Technische Universiteit Delft

Faculty of aerospace engineering

Master of Science Aerospace Engineering

Assignment - II

NON-LINEAR MODELING (AE4ASM505)

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Question-1

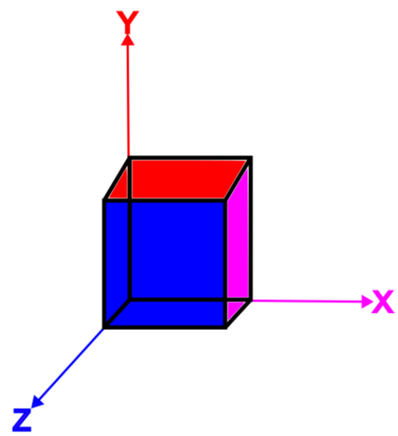

Figure 1.1: Cube of isotropic material, aligned with coordinate system

A cube of a homogenous isotropic material, shown in figure 1.1, is being deformed cyclically along the global coordinate axes, with displacement control. The cube edge length is L = 0.7 m. The cyclic loading is sinusoidal with a mean of 0, with amplitudes in meters per loading axis defined by: $A_x = 0.45$ (tension), $A_y = -0.2$ (compression), and $A_z = 0$. Due to these loading cycles, the cube undergoes repeated tension and compression. The total number of loading cycles is N = 10. Use a Possion's ratio $v = 0$ for the material.

The stress-strain response of the material is shown in figure 1.2 and summarized in table 1.1.

Table 1.1: Material stress-strain response

Strain	**Stress [MPa]**
0	0
0.375	15
1.5	22
> 1.5	22

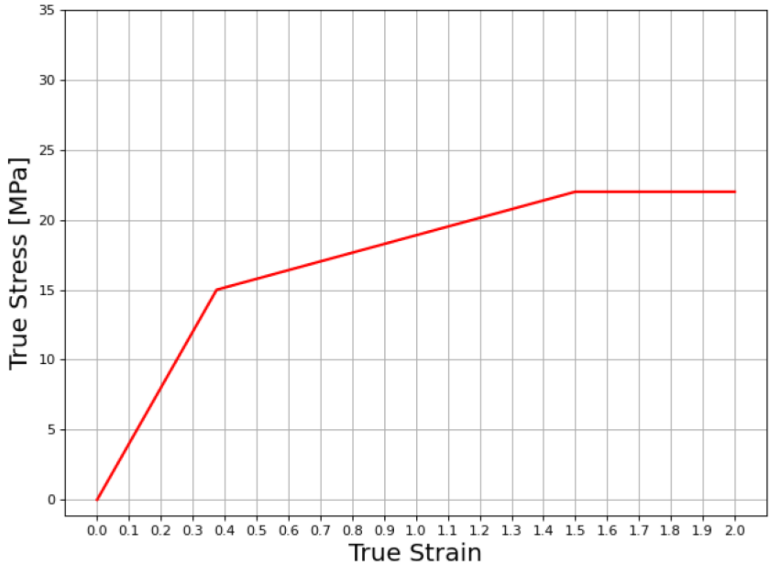

Figure 1.2: Material stress-strain response

Answer the following tasks in the report. Include any program that you make to help address these tasks in an appendix to the report.

1.1 Deformation gradient amplitude

Compute the deformation gradient at the maximum loading amplitude for the given cyclic loading. Use the total Lagrangian formulation. Show your calculations and report the matrix.

Let x be the current configuration. Let X be the reference configuration (here we are using the total Lagrangian formulation). Hence, X would be the initial reference configuration.

The deformation gradient is given by

$$F = \frac{\partial x}{\partial X}$$

In the question, the displacement is given to be in the sinusoidal form. Therefore,

$$u_1 = 0.45\,\frac{X_1}{0.7}\,\sin(2\pi f t)$$

$$u_2 = -0.2\,\frac{X_2}{0.7}\,\sin(2\pi f t)$$

$$u_3 = 0$$

Now calculating the mapping for the current configuration:

$$x_1 = X_1 + u_1$$

$$x_2 = X_2 + u_2$$

$$x_3 = X_3 + u_3$$

Therefore, the deformation gradient would be given by,

$$F = \begin{pmatrix} \frac{\partial x_1}{\partial X_1} & \frac{\partial x_1}{\partial X_2} & \frac{\partial x_1}{\partial X_3} \\ \frac{\partial x_2}{\partial X_1} & \frac{\partial x_2}{\partial X_2} & \frac{\partial x_2}{\partial X_3} \\ \frac{\partial x_3}{\partial X_1} & \frac{\partial x_3}{\partial X_2} & \frac{\partial x_3}{\partial X_3} \end{pmatrix}$$

For this question, only the maximum amplitudes are to be considered. Therefore,

$$u_1 = 0.45\,\frac{X_1}{0.7}$$

$$u_2 = -0.2\,\frac{X_2}{0.7}$$

$$u_3 = 0$$

Substituting we get,

$$F = \begin{pmatrix} 1 + \frac{0.45}{0.7} & 0 & 0 \\ 0 & 1 - \frac{0.2}{0.7} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$F = \begin{pmatrix} 1.6428 & 0 & 0 \\ 0 & 0.7142 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1.2 Nominal strain amplitude

Compute the nominal total strain tensor using small strain assumptions (i.e., the linear strain) at the maximum positive amplitude of loading. Show your calculations and report the final nominal strain tensor in the principal coordinate system.

For this part of the question, at the maximum positive amplitude of loading, the amplitudes of the loads are as follows:

$$u_1 = 0.45\,\frac{X_1}{0.7}$$

$$u_2 = -0.2\,\frac{X_2}{0.7}$$

$$u_3 = 0$$

The Green-Lagrange strain for linear cases is given as:

$$E = \frac{1}{2}(\frac{\partial u}{\partial X} + \frac{\partial u}{\partial X}^T)$$

In terms of matrix form, this can be written as:

$$E = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}) & \frac{1}{2}(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}) \\ \frac{1}{2}(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}) \\ \frac{1}{2}(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}) & \frac{1}{2}(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}) & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

$$E = \begin{pmatrix} 0.6428 & 0 & 0 \\ 0 & -0.2857 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1.3 True total strain amplitude

Compute the true total strain tensor using small strain assumptions at the maximum positive amplitude of loading and the maximum negative amplitude of loading. Show your calculations and report the final true strain tensor in the principal coordinate system. (Tip: use this formula for compressive strain conversion $\epsilon_t = -ln(1 - \epsilon_n)$.

We have already calculated the total strain tensor at the maximum positive amplitude of loading. It was obtained as:

$$E = \begin{pmatrix} 0.6428 & 0 & 0 \\ 0 & -0.2857 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

To calculate the true stress from the nominal stress, we use the equation for tensile stresses:

$$\epsilon_t = \ln(1 + \epsilon_n)$$

and the following equation for compressive stresses:

$$\epsilon_c = -\ln(1 - \epsilon_n)$$

For our Green-Lagrange strain tensor, there are only two components, one of which is positive while the other is negative. Applying the above formulas, we get:

$$\epsilon_t = \ln(1 + 0.6428)$$

$$\epsilon_t = 0.4964$$

$$\epsilon_c = -\ln(1 + 0.2857)$$

$$\epsilon_c = -0.2513$$

Therefore, the Green Lagrange Strain tensor for the true total strain is

$$E_{true} = \begin{pmatrix} 0.4964 & 0 & 0 \\ 0 & -0.2513 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For the case of maximum negative amplitude of loading, the displacements would be:

$$u_1 = -0.45\,\frac{X_1}{0.7}$$
$$u_2 = 0.2\,\frac{X_2}{0.7}$$
$$u_3 = 0$$

The Green-Lagrange strain in matrix form is written as:

$$E = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}) & \frac{1}{2}(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}) \\ \frac{1}{2}(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}) \\ \frac{1}{2}(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}) & \frac{1}{2}(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}) & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

$$E = \begin{pmatrix} -0.6428 & 0 & 0 \\ 0 & 0.2857 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

As mentioned earlier, now converting them into true strain tensor, we get:

$$\epsilon_c = -\ln(1 + 0.6428)$$

$$\epsilon_t = -0.4964$$

$$\epsilon_c = \ln(1 + 0.2857)$$

$$\epsilon_c = 0.2513$$

$$E_{true} = \begin{pmatrix} -0.4964 & 0 & 0 \\ 0 & 0.2513 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1.4 Forward Euler Integration

Use the forward Euler integration algorithm from the lecture notes to compute the true stress response to the true total strains imposed on the material cube. Make a plot for each principal true stress component versus the signed true Von Mises strain. You can compute the sign of the true total Von Mises strain from the sign of the true hydrostatic strain. Include the three plots in the report, with the stress-strain relationships clearly visible.

For solving this particular problem, apart from the normal parameters defined in the class, a few different things have been modified to get the results. These are explained below:

1. $\Delta\epsilon_A$: For calculating this parameter various ways were sought for. The most reliable one I found was to calculate for every point from zero iteration but as the step size increases, the error in Forward Euler also increases (square of the step size). Due to this, I was getting very bad results.
   Instead of calculating this, I made my step size itself very small. The total time steps were divided into 50000 individual ones and it was assumed that whenever it yields, that would itself be the $\Delta\epsilon_A$ due to the small size of the step.**This increased the compute time tremendously though**

2. The yield function: As no particular relationship between k and s_{yield} was given, initially as k and s_{yield} were in the same order, I tried adding them up as $s_{yield} + k$. But this didn't work out. With the help of a colleague, I then changed the yield function to be the von_mises stress of the current stress after which it yielded.

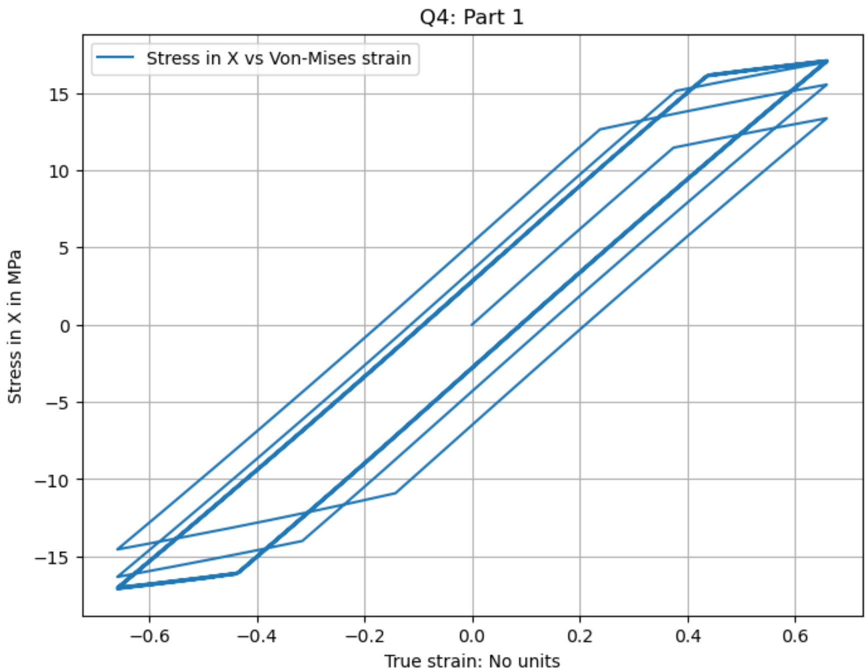

Figure 1.3: Principal stress in X vs Signed True Von-Mises Strain

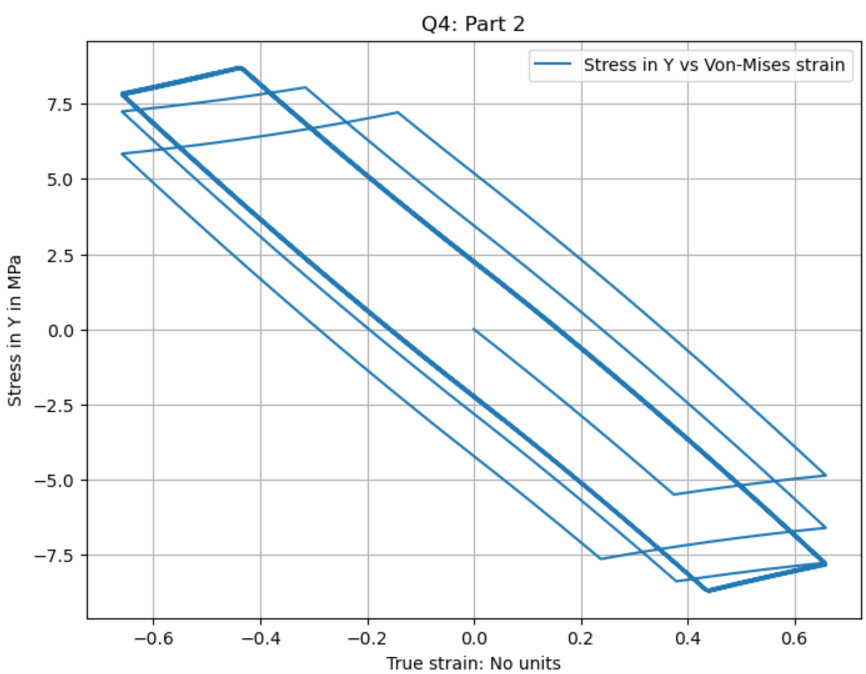

Figure 1.4: Principal stress in Y vs Signed True Von-Mises Strain

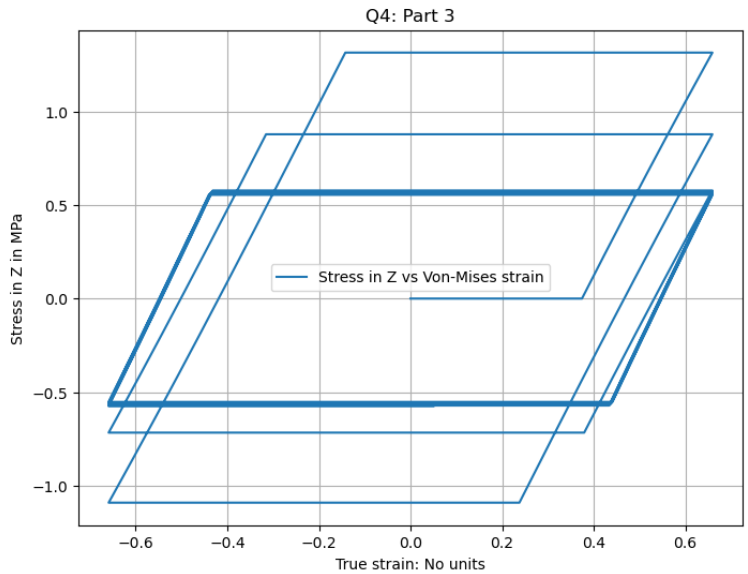

Figure 1.5: Principal stress in Z vs Signed True Von-Mises Strain

1.5 Strain along the Z direction

Explain why the strain along the Z direction occurs (remember that the Poisson ratio of the material is 0), and explain what needs to change in the stress-strain response of the material for the strain along the Z direction to converge to 0 as the number of loading cycles increases.

Initially, the strain along the Z-direction is zero but as we keep increasing the loading and the number of cycles, we start getting plastic strain which keeps adding up into the total true strain of the material. This brings in an additional strain that was not initially present.

This is caused by m_c which is $\frac{\partial f}{\partial \tau}$ evaluated on the yield curve. Since the yield function is being differentiated with respect to τ_3 even though it is zero, before being substituted with the value of zero τ_3 we are getting the residue value which adds from the next iteration in τ_c which in turn produces the plastic strain component in z-direction.

To overcome this, one solution that was found was to change the Von-Mises stress from

$$\sigma_{von} = \sqrt{\frac{1}{2} * ((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)}$$

to

$$\sigma_{von} = \sqrt{\frac{1}{2} * ((\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2)}$$

Other ways that would make the strain converge to zero as the number of cycles increases would be:

1. To change the material response after the initial yielding where it becomes zero slope. Instead of no longer hardening being possible, the material just deforms plastically. To make the stress converge to zero, it would be better to have continuous hardening.

2. Increasing the Young's Modulus would facilitate in making the material more brittle and hence, lesser plasticity.

1.6 Signed Von Mises stress

Make a plot of the signed true Von Mises stress versus the signed true total Von Mises strain, for the material stress-strain response given the problem statement. You can compute the sign of the true Von Mises strain from the sign of the true hydrostatic stress. What needs to change in the true stress - true strain input curve 1.2 of the material for the signed true Von Mises stress versus the signed true total Von Mises strain response to become linear eventually after the cyclic loading?

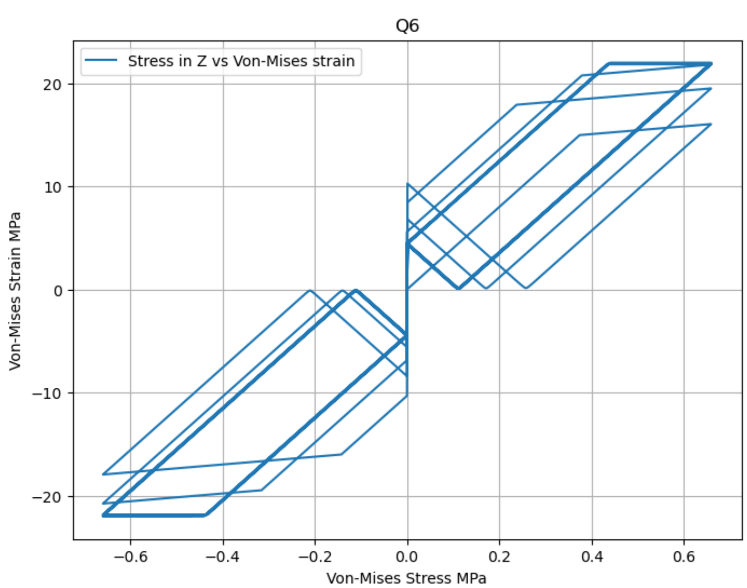

Figure 1.6: Signed Von-Mises strain vs signed Von-Mises stress

The plot requested in the question is shown in the figure 1.6. To make it linear eventually, remove the parameter $h_c = 0$ for stresses greater than 22 and keep the usual $h_c = 56/9$.

1.7 Influence of Possion ratio

What would happen to the stress response in the Z direction if the Poisson ratio for the material was v = 0.3? Comment on the expected differences in the elastic regime of the response.

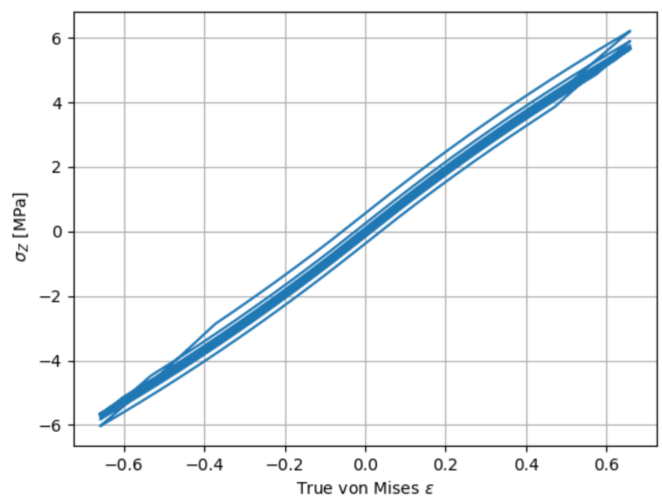

Figure 1.7: Stress in the Z-direction with Poisson Ratio = 0.3

As having a non-zero Poisson ratio would make the stresses from x and y also a matter for Z direction, unlike the earlier curve in 1.5, the stresses would no longer be small. In the elastic region, the stresses were either zero or near zero in Q4 1.1, but here we can see a linear increase in the strain as the number of cycles increases. However, there is no big envelope that is formed like earlier. The curve shrinks and stays together for most of the cycles.

1.8 Plasticity formulation limitations

Is the small strain theory of plasticity appropriate for this loading condition? Why/why not?

In my opinion, the deformations are high and the strain that is evaluated is still high enough. The assumption of small strain large deformations theory of plasticity is not appropriate here as we are calculating everything in terms of the initial configuration which is not representative enough for the problem.