GATE: CE - 30.2023

EE23BTECH11010 - Venkatesh D Bandawar *

Question: In the differential equation $\frac{dy}{dx} + \alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is (rounded off to three decimal places). (GATE CE 2023)

Solution:

Parameter	Value
x	0.0
	1.0
y	1.0
	0.8

TABLE I: Given parameters

let, $t = x$

$$\frac{dy}{dt} + \alpha ty = 0 \tag{1}$$

Taking fourier transform, where,

$$\frac{dy}{dt} \overset{\mathcal{F}}{\longleftrightarrow} j\omega Y(\omega) \tag{2}$$

$$a \cdot t \cdot y(t) \overset{\mathcal{F}}{\longleftrightarrow} a \cdot j\frac{d}{d\omega}Y(\omega) \tag{3}$$

From equation (2) and (3):

$$\frac{\omega}{\alpha}Y(\omega) + \frac{d}{d\omega}Y(\omega) = 0 \tag{4}$$

$$Y(\omega) = Ke^{-\frac{\omega^2}{2\alpha}} \tag{5}$$

Taking inverse fourier transform, Using gaussian integral, WKT,

$$e^{-a\omega^2} \overset{\mathcal{F}^{-1}}{\longleftrightarrow} \frac{1}{\sqrt{4\pi a}}e^{-\frac{t^2}{4a}} \tag{6}$$

From Table I:

$$y(t) = K\frac{\alpha}{\sqrt{2\pi}}e^{-\frac{\alpha t^2}{2}} \tag{7}$$

$$\frac{y(0)}{y(1)} = \frac{1}{e^{-\frac{\alpha}{2}}} \tag{8}$$

$$\ln\frac{5}{4} = \frac{\alpha}{2} \tag{9}$$

$$\alpha = 0.446 \tag{10}$$

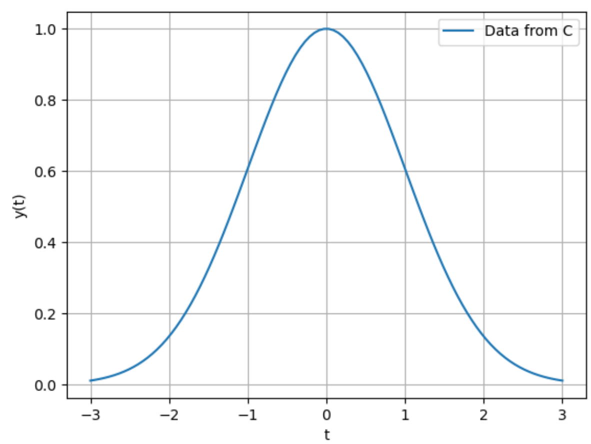

Fig. 1: Graph of y(t)