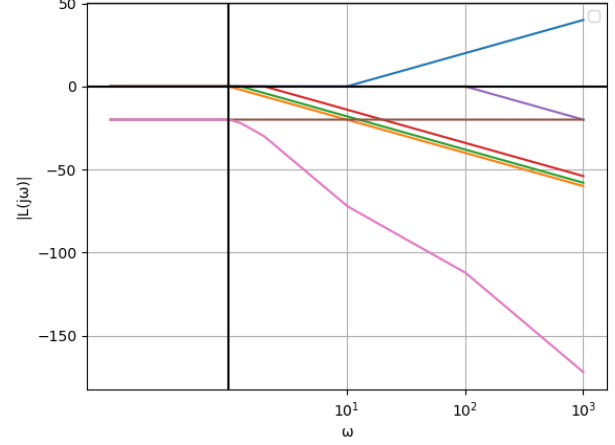


Fig. 1: Pink Line = Bode magnitude plot of loop gain

Slope of Bode magnitude plot (at $\omega = 3$) = -60 dB/decade