Assignment 18

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AI20MTECH14005

Download latex-tikz codes from

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https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/
    Assignment18_Matrix_Theory
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1 Problem

If $\mathbf{A}$ be the $n \times n$ matrix(with $n > 1$) satisfying $\mathbf{A}^2 - 7\mathbf{A} + 12\mathbf{I}_{n\times n} = \mathbf{0}_{n\times n}$ where $\mathbf{I}_{n\times n}$ and $\mathbf{0}_{n\times n}$ denotes the identity matrix and zero matrix of order n respectively. Then which of the following statements are true ?

1) $\mathbf{A}$ is invertible
2) t^2-7t+12n=0 where t=Tr($\mathbf{A}$)
3) d^2-7d+12=0 where d=Det($\mathbf{A}$)
4) λ^2-7λ+12=0 where λ is an eigen value of $\mathbf{A}$

2 Solution

Given	$\mathbf{A}$ be the $n \times n$ matrix where $n > 1$ satisfying the following equation $$\mathbf{A}^2 - 7\mathbf{A} + 12\mathbf{I}_{n\times n} = \mathbf{0}_{n\times n} \quad (2.0.1)$$
Explanation	The Cayley Hamilton Theorem states that every square matrix satisfies its own characteristic equation. Using this theorem the given equation (2.0.1) can be written as , $$\lambda^2 - 7\lambda + 12 = 0 \quad (2.0.2)$$ $$(\lambda - 4)(\lambda - 3) = 0 \quad (2.0.3)$$ $$\lambda_1 = 3 \quad (2.0.4)$$ $$\lambda_2 = 4 \quad (2.0.5)$$ Here λ_1 and λ_2 were eigen values of matrix $\mathbf{A}$ We know that determinant is product of eigen values. $$d = Det(\mathbf{A}) \quad (2.0.6)$$ $$\implies d = \lambda_1\lambda_2 \quad (2.0.7)$$ $$\implies d = 12 \neq 0 \quad (2.0.8)$$
Statement 1	A is invertible
	From equation (2.0.8), since $d \neq 0$ the given matrix $\mathbf{A}$ is Invertible. **True Statement**
Statement 2	$$t^2 - 7t + 12n = 0 \quad (2.0.9)$$

	We know that the trace is the sum of the eigen values. $$t = Tr(\mathbf{A}) \quad (2.0.10)$$ $$\implies t = \lambda_1 + \lambda_2 \quad (2.0.11)$$ $$\implies t = 7 \quad (2.0.12)$$ Substituting the equation (2.0.12) in (2.0.9) we get, $$7^2 - 7(7) + 12n = 0 \quad (2.0.13)$$ $$12n = 0 \quad (2.0.14)$$ Since given that $n > 1$ the equation (2.0.17) is not possible i.e $12n \neq 0$. Therefore, $t^2 - 7t + 12n = 0$ is a **False Statement**
Statement 3	$$d^2 - 7d + 12 = 0 \quad (2.0.15)$$
	Substituting the equation (2.0.8) in (2.0.15), we get, $$12^2 - 7(12) + 12 = 0 \quad (2.0.16)$$ $$72 = 0 \quad (2.0.17)$$ From equation (2.0.17) it is clear that the above statement 3 is invalid. **False Statement**
Statement 4	$$\lambda^2 - 7\lambda + 12 = 0 \quad (2.0.18)$$
	By Cayley Hamilton Theorem, equation (2.0.2) shows that the above statement 4 is valid. **True Statement**

TABLE 1: Explanation