NCERT 11.15. Q10

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Question: For the travelling harmonic wave y(x,t) = 2.0 cos 2π $(10t - 0.0080x + 0.35)$ where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4 m
(b) 0.5 m
(c) $\lambda/2$
(d) $3\lambda/4$

Solution: Harmonic wave :

symbol	parameter	value
k	angular wave number	$2\pi(0.008)$
λ	wavelength	125cm
ω	angular frequency	$2\pi(10)$
A	amplitude	2.0
ϕ	phase	$2\pi(0.35)$

$$y(x,t) = 2.0\cos 2\pi\,(10t - 0.0080x + 0.35) \tag{1}$$

Phase of harmonic wave (at x):

$$= 2\pi\,(10t - 0.0080x + 0.35) \tag{2}$$

$$k = 2\pi(0.008) \tag{3}$$

$$\because k = \frac{2\pi}{\lambda} \tag{4}$$

$$\lambda = \frac{2\pi}{k} \tag{5}$$

$$\lambda = \frac{2\pi}{2\pi \times 0.008} \tag{6}$$

$$\lambda = 125\text{cm} \tag{7}$$

(a) phase of harmonic wave at (x+400cm) :

$$= 2\pi\,(10t - 0.0080(x + 400) + 0.35) \tag{8}$$

$(from equation\,(2))$

$\Rightarrow$ phase difference :

$$\begin{aligned} &= 2\pi\,(10t - 0.0080\,(x + 400) + 0.35) \\ &\quad - 2\pi\,(10t - 0.0080x + 0.35) \end{aligned} \tag{9}$$

$$= 2\pi \times 0.0080 \times 400 \tag{10}$$

$$= 6.4\pi\,\text{radians} \tag{11}$$

(b) phase of harmonic wave at $(x + 50cm)$:

$$= 2\pi\,(10t - 0.0080\,(x + 50) + 0.35) \tag{12}$$

$(from equation\,(2))$

$\Rightarrow$ phase difference :

$$\begin{aligned} &= 2\pi\,(10t - 0.0080\,(x + 50) + 0.35) \\ &\quad - 2\pi\,(10t - 0.0080x + 0.35) \end{aligned} \tag{13}$$

$$= 2\pi \times 0.0080 \times 50 \tag{14}$$

$$= 0.8\pi\,\text{radians} \tag{15}$$

(c) phase of harmonic wave at $\left(x + \frac{\lambda}{2}\right)$:

$$= 2\pi(10t - 0.0080\left(x + \frac{\lambda}{2}\right) + 0.35) \tag{16}$$

(from equation (2))

$\Rightarrow$ phase difference :

$$\begin{aligned} &= 2\pi\left(10t - 0.0080\left(x + \frac{\lambda}{2}\right) + 0.35\right) \\ &\quad - 2\pi\,(10t - 0.0080x + 0.35) \end{aligned} \tag{17}$$

$$= 2\pi \times 0.0080 \times \frac{\lambda}{2} \tag{18}$$

$$= 2\pi \times 0.0080 \times \frac{125}{2}\,(\because \lambda = 125) \tag{19}$$

$$= \pi\text{radians} \tag{20}$$

(d) phase of harmonic wave at $\left(x + \frac{3\lambda}{4}\right)$:

$$= 2\pi\left(10t - 0.0080\left(x + 3 \times \frac{\lambda}{4}\right) + 0.35\right) \tag{21}$$

$(from equation\,(2))$

$\Rightarrow$ phase difference :

$$\begin{aligned} &= 2\pi\left(10t - 0.0080\left(x + 3 \times \frac{\lambda}{4}\right) + 0.35\right) \\ &\quad - 2\pi\,(10t - 0.0080x + 0.35) \end{aligned} \tag{22}$$

$$= 2\pi \times 0.0080 \times 3 \times \frac{\lambda}{4} \tag{23}$$

$$= 2\pi \times 0.0080 \times 3 \times \frac{125}{4}\,(\because \lambda = 125) \tag{24}$$

$$= \frac{3\pi}{2}\text{radians} \tag{25}$$