# GATE: CE - 30.2023

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**Question:** In the differential equation $\frac{dy}{dx} + \alpha xy = 0, \alpha$ is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of $\alpha$ is (rounded off to three decimal places). (GATE CE 2023)

**Solution:**

| Parameter | Value |
|---|---|
| $x$ | 0.0 |
| | 1.0 |
| $y$ | 1.0 |
| | 0.8 |

TABLE I: Given parameters

let, $t = x$

$$\frac{dy}{dt} + \alpha ty = 0 \tag{1}$$

Taking fourier transform, where,

$$\frac{dy}{dt} \xleftrightarrow{\mathcal{F}} j2\pi f Y(f) \tag{2}$$

$$a \cdot t \cdot y(t) \xleftrightarrow{\mathcal{F}} a \cdot \frac{j}{2\pi} \frac{d}{df} Y(f) \tag{3}$$

From equation (2) and (3):

$$\frac{4\pi^2 f}{\alpha} Y(f) + \frac{d}{df} Y(f) = 0 \tag{4}$$

$$Y(f) = Ke^{-\frac{4\pi^2 f^2}{2\alpha}} \tag{5}$$

Taking inverse fourier transform, Using gaussian integral, WKT,

$$e^{-a(2\pi f)^2} \xleftrightarrow{\mathcal{F}^{-1}} \frac{1}{\sqrt{4\pi a}} e^{-\frac{t^2}{4a}} \tag{6}$$

From Table I:

$$y(t) = K \frac{\alpha}{\sqrt{2\pi}} e^{-\frac{\alpha t^2}{2}} \tag{7}$$

$$\frac{y(0)}{y(1)} = \frac{1}{e^{-\frac{\alpha}{2}}} \tag{8}$$

$$\ln \frac{5}{4} = \frac{\alpha}{2} \tag{9}$$

$$\alpha = 0.446 \tag{10}$$

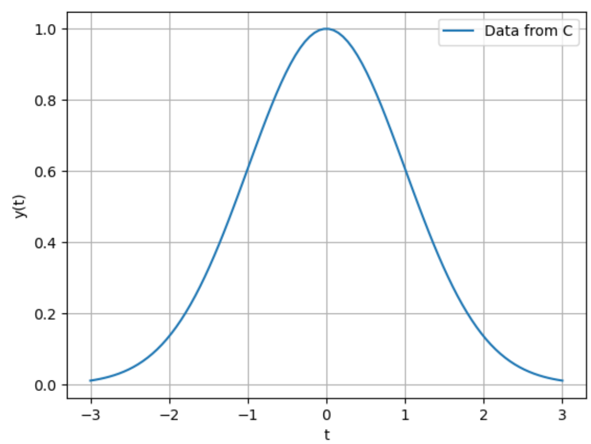


Fig. 1: Graph of y(t)