GATE 2021 EE.20

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I. LAPLACE TRANSFORM

Laplace transform of integrals: Let the function defined as $y(t) = \int_0^t f(u)du$ for all

Laplace transform of y(t) in t

$$\mathcal{L}(y(t)) = \int_0^\infty e^{-st} y(t) dt$$

$$= \int_0^\infty e^{-st} \int_0^t f(u) du dt$$

$$= \int_0^t f(u) du \left[-\frac{e^{-st}}{s} \right]_0^\infty + \int_0^\infty \frac{e^{-st}}{s} f(t) dt$$

$$= \frac{F(s)}{s}$$
(4)

II. RLC Low Pass Filter

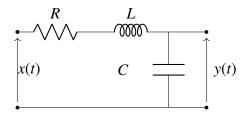


Fig. 1: RLC Low pass filter

Parameter	Description	
Z_C	Reactance of Capacitor	
Z_R	Reactance of Resistor	
Z_L	Recatance of Inductor	
x(t) = u(t)	Input Response	
y(t)	Output across capacitor	
ω_0	Angular resonant frequency	

TABLE I: Input Parameters

$$Y(s) = I(s)Z_C (5)$$

$$=\frac{X(s)}{Z_L + Z_R + Z_C} Z_C \tag{6}$$

$$H(s) = \frac{Y(s)}{X(s)} \tag{7}$$

$$=\frac{Z_C}{Z_L+Z_R+Z_C} \tag{8}$$

$$=\frac{\frac{1}{sC}}{sL+R+\frac{1}{sC}}\tag{9}$$

$$= \frac{1}{s^2 LC + sRC + 1} \tag{10}$$

$$= \frac{1}{s^2LC + sRC + 1}$$

$$\Longrightarrow H(s) = \omega_0^2 \frac{1}{(s - p_1)(s - p_2)}$$
(10)

where,

(4)

$$\omega_0 = \frac{1}{\sqrt{IC}} \tag{12}$$

$$p_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$
 (13)

$$= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \tag{14}$$

where

$$\alpha = \frac{R}{2L} \tag{15}$$

Damping Factor is given by,

$$\zeta = \frac{\alpha}{\omega_0} \tag{16}$$

$$=\frac{R}{2}\sqrt{\frac{C}{L}}\tag{17}$$

ζ	Pole Location	Referred to as	Condition
$\zeta > 1$	Different locations on		
	the negative real axis	Overdamped	$R > 2\sqrt{\frac{L}{C}}$
$\zeta = 1$	Coincide on		
	the negative real axis	Critically Damped	$R = 2\sqrt{\frac{L}{C}}$
ζ < 1	Complex Conjugate poles in		
	the left half of s-plane	Underdamped	$R < 2\sqrt{\frac{L}{C}}$

TABLE II: Effect of Damping Coefficient ζ on system behaviour

1) Overdamped Response

$$Y(s) = X(s)H(s) \tag{18}$$

$$=\omega_0^2 \frac{1}{s(s-p_1)(s-p_2)}$$
 (19)

$$=\frac{c_0}{s} + \frac{c_1}{s - p_1} + \frac{c_2}{s - p_2} \tag{20}$$

where,

$$c_0 = 1 \tag{21}$$

$$c_1 = \frac{p_2}{p_1 - p_2} \tag{22}$$

$$c_2 = \frac{p_1}{p_2 - p_1} \tag{23}$$

Taking inverse Laplace,

$$y(t) = c_0 + c_1 e^{p_1 t} + c_2 e^{p_2 t} (24)$$

$$= \left(1 + \frac{p_2}{p_1 - p_2}e^{p_1t} + \frac{p_1}{p_2 - p_1}e^{p_2t}\right)u(t)$$
(25)

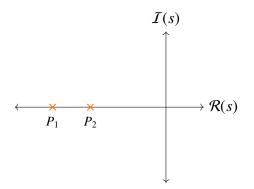


Fig. 2: s-Plane for Overdamped case

2) Critically Damped Response

$$Y(s) = X(s)H(s) \tag{26}$$

$$=\omega_0^2 \frac{1}{s(s-p)^2}$$
 (27)

$$= \frac{c_0}{s} + \frac{\hat{c}_1}{(s-p)^2} + \frac{c_2}{s-p}$$
 (28)

where

$$c_0 = 1 \tag{29}$$

$$c_1 = p \tag{30}$$

$$c_2 = -1 \tag{31}$$

Taking Inverse Laplace,

$$y(t) = c_0 + (c_1 t + c_2)e^{pt}$$
 (32)

$$= (1 + (pt - 1)e^{pt}) u(t)$$
 (33)

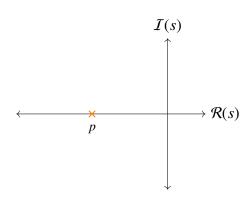


Fig. 3: s-Plane for Critically damped case

3) Underdamped Response

$$Y(s) = X(s)H(s)$$
 (34)

$$=\omega_0^2 \frac{1}{s(s-p)(s-p^*)}$$
 (35)

$$= \frac{c_0}{s} + \frac{c_1}{s - p} + \frac{c_2}{s - p^*} \tag{36}$$

where,

$$c_0 = 1 \tag{37}$$

$$c_1 = \frac{p^*}{p - p^*} \tag{38}$$

$$c_2 = \frac{p}{p^* - p} \tag{39}$$

Taking Inverse Laplace,

$$y(t) = c_0 + c_1 e^{pt} + c_2 e^{p^*t} (40)$$

$$=1+\frac{|p|}{\omega_d} e^{-\sigma t} \frac{e^{j(\omega_d t+\varphi)}+e^{-j(\omega_d t+\varphi)}}{2} \quad (41)$$

$$= \left(1 + \frac{|p|}{\omega_d} e^{-\sigma t} \cos(\omega_d t + \varphi)\right) u(t)$$
 (42)

where,

$$|p| = \sqrt{\omega_d^2 + \sigma^2} \tag{43}$$

$$\omega_d = \omega_0 \sqrt{1 - \zeta^2} \tag{44}$$

$$\sigma = \omega_0 \zeta \tag{45}$$

$$|p| = \sqrt{\omega_d^2 + \sigma^2}$$

$$\omega_d = \omega_0 \sqrt{1 - \zeta^2}$$

$$\sigma = \omega_0 \zeta$$

$$\varphi = \pi - \tan^{-1} \frac{\sigma}{\omega_d}$$
(43)
(44)
(45)

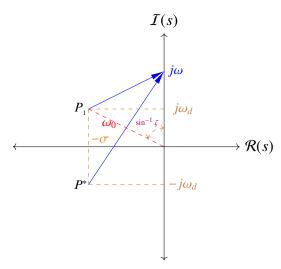


Fig. 4: s-Plane for Under damped case

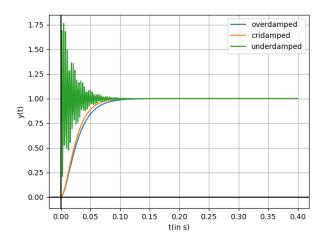


Fig. 5: Step response in all three cases