Problem 1:

① show that regular languages are closed under concatenation.

Theorem: If L_1 and L_2 are regular languages, then contatenation of L_1 and L_2 is also regular.

Proof: If L_1 and L_2 are regular, there are two DFAS $M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$ such that $L_1 = L(M_1)$ and $M_2 = (R, \Sigma, \delta_2, r_0, F_2)$ such that $L_2 = L(M_2)$. We construct a new DFA $M_3 = (Q \times R, \Sigma, \delta_3, (q_0, r_0), F_1 \times F_2)$, where $\delta_3((q,r),a) = (\delta_1(q,a), \delta_2(r,a))$, where $q \in Q, r \in R, a \in \Sigma$.

For any $x \in \Sigma^*$, $x \in L_1 \cancel{x} L_2 \Leftrightarrow$
$\delta_1^*(q_0, x) \in F_1$ and $\delta_2^*(r_0, x) \in F_2 \Rightarrow$
$(\delta_1^*(q_0,x), \delta_2^*(r_0,x)) \in F_1 \times F_2 \Leftrightarrow$
$\delta_3^*((q_0, r_0), x) \in F_1 \times F_2 \Leftrightarrow x \in L_1 \times L_2$.

③ show that regular languages are closed under difference.

Theorem: If L_1, L_2 are regular, then $L_1 - L_2$ are also regular.

Proof: $M_1 = (Q, \Sigma, \delta_1, q_0, F_1)$
$M_2 = (R, \Sigma, \delta_2, r_0, F_2)$
$M_3 = (Q \times R, \Sigma, \delta_3, (q_0, r_0), F_1 - F_2)$

$\delta_3((q,r),a) = (\delta_1(q,a), \delta_2(r,a))$

$q \in Q,\ r \in R,\ a \in \Sigma \quad x \in \Sigma^* \quad x \in L_1 - L_2$

$\delta_1^*(q_0, x) \in F_1$ and $\delta_2^*(r_0, x) \in F_2 \Rightarrow$

$(\delta_1^*(q_0,x), \delta_2^*(r_0,x)) \in F_1 - F_2 \Leftrightarrow$

$\delta_3^*((q_0, r_0), x) \in F_1 - F_2 \Leftrightarrow L_1 - L_2$

(4) Theorem: Show that regular languages are closed under quotients with final languages.

Proof:

Assume that L_1 and L_2 are regular, and let DFA $M = (Q, \Sigma, \delta, q_0, F)$ accept L_1.

We construct DFA $\overline{M} = (Q, \Sigma, q_0, \overline{F})$ as follows.

(a) For each $q_i \in Q$, determine if there is a $y \in L_2$ such that $\delta^*(q_i, y) \in F$.

(b) This can be done by following:

construct $M_i = (Q, \Sigma, \delta, q_i, F)$

if $L_2 \cap L(\overline{M}) \neq \phi$ then $q_i \in \overline{F}$.

If $x \in L_1/L_2$ then $x \in L(\overline{M})$

if $x \in L_1/L_2$, there exits a $y \in L_2$ such that $xy \in L_1$.

If $xy \in L_1$, then:

$\delta(q_0, x) = q$, for some $q \in Q$

$\delta(q, y) \in F$

By construction, $q \in \overline{F}$, so $\overline{M}$ accepts x.

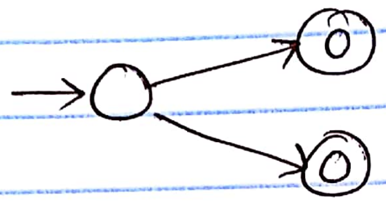
If similarly is easy to show that if $x \in L(\overline{M})$ then $x \in L_1/L_2$.

problem ②:

Theorem: show that regular languages are closed under the Kleene closure.

Proof: Let L is a regular language, then there exists an NFA 'N' such that.

N

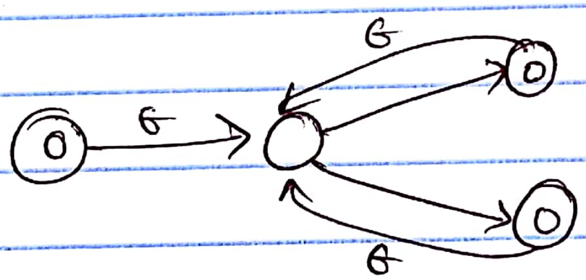


Now $L^* = \{x_1 x_2 x_3 \text{---} x_n \mid n \geq 0,\ x_i \in L,\ 0 \leq i \leq n\}$

which means L^* set of strings $\{\epsilon, x, x_1, x_2 x_1, x_2 x_3 \text{---}\}$

Now we can construct an NFA N^* as follows.

N^*



by the NFA, N^*. it can accept 'ε' string, and concatenated strings $x_1, x_1 x_2, x_1 x_2 x_3$ --- etc

So, N^* will accept the language L^*

Since, we can construct an NFA 'N^*' to accept language L^* we can say L^* is regular.

Problem 2:

$M = (Q, \Sigma, \delta, q_0, F) \quad \Sigma = (0,1)$

$Q = \{\{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

$F = \{\{q_1\}, \{q_0, q_1\}, \{q_1, q_2\}, \{q_0, q_1, q_2\}\}$

$\delta(q_0, 0) = \{q_0, q_1\}$

$\delta(q_1, 0) = \{q_2\}$

$\delta(q_1, 1) = \{q_2\}$

$\delta(q_0, 1) = \{q_1\}$

$\delta(q_2, 1) = \{q_2\}$

$\delta(\{q_0, q_1\}, 0) = \{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1\}, 1) = \{q_1, q_2\}$

$\delta(\{q_0, q_1, q_2\}, 0) = \{q_0, q_1, q_2\}$

$\delta(\{q_0, q_1, q_2\}, 1) = \{q_0, q_1\}$

$\delta(\{q_1, q_2\}, 1) = \{q_2\}$

$\delta(\{q_1, q_2\}, 0) = \{q_2\}$

$\delta(\{q_2\}, 1) = \{q_2\}$