Assignment 16

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Download latex-tikz codes from

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https://github.com/venkateshelangovan/IIT−Hyderabad−Assignments/tree/master/
    Assignment16_Matrix_Theory
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1 Problem

If $\mathbf{P}$ and $\mathbf{Q}$ are invertible matrices such that $\mathbf{PQ} = -\mathbf{QP}$,then we can conclude that

$$1.Tr(\mathbf{P}) = Tr(\mathbf{Q}) = 0 \tag{1.0.1}$$
$$2.Tr(\mathbf{P}) = Tr(\mathbf{Q}) = 1 \tag{1.0.2}$$
$$3.Tr(\mathbf{P}) = -Tr(\mathbf{Q}) \tag{1.0.3}$$
$$4.Tr(\mathbf{P}) \neq Tr(\mathbf{Q}) \tag{1.0.4}$$

2 Explanation with Proof

Given	$\mathbf{P}$ and $\mathbf{Q}$ are invertible matrices. Therefore $\mathbf{P}^{-1}$ and $\mathbf{Q}^{-1}$ exists. $$\mathbf{PQ} = -\mathbf{QP} \tag{2.0.1}$$
To Prove	Tr($\mathbf{P}$)=0
Proof 1	Post multiplying equation (2.0.1) by $\mathbf{Q}^{-1}$ we get, $$\mathbf{PQQ}^{-1} = -\mathbf{QPQ}^{-1} \tag{2.0.2}$$ $$\implies \mathbf{PI} = -\mathbf{QPQ}^{-1} \tag{2.0.3}$$ $$\implies \mathbf{P} = -\mathbf{QPQ}^{-1} \tag{2.0.4}$$ Taking trace on both sides for the equation (2.0.4), $$Tr(\mathbf{P}) = Tr(-\mathbf{QPQ}^{-1}) \tag{2.0.5}$$ $$\implies Tr(\mathbf{P}) = -Tr(\mathbf{QPQ}^{-1}) \tag{2.0.6}$$ We know that Tr($\mathbf{AB}$)=Tr($\mathbf{BA}$) Let $\mathbf{A}$=$\mathbf{Q}$ and $\mathbf{B}$=$\mathbf{PQ}^{-1}$ From the above property of trace equation (2.0.6) can be modified as $$Tr(\mathbf{P}) = -Tr(\mathbf{PQ}^{-1}\mathbf{Q}) \tag{2.0.7}$$ $$\implies Tr(\mathbf{P}) = -Tr(\mathbf{PI}) \tag{2.0.8}$$

	$\implies Tr(\mathbf{P}) = -Tr(\mathbf{P}) \quad (2.0.9)$ $\implies 2Tr(\mathbf{P}) = 0 \quad (2.0.10)$ $\implies Tr(\mathbf{P}) = 0 \quad (2.0.11)$
To Prove	Tr($\mathbf{Q}$)=0
Proof 2	Post multiplying equation (2.0.1) by $\mathbf{P}^{-1}$ we get, $\mathbf{PQP}^{-1} = -\mathbf{QPP}^{-1} \quad (2.0.12)$ $\implies \mathbf{PQP}^{-1} = -\mathbf{QI} \quad (2.0.13)$ $\implies \mathbf{PQP}^{-1} = -\mathbf{Q} \quad (2.0.14)$ Taking trace on both sides for the equation (2.0.14), $Tr(\mathbf{PQP}^{-1}) = Tr(-\mathbf{Q}) \quad (2.0.15)$ $\implies Tr(\mathbf{PQP}^{-1}) = -Tr(\mathbf{Q}) \quad (2.0.16)$ We know that Tr($\mathbf{AB}$)=Tr($\mathbf{BA}$) Let $\mathbf{A}$=$\mathbf{P}$ and $\mathbf{B}$=$\mathbf{QP}^{-1}$ From the above property of trace equation (2.0.16) can be modified as $Tr(\mathbf{QP}^{-1}\mathbf{P}) = -Tr(\mathbf{Q}) \quad (2.0.17)$ $\implies Tr(\mathbf{QI}) = -Tr(\mathbf{Q}) \quad (2.0.18)$ $\implies Tr(\mathbf{Q}) = -Tr(\mathbf{Q}) \quad (2.0.19)$ $\implies 2Tr(\mathbf{Q}) = 0 \quad (2.0.20)$ $\implies Tr(\mathbf{Q}) = 0 \quad (2.0.21)$
Statement 1	Tr($\mathbf{P}$)=Tr($\mathbf{Q}$)=0
Explanation	From equation (2.0.11) and (2.0.21) we could say that, $Tr(\mathbf{P}) = Tr(\mathbf{Q}) = 0 \quad (2.0.22)$ Valid Conclusion
Statement 2	Tr($\mathbf{P}$) $= Tr(\mathbf{Q}) = 1$
Explanation	From equation (2.0.11) and (2.0.21) we could say that, $Tr(\mathbf{P}) = Tr(\mathbf{Q}) \neq 1 \quad (2.0.23)$ Invalid Conclusion

Statement 3	Tr(**P**) = $-Tr(\mathbf{Q})$
Explanation	Substituting the conclusion 1 result equation (2.0.22) in equation (2.0.9) we get, $$Tr(\mathbf{P}) = -Tr(\mathbf{Q}) \tag{2.0.24}$$ Valid Conclusion
Statement 4	Tr(**P**) $\neq Tr(\mathbf{Q})$
Explanation	From equation (2.0.11) and (2.0.21) we could say that, $$Tr(\mathbf{P}) = Tr(\mathbf{Q}) \tag{2.0.25}$$ Invalid Conclusion

TABLE 1: Explanation with Proofs