GATE 2022 IN.53

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Question: In a unity-gain feedback control system, the plant $P(s) = \frac{0.001}{s(2s+1)(0.01s+1)}$ is controlled by a lag compensator $C(s) = \frac{s+10}{s+0.1}$ The slope (in dB/decade) of the asymptotic Bode magnitude plot of the loop gain at $\omega = 3$rad/s is _ _ _ _ _ _ _ _ (in integer) (GATE 2022 IN 53)

Solution:

Parameter	Description	Value
$P(s)$	Plant Transfer Function	$\frac{0.001}{s(2s+1)(0.01s+1)}$
$C(s)$	Lag Compensator	$\frac{s+10}{s+0.1}$
$L(s)$	Loop gain$=P(s) \times C(s)$	$\frac{0.001(s+10)}{s(2s+1)(0.01s+1)(s+0.1)}$
ω	Angular Frequency	3rad/s

TABLE I: Given Parameters list

let $j\omega = s$

The slope of the asymptotic Bode magnitude plot of the loop gain$= 20\log_{10} |L(j\omega)|$

$$\omega = 3 \tag{1}$$

$$L(j\omega) = \frac{0.001\,(3j+10)}{3j\,(6j+1)\,(0.03j+1)\,(3j+0.1)} \tag{2}$$

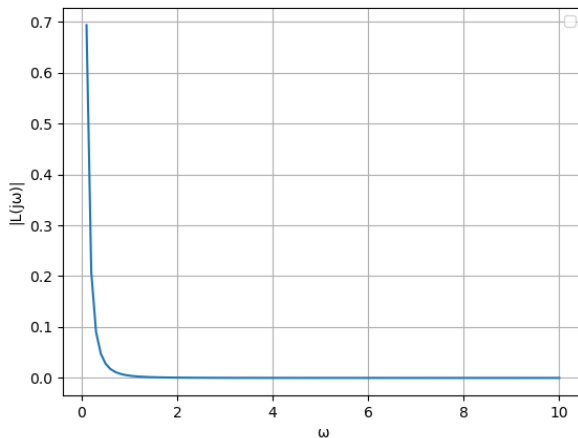$$slope \approx 20\log_{10} \frac{0.001 \times \sqrt{109}}{3 \times \sqrt{37} \times 1 \times 3} \tag{3}$$

$$\approx -74dB/decade \tag{4}$$

Fig. 1: Bode magnitude plot of loop gain