problem 1 :

yes, there is a regular language over an alphabet of 3 symblos i.e., $\{a, b, c\}$ (or) $\{0, 1, 2\}$ that has non-regular proper subset.

Examples

~~Let~~ $L = \{a, b, c\}^*$ is a regular language.

It has a non regular subset :

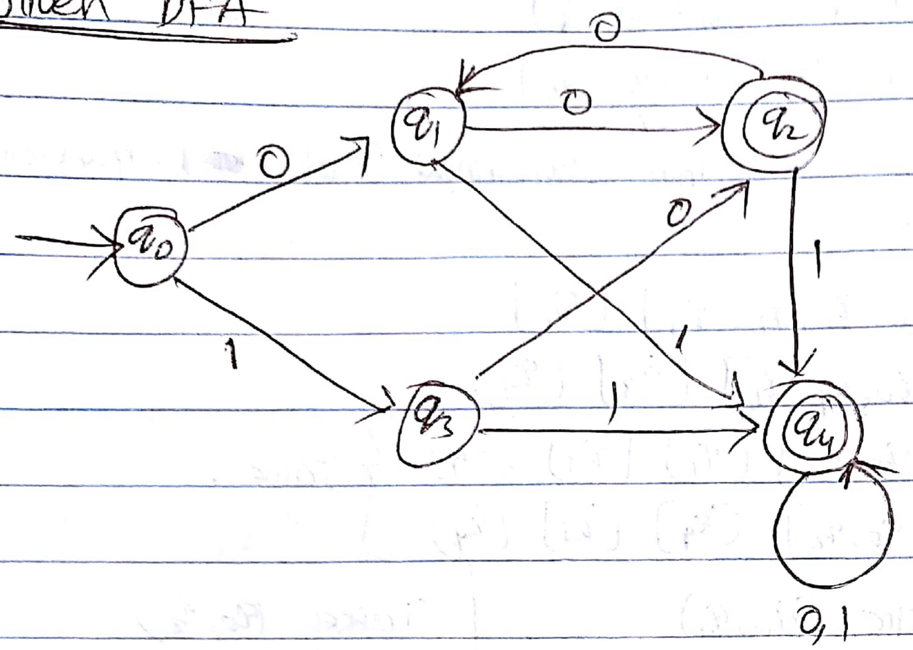
$$\{a^n b^n c^n \mid n \in N\}$$

I proved $a^n b^n c^n$ is not a regular subset in problem 2.1.

Hence proved.

~~Example~~ problem : 3

Given DFA



1. There are no states which cannot reach from initial state.
2. State Transition Table

	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_2^*	q_4^*
$* q_2$	q_1	q_4^*
q_3	q_2^*	q_4^*
$* q_4$	q_4^*	q_4^*

'*' → final state
'→' → initial state

3. Find Equivalent sets

(a) 0 Equivalent sets : seperate final states & non-final states.

$[q_0, q_1, q_3]$ $[q_2, q_4]$

(b) ~~1~~ Equivalent sets :

→ Take (q_0, q_1)

on 0 → (q_1, q_2^*) → q_1 & q_2 are diff states

on 1 → (q_3, q_4^*) → q_3 & q_4 are diff states,

So, q_0 & q_1 are not equivalent.

→ Take (q_0, q_3)

On 0 → (q_1, q_2^*) → q_1 & q_2 are diff states

On 1 → (q_3, q_4^*) → q_3 & q_4 are diff states

So, q_0 & q_3 are not equivalent.

~~$[q_0]$~~ ~~&~~

→ Take (q_2, q_4)

on 0 → (q_1, q_4^*) → q_1 & q_4 are diff states

on 1 → (q_4^*, q_4^*) → q_4^* & q_4^* are same states

So, q_2 and q_4 are not equivalent.

~~$[q_0]$ $[q_1]$ $[q_3]$ $[q_2]$ $[q_4]$~~

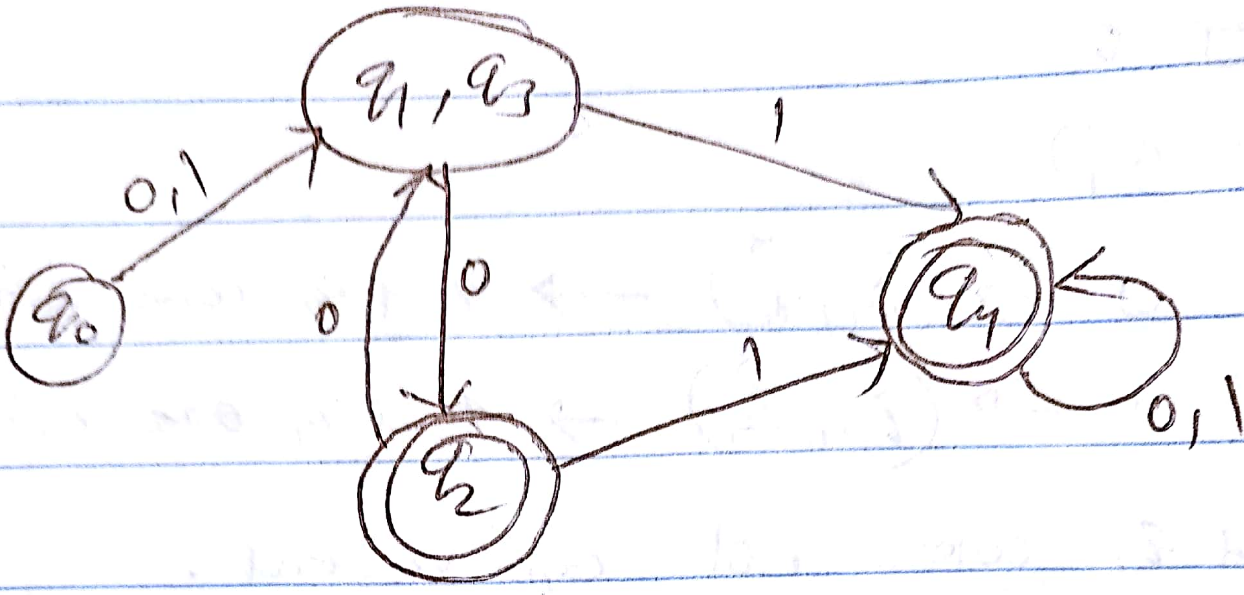
→ Take (q_1, q_3)

On 0 → (q_2^*, q_2^*) → q_2 in same state (final)

On 1 → (q_4^*, q_4^*) → q_4 in same state (final)

So, q_1 & q_3 are equivalent.

$[q_0]$ $[q_1, q_3]$ $[q_2]$ $[q_4]$



Minimized DFA

Problem 2 :

① $L = \{a^n b^n c^n \mid n \geq 0\}$ ⇒ Not regular, because b & c have to count n value.

ⓐ Assume that L is regular

ⓑ It has a pumping length P.

ⓒ Now there exists a pumping lemma such that $S = a^n b^n c^n$ & $|S| \geq P$.

$x \; y \; z$

Divide s in to 3 parts xyz

ⓓ Now show that $xy^iz \notin S$ for some i

ⓔ Now consider different cases ~~with~~ of s with a pumping length $p = 3$.

Then $s = a^P b^P c^P = a^3 b^3 c^3 = aaabbbccc$

Case 1: $\underbrace{a}_{x}\underbrace{aa}_{y}\underbrace{bbbccc}_{z} \Rightarrow xy^iz$ $[y = \{a^+\}]$

let ~~...~~ $i = 2$

$\Rightarrow a\,aaaa\,bbb\,cccc \notin \cancel{S}$ Count(a) = 5

Count(b) = 3

Count(c) = 4

Count(a, b, c) are not equal.

Hence proved that $xy^iz \notin L$

Case 2: $\underbrace{aaa}_{x}\underbrace{bbb}_{y}\underbrace{ccc}_{z}$ $[y = \{b\}^+]$

$i = 2$ in xy^iz

$\Rightarrow aaabbbbbbccc \notin L$ count(a) = 3
count(b) = 6
count(c) = 3

count(a,b,c) are not equal
Hence proved that $xy^iz \notin L$.

Case 3: $\underbrace{aaabbbcc}_{x}\underbrace{c}_{y}\underbrace{c}_{z}$ $[y = \{c\}^+]$

$i = 2$ in xy^iz

$\Rightarrow aaabbbccccc$ count(a) = 3
count(b) = 3
count(c) = 5

count(a,b,c) are not equal
Hence, proved that $xy^iz \notin L$.

Case 4: $\underbrace{aa}_{x}\underbrace{abb}_{y}\underbrace{bccc}_{z}$ $[y = abb]$

$i = 2$ in xy^iz

$\Rightarrow aaabbabbccc$ count(a) = 4
count(b) = 4
count(c) = 3

count(a,b,c) are not equal
Hence proved that $xy^iz \notin L$.

Case 5: $\underbrace{aaab}_{x}\underbrace{bbcc}_{y}\underbrace{c}_{z}$ $[y = bbcc]$
count(a) = 3
count(b) = 5
count(c) = 5

$i = 2$ in xy^iz

$\Rightarrow aaabbbccbbccc$
count(a,b,c) are not equal
Hence proved that $xy^iz \notin L$

Case 6: $\underline{aaa}\,\underline{abbbcc}\,\underline{c}$ [$y = aabbbcc$]

$x \quad y \quad z$

count(a) = 5

$i = 2$ in xy^iz count(b) = 6

$\Rightarrow$ aaabbbcc aabbbcc c count(c) = 5

count(a, b, c) are not equal.

Hence proved that $xy^iz \notin L$.

(f) Hence proved that none of these six (6) cases satisfy all 3 pumping conditions.

(g) Hence S cannot be pumped == CONTRADICTION.

Hence our statement L is regular is <u>not</u> true.

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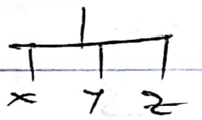
(2) $L = \{xx^R \mid \Sigma = \{a\}\}$ $\Rightarrow$ <u>Not regular</u>, x^R have to store x value

(a) Assume that L is regular

(b) It has a pumping length P.

(c) Now there exists a pumping lemma such that

$S = xx^R$ & $|S| \geq P$



(d) Now show that $xy^iz \notin S$ for some i

pumping length = 3.

Let $x = aaa$, then $S = aaa\ aaa$

Case 1: $x = aaa$

$x^R = aaa$

$s = \underbrace{aa}_{x}\underbrace{aa}_{y}\underbrace{aa}_{z} \Rightarrow xy^iz$

let $i = 3$ $\Rightarrow$ aa aaaaaa a count(a) = 9

count(a) are not equal.

Hence proved that $xy^iz \notin L$.

Hence our statement L is regular is not true.

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③ $L = \{xcx^R \mid x \in \Sigma^*\}$ where $\Sigma = \{a, b\}$.

ⓐ Assume that L is regular

ⓑ It has a pumping length p

ⓒ Now there exists a pumping lemma such that $s = xcx^R$ & $|s| \geq p$

Not regular

x^R have to store x value

ⓓ Now show that $xy^iz \notin S$ for some i

pumping length = 2, let $x = a^2b^2$

then ~~aabba~~

Case 1: $s = aabbcbbaa$

$\underbrace{a}_{x}\underbrace{a}_{y}\underbrace{bbcbbaa}_{z} \Rightarrow xy^iz$

let $i = 2$

aaabbcbbaa

count(a) = 5

count(b) = 4

x + x^R are not perfect c

Hence proved $xy^iz \notin L$.

Case 2: $\underbrace{aab}_{x}\underbrace{b}_{y}\underbrace{c\,bbaa}_{z}$

$xy^iz \Rightarrow i=2$

aabbbb c bbaa

$x =$ aabbbb
$x^R =$ bbaa } not reverse   count(a) = 4, count(b) = 6

count(a,b) are also not equal.

Hence $xy^iz \notin L$. proved.

Hence proved that none of these cases satisfy all conditions.

Hence S cannot be pumped == CONTRADICTION

Hence our statement L is regular is not true

④ $L = \{a^n c^m b^p \mid n+m = p\}$

ⓐ Assume That L is regular

ⓑ It has a pumping length P.

ⓒ Now there exists a pumping lemma such that $s = a^n c^m b^p$ & $|s| \geq P$

ⓓ Now show that $xy^iz \notin S$ for some i

pumping length = 3

Then $s = a^3 c^3 b^6$

Case 1: $a^3 c^3 b^6 \Rightarrow xy^iz$ [y = aa]

$\underbrace{a}_{x}\underbrace{aa}_{y}\underbrace{cccbbbbbb}_{z}$

let i = 2 ⇒ a aa aa cccbbbbbb

count(a) = 5 ; 5+3 = 8 ≠ 6

count(c) = 3

count(b) = 6

Hence proved that $xy^iz \notin L$.

Case 2: $a^3 c^3 b^6 \Rightarrow xy^iz$

$\underbrace{aaa}_{x}\underbrace{ccc}_{y}\underbrace{bbbbbb}_{z} \Rightarrow xy^iz$

let i=2 ⇒ aaa cccccc bbbbbb

count(a) = 3 ; 3+6 = 9 ≠ 6

count(c) = 6

count(b) = 6

Hence proved that $xy^iz \notin L$.

Hence our statement L is regular is <u>not</u> true.

(5) $L = \{0^n 1^m \mid n \geq m\}$

(a) Assume that L is regular

(b) It has a pumping length p.

(c) Now there exists a pumping lemma such that

$s = 0^n 1^m$ & $|s| \geq p$

(d) Now show that $xy^iz \notin s$ for some i.

(e) pumping length = 2

Then $s = 0^2 1^2$

~~case 1:~~ ~~s ε 0 0 1 1 x y z~~ ~~s = 0011 x y z~~

~~let i=2 in xy^iz~~ ~~s = 001011~~

~~s = 00011~~

case 1: $S = \underbrace{0}_{x}\underbrace{0}_{y}\underbrace{11}_{z}$

let $i=2$ in xy^iz

$S = 00111$

$\left.\begin{array}{l} count(0) = 2 = n \\ count(1) = 3 = m \end{array}\right\} \boxed{n < m}$

Hence proved that $xy^iz \notin L$

Hence proved that our statement L is regular is <u>not</u> true.