

NCERT 10.5.3 10Q

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Question: Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where an is defined as below :

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer:

(i) Given: $a_n = 3 + 4n$

we know that, The AP has constant common difference between two consecutive terms

$$\begin{aligned} \therefore \text{common difference } (d) &= a_{n+1} - a_n \quad (1) \\ &= (3 + 4(n+1)) \\ &\quad - (3 + 4n) \\ &= 4 \quad (2) \end{aligned}$$

\therefore Given equation has common difference between any two consecutive terms is 4 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
a_n	n^{th} term	$3 + 4n$
a_0	first term	3
d	common difference	4
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	465

TABLE (i): Given parameters in 1st AP

Z - Transformation of a_n :

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} [3 + 4n].u(n) z^{-n} \quad (3)$$

$$\Rightarrow X(z) = 3 \sum_{n=0}^{\infty} 1.z^{-n} + 4 \sum_{n=0}^{\infty} 1.n.z^{-n} \quad (4)$$

$$\Rightarrow X(z) = \frac{3}{1 - z^{-1}} + \frac{4.z^{-1}}{(1 - z^{-1})^2} \quad (5)$$

{Where, $|z| > 1$ }

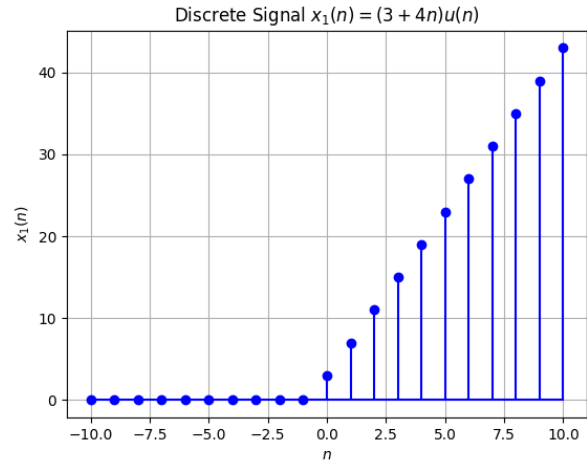


Fig. (i)

(ii) Given: $a_n = 9 - 5n$

we know that, The AP has constant common difference between two consecutive terms

$$\begin{aligned} \therefore \text{common difference } (d) &= a_{n+1} - a_n \quad (6) \\ &= (9 - 5(n+1)) - (9 - 5n) \\ &= -5 \quad (7) \end{aligned}$$

\therefore Given equation has common difference between any two consecutive terms is -5 i.e. independent of 'n'

Hence given sequence is in AP.

parameter	description	value
a_n	n^{th} term	$9 - 5n$
a_0	first term	9
d	common difference	-5
S_{15}	sum of first 15 terms : $\frac{n}{2}[2a_0 + (n-1)d]$	-390

TABLE (ii): Given parameters in 2nd AP

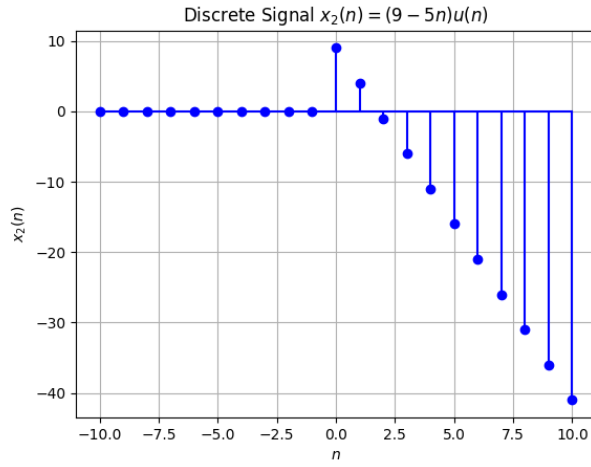


Fig. (ii)

Z - Transformation of a_n :

$$\Rightarrow X(Z) = \sum_{n=-\infty}^{\infty} [9 - 5n].u(n) z^{-n} \quad (8)$$

$$\Rightarrow X(z) = 9 \sum_{n=0}^{\infty} 1.z^{-n} - 5 \sum_{n=0}^{\infty} 1.n.z^{-n} \quad (9)$$

$$\Rightarrow X(z) = \frac{9}{1 - z^{-1}} - \frac{5.z^{-1}}{(1 - z^{-1})^2} \quad (10)$$

{Where, $|z| > 1$ }