Assignment 17

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AI20MTECH14005

Download all latex-tikz codes from

https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/Assignment17_Matrix_Theory

1 Problem

If $\mathbf{E}$ is the projection and f is a polynomial,then $f(\mathbf{E}) = a\mathbf{I} + b\mathbf{E}$. What are a and b in terms of the coefficients of f?

2 Solution

Given ,

E is the projection
f is a polynomial

$$f(\mathbf{E}) = a\mathbf{I} + b\mathbf{E} \tag{2.0.1}$$

Let ,

$$f(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n \tag{2.0.2}$$

Then,$f(\mathbf{E})$ can be written as,

$$f(\mathbf{E}) = c_0\mathbf{I} + c_1\mathbf{E} + c_2\mathbf{E}^2 + \cdots + c_n\mathbf{E}^n \tag{2.0.3}$$

Since $\mathbf{E}$ is the projection,

$$\mathbf{E}^2 = \mathbf{E} \tag{2.0.4}$$

$$\mathbf{E}^k = \mathbf{E} \quad \text{for any } k > 1 \tag{2.0.5}$$

Using equations (2.0.4) and (2.0.5), equation (2.0.3) can be modified as ,

$$f(\mathbf{E}) = c_0\mathbf{I} + c_1\mathbf{E} + c_2\mathbf{E} + \cdots + c_n\mathbf{E} \tag{2.0.6}$$

$$f(\mathbf{E}) = c_0\mathbf{I} + (c_1 + c_2 + \cdots + c_n)\mathbf{E} \tag{2.0.7}$$

Comparing the equations (2.0.1) and (2.0.7) we get,

$$a = c_0 \tag{2.0.8}$$

$$b = c_1 + c_2 + \cdots + c_n \tag{2.0.9}$$

Here, a is the constant term of f and b is the sum of all other co-efficients