TOC – Asn 6

problem ① :

① $S \rightarrow ABa$

② $A \rightarrow aab$

③ $B \rightarrow Ac$

step①: There are no productions which are in a CNF to remove (Productions P of grammar G)

step②: For $S \rightarrow ABa$

- add $A_a \rightarrow a$ to P';
- rewrite $S \rightarrow ABa$ as $S \rightarrow AB A_a$

For $A \rightarrow aab$

- add $B_b \rightarrow b$ to P';
- rewrite $A \rightarrow aab$ as $A \rightarrow A_a A_a B_b$

For $B \rightarrow Ac$

- add $C_c \rightarrow c$ to P';
- rewrite $B \rightarrow Ac$ as $B \rightarrow AC_c$

Now

① $S \rightarrow ABA_a$	④ $A_a \rightarrow a$
② $A \rightarrow A_a A_a B_b$	⑤ $B_b \rightarrow b$
③ $B \rightarrow AC_c$	⑥ $C_c \rightarrow c$

step③: production ~~set~~ $B \rightarrow AC_c$, $A_a \rightarrow a$, $B_b \rightarrow b$ & $C_c \rightarrow c$ are already in CNF. So add these productions to P'.

Replace $S \longrightarrow ABA_a$ with

$S \longrightarrow D_1 A_a$;

$D_1 \longrightarrow AB$

Replace $A \longrightarrow A_a A_a B_b$ with

$A \longrightarrow D_2 B_b$;

$D_2 \longrightarrow A_a A_a$

Result :

$S \longrightarrow D_1 A_a$

$A \longrightarrow D_2 B_b$

$B \longrightarrow A C_c$

$D_1 \longrightarrow AB$

$D_2 \longrightarrow A_a A_a$

$A_a \longrightarrow a$

$B_b \longrightarrow b$

$C_c \longrightarrow c$

problem ② :

① $S \longrightarrow ABC$

② $C \longrightarrow BaB \mid c$

③ $B \longrightarrow b \mid bb$

④ $A \longrightarrow a$

step① : we iterate through the productions and look which ones are already in CNF. There are only three so we place them into P':

$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow c$

step ② :

For $S \rightarrow ABC$

- add $D_1 \rightarrow AB$ to P';
- rewrite $S \rightarrow ABC$ as $S \rightarrow D_1C$

For $C \rightarrow BaB$

- add $C_a \rightarrow a$ to P';
- rewrite $C \rightarrow BaB$ as $C \rightarrow BC_aB$

For $B \rightarrow bb$

- add $C_b \rightarrow b$ to P';
- rewrite $B \rightarrow bb$ as $B \rightarrow C_bC_b$

Now

$S \rightarrow D_1C$	$D_1 \rightarrow AB$
$C \rightarrow BC_aB$	$C_a \rightarrow a$
$B \rightarrow C_bC_b$	$C_b \rightarrow b$

step ③ : productions $S \rightarrow D_1C$, $D_1 \rightarrow AB$, $B \rightarrow C_bC_b$, $C_a \rightarrow a$ & $C_b \rightarrow b$ are in CNF.

So add these productions to P'.

Replace $C \rightarrow BC_aB$ with

$C \rightarrow D_2B$;

$D_2 \rightarrow BC_a$

Result:

$S \rightarrow D_1C$

$C \rightarrow D_2B \mid c$

$B \rightarrow C_bC_b \mid b$

$A \rightarrow a$

$D_1 \rightarrow AB$

$D_2 \rightarrow BC_a$

$C_a \rightarrow a$

$C_b \rightarrow b$

Problem (3):

$G = (V, T, S, P)$

(1) Consider a production rule $S \rightarrow ABCD$

The corresponding CFG is

$S \rightarrow D_1D$

$D_1 \rightarrow D_2C$

$D_2 \rightarrow AB$

$k = 4$, $|P| = 1$ & $|T| = 0$

$(k-1)|P| + |T| = 3(1) = \underline{\underline{3}}$

Number of productions in CFG are 3 which is equal to $(k-1)|P| + |T|$.

② Consider another production rule $S \rightarrow ABcc$
Corresponding grammar in CFG is

~~$S \rightarrow D_1$~~ $S \rightarrow AD_1$

$D_1 \rightarrow BD_2$

$D_2 \rightarrow D_3 D_3$

$D_3 \rightarrow c$

$k = 4$, $|P| = 1$ & $|T| = 2$

$(k-1)|P| + |T| = (4-1)1 + 2 = 5$

Number of productions in CFG are 4 which is less than $(k-1)|P| + |T|$. Hence proved that, for any given CFG, there is an equivalent CNF grammar with no more than $(k-1)|P| + |T|$ productions.

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