Assignment 5

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***Abstract*—This document explains the the concept of equation of circle given the radius and point through which the circles passes through**

Download all latex-tikz codes from

https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/Assignment5_Matrix_Theory

1 Problem

Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point $\begin{pmatrix}2\\3\end{pmatrix}$

2 Equation of Circle

Equation of the circle with radius r and centre(h,k) is given by,

$$x^T x + 2u^T x + f = 0 \tag{2.0.1}$$

where,

$$f = \mathbf{u}^T\mathbf{u} - r^2 \tag{2.0.2}$$

3 Solution

The radius and centre are respectively given by,

$$r = 5 \tag{3.0.1}$$

$$\mathbf{c} = -u = k\mathbf{e} \tag{3.0.2}$$

Where ,

$$\mathbf{e} = \begin{pmatrix}1\\0\end{pmatrix} \tag{3.0.3}$$

$$\mathbf{x_1} = \begin{pmatrix}2\\3\end{pmatrix} \tag{3.0.4}$$

From the given data , we modify equation 2.0.1 as,

$$\mathbf{x_1}^T\mathbf{x_1} + 2\begin{pmatrix}-k & 0\end{pmatrix}\begin{pmatrix}-k\\0\end{pmatrix} + f = 0 \tag{3.0.5}$$

$$\|\mathbf{x_1}\|^2 + 2\left(k^2\right) + f = 0 \tag{3.0.6}$$

$$2k^2 + f = -\|\mathbf{x_1}\|^2 \tag{3.0.7}$$

Subsituting u in equation 2.0.2 , we get ,

$$f = \begin{pmatrix}-k & 0\end{pmatrix}\begin{pmatrix}-k\\0\end{pmatrix} - r^2 \tag{3.0.8}$$

$$f = \left(k^2\right) - r^2 \tag{3.0.9}$$

$$k^2 - f = r^2 \tag{3.0.10}$$

From equations 3.0.7 and 3.0.10,

$$\begin{pmatrix}2 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}k^2\\f\end{pmatrix} = \begin{pmatrix}-\|\mathbf{x_1}\|^2\\r^2\end{pmatrix} \tag{3.0.11}$$

Here ,$\|\mathbf{x_1}\|$ is given by ,

$$\|\mathbf{x_1}\| = \sqrt{2^2 + 3^2} \tag{3.0.12}$$

$$\|\mathbf{x_1}\| = \sqrt{13} \tag{3.0.13}$$

Subsituting equation 3.0.13,3.0.1 in equation 3.0.11 we get ,

$$\begin{pmatrix}2 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}k^2\\f\end{pmatrix} = \begin{pmatrix}-13\\25\end{pmatrix} \tag{3.0.14}$$

The augumented matrix of 3.0.14 is given by ,

$$\left(\begin{array}{cc|c}2 & 1 & -13\\1 & -1 & 25\end{array}\right) \tag{3.0.15}$$

By using row reduction technique, we get ,

$$\left(\begin{array}{cc|c}2 & 1 & -13\\1 & -1 & 25\end{array}\right) \xleftrightarrow{R_2\leftrightarrow R_1} \left(\begin{array}{cc|c}1 & -1 & 25\\2 & 1 & -13\end{array}\right) \tag{3.0.16}$$

$$\left(\begin{array}{cc|c}1 & -1 & 25\\2 & 1 & -13\end{array}\right) \xleftrightarrow{R_2=R_2-2R_1} \left(\begin{array}{cc|c}1 & -1 & 25\\0 & 3 & -63\end{array}\right) \tag{3.0.17}$$

$$\left(\begin{array}{cc|c}1 & -1 & 25\\0 & 3 & -63\end{array}\right) \xleftrightarrow{R_2=\frac{R_2}{3}} \left(\begin{array}{cc|c}1 & -1 & 25\\0 & 1 & -21\end{array}\right) \tag{3.0.18}$$

$$\left(\begin{array}{cc|c}1 & -1 & 25\\0 & 1 & -21\end{array}\right) \xleftrightarrow{R_1=R_1+R_2} \left(\begin{array}{cc|c}1 & 0 & 4\\0 & 1 & -21\end{array}\right) \tag{3.0.19}$$

Equation 3.0.14 can we rewritten as ,

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} k^2 \\ f \end{pmatrix} = \begin{pmatrix} 4 \\ -21 \end{pmatrix} \tag{3.0.20}$$

Expanding the above equation 3.0.20 we get ,

$$k^2 = 4 \tag{3.0.21}$$

$$k = \pm 2 \tag{3.0.22}$$

$$f = -21 \tag{3.0.23}$$

To get the centre substitute equation 3.0.22 in equation 3.0.2 To verify the above results we plot the circle with centre c as $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$,

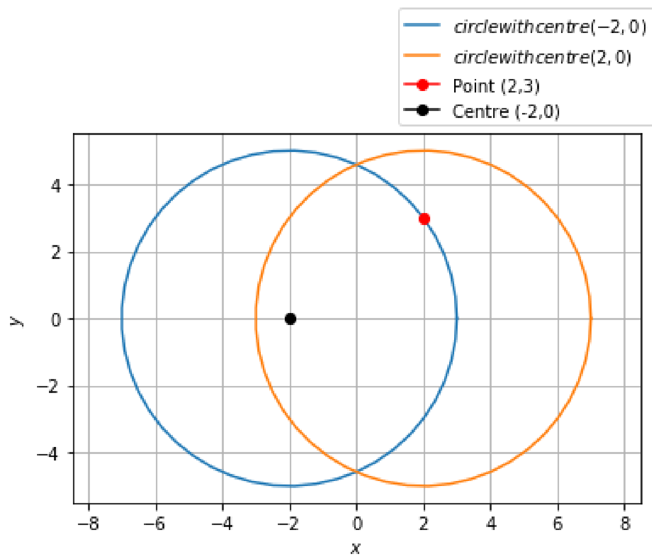

Fig. 1: Circle of radius 5 centre lies on x-axis and passing through the point(2,3)

From the above figure 1 it is clear that circle with centre $\mathbf{c}$=$\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ passes through the point $\mathbf{x_1}$

Desired equation of circle is given by ,

$$c = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{3.0.24}$$

$$f = -21 \tag{3.0.25}$$