

# NCERT 11.9.4 8Q

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**Question:** Find the sum to n terms of series ,  
whose  $n^{th}$  term is :  $n(n+1)(n+4)$ .

**Solution**

Parameter	Description	Value
$x(n)$	$n^{th}$ term of series	$n(n+1)(n+4)u(n)$
$y(n)$	sum of n terms of series	

TABLE 0: Given parameters

from equation (??) to (??),

$$X(z) = \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^3} + \frac{4z^{-1}}{(1-z^{-1})^2} \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

$$= \frac{z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^5} + \frac{5z^{-1}(z^{-1}+1)}{(1-z^{-1})^4} + \frac{4z^{-1}}{(1-z^{-1})^3} \quad (3)$$

Using contour integration for inverse Z transformation,

$$\begin{aligned} y(n) &= \frac{1}{2\pi j} \oint_c Y(z)z^{n-1} dz \\ &= \frac{1}{2\pi j} \oint_c \frac{(z^2+4z+1)}{(z-1)^5} z^{n+1} dz \\ &\quad + \frac{1}{2\pi j} \oint_c \frac{5(z+1)}{(z-1)^4} z^{n+1} dz \\ &\quad + \frac{1}{2\pi j} \oint_c \frac{4}{(z-1)^3} z^{n+1} dz \\ \therefore R &= \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (4) \end{aligned}$$

$$\begin{aligned} R_1 &= \frac{1}{4!} \lim_{z \rightarrow 1} \frac{d^4}{dz^4} \left( (z-1)^5 \frac{(z^2+4z+1)z^{n+1}}{(z-1)^5} \right) \\ &= \frac{(n+3)(n+2)(n+1)(n)}{4!} \\ &\quad + \frac{4(n+2)(n+1)(n)(n-1)}{4!} \\ &\quad + \frac{(n+1)(n)(n-1)(n-2)}{4!} \\ R_2 &= \frac{1}{3!} \lim_{z \rightarrow 1} \frac{d^3}{dz^3} \left( (z-1)^4 \frac{5(z+1)z^{n+1}}{(z-1)^4} \right) \\ &= \frac{5(n+2)(n+1)(n)}{3!} + \frac{5(n+1)(n)(n-1)}{3!} \\ R_3 &= \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left( (z-1)^3 \frac{4z^{n+1}}{(z-1)^3} \right) \\ &= \frac{4n(n+1)}{2!} \\ \Rightarrow y(n) &= R_1 + R_2 + R_3 \\ &= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2} \quad (5) \end{aligned}$$

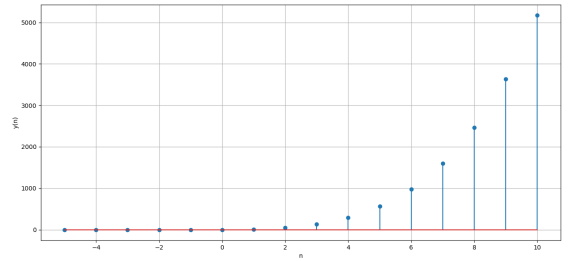


Fig. 0: sum of n terms of series