Technische Universiteit Delft

Faculty of aerospace engineering

Master of Science Aerospace Engineering

Assignment - II

DESIGN AND ANALYSIS OF COMPOSITE STRUCTURES - I (AE4ASM109)

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Question

As competition between aircraft manufacturers increases, it becomes necessary to have an efficient organization which can create new models rapidly in order to stay ahead of the competition. Among others, this implies the existence of a very fast and reliable design group which can create trade studies very quickly. You are to interview with the elite design organization of one of the big aircraft manufacturers. During the discussions, you made a good impression but the group leads are not sure if you have "what it takes". So, towards the end of the day, when you are tired and want to go home, they pose to you the following problem for which they give you two hours to complete.

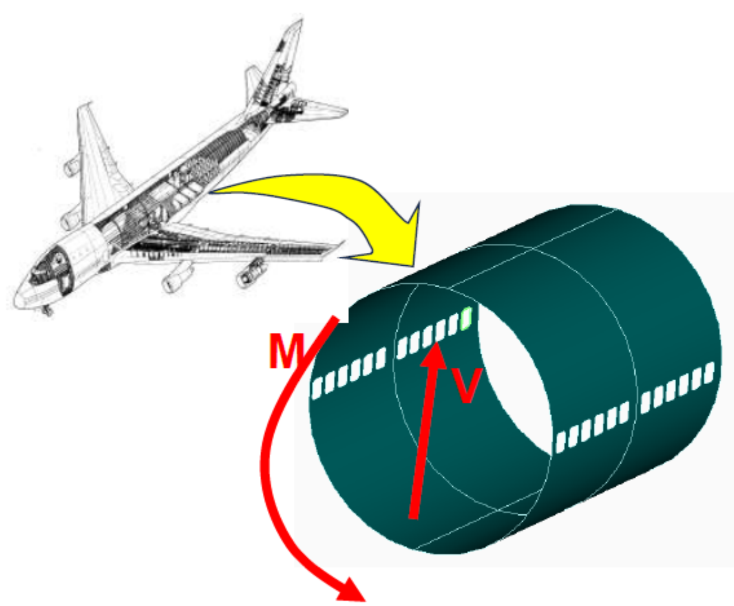

Figure 1.1: Descriptive Figure

The new model they are considering has a fuselage with 6 m diameter. At its most highly loaded location a shear load V=1.5 MN and a bending moment M=15 MNm are applied. (a) Design a monolithic cross-section assuming Aluminum is used. (b) Design the same monolithic cross-section but now assume it is made out of a composite material (c) Design the same cross-section but now use skin and T stiffeners. (d) Determine the % weight reduction from (a) to (b) to (c). Include sketches showing the geometry and layup(s) of skin and stiffeners.

Table 1.1: Material properties

Property	**Aluminum**	**Composite (UD tape)**
E_X	69	142
E_X	69	11.2
G_{XY}	26	5
v_{XY}	0.29	0.3
X^t	410 (yield)	2200
X^c	430 (yield)	1800
Y^t	400 (yield)	70
Y^c	430 (yield)	300
S	230 (yield)	100
t_{ply}	NA	0.135
ρ	2770	1610

Notes:

1. For the Al design you can assume the same thickness everywhere
2. For the monolithic composite design you must have at least three different thicknesses
3. For the skin-stiffened composite design you must have at least three different thicknesses for the skin and at least three different stiffener cross-sections. Also, in each stiffener, the web layup must be different than the flange layup.
4. It is up to you to decide how many stiffeners you will use and at what spacing. The spacing need not be constant.
5. No need to check for panel breaker condition, global buckling (skin + stiffeners together), no need to check for skin-stiffener separation. Only material covered so far in the course.
6. You can work in teams of up to four persons per team. Each team member submits his/her own report in his/her own words. On the first page, the names of the other group members must also be mentioned.

Basic parameters

For composites in general, we need to have some basic parameters that we can use to analyze the composite properly. In this chapter, these are described in little detail, and later for the analysis they are used when required. All these are coded in Python and are called functions in the code.

2.1 ABD Matrix

Generally, for a laminate which consists of n laminas in it, we need to determine three matrices in the form of A, B and D. These matrices are then assembled in the following way:

$$\begin{pmatrix} A & B \\ B & D \end{pmatrix}$$

This is a 6 x 6 matrix and this is used in combination with the given loads to find the strains and stresses described in the upcoming sections.

To find A, B, and D, the following formulas are used:

$$A_{ij} = \Sigma_{k=1}^{n} Q_{ij}^{k}(Z_k - Z_{k-1})$$

$$B_{ij} = \Sigma_{k=1}^{n} Q_{ij}^{k}(Z_k^2 - Z_{k-1}^2)$$

$$D_{ij} = \Sigma_{k=1}^{n} Q_{ij}^{k}(Z_k - Z_{k-1}^3)$$

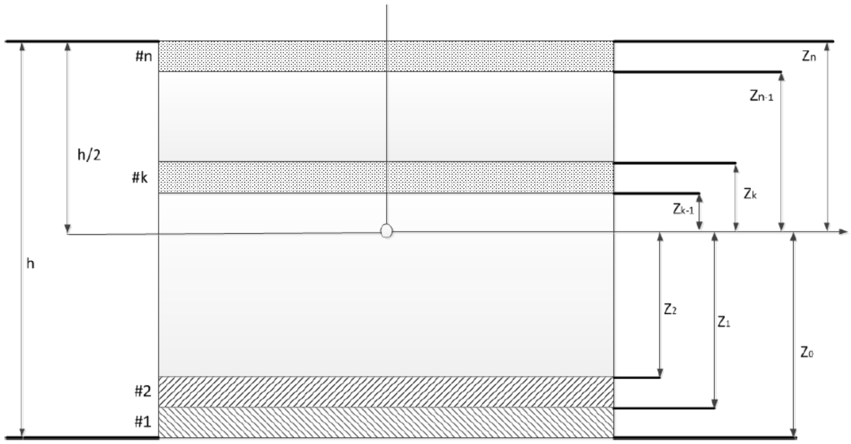

Figure 2.1: Way to determine Z

Figure above gives the way to find Z for a particular lamina. Basically, for an ij term (i,j vary from 1 to 3 so it is a 3 x 3 matrix), we add the compliance matrix of each of the layers and multiply it with a particular combination of Z values for that particular laminate to obtain A, B, and D. The compliance matrix is obtained for an orthotropic laminate as:

$$Q = \begin{pmatrix} \frac{1}{E_1} & \frac{-v_{12}}{E_1} & 0 \\ \frac{-v_{12}}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{pmatrix}$$

2.2 Equivalent properties of a laminate as a whole

Once we have the matrix for A and D, we can find the equivalent properties of the laminate as a whole. Using the A matrix we can find it out for the membrane properties while using the D matrix we can find the bending properties.

For the membrane properites, we obtain it by

$$Q = \frac{A}{total\ thickness}$$

$$compliance\ matrix = Q^{-1}$$

$$E_x = \frac{1}{complaince\ matrix[0][0]}$$

$$E_y = \frac{1}{complaince\ matrix[1][1]}$$

$$G_{xy} = \frac{1}{complaince\ matrix[2][2]}$$

$$v_{xy} = -E_x * complaince\ matrix[1][0]$$

$$v_{yx} = -E_y * complaince\ matrix[0][1]$$

These relations are just obtained by comparing them with the one in Q described earlier.
The bending properties are obtained by,

$$D_{inv} = D^{-1}$$

$$E_{xb} = \frac{12}{t^3 D_{inv}[0][0]}$$

$$E_{yb} = \frac{12}{t^3 D_{inv}[1][1]}$$

$$G_{xyb} = \frac{12}{t^3 D_{inv}[2][2]}$$

$$v_{xyb} = -\frac{D_{inv}[0][1]}{D_{inv}[0][0]}$$

$$v_{yxb} = -\frac{D_{inv}[1][0]}{D_{inv}[1][1]}$$

2.3 Stress in Laminas

This is very crucial as we use these values for every lamina and then use Puck's criteria of failure, later on, to see if a first-ply failure occurred or not for a particular thickness and layup.
We have,

$$\begin{pmatrix} N \\ M \end{pmatrix} = \begin{pmatrix} A & B \\ B & D \end{pmatrix} \begin{pmatrix} \epsilon \\ k \end{pmatrix}$$

where N and M are

$$N = \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix}$$

$$M = \begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix}$$

and ϵ and k are

$$\epsilon = \begin{pmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{pmatrix}$$

$$k = \begin{pmatrix} k_x \\ k_y \\ k_{xy} \end{pmatrix}$$

The first step would then be to take the inverse of ABD matrix and multiply with the loading to find the strains and the curvature.

ϵ is the strain at z =0 where as k is the curvature. To obtain strain in any lamina, we use the formula $\epsilon_{x_z} = \epsilon_x + z * k_x$, $\epsilon_{y_z} = \epsilon_y + z * k_y$ and $\epsilon_{xy_z} = \epsilon_{xy} + z * k_{xy}$. (z would be the average of the z for that laminate, therefore $Z = 0.5 * (Z_{k+1} + Z_k)$.

Once these strains are obtained in each of the lamina, we can determine the stress in each lamina by multiplying with the compliance matrix. Remember, the strains are in the global coordinate system(loading axis) while the compliance matrix is in the local/material coordinate system. For this purpose, we rotate the strain with the transformation matrix and get the stress in the local coordinate system.

$$stress = QT\epsilon$$

where $T = \begin{pmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & m^2 & m^2 - n^2 \end{pmatrix}$, $\epsilon = \epsilon + k * z$.

2.4 Puck criteria of failure

For this assignment, the Puck criterion of failure that was taught in the class was used.
There are four failure indices that we calculate. The first is for fiber failure in tension or compression, while the other three are for inter-fiber failure A, inter-fiber failure B, and inter-fiber failure C respectively. If any of these failure indices are more than 1, it is then assumed that first-ply failure occurs. The equations for this are given below:

$$f_{t/c} = \frac{1}{\pm X_{t,c}}(\sigma_1 - (v_{21} - v_{21f} m_{\sigma f} \frac{E_1}{E_{1f}}(\sigma_2 + \sigma_3))$$

where the σs are obtained from the stress in each lamina. E_{1f} (property of the fiber) was assumed to be 145.3 GPa. $m_{\sigma f}$ is taken to be 1.1 while v_{21f} (property of the fiber) is taken to be 0.2. Remaining properties are the one for the particular laminate.

$$f_{IFF-A} = \sqrt{(\frac{\sigma_{21}}{\sigma_{12}^u})^2 + (1 - \frac{p_{12}^t \sigma_{22_T}^u}{\sigma_{12}^u})^2 (\frac{\sigma_{22}}{\sigma_{22T}^u})^2} + p_{12}^t \frac{\sigma_{22}}{\sigma_{12}^u}$$

Here, the constant $p_{12}^t = 0.3$ and has been obtained experimentally by Puck. The σs with coefficient u are the yield strengths in that particular direction.

- Mode B ($\sigma_{22} < 0$)

$$0 \le \left|\frac{\sigma_{22}}{\sigma_{12}}\right| \le \frac{\sigma_{23}^{A}}{\left|\sigma_{12}^{c}\right|}$$

$$\frac{1}{\sigma_{12}^{u}}\left(\sqrt{\sigma_{12}^{2} + (p_{12}^{(-)}\sigma_{22})^{2}} + p_{12}^{(-)}\sigma_{22}\right) = 1$$

$$\sigma_{23}^{A} = \frac{\sigma_{12}^{u}}{2p_{12}^{(-)}}\left(\sqrt{1+2p_{12}^{(-)}\frac{\sigma_{22C}^{u}}{\sigma_{12}^{u}}} - 1\right)$$

$\sigma_{12}^{c} = \sigma_{12}^{u}\sqrt{1+2p_{23}^{(-)}}$ where $p_{12}^{(-)}$ is 0.2

Figure 2.2: IFF-B

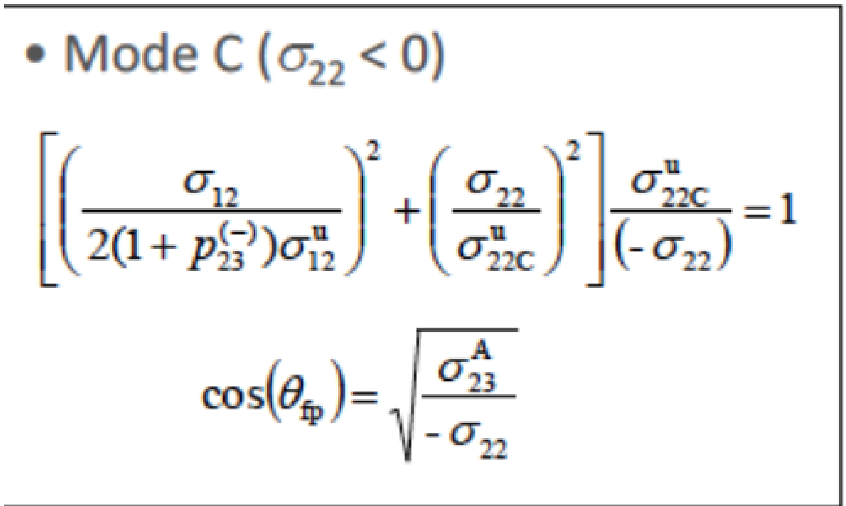

Figure 2.3: IFF-C

*The images and puck rules are taken from Dr. Dimitrios Zarouchas's slides for the Design and Analysis of Composite Structures-I course.

2.5 Von-mises stress

For Aluminium where there is interaction between shear and compression, we can't use Puck criteria. Instead, we use Von-Mises criteria as given below:

$$\sigma_{\text{vonMises}} = \sqrt{\sigma_X^2 - \sigma_X\sigma_Y + \sigma_Y^2 + 3\tau_{XY}^2}$$

$$\sigma_{\text{vonMises}} = \frac{1}{t}\sqrt{N_X^2 - N_X N_Y + N_Y^2 + 3N_{XY}^2}$$

$$N_{\text{interaction}} = \sqrt{N_X^2 - N_X N_Y + N_Y^2 + 3N_{XY}^2}$$

$$t_{\text{interaction}} = (1+\text{MS})\,\frac{N_{\text{interaction}}}{Y_t}$$

2.6 Ply-stacking rules

This has been taken from the prescribed textbook for the course and has been rephrased into my own words:

1. The final laminate must consist of atleast 3 different orientations to avoid matrix domination in any direction.

2. We limit the orientation to 0, 90, +45 and -45 degrees and by ensuring at least 10% of each of these orientations exist.

3. To avoid generation of B matrix and subsequently coupling between axial and bending forces, we make sure that the laminate remains symmetrix. Also, ensure that the laminate is balanced.

4. +45 and -45 must always be situated next to each other. This will minimize D_{16} and D_{26}. Also, these 45 degrees must be as far away from the neutral axis to maximize D_{66}

5. 0 degree lamina must be away from the neutral axis to improve the bending stiffness of the laminate.

6. At most four laminas of the same orientation must be next to each other. Otherwise, matrix cracks can be generated.

2.7 Bending analysis

For bending forces, we know the equation:

$$\sigma = \frac{My}{I}$$

where σ is the bending stress at a particular location y.
y is the distance from the neutral axis.
I is the moment of inertia. It is given by the formula, $I = \pi r^3 t$. where r is the mean radius, while t is the thickness of the section. For this question, r is assumed to be equal to the outer radius which is 3000 mm. t is an unknown and is taken to the LHS to make it Nx.

Therefore,

$$N_x = \frac{My}{\pi r^3}$$

In terms of radius, y can be substituted as $r \sin\theta$.

2.8 Shear analysis

For shear stress at any point, we know it is given by the equation:

$$\tau = \frac{FAy}{Ib}$$

where,

F = The shear Force acting at the section

Ay = The moment of the area above the section

A = Area above the section where we want to measure the shear stress

y = Centroid of the area

I = Moment of inertia

b = width of the section where we want to measure the shear stress

To obtain shear force at any point, we will need to derive it for a circular hollow cross-section.

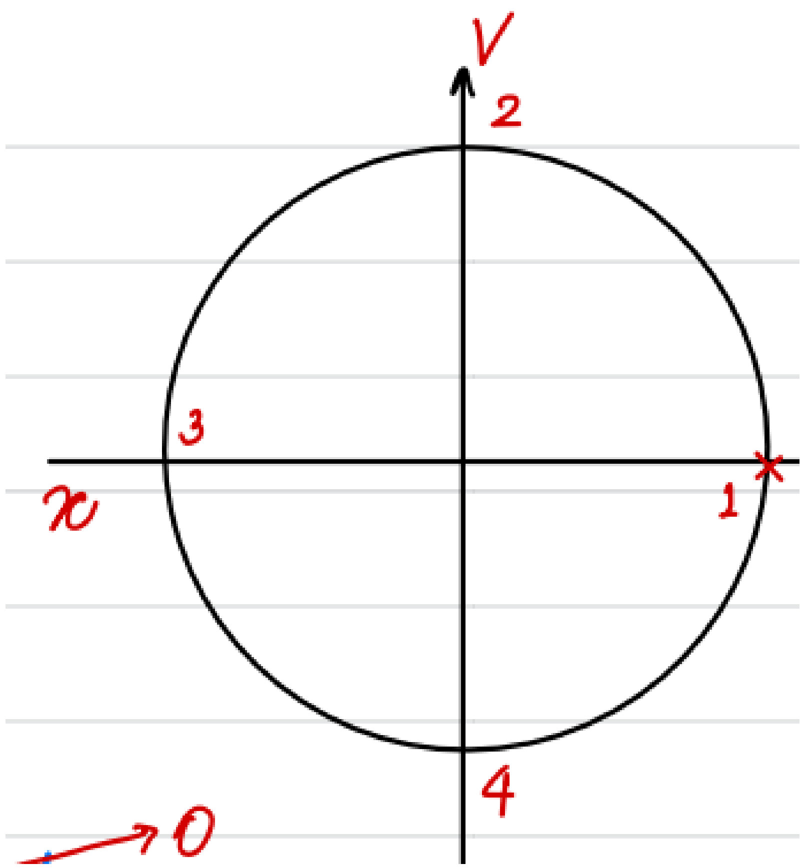

Figure 2.4: Diagram used for N_{xy} derivation

As per figure 2.4, x is the axis of symmetry. $I_{xy} = 0$, $S_x = 0$ and $S_y = V$. Assuming that the origin is at 1, the shear flow would be $q_{s,0}$.

$$q_s = -\frac{(S_x I_{xx} - S_y I_{xy})}{(I_{xx}I_{yy} - I_{xy}^2])}\int_0^s t_x ds - \frac{(S_y I_{yy} - S_x I_{xy})}{(I_{xx}I_{yy} - I_{xy}^2])}\int_0^s t_y ds + q_{s,0}$$

$$q_s = -(\frac{S_y}{I_{xx}}\int_0^s t_y ds) + q_{s,0}$$

$$I_{xx} = \pi r^3 t$$

$$yds = r\sin\theta r d\theta$$

$$q_{b1,2} = -\frac{V}{\pi r^3 t}\int_0^\theta tR\sin\theta R d\theta$$

$$q_{b1,2} = -\frac{V}{\pi R}(-\cos\theta)_0^\theta = \frac{V}{\pi R}(\cos\theta - 1)$$

For all the 4 sections it is identical and we are choosing the moment center so as it coincides with S_x and S_y.

Net shear force in the section is zero.

$$0 = \int pq_b ds + 2Aq_{s,0}$$

$$0 = \int_0^{\pi/2}\frac{V}{\pi R}(\cos\theta - 1)R^2 d\theta + 2\pi R^2 q_{s,0} + \int_{\pi/2}^{\pi}\frac{V}{\pi R}(\cos\theta - 1)R^2 d\theta + \int_p i^{3\pi/2}\frac{V}{\pi R}(\cos\theta - 1)R^2 d\theta$$

$$+\int_{3\pi/2}^{2\pi}\frac{V}{\pi R}(\cos\theta - 1)R^2 d\theta$$

$$-2^2 q_{s,0} = \frac{VR}{\pi}((\sin\theta - \theta)_0^{\pi/2} + (\sin\theta - \theta)_{\pi/2}^{\pi} + (\sin\theta - \theta)_{\pi}^{3\pi/2}) + (\sin\theta - \theta)_{3\pi/2}^{\pi/2})$$

$$q_{s,0} = \frac{-V}{2\pi R^2}(1 - \pi/2 - /pi - 1 + \pi/2 - 1 - 3\pi/2 + \pi - 2\pi + 1 + 3\pi/2])$$

$$q_{s,0} = \frac{V}{\pi R}$$

$$q_s = \frac{V}{\pi R}(\cos\theta - 1) + \frac{V}{\pi R}$$

This is in N/mm which is exactly equal to our required quantity N_{xy}

2.9 Buckling analysis for monolithic composite and Aluminum section

For buckling, the following equations are used:
We try to optimize the design for a margin of safety. For any thickness that is selected for Aluminum, the reserve factor is calculated, and based on this the margin of safety = reserve factor - 1. This margin of safety at any of the y's is required to be greater than 1 and the entire design for buckling revolves around this. Also, the buckling factor must be less than 1.

$$\frac{N_x}{N_{x_{critical}}} + \frac{N_{xy}}{N_{xy_{critical}}}^2 = bf$$

where bf is the buckling factor and this must be less than 1.

$$\frac{N_x * RF}{N_{x_{critical}}} + \frac{N_{xy} * RF}{N_{xy_{critical}}}^2 = 1$$

where RF is the reserve factor and

$$mos = RF - 1$$

where mos is the margin of safety.

N_x and N_{xy} are straight forward. We can directly find them from the equations mentioned earlier in bending and shear respectively.

On the other hand, $N_{x_{critical}}$ is the load that it buckles under when only pure compression acts. $N_{xy_{critical}}$ is the load that it buckles under when only pure shear acts. The equations to find these are given below:

$$N_{x_{critical}} = \frac{N_{E_{critical}}}{1 + \frac{kN_{E_{critical}}}{tG_c}}$$

here, k was taken to be 5/6 and $G_C = 0.7G_{xy}$

$$N_{E_{critical}} = \frac{\pi^2}{a^2}[D_{11}m^2 + 2(D_{12} + 2D_{66})(AR)^2 + D_{22}\frac{(AR)^4}{m^2}]$$

We need to find the right value of m that minimizes $N_{E_{critical}}$. This was achieved using the python scipy module.
Lets now move onto N_{xy}:

$$N_{xy_{critial}} = \frac{N_{xy_c}}{1 + \frac{N_{xy_c}}{t_c G_c}}$$

where

$$N_{xy_c} = \pm\frac{9\pi^4 b}{32a^3}(D_{11} + 2(D_{12} + 2D_{66}\frac{a^2}{b^2} + D_{22}\frac{a^4}{b^4})$$

As you can notice, for an Aluminum structure, how do we get the D matrix and then extract the corresponding values for D_{11},D_{22},D_{12} and D_{66}. The values of this D are obtained using the same equations for composite, but instead, we use a single 0-degree ply of thickness (that has been calculated for this iteration). Based on this we get a D matrix and then we extract the corresponding values of D to obtain D_{11},D_{22},D_{12}, and D_{66}.

Part - A: Aluminum Section

For the Aluminum section analysis, we are taking the thickness to be constant throughout. Therefore, the neutral axis would be at the geometric center of the circle. At the Neutral axis, the shear force would be maximum whereas we keep going away from the neutral axis, the bending forces become critical and the shear force reduces. At the topmost point of the section, forces due to tension will be maximum and shear will be zero, whereas at the bottom-most point of the section, forces due to compression will be maximum, and again shear will be zero.

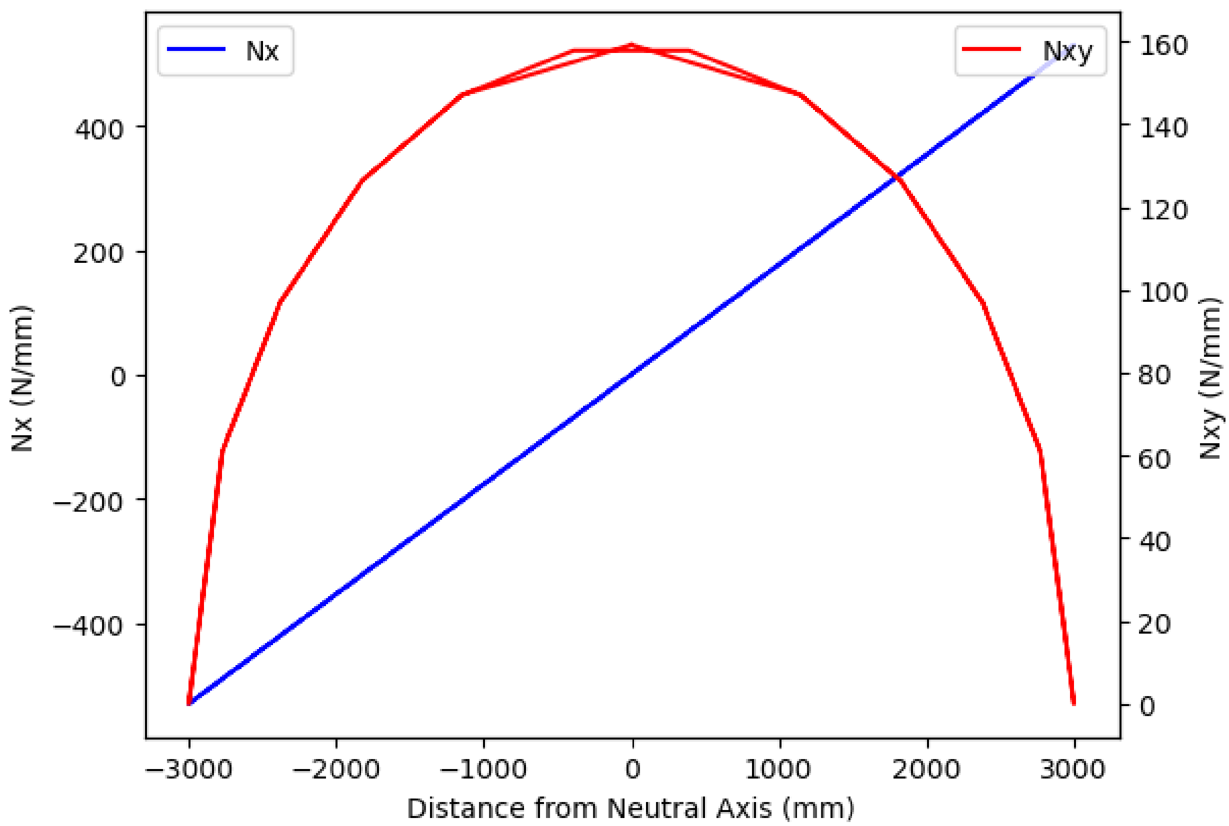

Figure 3.1: Loading in bending and shear along neutral axis

The equations used to plot the graph in 3.1, are derived in the sections of bending and shear earlier.

3.1 Properties of Aluminum

For Aluminum, the given properties were multiplied by a safety factor. This was in the same range as the composite sections too, and the factor was 0.416 (It was a product of knockdown factors due to material scatter, environment, and damage). Ideally, material scatter and damage are less prevalent for Aluminium but usually, a safety factor of 2 is taken which translates to a 0.5 knockdown factor. As they are pretty close, this was an acceptable assumption. These knockdown factors were applied to the strengths alone and not the moduli.

Table 3.1: Aluminum properties

Property	**Aluminum**	**Al with safety factor**
E_X	69	69
E_Y	69	69
G_{XY}	26	26
v_{XY}	0.29	0.29
X^t	410 (yield)	170.56
X^c	430 (yield)	178.88
Y^t	400 (yield)	166.4
Y^c	430 (yield)	178.88
S	230 (yield)	95.68
t_{ply}	NA	NA
ρ	2770	2770

A brief of the forces and axis taken for consideration are given in the figure 3.2. As can be seen, the X direction is assumed to be along the length axis while Y and Z are in the plane of the circular cross-section. The tensile and compressive forces due to bending occur along the X-direction and hence for all analysis, the properties of compression and tension are taken in the X-direction. Shear force is a parabolic distribution where as the bending forces are linearly distributed. Shear is maximum at y = 0, while bending is maximum at y = 3 (tensile) and y = -3 compressive forces.

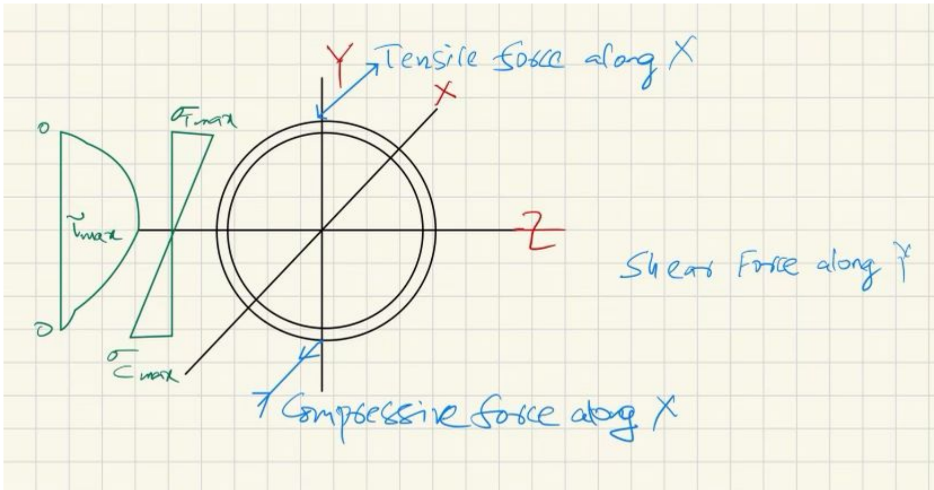

Figure 3.2: Summary of the directions of the forces

The following parameters were used:
Shear Force, $V = 1.5 \times 10^6$ N
Bending Moment, $M = 15 \times 10^9$ N-mm
Diameter = 6000 mm Radius = Diameter/2 = 3000 mm The location of interest is described in terms of theta (theta zero being along the Z-axis and theta 90 along the Y-axis).
These theta's are [0, 22.5, 37.5, 52.5, 67.5, 90, 90, 112.5, 127.5, 142.5, 157.5, 172.5, 187.5, 202.5, 217.5, 232.5, 247.5, 270, 270, 292.5, 307.5, 322.5, 337.5, 360]. Therefore, for every quadrant, we will have 6 zones of interest.
Based on these zones of interest, we can plot the N_x and N_{xy} along the circumference. This is shown in 3.1.

Using the equations mentioned in section 2.6 (Buckling analysis for monolithic composite and Aluminium section), we get the thickness for margin of safety ¿= 1 and buckling factor less than 1.

Once the thicknesses are obtained, we then check if the Von-Mises criterion is satisfied for the given N_x and N_{xy} loads at that particular location. After designing for buckling, I haven't observed any failure for the Von-Mises criteria so far.

3.2 Results

The final thickness that has been obtained for the Aluminum section is 14.0 mm. The minimum margin of safety factor obtained was 1.20 while the maximum buckling factor obtained was 0.45. This was obtained at the bottom-most point where pure compression occurs. No failure occurs based on Von-Mises criteria anywhere.

$$mass = Area * density$$

$$mass = \pi * (r^2 - (r - t)^2) * \rho$$

Only the maximum thickness is taken and the mass is calculated.

$$mass = \pi * (3000^2 - (3000 - 14)^2) * 2.77 \times 10^{-6}$$

$$mass = 0.72928 kg/mm$$

$$mass = 729.28 kg/m$$

Part - B: Composite Section

For all layups from here on, they are defined radially inwards. That means the higher the number of plies, the higher would be the thickness and the lower the inner diameter. For all the properties, we apply a knockdown factor for the environment (0.8), a knockdown factor for damage (0.65), and a knockdown factor for the material scatter of about 0.8 is applied. Therefore, the total knockdown factor would be $0.8 \times 0.65 \times 0.8 = 0.42$.

Next, having a look at the cross-section of the fuselage that we are analyzing is important. Because some of the parameters are dependent on this.

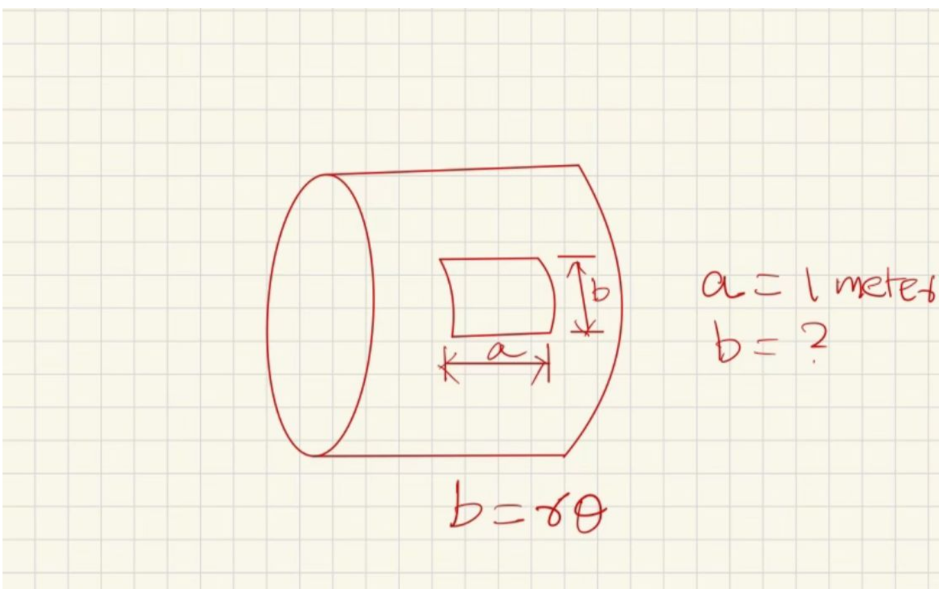

Figure 4.1: a and b values

The rest of the procedure is similar to the one done for Aluminum. Only here we will have theta and thickness for each of the layers.

Here, an initial layup for the monolithic composite is assumed to be [45, -45, 45, -45, 0, 90, 90, 0, -45, 45, -45, 45, 0, 90]. This follows all the rules mentioned earlier and as the parameters for margin of safety are not met, we keep incrementing the layers.

4.1 Results

The layup at the most critical section was found to be consisting of 114 layers. The layup was $[45, -45, 45, -45, 45, -45, 0, 90, 90, 0, -45, 45, -45, 45, 0, 0, 90, 0, 90]_{3s}$. Ply layups are constructed by replicating a sequence of plies n times and then reflecting it. This method ensures a symmetric, well-balanced layup, with the majority of 0° and 45° plies positioned on the outermost layers.

For the Aluminium section, the crucial point of the design was where compression was maximum. But here for monolithic composite, it is happening slightly away from y = -3 mm where the interaction between shear and bending is maximum.

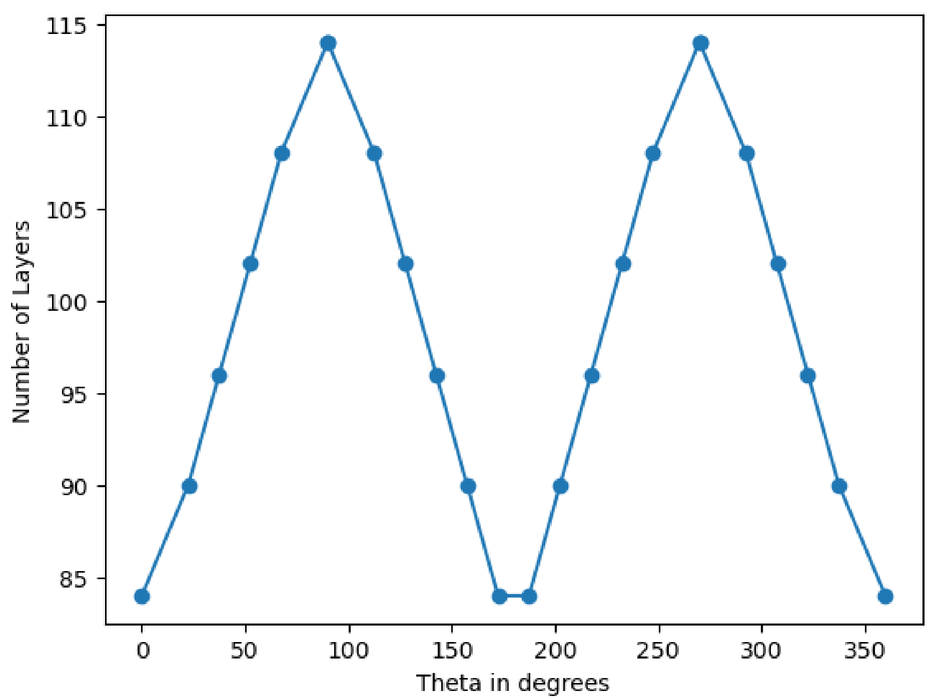

Figure 4.2: Number of layers required along theta

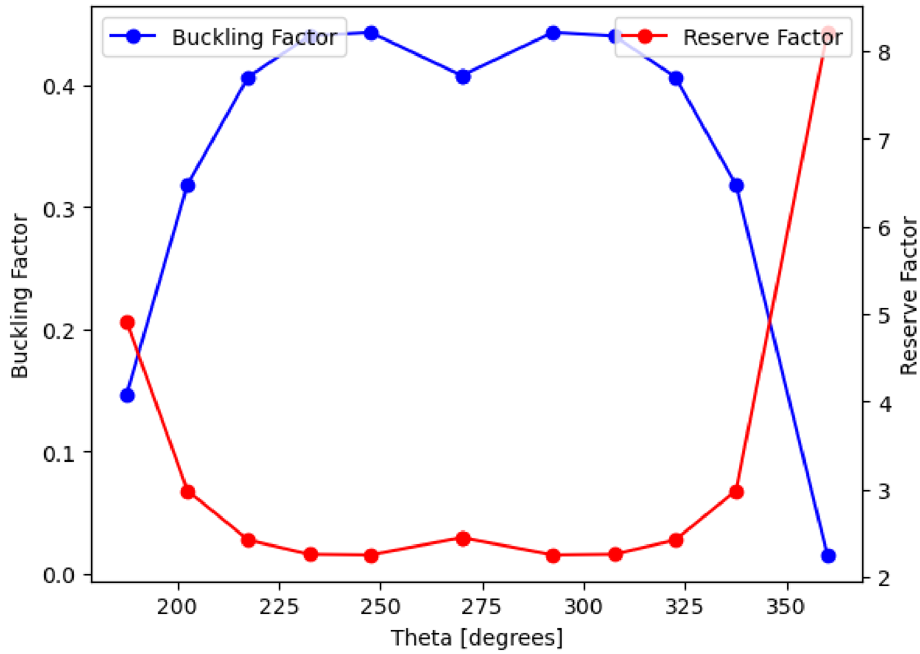

Figure 4.3: Reserve factor and buckling factor for Quadrant - 3 and 4 (compressive regions)

Therefore, the maximum thickness would be 0.135×114 mm. Calculating the mass of the monolithic section:

$$mass = Area * density$$

$$mass = \Sigma\pi(r^2 - (r - t_i)^2) * \rho * \theta/360$$

$$mass = 404.68 kg/m$$

Table 4.1: Laminate Stack-up for Each Zone

Zone	Stackup	t(mm)
1, 12, 13 & 24	$[\pm45, 45, 0, 90, 0, (\pm45)_2, (0, 90)_s, (\pm45)_2, 0, 90, 0, 45, \pm45]_3$	9.72
2, 11, 14 & 23	$[(\pm45)_2, 0, 90, 0, (\pm45)_2, (0, 90)_s, (\pm45)_2, 0, 90, 0, (\pm45)_2]_3$	10.53
3, 10, 15 & 22	$[(\pm45)_2, 45, 0, 90, 0, (\pm45)_2, (0, 90)_s, (\pm45)_2, 0, 90, 0, 45, (\pm45)_2]_3$	11.34
4, 9, 16 & 21	$[(\pm45)_2, 45, 0, 90, 0, (\pm45)_2, (0, 90, 0)_s, (\pm45)_2, 0, 90, 0, 45, (\pm45)_2]_3$	12.15
5, 8, 17 & 20	$[(\pm45)_2, 45, 0, 90, 0, (\pm45)_2, (0, 90)_2, (90, 0)_2, (\pm45)_2, 0, 90, 0, 45, (\pm45)_2]_3$	12.96
6, 7, 18 & 19	$[(\pm45)_2, 45, 0, 90, 0, (\pm45)_2, 0, (0, 90)_2, (90, 0)_2, 0, (\pm45)_2, 0, 90, 0, 45, (\pm45)_2]_3$	13.77

Part - C: Composite Section with stiffener

For the composite section with stiffeners included, The loads are evaluated this time at y = -3, -2, -1, 0, 1, 2 and 3 meters. y is the distance of the neutral axis from the point of interest. This is because I wanted to have sufficient distance for a section to be able to hold a few stiffeners. Therefore, here it will be 3 zones of interest per quarter.

Also, compared to composites, apart from bending, shear, and skin buckling (here buckling considered only between the stiffeners), we need to look for failure in buckling of the stiffeners (Euler buckling load: assuming it is simply supported) and crippling failure. The sequence of performing these operations is as follows:

1. We first try to find the skin layup at each of these points. (The layup would remain constant until it reaches a new value of y). For the top half of the skin, this is done by just using the 'Puck criterion' for first-ply failure. For the bottom half of the skin where the loading is compressive, this is done through buckling equations and then we check for first ply failure using the 'Puck Criterion'.

2. Once the skin layup is obtained, next the axial and bending stiffness (EA and EI) that is required for each of the stiffeners are obtained for that particular y value.

3. Based on these values, a suitable layup, and suitable dimensions for the stiffener are obtained. For this problem, a J stiffener is shortlisted.

4. Once the stiffener's dimensions and layup are finalized, next we need to verify whether the stiffener would buckle (based on Euler's condition) or cripple. If the stiffeners do buckle or cripple then again the layups are suitably adjusted to avoid this failure.

For the skin, six different thicknesses are considered at y values of -3,-2,-1,0,1,2,3 meters. For the stiffeners, three different cross-sections are selected. This is achieved by varying the widths of the T-cross-section and correspondingly different thicknesses/layups are obtained. Also, it is ensured that the layup in the flange is different from that of the layup in the web.

Lastly, when we add different stiffener combinations in the top half and bottom half, the neutral axis might shift, and the moment of inertia will change. However, this variation in the neutral axis and moment of inertia has been ignored for this report.

5.1 Design of Skin

As mentioned earlier, skin would be designed at specified y values and the design would remain constant till the next y-change occurs. For each of these y-values, we would be required to calculate the Force per unit length, N_x and N_{xy}. This is done by using a similar calculation procedure mentioned in Part B.

Once these loads are available, next we will need to assume a layup and calculate the A, B, and D matrices, and check for the stresses in each of the laminas. Again, the detailed procedure to do this was covered in Part B. However, as we have noticed for both the Monolithic section and the Aluminum section, it is buckling which is the critical parameter based on which we need to design. Checking for first-ply failure can be limited to the end of the step after calculating the buckling load. If it fails after the buckling condition is satisfied, we add additional layers to compensate. If failure occurs according to Puck criteria, we add 45-degree layers where shear is the dominant force (between y = -1 and y = 1) and 0-degree layers when compression/tension is the dominant force.

Now for the design of the skin, basically the fuselage can be analyzed in two different halves. From the neutral axis to the bottom (y=0 to y=-3 meters), where compression is the dominant force. Here, we will have to carry buckling analysis and the thickness must be designed based on buckling. For the top half (y=0 to y =3), tension is the dominant force. But still we are going to analyse it based on compression only because in other loading conditions we might need compression capability.

By following the guidelines for layup, I have gone ahead with a balanced and symmetric layup for the first iteration. This is the layup that has been chosen,

$$\theta = [45, -45, 0, 45, -45, 45, -45, 90, -45, 45, -45, 45, 0, -45, 45]$$

If failure occurs, then as mentioned earlier, we add layers at the center.

Value of y (m)	Critical_failure	Nx (N/mm)	Nxy (N/mm)	Final No. of Layers	Buckling factor	Reserve factor	Margin of Safety
-3	Buckling	-530.58	0	39	0.477	2.09	1.09
-2	Buckling	-353.72	88.41	59	0.297	2.03	1.03
-1	Buckling	-176.86	141.47	67	0.267	2	1
0	Puck Criterion (Fiber/IFF)	0	159.16	15	NA	NA	NA
1	Puck Criterion (Fiber/IFF)	176.86	141.47	15	NA	NA	NA
2	Puck Criterion (Fiber/IFF)	353.72	88.41	15	NA	NA	NA
3	Puck Criterion (Fiber/IFF)	530.58	0	15	NA	NA	NA

Figure 5.1: Optimization for skin thickness

As shown in figure 6.1, for each value of y, calculations were done to optimize the setup. Margin of safety was the condition that was given to the program, to optimize the number of layers. If the margin of safety was less than 1, then layers would be added depending on where the analysis is being done. If shear is the crucial one, then 45-degree layers are added at the center. If bending is the crucial one, then 0-degree layers are added at the center. These were the equations that were used to obtain the results:

$$\frac{N_{x_s}kin}{N_{x_{critical_{skin}}}} + (\frac{N_{xy}}{N_{xy_{critical}}})^2 = bf$$

where bf is the buckling factor.
Similar to earlier, the determination of the reserve factor can be done through:

$$\frac{N_{x_s}kin * R}{N_{x_{critical_{skin}}}} + (\frac{N_{xy} * R^2}{N_{xy_{critical}}})^2 = 1$$

$$mos = R - 1$$

$$N_{x_{skin}} = \frac{N_x}{\lambda}$$

$$N_{x_{critical_{skin}}} = \frac{\pi^2}{a^2}(D_{11}m^2 + 2(D_{12} + 2D_{66})\bar{AR}^2 + D_{22}\frac{\bar{AR}^4}{m^2})$$

N_{xy} on the other hand is the entire shear load as this would be taken completely by the skin.

$$N_{xy_{critical}} = \frac{\pi^4 \frac{b}{a^3}}{\sqrt{\frac{14.28}{D_1^2} + \frac{40.96}{D_1 D_2} + \frac{40.96}{D_1 D_3}}}$$

$$D_1 = D_{11} + D_{22}(\frac{a}{b})^4 + 2(D_{12} + 2D_{66})(\frac{a}{b})^2$$

$$D_2 = D_{11} + 81D_{22}(\frac{a}{b})^4 + 18(D_{12} + 2D_{66})(\frac{a}{b})^2$$

$$D_3 = 81D_{11} + D_{22}(\frac{a}{b})^4 + 18(D_{12} + 2D_{66})(\frac{a}{b})^2$$

5.2 Determination of bending and axial stiffness for the stiffeners

First, we need to be able to find the values of the axial and bending stiffness for our stiffener. The stiffener taken is a T-stiffener. Figure 5.5 shows the geometry for the T-stiffener and the procedure to determine the neutral axis for the siffener.
The stiffness of the T-stiffener are calculated using the following equations:

$$EA = E_1 A_1 + E_2 A_2$$

$$EI = E_1 I_1 + E_2 I_2$$

I_1 and I_2 would need to be calculated with respect to the neutral axis.

$$I_1 = \frac{t_1 b_1^3}{12} + t_1 b_1 * (\frac{b_1}{2} - \bar{y})^2$$

$$I_2 = \frac{t_2^3 b_2}{12} + t_2 b_2 * (\bar{y} - \frac{t_2}{2})^2$$

Now we have a way to compare the stiffness that is required for our stiffener with the one that is existing based on geometry. We can proceed ahead. As the skin is finalized, we now proceed to the design of stiffeners. When there are both skin and stiffeners in place, the compressive force is assumed to be distributed between the skin and the stiffener with the majority of it being carried by the stiffener. On the other hand, the shear force is taken completely by the skin and is not transmitted in any way to the stiffener.

In the design of the stiffeners, it is assumed that we are attaching the skin to the stiffener through co-curing. The first step here would be to determine the required bending and axial stiffness for the stiffeners. This is obtained through the following equation:

$$EI = D_{11} d_s [\sqrt{\frac{D_{22}}{D_{11}}}(2\lambda \bar{AR}^2 - \sqrt{\frac{D_{22}}{D_{11}}}(AR^4)) + \frac{2(D_{12} + 2D_{66})}{D_{11}}(\lambda \bar{AR}^2 - (AR)^2 - 1]$$

A value of λ was assumed to be 8 for this calculation. $\frac{1}{\lambda}$ is the fraction of the total compressive load applied to the skin alone. AR here is the aspect ratio given by $\frac{a}{b}$ and $\bar{AR} = \frac{a}{d_s}$. a and b are the same as earlier, d_s is the distance between two stiffeners. Here, d_s is assumed to be 125 mm (if d_s is more than 100 we can have cost savings by not using too many of the stiffeners). The values with subscript to D are the values of the D matrix from the ABD matrices.

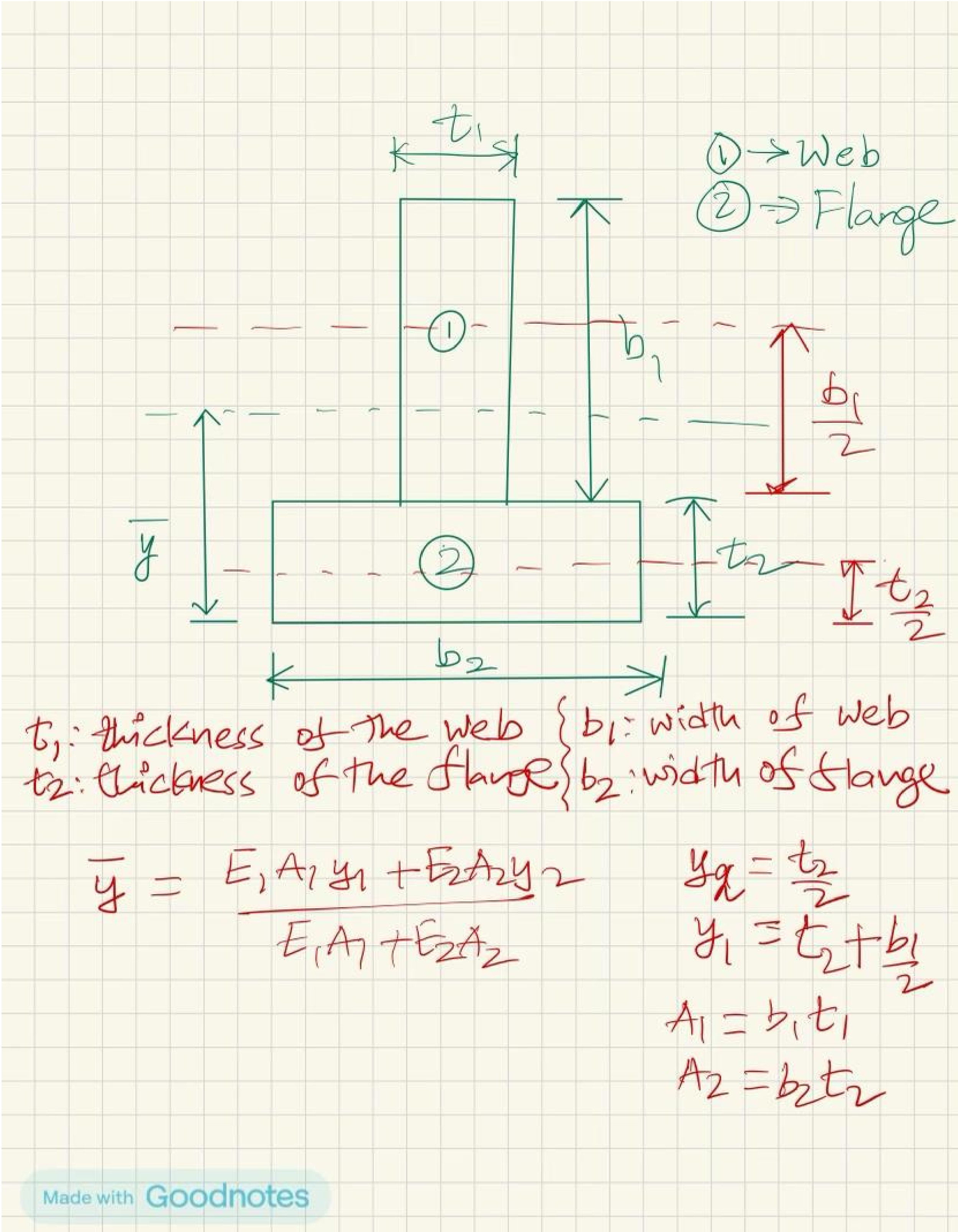

Figure 5.2: T-stiffener geometry

For the axial stiffness, the following equation is used:

$$EA = A_{11}(\lambda - 1)d_s$$

Once we have the value of b and d_s, we can determine the number of stiffeners for a particular b.

$$n_s = \frac{b}{d_s} + 1$$

This is assuming there are stiffeners at both ends. However, this can only be used for the starting section. After that, we would not need to place at both ends. So for y $\geq$ -3, we can just use

$$n_s = \frac{b}{d_s}$$

Once these values are finalized, then we need to design a stiffener, which can provide at least values equivalent to this. For our stiffener, the required values of EA and EI are:

Table 5.1: Axial and bending stiffness

y (m)	**EA kN**	**EI N - mm^2**	**EA kN obtained**	**EI N - mm^2 obtained**
-3	158854	2.28 x 10^{11}	402134	3.42 x 10^{11}
-2	826046	3.87 x 10^{11}	968594	4.62x 10^{11}
-1	1366643	5.11 x 10^{11}	1573794	6.31 x 10^{11}

Note: The table above consists of the final values of EA and EI obtained even after considering the buckling and crippling layups. That is the reason why it is significantly higher.

5.3 Determination of layup for stiffeners

Apart from obtaining the stiffness for EA and EI, we also need to satisfy two other conditions. That is stiffener buckling and crippling. The equations are mentioned below. But before that, let us give the final values of the cross-section for all the stiffeners we are considering.

Table 5.2: Cross-section of the stiffeners

y (m)	depth of the section 1 (b_1)	width of the section 2 (b_2)
-3	54	54
-2	64	64
-1	74	74
0	84	84
1	74	74
2	64	64
3	54	54

Based on these lengths, we try to optimize the lay-ups.

5.3.1 Stiffener buckling

The buckling equation for the stiffener is given by:

$$F_{critical} = \frac{\pi^2 EI}{L^2}$$

However, here we need to determine whether we need to use $E_{bending}$ or $E_{membrane}$. A condition was used that if t is greater than 1 cm then E would be $E_{membrane}$, if it is less than 1 cm then E would remain the bending one. The formula for determining 'I' was already provided in the earlier section. (Source: Slides)

Next would be the determination of the margin of safety. Again, the margin of safety is set to be at least equal to 1. For determining margin of safety, the formula used was

$$MOS = \frac{F_{critical}}{F_{stiffener}} - 1$$

where,

$$F_{stiffener} = (\lambda - 1)\frac{F_{total}}{\lambda}$$

where,

$$F_{total} = b * N_x$$

5.3.2 Crippling

Two types of crippling can occur here:

1. OEF crippling (one edge free)
$$\sigma_{cripling} = \sigma_c^u \frac{2.151}{(\frac{b}{t})^{0.717}}$$
2. NEF crippling (No edge free)
$$\sigma_{cripling} = \sigma_c^u \frac{14.92}{(\frac{b}{t})^{1.124}}$$

Crippling analysis needs to be performed for each of the members separately. So for this, the forces in each of the members of the stiffener are required. This is obtained by using the equation:

$$F_{mem_i} = \frac{E_i A_i}{\Sigma E_i A_i} * F_{stiffener}$$

Next, the stress in the following member is found to be,

$$\sigma_i = \frac{F_{mem_i}}{A_i}$$

This stress must be less than the crippling stress obtained earlier. For all our analysis, I have taken No edge free. This results in a conservative analysis compared to one-edge free. Again margin of safety is calculated here and this must be greater than 1 for both the members at a selected y,

$$MOS = \frac{\sigma_{cripling}}{\sigma_i} - 1$$

5.4 Results

The mass of the composite section with stiffener is calculated using the formula:

$$mass = mass_{skin} + mass_{stiffeners}$$

$$mass_{skin} = \Sigma\pi/4(r^2 - (r - t_i)^2) * 30/360$$

$$mass_{skin} = 199.1kg/m$$

$$mass_{stiffener} = \sigma(Area_{stiffener_1} + Area_{stiffener_2}) * n(i)\rho$$

where,$n(i)$ = no. of stiffeners for that particular y.

$$mass_{stiffener} = 54.31kg/m$$

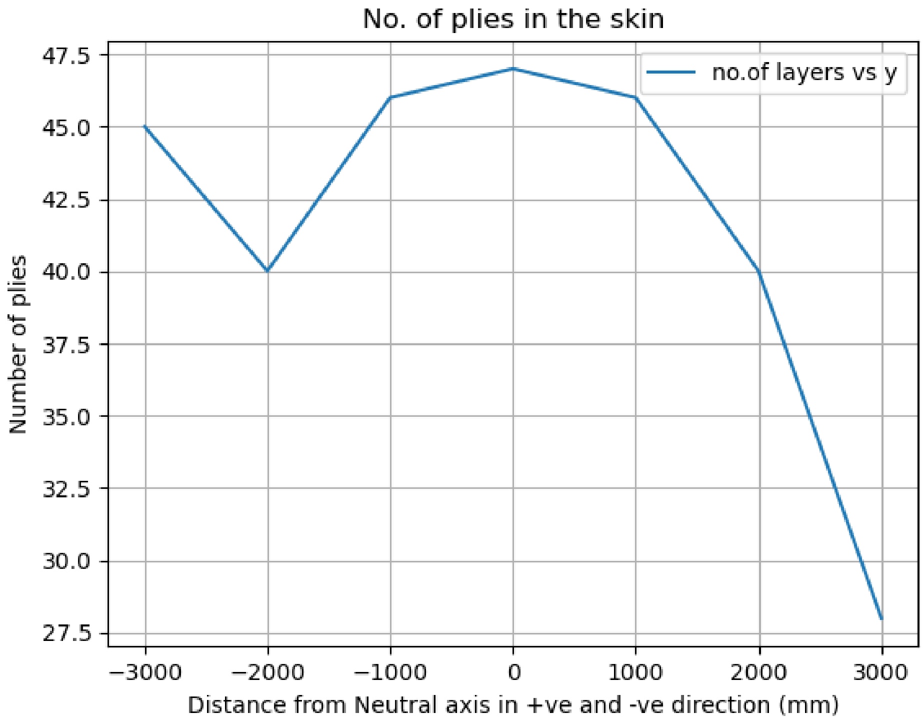

Figure 5.3: No. of plies in the skin vs distance away from neutral axis

$$mass = 253.5kg/m$$

Final cross-section of the stiffeners is given in table 5.3

Table 5.3: Cross-section of the stiffeners

y (m)	**Depth (b_1)(mm)**	**Thickness sec 1 (mm)**	**Width(b_2)(mm)**	**Thickness sec 2 (mm)**
-3	54	10.66	54	9.45
-2	64	8.23	64	7.02
-1	74	5.67	74	4.45
0	84	2.7	84	1.62
1	74	5.67	74	4.45
2	64	8.23	64	7.02
3	54	10.66	54	9.45

Figure 5.4: No. of plies in section 1 of the T-stiffener vs distance away from neutral axis

Table 5.4: Laminate Stack-up for Each Zone of the Skin

Zone	Stackup	t(mm)
1, 12, 13 & 24	$[(\pm 45)_2, 0, 90, 0, (\pm 45)_2, (0, 90)_s, (\pm 45)_2, 0, 90, 0, (\pm 45)_2]_2$	7.02
2, 11, 14 & 23	$[(\pm 45)_2, 45, 0, 90, 0, (\pm 45)_2, (0, 90)_s, (\pm 45)_2, 0, 90, 0, 45, (\pm 45)_2]_2$	7.56
3, 10, 15 & 22	$[(\pm 45)_3, 0, 90, 0, (\pm 45)_2, (0, 90)_s, (\pm 45)_2, 0, 90, 0, (\pm 45)_3]_2$	7.56
4, 9, 16 & 21	$[(\pm 45)_2, 45, (0, 90)_2, (\pm 45)_2, (0, 90, 0)_s, (\pm 45)_2, (90, 0)_2, 45, (\pm 45)_2]_3$	8.64
5, 8, 17 & 20	$[(\pm 45)_2, 45, 0, 90, 0, (\pm 45)_2, (0, 90)_2, (90, 0)_2, (\pm 45)_2, 0, 90, 0, 45, (\pm 45)_2]_3$	8.64
6, 7, 18 & 19	$[(\pm 45)_2, 45, 0, 90, 0, (\pm 45)_2, 0, (0, 90)_2, (90, 0)_2, 0, (\pm 45)_2, 0, 90, 0, 45, (\pm 45)_2]_3$	9.18

Zones in table 5.3, would move from 1 to 24 as theta changes from 0 degrees to 360 degrees.

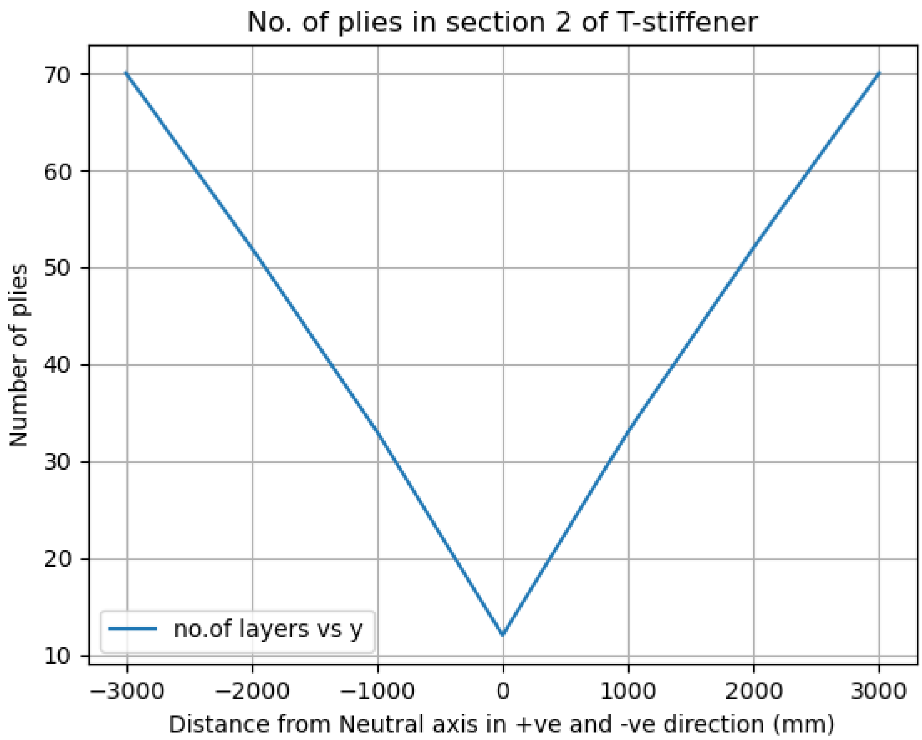

Figure 5.5: No. of plies in section 2 of the T-stiffener vs distance away from neutral axis

Part - D: Comparison

The comparison of the variation in mass variation for all three designs is summarised in table 6.1.

Table 6.1: Mass comparison for all three cases

Case	**mass (kg/m)**	**% reduction in mass**
Al	729.28	-
Monolithic	404.68	44.5
With Stiffeners	253.5	37.3

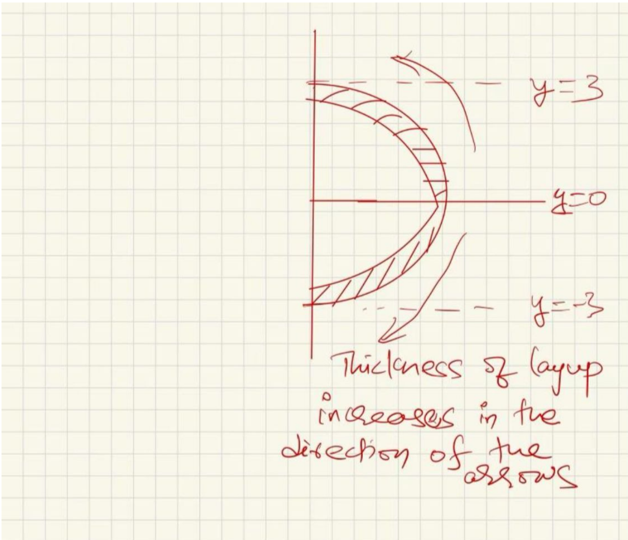

Figure 6.1: Skin Thickness increasing direction for monolithic composite