Assignment 9

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***Abstract*—This document performs QR decomposition on a given 2X2 matrix.**

Download all latex-tikz codes from

https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/Assignment9_Matrix_Theory

1 Problem

Find QR decomposition of $\begin{pmatrix} 4 & 3 \\ 5 & -2 \end{pmatrix}$

2 QR Decomposition

The QR decomposition of a matrix is a decomposition of the matrix into an orthogonal matrix and an upper triangular matrix. A QR decomposition of a real square matrix A is a decomposition of A as

$$\mathbf{A} = \mathbf{QR} \tag{2.0.1}$$

where $\mathbf{Q}$ is an orthogonal matrix and $\mathbf{R}$ is an upper triangular matrix

3 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 5 & -2 \end{pmatrix} \tag{3.0.1}$$

Let $\mathbf{a}$ and $\mathbf{b}$ be the column vectors of the given matrix

$$\mathbf{a} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{3.0.2}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{3.0.3}$$

The above column vectors (3.0.2) ,(3.0.3) can be expressed as ,

$$\mathbf{a} = t_1 \mathbf{u}_1 \tag{3.0.4}$$

$$\mathbf{b} = s_1 \mathbf{u}_1 + t_2 \mathbf{u}_2 \tag{3.0.5}$$

Where,

$$t_1 = \|\mathbf{a}\| \tag{3.0.6}$$

$$\mathbf{u}_1 = \frac{\mathbf{a}}{t_1} \tag{3.0.7}$$

$$s_1 = \frac{\mathbf{u}_1^T \mathbf{b}}{\|\mathbf{u}_1\|^2} \tag{3.0.8}$$

$$\mathbf{u}_2 = \frac{\mathbf{b} - s_1 \mathbf{u}_1}{\|\mathbf{b} - s_1 \mathbf{u}_1\|} \tag{3.0.9}$$

$$t_2 = \mathbf{u}_2^T \mathbf{b} \tag{3.0.10}$$

The (3.0.4) and (3.0.5) can be written as,

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} t_1 & s_1 \\ 0 & t_2 \end{pmatrix} \tag{3.0.11}$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} = \mathbf{QR} \tag{3.0.12}$$

Here, $\mathbf{R}$ is an upper triangular matrix and $\mathbf{Q}$ is an orthogonal matrix such that

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{3.0.13}$$

Now using equations from (3.0.6) to (3.0.10) we get,

$$t_1 = \sqrt{4^2 + 5^2} = \sqrt{41} \tag{3.0.14}$$

$$\mathbf{u}_1 = \frac{1}{\sqrt{41}} \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{3.0.15}$$

$$s_1 = \begin{pmatrix} \frac{4}{\sqrt{41}} & \frac{5}{\sqrt{41}} \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \frac{2}{\sqrt{41}} \tag{3.0.16}$$

$$\mathbf{u}_2 = \frac{1}{\sqrt{41}} \begin{pmatrix} 5 \\ -4 \end{pmatrix} \tag{3.0.17}$$

$$t_2 = \begin{pmatrix} \frac{5}{\sqrt{41}} & \frac{-4}{\sqrt{41}} \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \frac{23}{\sqrt{41}} \tag{3.0.18}$$

Substituting the values from (3.0.14) to (3.0.18) in (3.0.12) we obtain QR decomposition as,

$$\begin{pmatrix} 4 & 3 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} \frac{4}{\sqrt{41}} & \frac{5}{\sqrt{41}} \\ \frac{5}{\sqrt{41}} & \frac{-4}{\sqrt{41}} \end{pmatrix} \begin{pmatrix} \sqrt{41} & \frac{2}{\sqrt{41}} \\ 0 & \frac{23}{\sqrt{41}} \end{pmatrix} \tag{3.0.19}$$