Assignment 4

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***Abstract*—This document explains the proof of congruent triangles**

Download all latex-tikz codes from

https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/Assignment4_Matrix_Theory

1 Problem

In $\triangle ABC$, $\mathbf{D}$,$\mathbf{E}$ and $\mathbf{F}$ are respectively the mid-points of sides AB,BC and CA. Show that $\triangle ABC$ is divided into four congruent triangles by joining D,E and F.

2 Congruent Triangles

When two triangles are congruent they will have exactly the same three sides and exactly the same three angles.The one triangle may be a mirror image of the other triangle.

3 Solution

Given : $\triangle ABC$, $\mathbf{D}$,$\mathbf{E}$ and $\mathbf{F}$ are the midpoints of AB,BC and CA respectively

$$\mathbf{D} = \frac{A+B}{2} \tag{3.0.1}$$
$$\mathbf{E} = \frac{B+C}{2} \tag{3.0.2}$$
$$\mathbf{F} = \frac{A+C}{2} \tag{3.0.3}$$

The direction vector $\mathbf{m}_{DF}$ is given by ,

$$\mathbf{m}_{DF} = \mathbf{D} - \mathbf{F} \tag{3.0.4}$$

Since $\mathbf{D}$ is the mid-point of AB and $\mathbf{F}$ is the mid-point of AC,from equation 3.0.4,

$$\mathbf{m}_{DF} = \frac{\mathbf{A}+\mathbf{B}}{2} - \frac{\mathbf{A}+\mathbf{C}}{2} \tag{3.0.5}$$
$$\mathbf{m}_{DF} = \frac{\mathbf{B}-\mathbf{C}}{2} \tag{3.0.6}$$
$$\mathbf{m}_{DF} = \frac{\mathbf{m}_{BC}}{2} \tag{3.0.7}$$

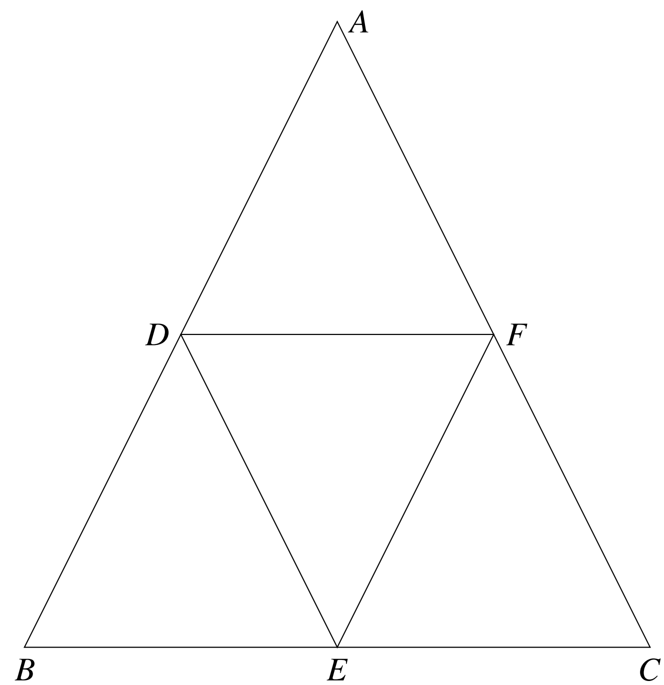

Fig. 1: Right Angled Triangle by Latex-Tikz

where $\mathbf{B} - \mathbf{C}$ is direction vector of line segment BC From equation 3.0.6 we could say that,

$$DF \parallel BC \tag{3.0.8}$$

Similarly we could show that ,

$$DE \parallel AC \tag{3.0.9}$$
$$EF \parallel AB \tag{3.0.10}$$

Since given $\mathbf{E}$ is the mid-point of BC,

$$\mathbf{m}_{BE} = \mathbf{m}_{EC} = \frac{\mathbf{m}_{BC}}{2} \tag{3.0.11}$$

$$\mathbf{m}_{BC} = 2\mathbf{m}_{BE} \tag{3.0.12}$$

Substituting equation 3.0.12 in equation 3.0.7,we get,

$$\mathbf{m}_{DF} = \mathbf{m}_{BE} \tag{3.0.13}$$

From equation 3.0.13,since the opposite sides are equal and parallel (DF=BE and DF|| BE),we could say that BDFE is parallelogram From Fig.1, Consider parallelogram BDFE, where DE is the diagonal of the parallelogram BDFE as shown in Fig.2

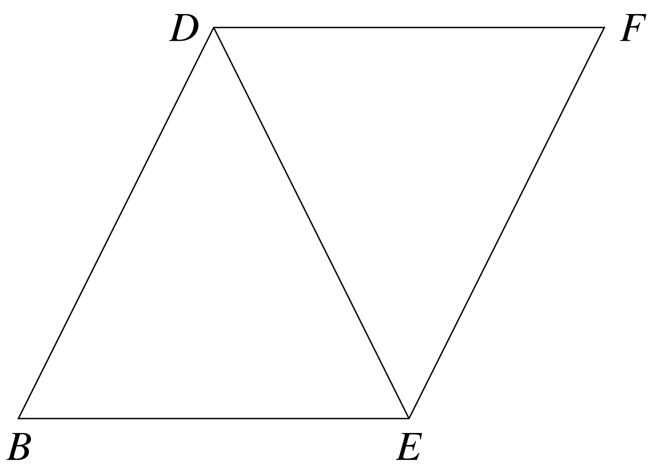

Fig. 2: Parallelogram by Latex-Tikz

Since BDFE is a parallelogram,

$$EF = BD \tag{3.0.14}$$

From the above equations 3.0.13,3.0.14 and DE is common side to both the $\triangle BDE$ and $\triangle DEF$, by Side-Side-Side (SSS) rule, if all the three sides of one triangle are equivalent to the corresponding three sides of the second triangle, then the two triangles are said to be congruent.

$$\triangle DBE \cong \triangle DEF \tag{3.0.15}$$

Similarly,

$$\triangle ADF \cong \triangle DEF \tag{3.0.16}$$

$$\triangle CEF \cong \triangle DEF \tag{3.0.17}$$

From equations 3.0.15,3.0.16 and 3.0.17,we could conclude that,

$$\triangle DBE \cong \triangle ADF \cong \triangle CEF \cong \triangle DEF \tag{3.0.18}$$

From equation 3.0.18 we could say that all four triangles are congruent which is obtained by joining the midpoints of the $\triangle ABC$

Hence Proved