Assignment 20

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AI20MTECH14005

Download latex-tikz codes from

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https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/
    Assignment20_Matrix_Theory
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1 Problem

Let $\mathbf{A}$ be an invertible 4×4 real matrix. Which of the following are NOT true ?

1) Rank $\mathbf{A}$ =4
2) For every vector $\mathbf{b} \in \mathbb{R}$, $\mathbf{A}\mathbf{x} = \mathbf{b}$ has exactly one solution.
3) dim(nullspace $\mathbf{A}$)$\geq$ 1
4) 0 is an eigenvalue of $\mathbf{A}$

2 Solution

Given	$\mathbf{A}$ is an invertible real matrix of order 4×4
Solution	Since given $\mathbf{A}$ is an invertible matrix, A has full rank. $$det(\mathbf{A}) \neq 0 \quad (2.0.1)$$ $$Rank(\mathbf{A}) = 4 \quad (2.0.2)$$ Let $\lambda_1, \lambda_2, \lambda_3$ and λ_4 be the eigenvalues of matrix $\mathbf{A}$. We know that determinant of matrix $\mathbf{A}$ is the product of eigenvalues of $\mathbf{A}$. $$\lambda_1\lambda_2\lambda_3\lambda_4 \neq 0 \quad (2.0.3)$$
Statement 1	$$Rank(\mathbf{A}) = 4$$
	Since $\mathbf{A}$ is an invertible matrix,it has full rank as shown in equation (2.0.2). **True Statement**
Statement 2	For every vector $\mathbf{b} \in \mathbb{R}$, $\mathbf{A}\mathbf{x} = \mathbf{b}$ has exactly one solution.
	For every $\mathbf{b}$, $$x = A^{-1}b$$ $\mathbf{x}$ will be unique solution for every $\mathbf{b}$. **True Statement**
Statement 3	dim(nullspace $\mathbf{A}$)$\geq$ 1.
	Using Rank Nullity Theorem, $$Rank(\mathbf{A}) + dim(nullspace\mathbf{A}) = n$$ $$\implies 4 + dim(nullspace\mathbf{A}) = 4$$ $$\implies dim(nullspace\mathbf{A}) = 0 \ngeq 1 \quad (2.0.4)$$

	where n is the number of columns in **A** Equation (2.0.4) proves that the given statement is **NOT True**.
Statement 4	0 is an eigenvalue of **A**
	From equation (2.0.3), we could say that no eigenvalue of **A** could be 0. **NOT True Statement**

TABLE 1: Explanation