Technische Universiteit Delft

Faculty of aerospace engineering

Master of Science Aerospace Engineering

Assignment - 1

DESIGN AND ANALYSIS OF COMPOSITE STRUCTURES - I (AE4ASM109)

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Introduction

For the analysis of the assignment, the following UD material properties were used:

Table 1.1: Properties

Property	**Mean**	**Std**	**Units**
E_1	145.3	3.28	GPa
E_2	8.5	1.28	GPa
v_{12}	0.31	0.018	-
G_{12}	4.58	0.83	GPa
X_t	1932	128.3	MPa
Y_t	108	8.2	MPa
S	132.8	6.21	MPa

Furthermore, the following values of the compression strengths will be used where applicable:
$X_c = 1480$ MPa
$Y_c = 220$ MPa

1.1 Summary of the assumptions

1. The thickness of the laminates were not provided and it was assumed to be **0.125 mm** as used for the in-class problems.

2. For Puck criteria: E_{f_1} and $v_{f_{12}}$ for the fiber were not given. From the lecture slides, they were assumed to be $E_{f_1} = 225GPa$ and $v_{f_{12}} = 0.2$. The value of $m_{\sigma,f} = 1.1$ was also taken from the lecture slides.

3. The variation of stress along a lamina was assumed to be constant. It was calculated based on the average value of Z for that particular laminate.

4. X_c and Y_c along with their standard deviation were not provided in the question. The following values were assumed as shown in the table 1.2.

Table 1.2: Properties for compressive load

Property	**Mean**	**Std**	**Units**
X_c	1480	128.3	MPa
Y_c	220	8.2	MPa

5. For the purpose of reliability analysis, all the laminas of a single laminate were assumed to have the same random properties for a particular iteration. This was done just to improve the computational speed.

6. First ply failure is assumed to mean when any ply fails in any mode for the first time (including the degradation).

7. Last ply failure is assumed to have happened when all the plies have degraded and subsequently failed.

Question-1

2.1 Question-1a

Calculate the engineering constants (in-plane and flexural) of the following laminate $[15/\pm\theta/75_2]_{ns}$ as a function of angle θ. Present your results in plots. How does n affect the results? (*Make sure you select the appropriate number of θ and n in order to draw your conclusions.*)

2.1.1 Expected outcomes from prior knowledge

Before doing the actual analysis, it is always better to have an initial expectation of the behavior of the composite structure so that any anomalies can be identified or explained in a better way.

The given laminate is in the form of $[15/\pm\theta/75_2]_{ns}$ with two variables θ and n. For the inplane properties, the expected variation with theta for a laminate for Young's modulus along the loading direction is expected to be maximum with theta zero degrees and the least for theta 90 degrees. Similarly, for Young's modulus perpendicular to the loading direction, it is expected to be maximum for theta of 90 degrees and minimum for theta of 0 degrees. For the Rigidity modulus, it is expected to have a maximum at 45 degrees theta and taper off towards 0 and 90 degrees. (Theta is with respect to the loading axis).

For the second variable n, as we keep increasing the number of sequences of the standard stack given, the properties in the A matrix would increase proportionally but at the same time, the thickness of the entire laminate also increases proportionally, thereby the in-plane properties should remain the same no matter how many stacks are added on top of each other.

The bending properties which are dependent on the D matrix don't increase linearly but in a cubic manner with the increase in thickness (difference of cubes of the z values in reality). So this would mean that initially there should be an increase in the values of the flexural elastic properties but as we keep increasing the no.of stacks, the difference would taper off and should ideally reach an

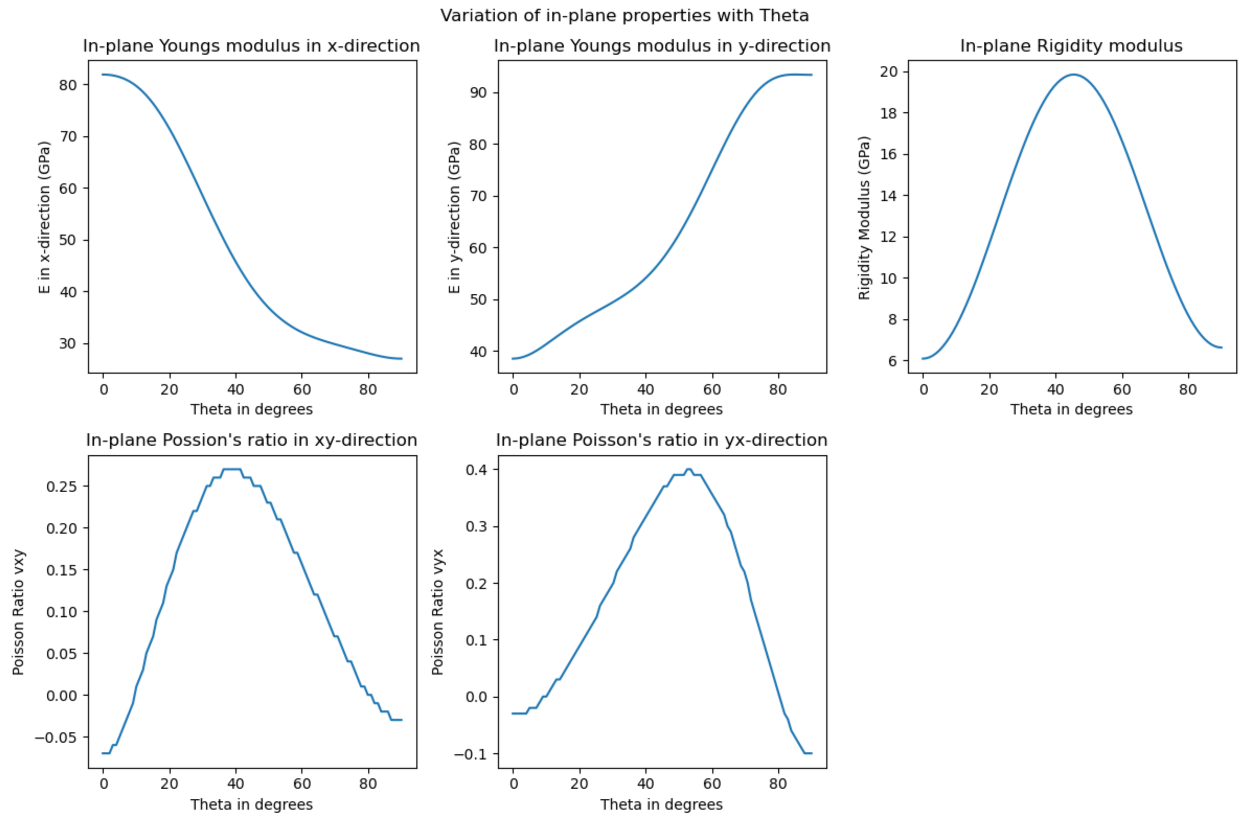


Figure 2.1: Variation of in-plane properties with Theta

asymptote for the properties.

For the bending properties with respect to theta variation, the greater the number of plies closer to zero degrees and away from the neutral axis, the better the properties must be for the flexural Youngs modulus along the loading direction. Similarly for the Youngs modulus along the direction perpendicular to the loading direction, it is better to have 90 degrees plies away from the neutral axis. For the Shear Modulus, the shear stress is maximum at the neutral axis and zero at the top and bottom. Hence, the preference should be having a 45-degree ply closer to the neutral axis to maximize the Rigidity modulus.

For this question, a three-step approach has been taken to analyze the variation of the properties of the laminate. These are:

1. The variation of properties with theta when the value of n is constant.

2. The variation of properties with n when the value of theta is constant.

3. The variation of properties with both n and theta varying simultaneously to check for any outliers.

2.1.2 Variation with theta (n =1)

For this analysis, theta was plotted from 0 to 90 degrees. This is because the variation with theta is symmetric (mirror image) about the theta = 90 degrees plane and having the data for this half would be sufficient (only for in-plane, for Flexural it was insufficient as explained below). For the increments, incrementing it by 5 degrees every time would be sufficient but just because we have the computational power, the increments were done for every degree.

As seen in the figure 2.1, the in-plane properties show an expected trend. As the value of theta increases from zero to 90 degrees, the Young's modulus along the loading direction (x), decreases. This is intuitive as having more fibers in the direction of the loading would increase the value of Young's modulus in this direction. On the other hand, Young's modulus in the Y-direction (perpendicular to the loading) increases from zero to 90 degrees. Again, this is because having the fibers in a 90-degree orientation would help in maximizing the Young's modulus for that direction.

On the other hand, for Rigidity modulus, it is best to have a lay-up of 45 degrees to maximize resistance against shear loads. This can be observed in the plot as we tend to reach a 45-degree lay-up the Rigidity modulus shoots up but at the same time as we move away from it, the value decreases (both towards 0 and 90 degrees). For the Poisson's ratio, a variation with theta is observed probably indicating the principal planes for the given loading direction at those maximums.

Quite similar variation is also seen for the properties pertaining to the flexural data for E_{xb}, E_{yb}, and G_{xyb} in figure 2.2. However, it is observed to be non-symmetric about 90 degrees for the Poisson's ratios. Hence, the theta was changed to 180 degrees for the flexural plot and replotted as shown in figure 2.3. In this plot, it could be observed that the plots are no longer symmetric (Except maybe for E_{yb}. Again this could normalize when the values of n are changed but that would be analyzed in the later section.

Initial impressions agree with the excepted behavior as explained earlier. The only issue seems to be that of the symmetrical behavior.

2.1.3 Variation with n (theta = 0)

For analyzing the variation of the properties with n, theta was kept constant at 0 degrees. N was varied from 0 to 20 to find out how the properties in both in-plane and flexural vary and what

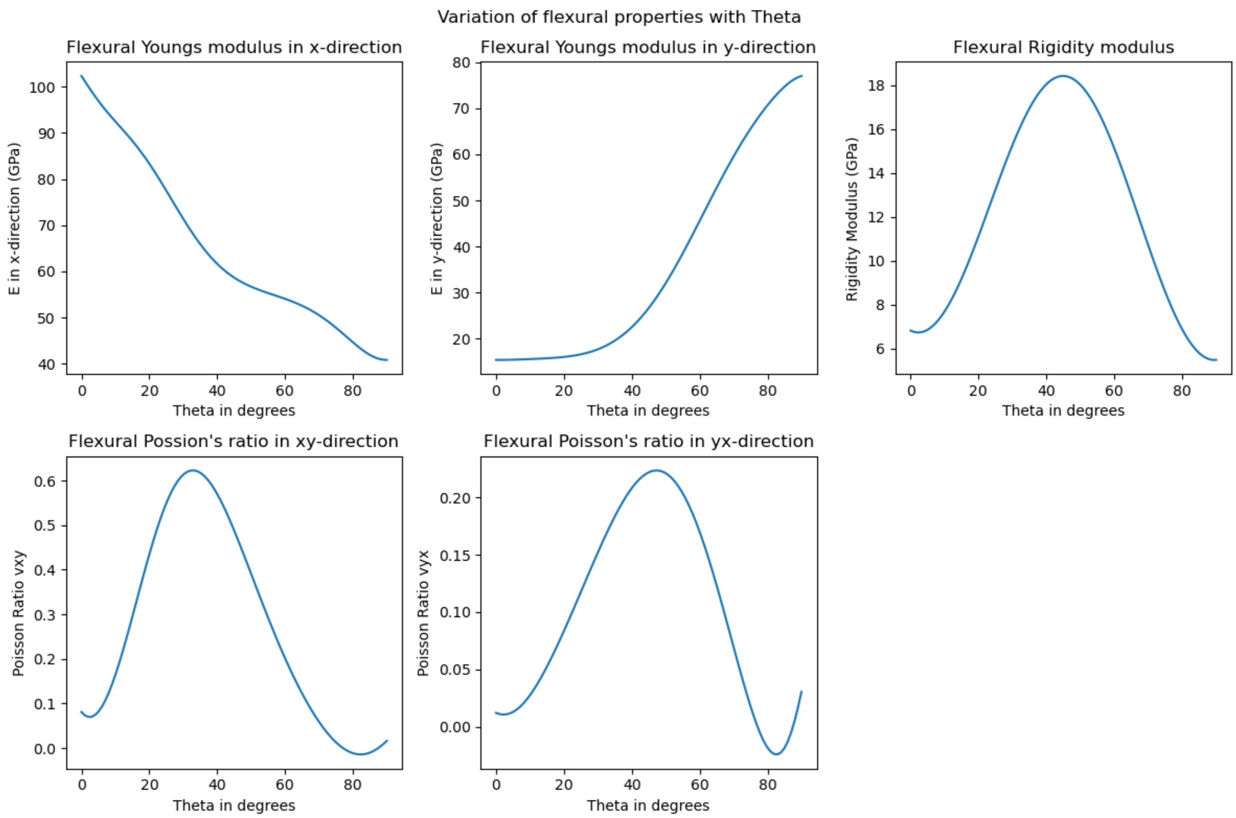


Figure 2.2: Variation of flexural properties with Theta

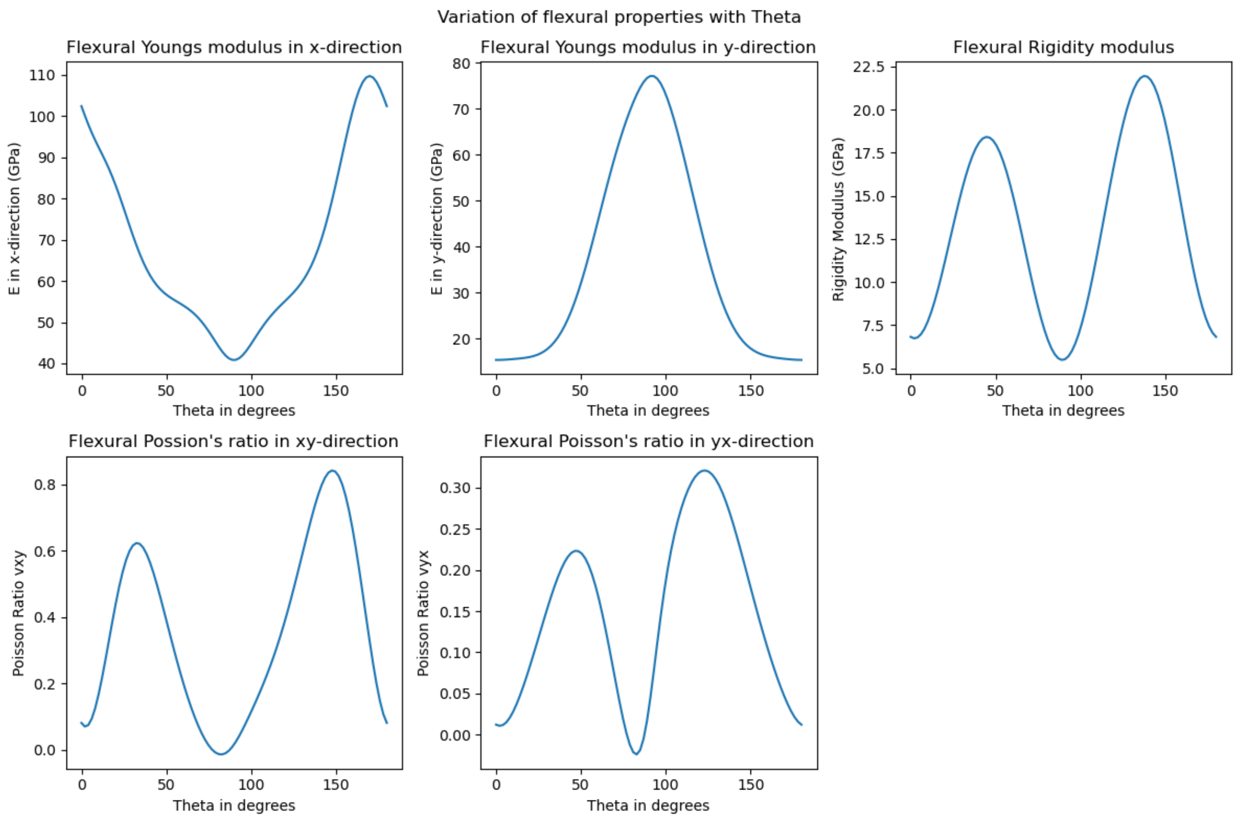


Figure 2.3: Variation of flexural properties with Theta for 180 degrees

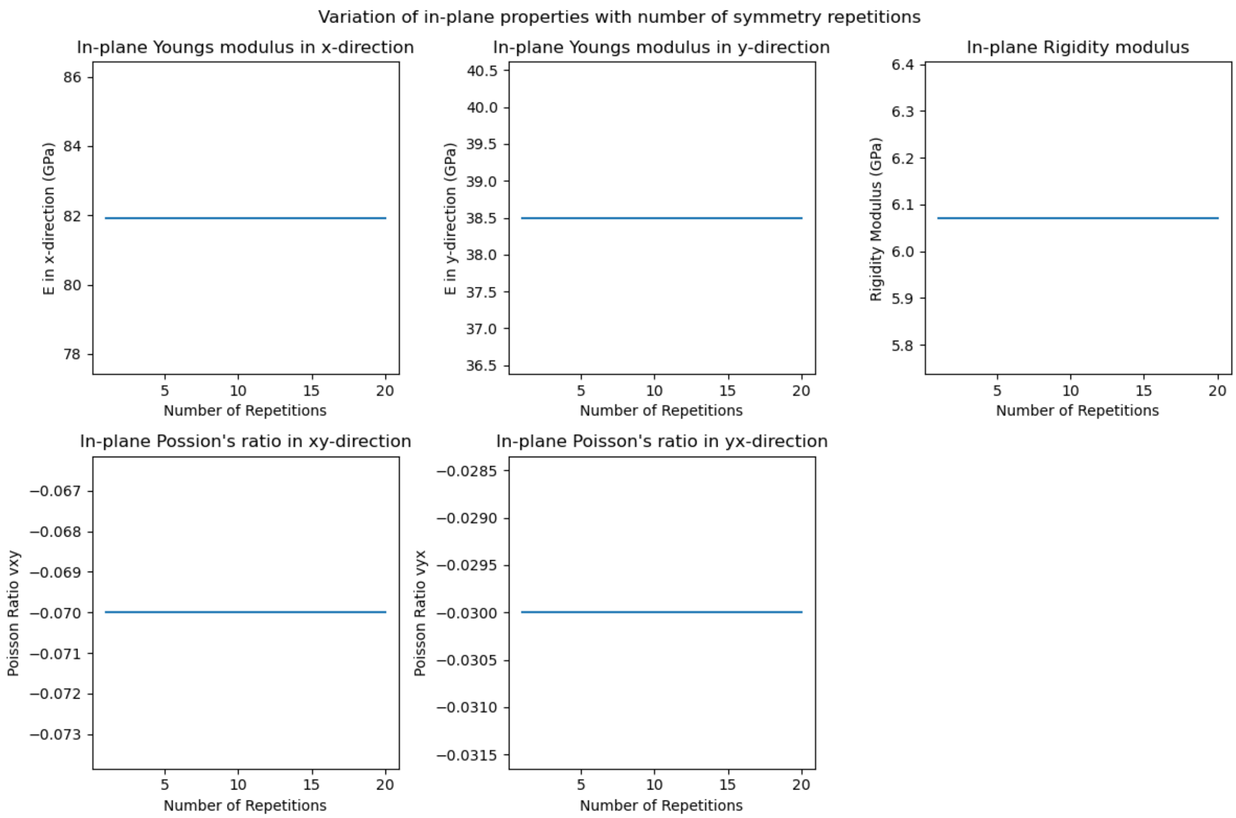


Figure 2.4: Variation of in-plane properties with n

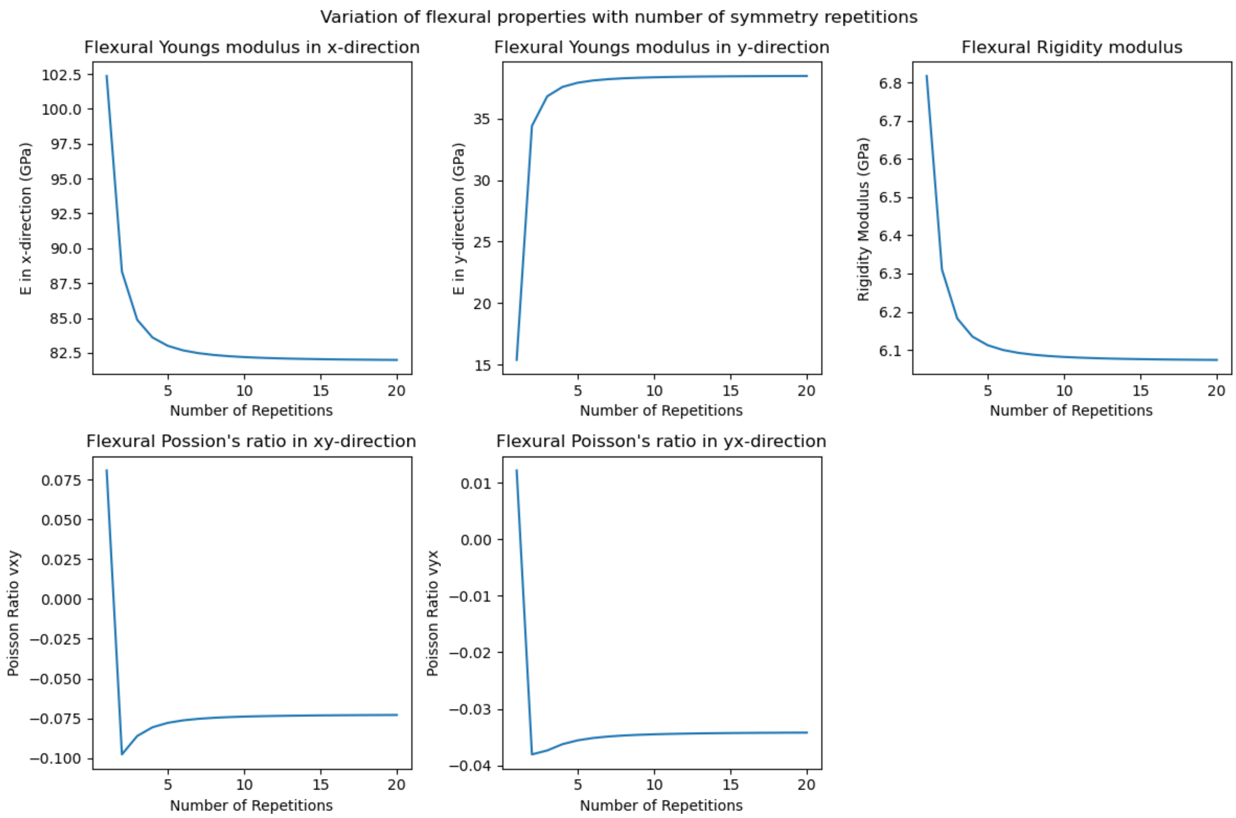


Figure 2.5: Variation of flexural properties with n

number of values of n is sufficient to make conclusions for the laminate.

The in-plane properties don't change with the increase of n. This tallies well with our initial impressions of the problem as both the thickness and the A-matrix scale equally with thickness and this would keep them constant. Figure 2.4 proves this.

For the flexural properties, as we keep increasing the value of n, the modulus E_{xb} would be the highest with n = 1. This is because the preferred layers for bending (closer to 0 degrees) are all situated at the top and bottom while all the 75 plies are closer and closer to the neutral axis. Once we start increasing the number of plies, this effect diminishes as the plies get more and more evenly distributed. Hence at around 7 plies, it reaches an asymptote and remains constant.

The trends described above for the membrane and flexural properties were tested and plotted for theta = 0 degrees but similar patterns were also noticed for different thetas. The only difference is the initial strength at n = 1, which depends on the value of theta and then it reaches an asymptote at about n=7 value.

2.1.4 Variation with n and theta

We have just seen that for the flexural properties, the curves are not symmetric around the 90-degree plane. Hence, for this part, all the curves are plotted for 180 degrees to account for this minor variation too.

As we kept increasing the values of n, we found out that at around the value of n=7, the properties start to reach an asymptote and further variation of n doesn't seem to affect the properties. So for the value of n =7 and theta from 0 to 180 degrees plots were made to see the variation of properties with theta. The in-plane property variation is shown in figure 2.6 and the variation of flexural properties is shown in 2.7

We observe the exact variation we observed initially with n=1 and theta being varied for all the in-plane properties. For the flexural properties as we kept increasing n the issue with symmetry disappeared and what is more noteworthy is that the flexural properties more and more started to represent the membrane properties with an increase in n. As can be seen in the figures, the plots match very closely for both in-plane and the flexural properties for n = 7, and as this n increases further they become practically identical.

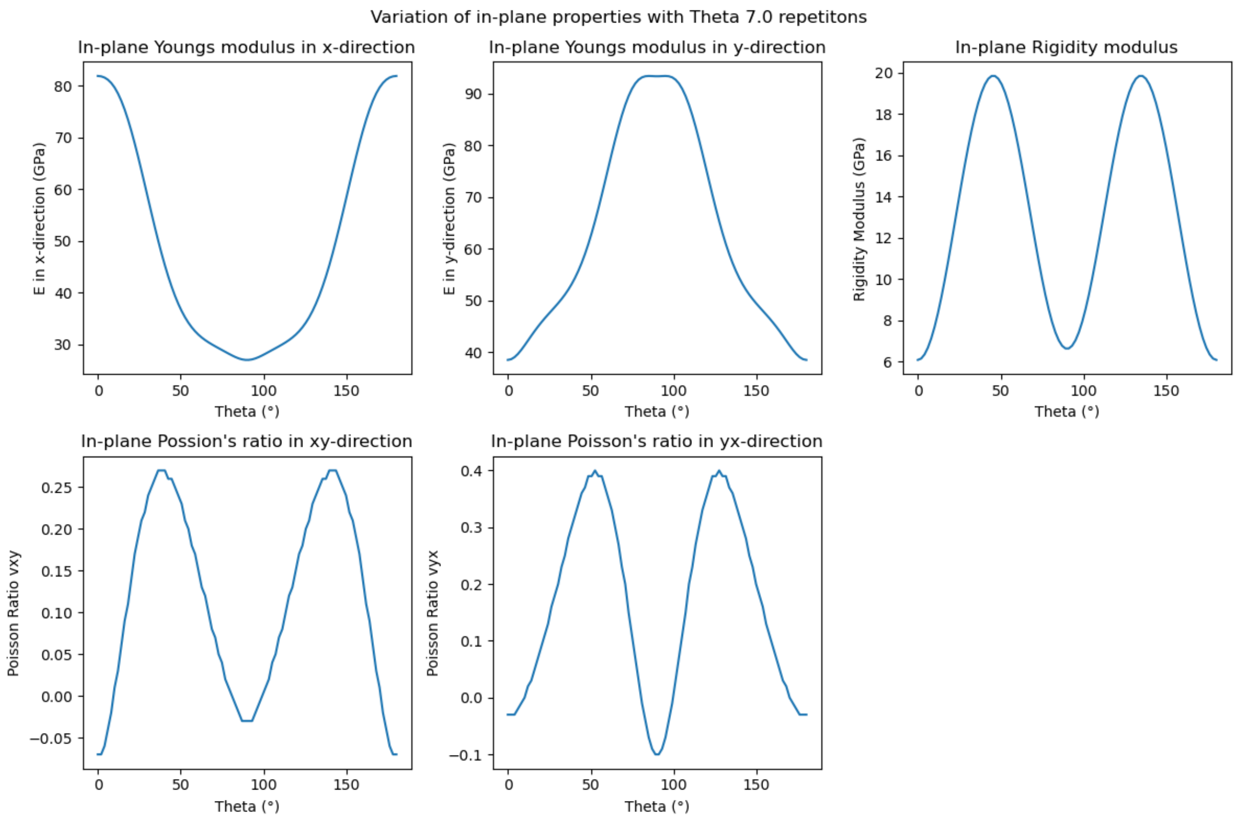


Figure 2.6: Variation of in-plane properties with theta for n = 7

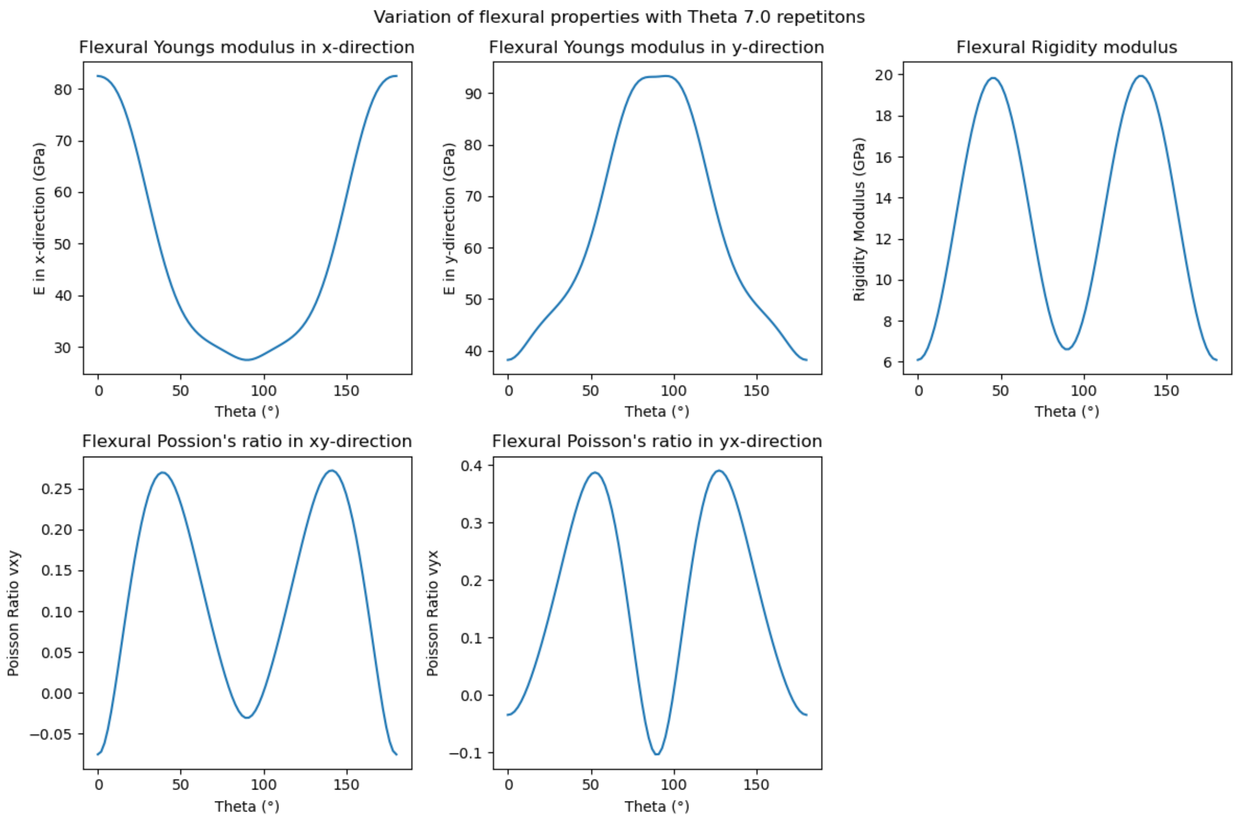


Figure 2.7: Variation of flexural properties with theta for n = 7

2.2 Question-1b

A composite laminate $[0_2/90/30/90]_T$ **is subjected to the following in-plane and flexural loadings:**

1. $N_x = 0.2 \times 10^2$ **[N/m],** $N_y = 1.8 \times 10^4$ **[N/m] and** $M_x = 18 \times 10^3$ **[N]**

Calculate the stresses and strains for each ply through the thickness and plot the results for the principal coordinate system.

For the given problem, the thickness of each layer was considered to be 0.125 mm or 0.125×10^{-3} **meters. As the question mentions the loads in N/m notation, all the values have been converted to meters for processing. (GPa =** 10^9 **Pa).**

2.2.1 Abaqus results

Again, before plotting the results using the in-class theory, Abaqus was used to make a Finite Element Model and it was used to measure the stresses and strains along the different laminas (Middle point was used). The results are given in the table 2.1 and 2.2 and the model has been attached to the email.

Table 2.1: Stresses in each ply at the middle section point

Ply Number	**Orientation**	σ_1 GPa	σ_2 GPa	σ_{12} GPa
1	0°	-368.5	-6.0	0.315
2	0°	42.93	0.44	-9.63
3	90°	-14.98	26.18	19.59
4	30°	242.4	38.96	-39.4
5	90°	-35.03	73.73	39.49

Table 2.2: Strains in each ply at the middle section point

Ply Number	**Orientation**	ϵ_1	ϵ_2	ϵ_{12}
1	0°	-2.52	-0.08	0.07
2	0°	0.29	-0.039	-2.1
3	90°	-0.159	3.11	4.27
4	30°	1.59	4.066	-8.6
5	90°	-0.39	8.748	8.62

2.2.2 Strains in laminas

The given laminate is $[0_2/90/30/90]_T$ which is non-symmetric and the loads that are acting are $N_x = 0.2 \times 10^2$ N/m, $N_y = 1.8 \times 10^4$ N/m and $M_x = 18 \times 10^3$ N. These values bring into the picture the curvature and the value of z becomes crucial for calculating these. An assumption has been made for calculating strains using Z. Z for a laminate is taken as constant for that particular laminate and equal to $\frac{z(i+1)+z(i)}{2}$, i.e., the average value of z for that particular laminate.

Using this, the strains for the different laminas were calculated for their principal coordinate system. This is achieved by calculating the values of the strains ϵ_{x0}, ϵ_{y0}, ϵ_{xy_0} and the curvatures k_x, k_y and k_{xy}. Then, the individual strains for each lamina are calculated using the equation: $\epsilon_x = \epsilon_{x_0} + z * k_x$, $\epsilon_y = \epsilon_{y_0} + z * k_y$ and $\epsilon_{xy} = \epsilon_{xy_0} + z * k_{xy}$. These values are given in the table 2.3 for each of the laminas. (Note: If z was also assumed to be varying along the thickness, strains would vary linearly along the thickness of every laminate).

Table 2.3: Strains in each ply

Ply Number	**Orientation**	ϵ_1	ϵ_2	ϵ_{12}
1	0°	-2.52	-0.08	0.07
2	0°	0.29	-0.04	-2.1
3	90°	-0.159	3.11	4.28
4	30°	1.59	4.07	-8.6
5	90°	-0.39	8.75	8.62

2.2.3 Stresses in laminas

To find the stresses along the principle directions, the strains along the principal directions need to be multiplied by the respective compliance matrix. After doing this, the stresses for the different laminas obtained are given in the table 2.4.

Table 2.4: Stresses in each ply

Ply Number	**Orientation**	σ_1 GPa	σ_2 GPa	σ_{12} GPa
1	0°	-368.5	-5.9	-0.31
2	0°	42.9	0.4	-9.6
3	90°	-14.9	26.2	19.6
4	30°	242.4	38.9	-39.3
5	90°	-35.0	73.7	39.4

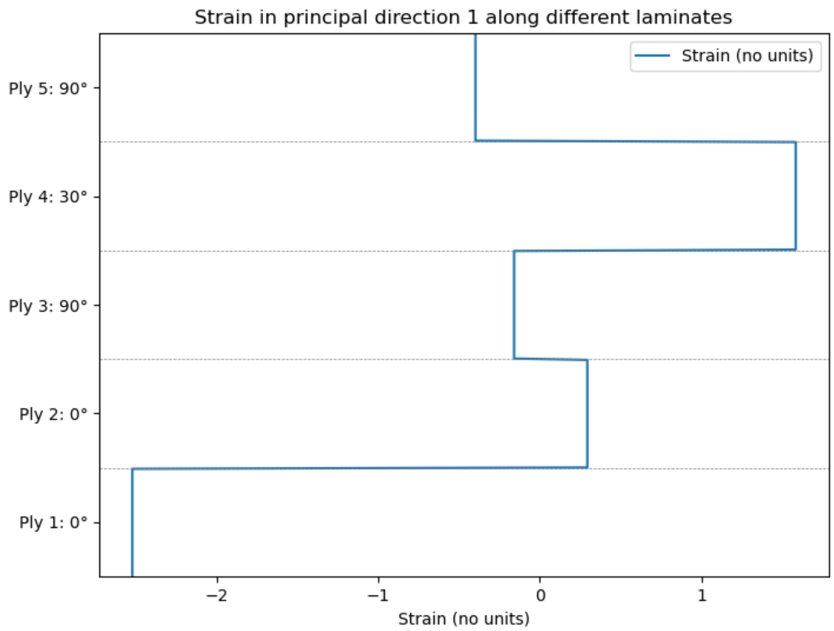


Figure 2.8: strain across the laminate in principal direction 1

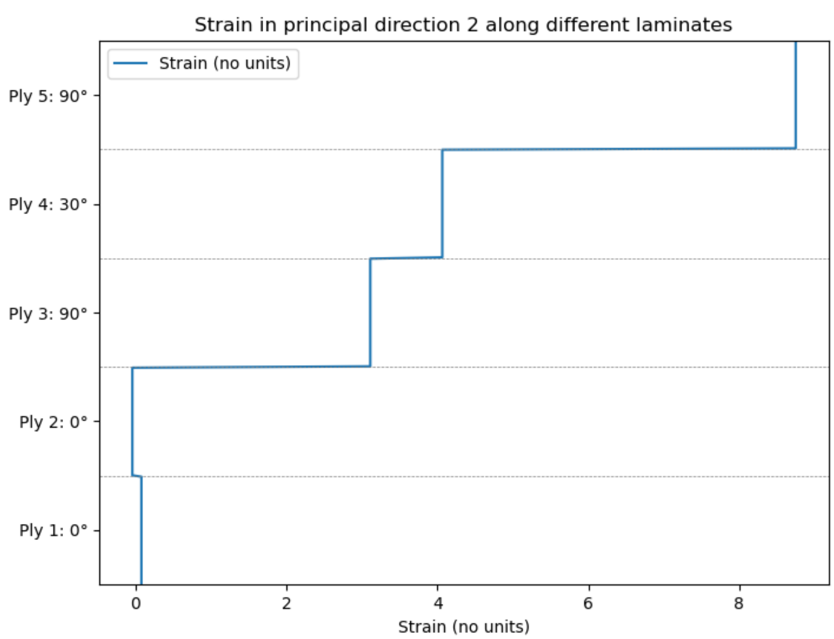


Figure 2.9: strain across the laminate in principal direction 2

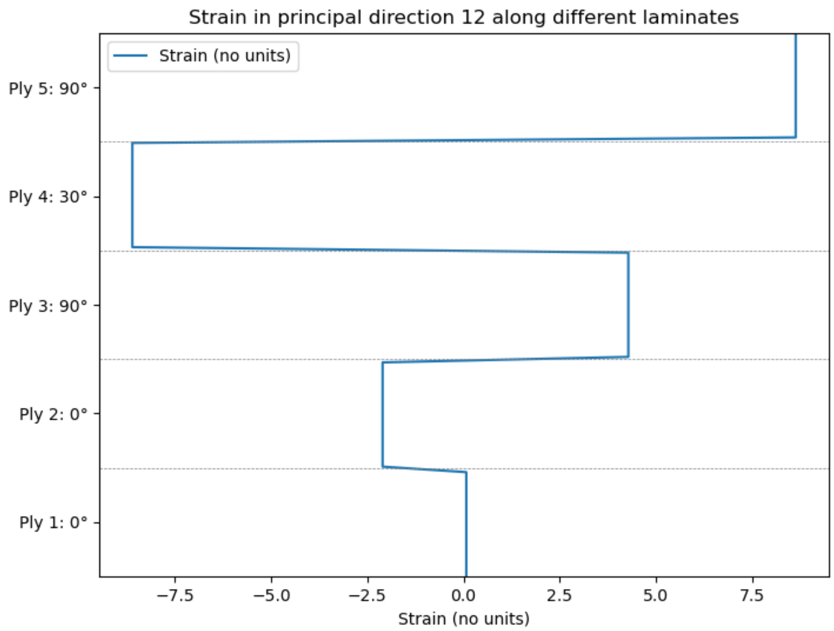


Figure 2.10: strain across the laminate in principal direction 12

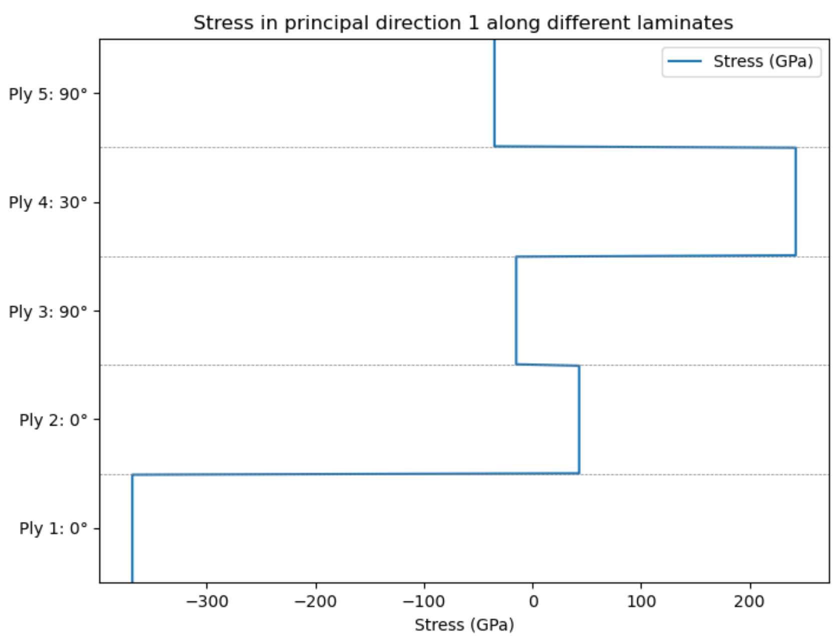


Figure 2.11: Stress across the laminate in principal direction 1

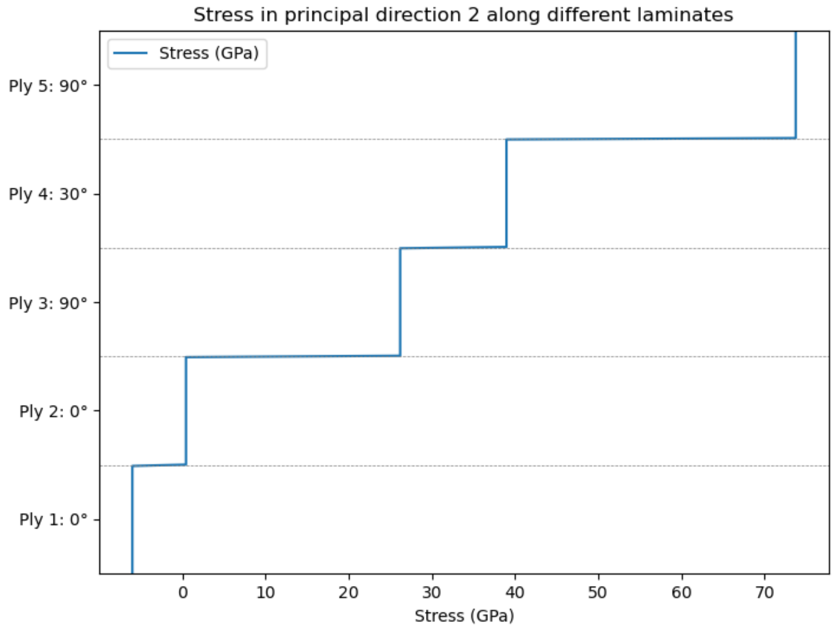


Figure 2.12: Stress across the laminate in principal direction 2

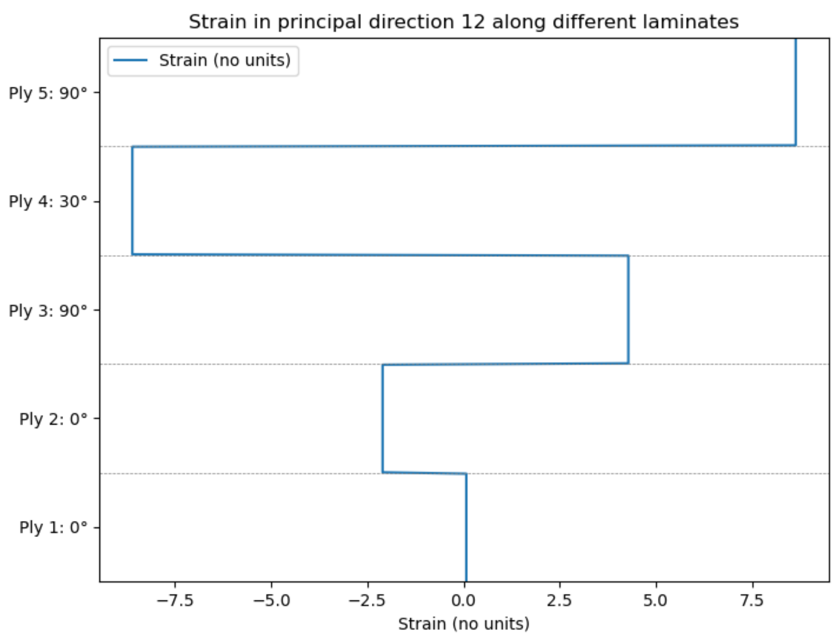


Figure 2.13: Stress across the laminate in principal direction 12

Question-2

3.1 Question-2a

For the quasi-isotropic laminate $[0/90/\pm 45]_{2s}$ plot the biaxial stress failure envelopes for axial loading N_y-N_s utilizing Puck and Max. Stress failure criteria. Indicate the FPF and LPF in each plot and for each loading ratio.
Report the global failure strains as well (for each loading step where failure occurs).

3.1.1 Expected Results

The given laminate layup is symmetric, implying the B matrix will be zero and hence, it should to have no extension-bending coupling. Since, the laminate is loaded in the transverse (N_y) and shear (N_{xy}) directions, the following behaviour is expected at different loading ratios,

1. $N_y/N_{xy} > 1$: 0° plies would fail first as they would be loaded transverse to the fiber direction. 45° plies would fail next as they are predominantly loaded in the transverse axial direction. 90° plies would be expected to be the last plies failed as they are loaded predominantly along the fiber direction. (A similar result was also seen in the in-class example when the laminate was loaded along the global X-axis).

2. $N_{xy}/N_y > 1$: 0° plies would fail first, followed by by 90° plies. 45° plies would be expected to fail last as they are loaded predominantly in shear.

From the ply strengths mentioned in Table 1.1 and the layup consisting of more 45° plies than 90° plies, it is expected that the limit of N_y on the failure envelope would be comparable to the limit of N_{xy}.

3.1.2 Degradation Rules

The following degradation and damage rules have been used in this analysis,

1. If a ply fails in Tension or Compression, it is considered to be failed. Its properties are made zero and it no longer contributes to the recalculated ABD matrix.

2. If a ply fails in shear, its transverse properties are degraded by a factor of 0.1. Transverse properties include $E_2, G_{12},\ and\ v_{12}$. The degraded properties are used in recalculating the ABD matrix.

3. If a ply that has previously failed in shear fails again, it is considered to be failed completely and all its properties are made zero.

3.1.3 Assumptions and Notes

The following assumptions were made regarding the procedures to carry out the progressive damage analysis,

1. The following values required for evaluating the Fiber Failure Indices using Puck criterion have been considered from [1], $E_{f1} = 225 GPa$ and $v_{f12} = 0.2$. Values chosen are for AS4/3501-6 as the on-axis strengths were closest to what was provided in Table 1.1. The value of $m_{\sigma,f} = 1.1$ for CFRP is taken from [2].

2. The value of inclination parameters (p) have been taken from [2], Table 1 for CFRP.

3. First ply failure is assumed to mean when any ply fails in any mode for the first time.

4. To get all possible ratios of N_y and N_{xy}, the loop was run to trace out a semi-circle. There an angle was set and the radius was extended along that angle. N_y and N_{xy} were considered as components of that radius. The points of failure along the radius were recorded as First Ply Failure and Last Ply Failure stresses. The angle was incremented to sweep over 180°. The failure envelope plots were found to be symmetric about their X-axis so this helps reduce computation time.

5. Global failure strains reported are for the whole laminate in the global coordinate axis. The failure strains are reported for both First Ply and Last Ply Failures.

6. For last ply failure reported, the limitation of the code written should be taken into consideration: At a given load, if multiple plies are reported to be failed, the code cannot determine which failed last. Hence, the last reported failure is dependent on the order in which the plies are checked for failure. Keeping this limitation in mind, the observations in the below sections have been written.

3.1.4 Failure Envelope for Max Stress Criterion

Max Stress criterion is implemented as per [3]. The failure envelope in terms of global laminate stresses is shown in Figure 3.1.

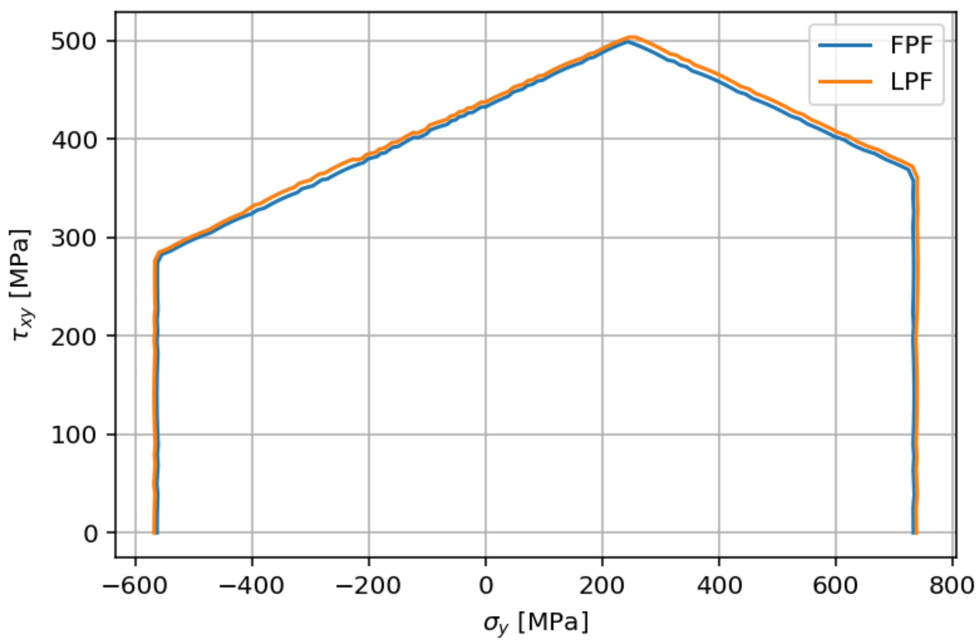


Figure 3.1: Failure Envelope for Max Stress Criterion

It is observed that all first ply failures are in longitudinal tension or compression. Results for first ply failure at different loading ratios are listed below,

1. $N_y/N_{xy} > 0.5$: The first plies to fail are 90° plies, which seems counter-intuitive to our initial expectations. This can be explained by considering the effect of stiffness on the ply stresses. 90° plies have the highest stiffness under N_y loading. This causes them to have the highest stress (σ_{11}) in this case, as they attract more load. 0° plies, on the other hand, experience compressive stress due to the Poisson ratio effect. They start to experience tensile stresses as N_{xy} increases, but at the same time 45° plies start to get loaded on-axis and take the majority of the stress, failing next. A similar trend is observed when the laminate is loaded in compression. There 90° plies start failing first in compression, followed by -45° plies. This is shown in Figure 3.2.

2. $N_{xy}/N_y > 0.5$: 45° plies fail as the loading is now predominantly through shear (N_{xy}). This is taken mostly as longitudinal stress (σ_{11}) by 45° plies as they are now loaded on-axis. Similarly, in compression, -45° plies fail first. This is shown in Figure 3.3.

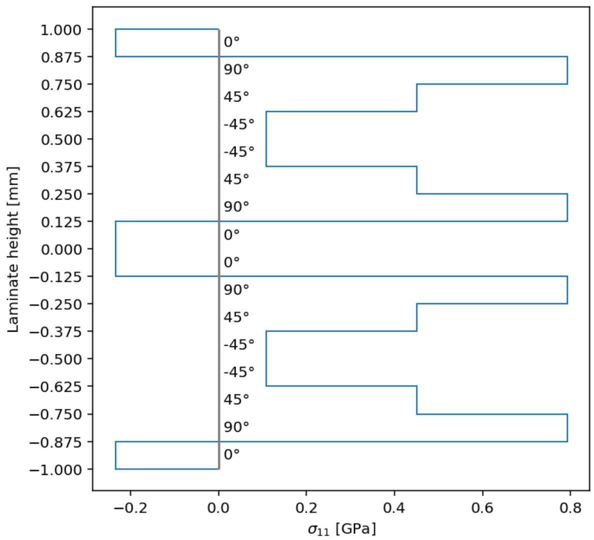


Figure 3.2: Ply-wise σ_{11} *at* $N_y/N_{xy} > 0.5$

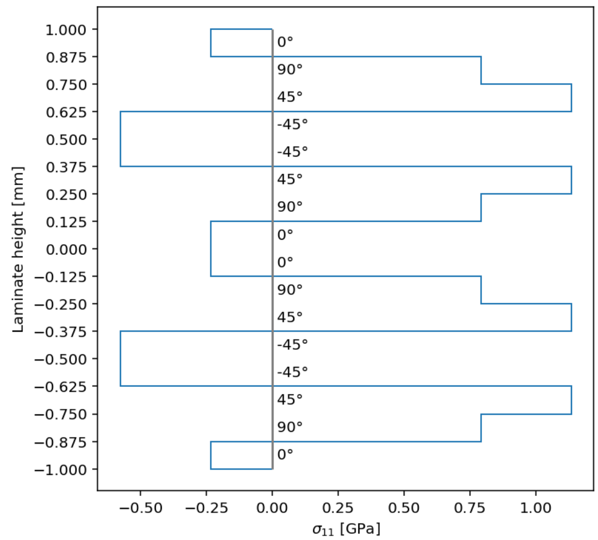


Figure 3.3: Ply-wise σ_{11} *at* $N_{xy}/N_y > 0.5$

(The stresses are plotted with arbitrary load values to show the effects of loading ratio.)

It was observed that 0° plies are the last plies to fail,

1. $N_y/N_{xy} > 1$: 0° plies start failing in longitudinal compression due to the effect of Poisson's ratio at high N_y loading. With increase in N_{xy}, -45° plies fail in transverse compression.As N_{xy} increases, 0° plies fail last in transverse tension.
2. $N_{xy}/N_y > 1$: 0° plies fail in shear as that is now the predominant loading condition.

Global failure strains are shown at first ply and last ply failures in Figures 3.4 and 3.5, respectively. Strains are plotted as sweeping counter-clockwise along the failure envelope through 180°.

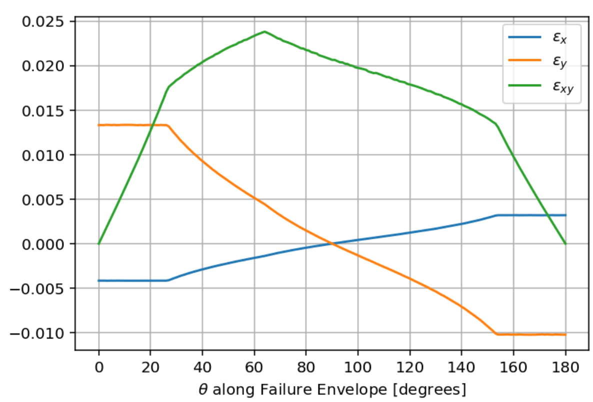


Figure 3.4: Global Strains at First Ply Failure

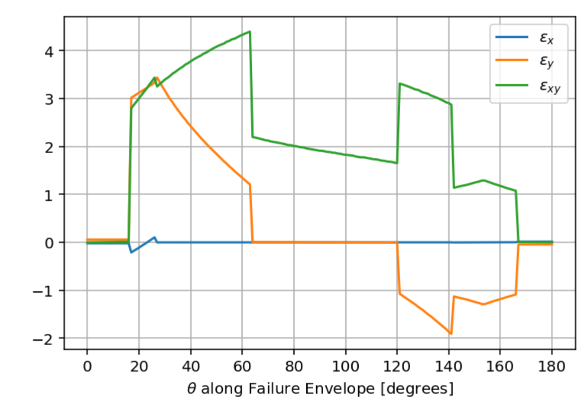


Figure 3.5: Global Strains at Last Ply Failure

At first ply failure, looking at all three strains, the the flat lines at the beginning and end are where only 90° are failing due to high transverse (N_y) loading of the laminate. The region in between is where 45° and -45° plies start failing due to loading in shear (N_{xy}).

At last ply failure, the failure strains are low at the beginning when the 90° plies have failed already. This is because they provide the majority of the laminate strength at high N_y loading. Once they fail, the other plies fail soon afterwards. As 45° plies start failing first, 90° and 0° plies can still take the shear loading so the laminate can withstand higher strain levels. However, as the shear load N_{xy} increases, this has to be taken predominantly by the 45° and -45° plies, which have failed, explaining the drop in between. As the transverse loading (N_y) becomes compressive (beyond 90°), a similar trend is observed but with lower strains as the strength of the lamina is lower in compression than in tension.

3.1.5 Failure Envelope for Puck Criterion

Puck criterion is implemented as per [2]. The failure envelope and global laminate stresses is shown in Figure 3.6.

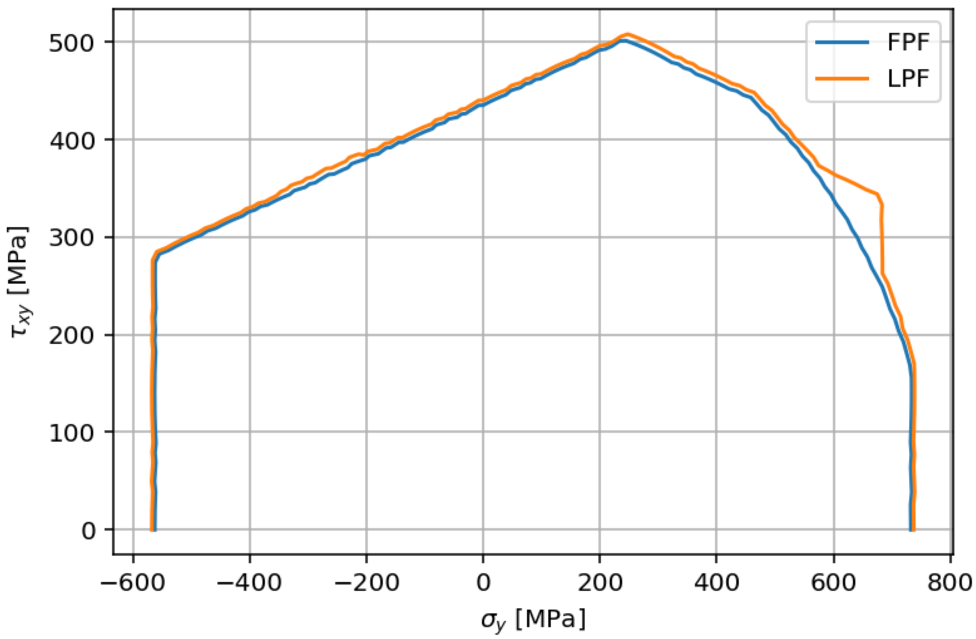


Figure 3.6: Failure Envelope for Puck Criterion

Similar to the Max Stress criterion, it is observed that almost all first ply failures with Puck criterion are in fiber tension or compression. However, the failure envelope deviates when inter-fiber failure (IFF) is observed, which is not accounted for in the Max Stress criterion. 0° plies fail in IFF Mode A when N_{xy} starts increasing. IFF Mode A defines failure of the matrix when it is loaded in a combination of transverse tension and shear load. Due to the low transverse tensile strength of the lamina, the σ_{22}/Y_t term contributes greatly to the failure index. This effect is lost when N_{xy} is high enough that 45° plies start getting loaded on-axis and attract the most load. At this point, the failure envelope is becomes similar to the Max Stress criterion's again. In compression, the failure envelope matches with the Max Stress criterion. Hence, the explanation in the previous section applies here as well.

For last ply failure, 0° plies are again observed to fail last through different inter-fiber failure modes. However, when loading is predominantly in N_y, fiber failure occurs. Observations for different loading ratios are listed below,

1. $N_y, N_{xy} > 0$: 45° plies fail last when in IFF Mode A when $N_y >> N_{xy}$. However, as the ratio of N_y/N_{xy} decreases, due to higher stiffness of 45° plies in that direction, they fail first,

causing 0° plies to fail last in IFF Mode A.

2. $N_y < 0$ *and* $N_{xy}/|N_y| > 1$: In compression, with high shear stresses, the 0° plies fail last in IFF Mode B.

3. $N_y < 0$ *and* $|N_y|/N_{xy} > 1$: When predominantly in transverse compression, the 0° plies fail last in IFF Mode C.

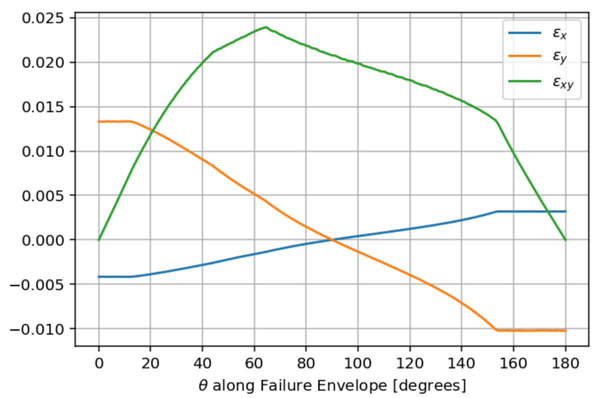


Figure 3.7: Global Strains at First Ply Failure

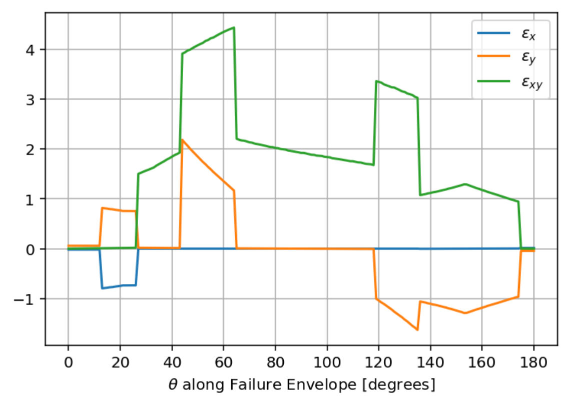


Figure 3.8: Global Strains at Last Ply Failure

Global strains are plotted at first ply and last ply failures in Figures 3.7 and 3.8, respectively. The strains observed at first ply failure are comparable to the strains in Max Stress criterion. The exception is the lower strains where plies fail in IFF Mode A.

At last ply failure, the low failure strains at high N_y loading are due to the failure of 90° plies that reduce the majority of the load carrying capacity of the laminate in the transverse direction. Hence, the remaining plies fail soon afterwards. The region of low failure strains between 25° to 45° in Figure 3.8 corresponds to where IFF Mode A failure happens. The rest is similar to what was observed with the Max Stress criterion in Figure 3.5. Hence, the explanation given in the previous section applies here as well.

3.2 Question-2b

Which regions of the failure envelopes have the smallest and largest damage tolerance, respectively? Which criterion provides more conservative results? Why? Motivate your answer by comparing the mathematical formulations of the criteria.

3.2.1 Damage Tolerance

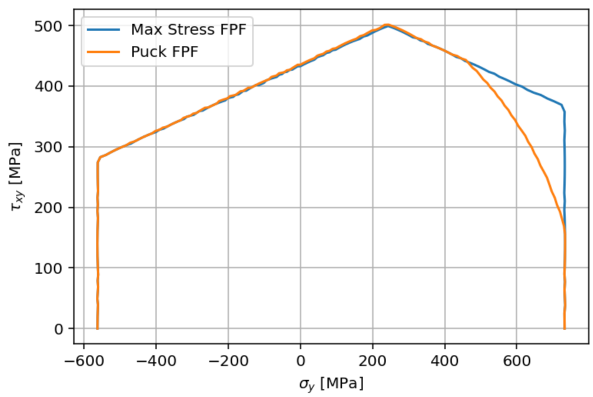
Damage tolerance could be defined as the difference in stress between the first ply and last ply failure, i.e. how much more stress the laminate could withstand after the initial damage. Looking at the failure envelopes for the two failure criteria,

1. Max Stress criterion: In Figure 3.1, the difference in stresses between first ply failure and last ply failure is negligible throughout the failure envelope. Hence, according to this criterion, the laminate has an overall low damage tolerance. Looking closely at the regions where $\tau_{xy} > 300\,MPa$, in tension ($\sigma_y > 0$) and in compression ($\sigma_y < -200MPa$), the last ply fails at a slightly higher stress i.e. more than one increment later. In these regions, $N_{xy}/N_y > 1$. These regions could be defined regions with the highest damage tolerance. The regions with the smallest damage tolerance is where $N_y >> N_{xy}$.

2. Puck criterion: In Figure 3.6, the region where $N_y/N_{xy} \approx 0.5$, highest damage tolerance is observed. In this 'bump', 0° plies fail first in IFF Mode A and 45° plies fail last, also in IFF Mode A. Here, after 0° plies have been degraded, 90° plies fail in Fiber Failure (FF), followed by total failure of 0° in FF. The last remaining plies are 45° and -45°, which fail in IFF Mode A. Since 45° plies are still remaining after the first ply failure, the stress that the laminate can take in this combination of loading is higher. In the region surrounding this 'bump', the first ply degraded is 0°, but 45° and -45° plies fail in FF after that. Since the degraded 0° plies and 90° plies cannot take this high shear load, they fail soon after. The regions with the smallest damage tolerance is where $N_y >> N_{xy}$.

3.2.2 Comparison of Failure Criteria

The failure envelopes for the two criteria are compared in Figures **??** and 3.9. From the comparison, it is can be observed that Puck criterion predicts a lower failure stress for both first and last ply failures, since it considers failure modes with respect to the matrix. Hence, Puck criterion is conservative in comparison to the Max Stress criterion.



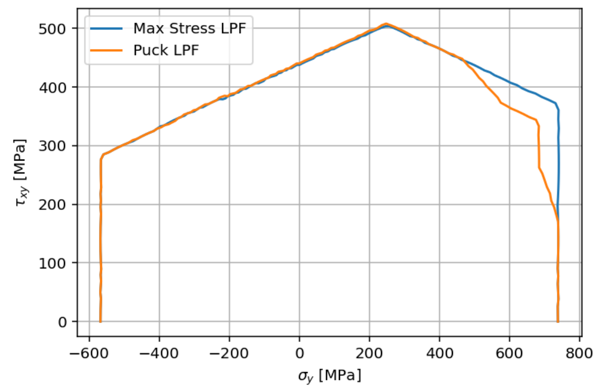


Figure 3.9: Comparison of First Ply (left) and Last Ply (right) Failure Envelopes

In the mathematical formulation of Puck's criterion, we can focus on IFF Mode A since that is the failure mode observed in the conservative region. This mode represents failure of the matrix parallel to the fibers or failure at the fiber-matrix interface [2]. Mode A failure is caused by a combination of transverse tensile stress (σ_y) and shear stresses (τ_{xy}). Mode A failure can happen along any ratio of these stresses, or when they act alone [2]. Here, since the fracture surfaces get separated, transverse properties are degraded. This is accounted for through the degradation rules listed earlier. In the equation to calculate failure index for Mode A ([2], Eq. 71), the interaction between shear stress and transverse stress is considered. The last term in the equation captures the effect of a portion of the transverse stress that would cause shearing in the matrix and is compared against the shear strength. This effect of interaction is not considered in the Max Stress criterion.

Failure stresses for fiber failure however, are comparable between the two criteria even though Puck criterion considers the effect of Poisson's ratio ([2], Eq. 21). The second term in the equation adds a fraction of the transverse stress which corresponds to the Poisson's effect. Calculating this factor with the values of $E_{f1} = 230\,GPa$ and $v_{f12} = 0.2$, it is -0.124. This means a negligible portion of the transverse stress is added to the longitudinal stress to calculate the fiber failure index. Since the first ply failure observed in most regions was due to fiber failure, it explains the negligible difference in the failure envelope between the two criteria.

Question-3

The $[0/90/\pm45]_{2s}$, is loaded with a resultant force N, at 30∘ angle respectively to X-axis. Calculate the probability of First Ply Failure failure for N=850 N/mm and N=1200 N/mm utilizing the Monte Carlo Simulation. Your limit state should be based on the Puck criterion. Select one of the two approaches presented during the class utilizing the theorem of large numbers to study the convergence of your calculations. Motivate your selection. The Elastic and strength properties should be considered random variables and independent.

4.0.1 Expected results

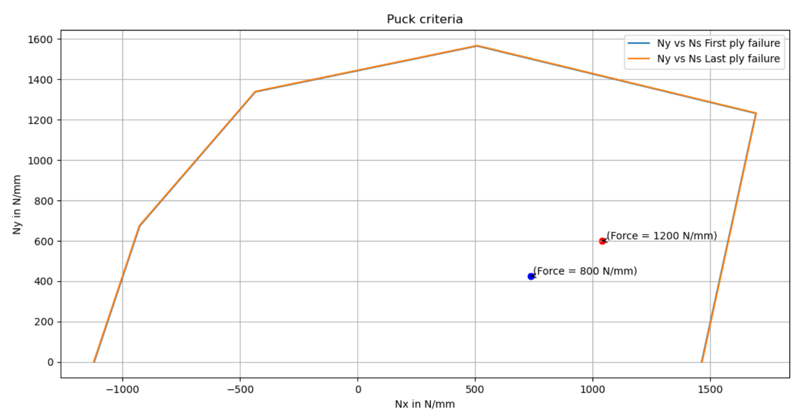


Figure 4.1: Failure envelope for Nx and Ny load case

An envelope was plotted based on the mean properties for the laminate. Two load cases of F = 850 N/mm and F = 1200 N/mm are given in the question. Per the envelope for the mean properties, both the load cases lie within the envelope with F = 1200 N/mm a little towards the edge of the failure envelope. This means that both the load cases should not fail based on the mean properties, but when fluctuations in the properties occur, F = 1200 N/mm should have a higher chance of failing than F = 850 N/mm meaning F = 850 N/mm would be more reliable.

4.0.2 Sampling procedure

The properties given in the table 1.1 are assumed to vary as per a Gaussian distribution. However, two of the remaining properties are missing (X_C and Y_C). The question mentions the compressive properties but their standard deviation is not given. For this question, the following assumption is made for the compressive properties as shown in 4.1. Additionally, the entire laminate (all the laminas) was assumed to have the same properties for a particular iteration (though it would have been better to assume different properties for different laminas, this has not been made because the running time for the convergence program was found to be too large (26 hrs)).

Table 4.1: Properties for compressive load

Property	**Mean**	**Std**	**Units**
X_c	1480	128.3	MPa
Y_c	220	8.2	MPa

For the sampling, the following procedure was followed to ensure that the samples remain representative of the whole population:

1. The PDFs and CDFs were generated using the inbuilt functions in Python. The corresponding mean and standard deviation for each property were used. The number of samples generated was 1 million for each of the properties.

2. Next from these 1 million samples, a predefined number of random values are selected. Once this selection is done, then the mean is evaluated for this number of samples and compared with the original mean. If the difference is of the order of standard deviation/10, then these random samples are accepted as a "Good enough" representative sample space. Otherwise, the iteration is repeated until the mean of this random number of samples are within the mentioned range.

3. Once these samples are obtained, they are stored for the Monte Carlo simulation in an array with each row representing a particular property and each column the variation of that particular property.

4.0.3 Monte-Carlo Simulation

The procedure followed for Monte-Carlo simulation is as follows:

1. As mentioned earlier, the samples were stored in an array with each row consisting of a particular property and each column being the variations of that particular property.

2. Sets were made within this array for further analysis. For example, column n would make a set of properties that would then be simulated for results. Column n would have the properties $[E_1(n),\ E_2(n),\ v_{12}(n),\ G_{12}(n),\ X_t(n),\ X_c(n),\ Y_t(n),\ Y_c(n),\ S(n)]$
3. So for each of the sets, Puck's criterion was used to calculate the first ply failure.
4. The probability of failure was then calculated as

$$Probability\ of\ failure = \frac{Total\ no.\ of\ failures}{Total\ simulations}$$

4.0.4 Convergence

The results from the Monte-Carlo simulation will never be identical for two successive runs. This is because different probabilities of failures will be encountered for different sets of properties and different initial sets that get generated for each run. For this question, convergence was explored for two different factors.

1. Convergence for the material properties (Following the law of large numbers)
2. Convergence for the outcome

Convergence for material properties:
As discussed earlier in the section on sampling procedure, this particular convergence was achieved by selecting a large enough number of samples from the population and then testing for the mean of all the properties with the actual mean given in the question. If this condition is not satisfied, then a different set of the same number of random samples were then selected and checked for closeness to the mean. This is repeated until the error with respect to the mean is within 10% of the standard deviation.

Convergence for the outcome:
Even though convergence of material properties was ensured, it was observed that the outcome varied drastically with differences between successive runs being nearly 250% in some runs. So, a better method was to be devised to ensure convergence and this was thought to be possible through the outcomes convergence.

Convergence for the outcome was not so straightforward. Initially, the error between successive runs was calculated as the sample sizes were increased. Whenever the error was less than 1% compared to the previous iteration, the program was terminated saying convergence was achieved. This could be observed in the figure 4.2. But this could not be replicated for successive runs as can be seen from 4.3. There is too much variation to suggest which sample size is the right one to run the

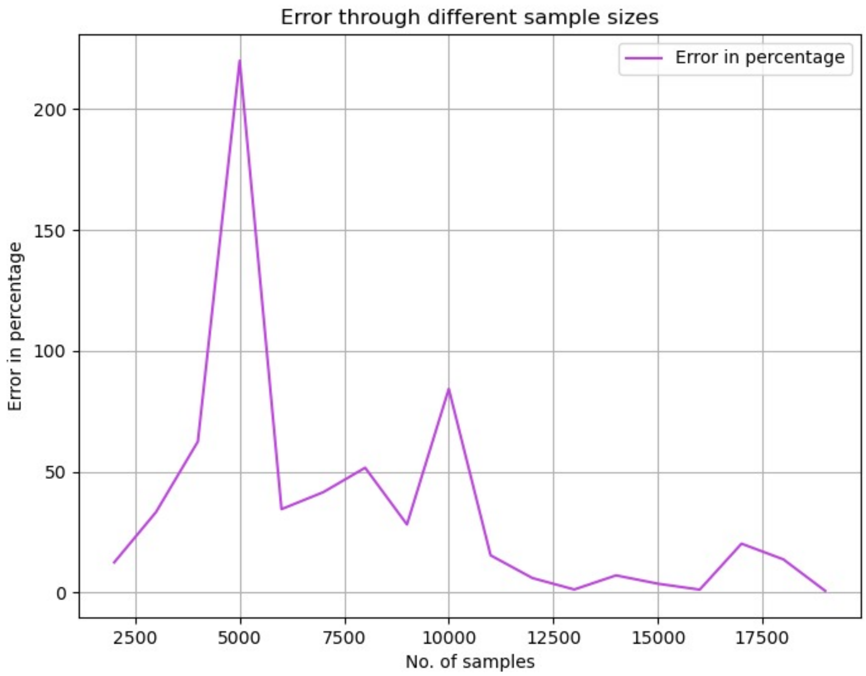


Figure 4.2: Convergence attempt through error measurement as sample sizes were increased - Attempt 1 (1200 N/mm)

Monte-Carlo simulation. A better way needs to be developed.

After a couple of deliberation between us, a new way of expressing the convergence for these Monte-Carlo simulations was thought of. The results after the iterations will be expressed in terms of confidence intervals. For example after n-runs:

$$Probability\ of\ failure = (Mean)\ [upper\ limit, lower\ limit](95\%\ confidence)$$

Now, the convergence is being done to keep the upper limit and the lower limit within a certain percentage of the mean. For example, within 5 % of the mean. Initial convergence for 15% error is shown in figure 4.4.
Based on this logic, the final convergence has been done to identify the best possible sample size and the minimum iterations required to maintain a certain confidence level for our Monte-Carlo simulations.

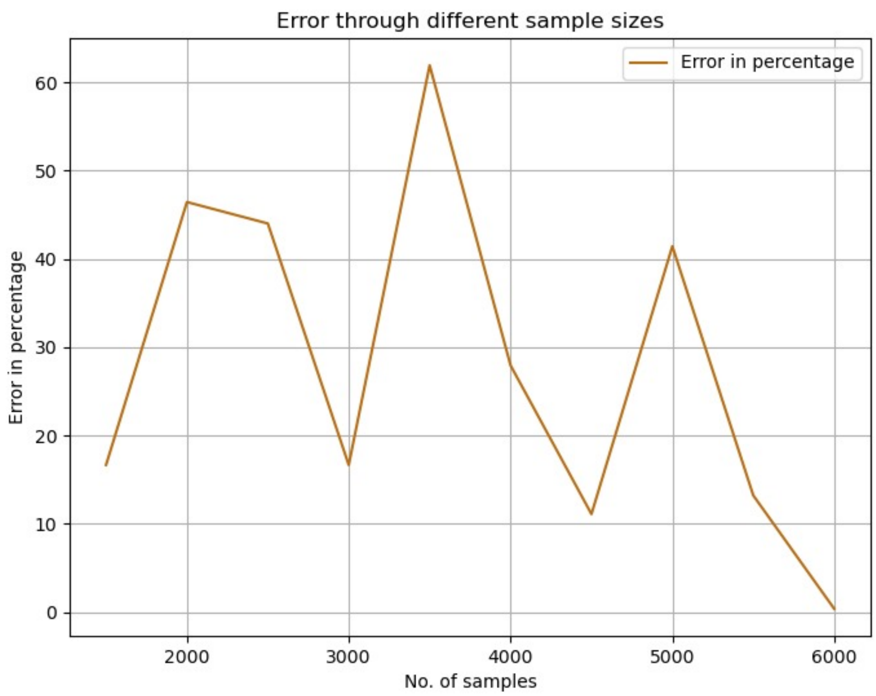


Figure 4.3: Convergence attempt through error measurement as sample sizes were increased - Attempt 2 (1200 N/mm)

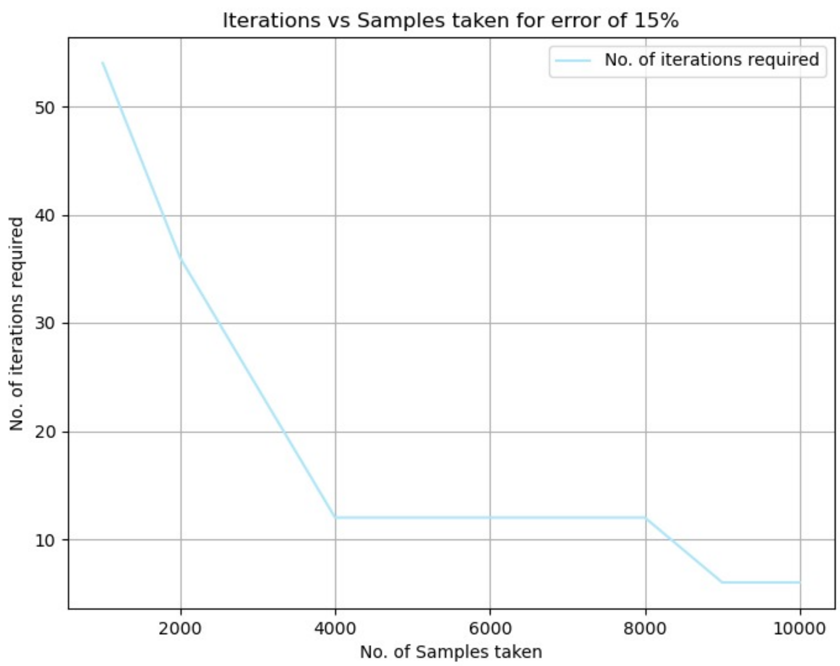


Figure 4.4: Convergence attempt stable confidence levels as sample sizes were increased - Attempt 3 (1200 N/mm)

4.0.5 Final results

Results for F = 850 N/mm

For F = 850 N/mm, there were no failures observed at any of the sample sizes. This should indicate that even for the worst possible material properties, F = 850 N/mm might lie within the failure envelope.

Results for F = 1200 N/mm

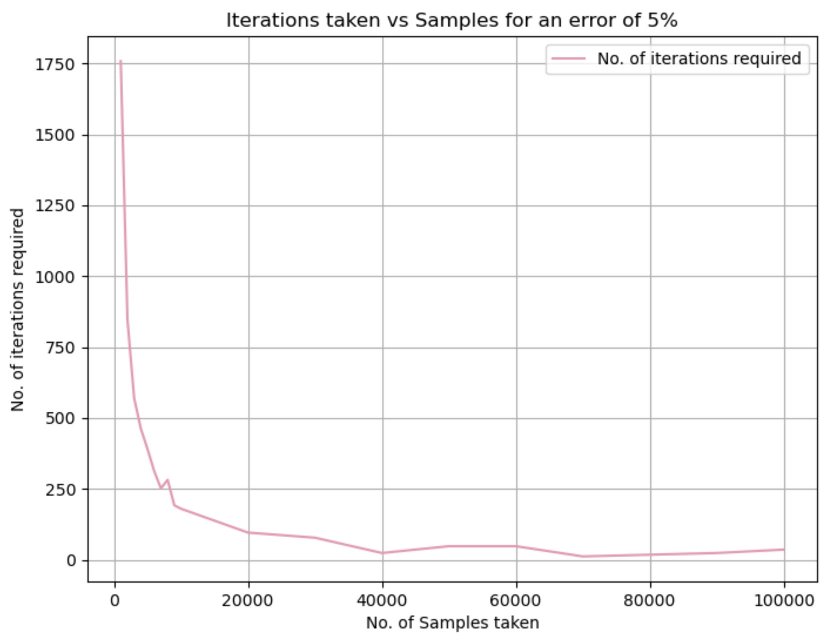


Figure 4.5: Convergence as sample sizes were increased for F = 1200 N/mm

As shown in figure 4.5, convergence was attempted at every 1000 intervals and as the number of samples increased, the number of iterations required to reach a given mean reduced. The mean value for N = 10^5 samples was 0.346% with the upper and lower bounds at 0.338% and 0.354% for a confidence level of 95%.

Contributions

As mentioned in the email, we discussed the possible solution paths for each question together but in the end coded it independently. If one of us was stuck at any point, we decoded it together. That is the reason why both the codes have been submitted in the email. However, only a single report is being submitted as we felt that the final results were still a byproduct of both of our efforts.

For the report, Mr. Zia Ansari wrote the complete solution/reasoning for Question 2 whereas Mr. Venkatesh wrote the complete solution/reasoning for Questions 1 and 3. There was a particular preference or reason why it was done this way. The assignment was divided into questions and based on the weightage we decided that Q1 and Q3 would have the same work as Q2 alone.

As for who does which of these "divided" parts, it was just decided based on a **coin toss** in the end.

Bibliography

[1] B. van Dongen, "Progressive damage modelling of frps using a blended stress-strain and fracture mechanics approach in fem," 2017. [Online]. Available: `https://repository.tudelft.nl/islandora/object/uuid%3Ac879ec90-a2bd-42a8-884d-d66e1af2102e`.

[2] "Puck's action plane fracture criteria," in *Analysis of Failure in Fiber Polymer Laminates: The Theory of Alfred Puck*. Berlin, Heidelberg: Springer Berlin Heidelberg, 2008, pp. 37–115, ISBN: 978-3-540-75765-8. DOI: `10.1007/978-3-540-75765-8_4`. [Online]. Available: `https://doi.org/10.1007/978-3-540-75765-8_4`.

[3] "Review of laminate strength and failure criteria," in *Design and Analysis of Composite Structures*. John Wiley Sons, Ltd, 2013, ch. 4, pp. 55–63, ISBN: 9781118536933. DOI: `https://doi-org.tudelft.idm.oclc.org/10.1002/9781118536933.ch4`. eprint: `https://onlinelibrary-wiley-com.tudelft.idm.oclc.org/doi/pdf/10.1002/9781118536933.ch4`. [Online]. Available: `https://onlinelibrary-wiley-com.tudelft.idm.oclc.org/doi/abs/10.1002/9781118536933.ch4`.