Delft University of Technology

STABILITY AND ANALYSIS OF STRUCTURES - 1

AE4ASM106

Assignment: 2

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Question 1:

Buckling load for a clamped-pinned beam/column. A slender beam/column of length L is subjected to an axial compressive load F. The beam is pinned at x = 0 and clamped at x = L. The beam is made of a linearly elastic material with Youngs Modulus E and moment of inertia I.

As derived in class, the pre-buckled state is $w_0 = 0$ and the buckling equation for the buckling mode w_1 is

$$EI\,\frac{d^4w_1}{dx^4} + F\,\frac{d^2w_1}{dx^2} = 0, for\ all\ 0 \le x \le L$$

The general solution to this buckling equation is

$$w_1(x) = C_1 \sin kx + C_2 \cos kx + C_3x + C_4$$

$$where, k = \sqrt{\frac{F}{EI}}$$

Sub question 1:

i. Consider the boundary conditions for the pinned (at x=0) and clamped at (x=L) beam/column:

$$w_1(0) = 0\ w_1''(0) = 0\ w_1(L) = 0\ w_1'(L) = 0$$

Find the smallest critical load for buckling.

(Hint: in this case the characteristic equation cannot be solved analytically, but you can use the fact that the smallest positive solution to the equation $z\cos z - \sin z = 0$, which can also be written as $\tan z = z$, is z = 4.493)

Given that the solution is,

$$w_1(x) = C_1 \sin kx + C_2 \cos kx + C_3x + C_4$$

With the boundary conditions,

$$w_1(0) = 0, w_1''(0) = 0, w_1(L) = 0, w_1'(L) = 0$$

$w_1(0) = 0 =>$

$$w_1(0) = C_1 \sin k * 0 + C_2 \cos k * 0 + C_3 * 0 + C_4 = 0$$

$$w_1(0) = C_2 + C_4 = 0 \rightarrow 1$$

$w_1(\mathrm{L}) = 0 =>$

$$w_1(L) = C_1 \sin kL + C_2 \cos kL + C_3 L + C_4 = 0 \rightarrow 2$$

$w_1{}'(\mathrm{L}) = 0 =>$

$$w'_1(x) = kC_1 \cos kx - kC_2 \sin kx + C_3$$

$$w'_1(L) = kC_1 \cos kL - kC_2 \sin kL + C_3 = 0 \rightarrow 3$$

$w_1{}''(0) = 0 =>$

$$w''_1(x) = -k^2 C_1 \sin kx - k^2 C_2 \cos kx$$

$$w''_1(0) = k^2 C_1 \sin k * 0 - k^2 C_2 \cos k * 0 = 0$$

$$C_2 = 0 \rightarrow 4$$

From 1,

$$C_4 = 0 \rightarrow 5$$

From 2,

$$C_1 \sin kL + C_3 L = 0 \rightarrow 6$$

From 3,

$$k\, C_1 \cos kL + C_3 = 0$$

$$C_3 = -k\, C_1 \cos kL \rightarrow 7$$

Substituting 7 in 6, we get

$$C_1 \sin kL - kC_1 L \cos kL = 0$$

$$\sin kL = kL \cos kL$$

$$\tan kL = kL$$

Using the relation given in the question,

$$kL = 4.493$$

$$k = \frac{4.493}{L}$$

Finding the force for the smallest buckling load:

$$F = k^2 EI$$

$$F = \frac{4.493^2}{L^2} EI$$

The general solution is given by:

$$w_1(x) = C_1 \sin(\frac{4.493x}{L}) + C_3 x$$

Finding C_1 and C_3 by equating the norm of w_1 to 1.

$$||w_1(x)||^2 = \int_0^L C_1 \sin(\frac{4.493x}{L}) + C_3 x \; dx = 1$$

Using python for finding the integral:

And using the equation 6,

We get,

$$C_1 = \sqrt{\frac{1}{0.794L}}$$

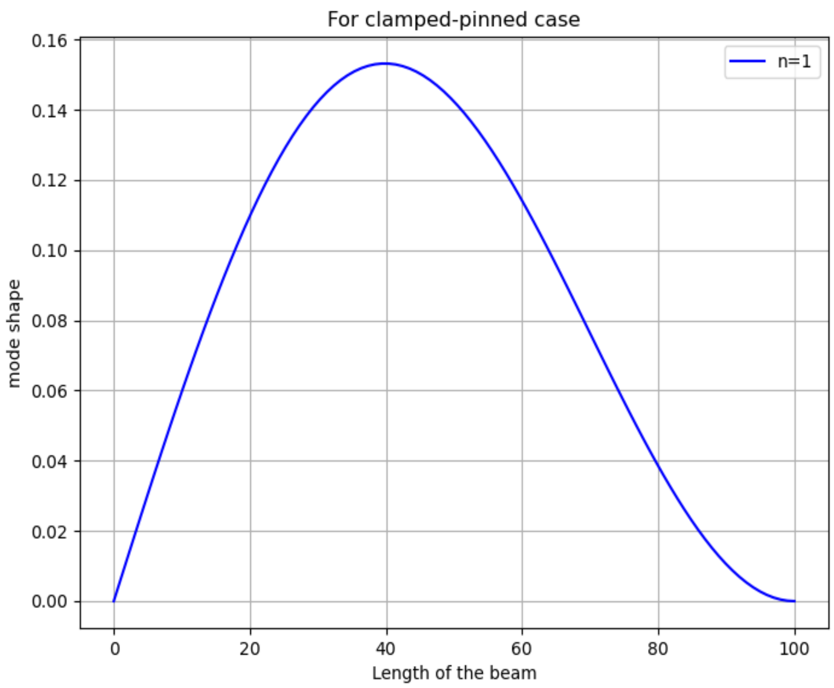
$$C_3 = \sqrt{\frac{6}{5L^3}}$$

$$w_1(x) = \sqrt{\frac{1}{0.794L}} \sin(\frac{4.493x}{L}) + \sqrt{\frac{6}{5L^3}} x$$

Sub question 2:

ii. Sketch the shape of the smallest buckling mode (A sketch with the key qualitative aspects is sufficient). Alternatively, you can also choose your own values for E,I,L and plot the curve with a program.

Since we have substituted the value of k in the form of L in the equation, we won't need to substitute E and I. Assuming a value of L = 100 mm and using python for plotting the mode shape we get,



Question 2:

Buckling of a plate under uniform uniaxial compression. Consider a flat rectangular a x b plate of thickness h. The plate is simply supported and uniformly loaded on the edge of width b with a compressive normal load intensity P_a (in N/m).

Sub question 1:

i. Using the formulas developed in class, determine the critical buckling factor k_b for a plate of dimensions a = 2m, b = 1m and for a plate of dimensions a = 1m, b = 2m. How many half-waves does the buckled mode have in each case?

For a uniaxial compression, the buckling factor k_b is given by,

$$k_b = \left(\frac{m}{r} + \frac{r}{m}\right)^2$$

Where,

$$r = \frac{a}{b}$$

Case 1:

a = 2 m and b = 1 m.

r = 2

Using the graph from the slides. For, r =2, m = 2.

The no. of half waves would also be 2.

$$k_b = \left(\frac{m}{r} + \frac{r}{m}\right)^2$$

$$k_b = \left(\frac{2}{2} + \frac{2}{2}\right)^2 = 4$$

Case 2:

a = 1 m and b = 2 m

r = 0.5

Using the graph from the slides. For, r =0.5, m = 1.

The no. of half waves would also be 1.

$$k_b = \left(\frac{1}{0.5} + \frac{0.5}{1}\right)^2$$

$$k_b = \left(2 + \frac{1}{2}\right)^2 = 6.25$$

Sub question 2:

ii. Suppose that the a = 2m, b = 1m plate should have a mass of 100 kg. One plate is made of ASTM A36 steel (E=200 GPa, ν = 0.3, ρ=7800 kg/m³) and another one of 2024 Al (E=73 GPa, v = 0.33, ρ=2780 kg/m³). Which plate has a higher critical buckling load for the same mass (total load in N)? What are the corresponding critical stresses in each case?

Given two plates with a = 2m and b = 1m

For ASTM A36 steel, given:

E = 200 GPa

v = 0.3

ρ = 7800 kg/m³

Finding the thickness using the mass and density:

$$a * b * h * \rho = mass$$

$$h = \frac{mass}{a * b * \rho} = \frac{100}{2 * 1 * 7800} m = 6.41\ mm$$

Finding the critical buckling stress,

$$\sigma_c = \frac{D\pi^2}{hb^2} k_b$$

$$D = \frac{Eh^3}{12(1-v^2)} = \frac{200 * 10^3 * 6.41^3}{12(1-0.3^2)} = 4832712.839\ N - mm$$

$$\sigma_c = \frac{4832712.839\ \pi^2}{6.41 * 1000^2} * 4$$

$$\sigma_c = 29.71\ MPa$$

For 2024 Al, given:

E = 73 GPa

v = 0.33

ρ = 2780 kg/m³

Finding the thickness using the mass and density:

$$a * b * h * \rho = mass$$

$$h = \frac{mass}{a * b * \rho} = \frac{100}{2 * 1 * 2780} m = 17.99\ mm$$

Finding the critical buckling stress,

$$\sigma_c = \frac{D\pi^2}{hb^2} k_b$$

$$D = \frac{Eh^3}{12(1-v^2)} = \frac{73 * 10^3 * 17.99^3}{12(1-0.33^2)} = 39718313.16\ N - mm$$

$$\sigma_c = \frac{39718313.16\ \pi^2}{17.99 * 1000^2} * 4$$

$$\sigma_c = 87.16\ MPa$$

Now, finding the critical buckling load:

For ASTM A36 steel:

$$Critical\ Buckling\ load = stress * area$$

Where $area = b * h$

$$Critical\ Buckling\ load = 29.71 * b * h$$

Where h = 6.41 mm

$$Critical\ Buckling\ load = 190.44\ kN$$

For 2024 Al:

$$Critical\ Buckling\ load = stress * area$$

Where $area = b * h$

$$Critical\ Buckling\ load = 87.16 * b * h$$

Where h = 17.99 mm, b = 1000 mm

$$Critical\ Buckling\ load = 1568\ kN$$

Sub question 3:

iii. The yield stress of A36 steel is 250 MPa and for the 2024 Al it is 324 MPa. What would be the limiting factor for the design (buckling load or plastic yield?)

The buckling stress for A36 steel was found to be 29.71 MPa and the given yield stress is 250 MPa. Hence, the limiting factor would be buckling load instead of yielding.

The buckling stress for 2024 Al was found to be 87.16 MPa and the given yield stress is 324 MPa. Hence, the limiting factor would be buckling load instead of yielding.

Annexure Python Code:

Question 1:

Integrating to find C1 and C3:

```
import sympy as sp
eq = sp.integrate((C1*sp.sin(4.4934*x/L)+C3*x)**2,(x,0,L))
eq = eq.subs({C3: -C1*np.sin(4.493)/L})
```

Output:

0.793953033675926*C1**2*L -> which is equal to 1.

Find C1 and substitute to find C2.

For the plot:

```
import numpy as np
import matplotlib.pyplot as plt

w1 = (1/np.sqrt(0.794*L))*np.sin(4.4934*x/L)+np.sqrt(6/(5*L**3))*x

plt.figure(figsize = (8,6))
plt.plot(x,w1,color='blue',label='n=1')
plt.xlabel('Length of the beam')
plt.ylabel('mode shape')
plt.title('For clamped-pinned case')
plt.legend()  # Show legend
plt.grid(True)  # Show grid
plt.show()
```

Output: Shown in the question itself in the form of the plot.