ASSIGNMENT 4

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1.

(a) $\forall x \; \forall y \quad (x+y = y+x) \wedge (x \times y = y \times x)$

(b) $\forall x \; \forall y \; \exists z \quad (x+y=z) \wedge (x \times y = z) \wedge Integer(z)$

(c) $\forall x \; \forall y \; \forall z \quad (x+y) \times z = (x \times z) + (y \times z)$

(d) $\forall x \; \forall y \; \forall z \quad ((x+y)+z = x+(y+z)) \wedge ((x \times y) \times z = x \times (y \times z))$

(e) $\forall x \quad (x+0 = x) \wedge (x \times 1 = x)$

2.

$\alpha = \forall x \, (P(x) \vee Q(x))$
$\beta = \forall x \, P(x) \vee \forall x \, Q(x)$

Let us create a truth table

$P(x)$	$Q(x)$	α	β
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

$\because$ if α is true, β is also true

$\therefore \alpha \models \beta$

3. Following are abbrevations for various words used in the question.

HS : Han Solo.	MF : Millenium Falcon
PL : Princess Leia	RA : Rebel Alliance
OW : Obi-Wan Kenobi	LS : Light Saber

a. Owns (HS, MF)

b. Unhappy (PL)

c. Loves (PL, HS)

d. $\forall x \; (Owns(x, MF) \vee Unhappy(x) \Rightarrow Visits(x, OW))$

e. $\forall x \; Visits(x, OW) \Rightarrow Wise(x)$

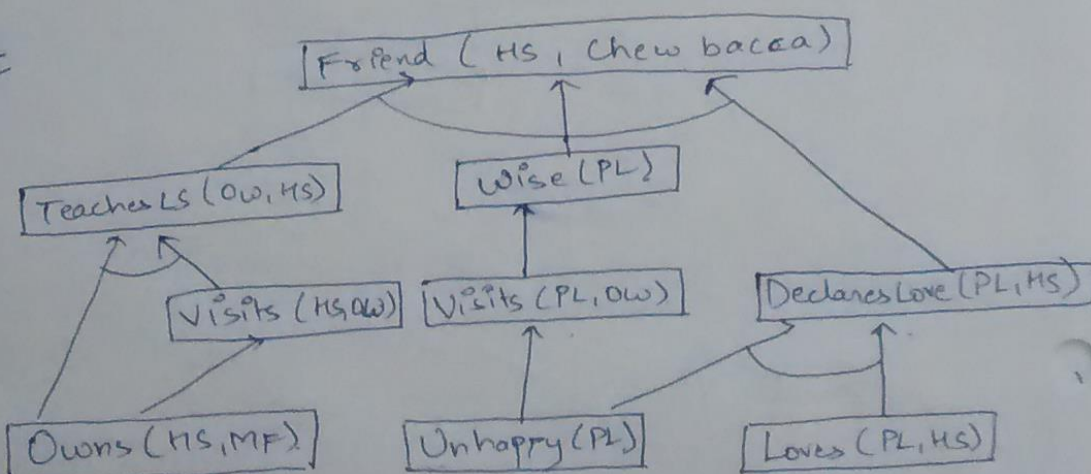
f. $\forall x \; (Owns(x, MF) \wedge Visits(x, OW) \Rightarrow TeachesLS(OW, x))$

g. $\forall x \; ((Unhappy(x) \vee Owns(x, MF)) \wedge TeachesLS(OW, x) \Rightarrow Joins(x, RA))$

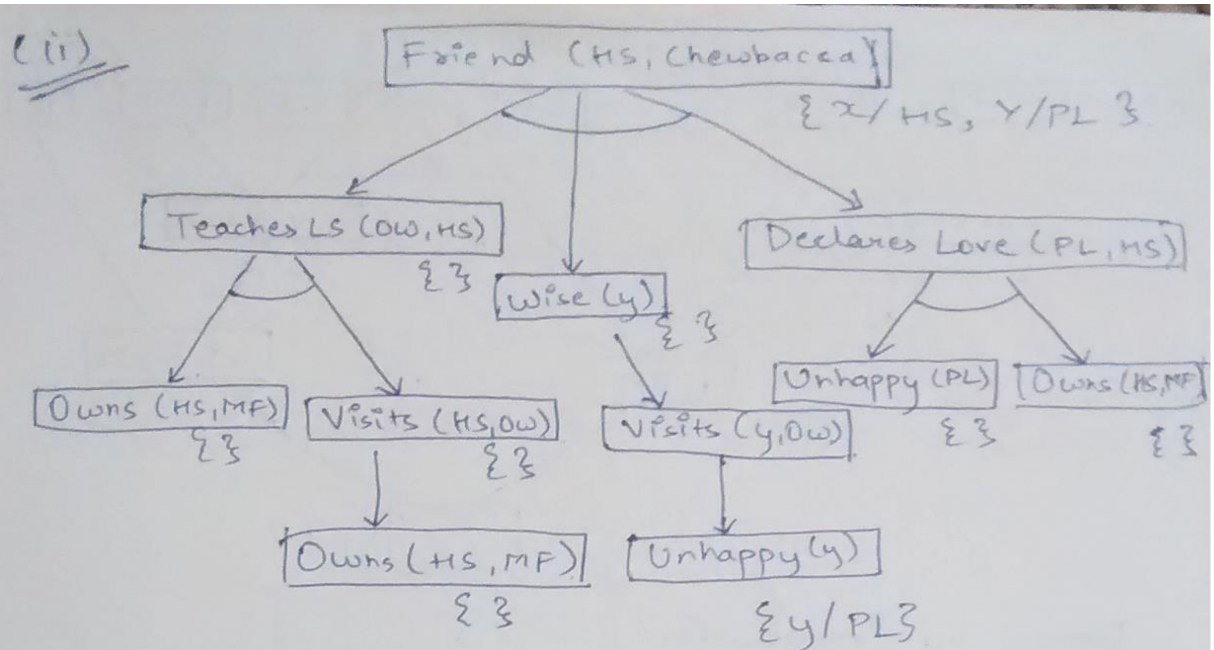
h. $\forall x \forall y \; Unhappy(x) \wedge Loves(x, y) \Rightarrow DeclaresLove(x, y)$

i. $\forall x \forall y \; (TeachesLS(OW, x) \wedge DeclaresLove(y, x) \wedge Wise(y) \Rightarrow Friend(x, Chewbacca))$

(i)



(ii)



4.

A : Zeus is able to prevent evil

W : Zeus is willing "

P : Zeus Prevents evil.

I : Zeus is Impotent.

M : Zeus is Malevolent.

E : Zeus exists

~~Conv~~ Converting all statements to propositional logic:-

a. $A \wedge W \Rightarrow P$

$-(A \wedge W) \vee P$

$-A \vee -W \vee P$

b. $-A \Rightarrow I$

$-(-A) \vee I$

$A \vee I.$

c. $-W \Rightarrow M$

$-(-W) \vee M$

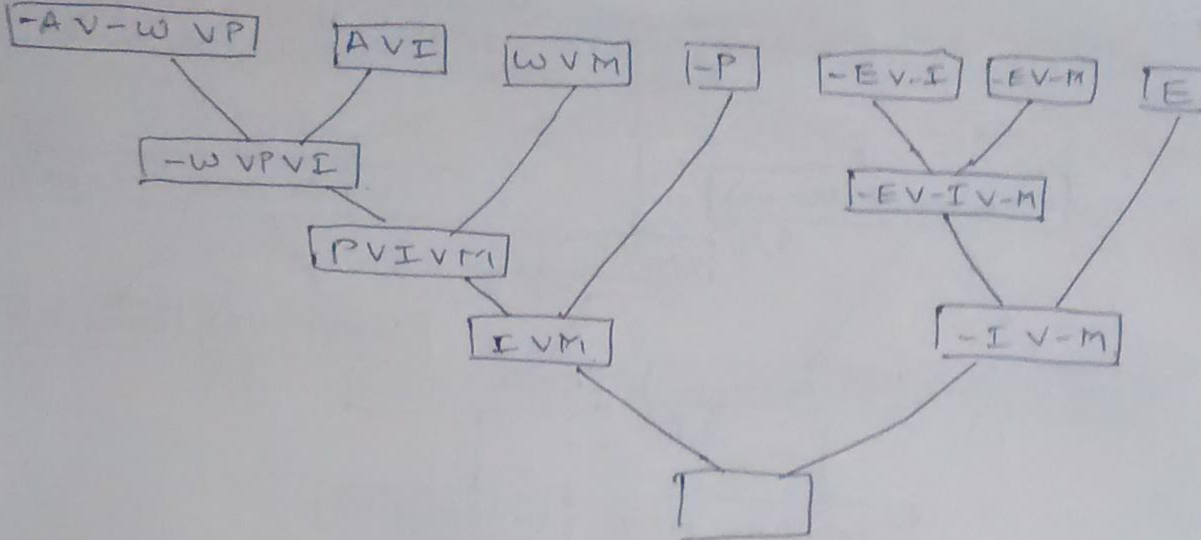
$W \vee M.$

d. $-P$

e. $E \Rightarrow -M \wedge -I$

$-E \vee (-M \wedge -I)$

$(-E \vee -M) \wedge (-E \vee -I)$



Hence by resolution.

$\bar{E}$: Zeus does not exist.

5. Using resolution to show negation leads to contradiction

(a) S: $\forall x\,(P(x) \Rightarrow P(x))$

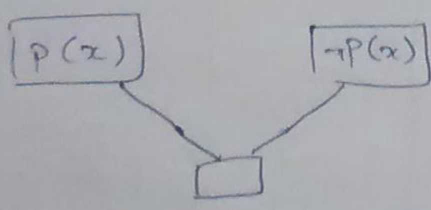
$\bar{S}$: $\neg \forall x\,(P(x) \Rightarrow P(x))$ (Removing biconditionals)

$\neg \forall x\,(\neg P(x) \vee P(x))$

$\exists x\ \neg(\neg P(x) \vee P(x))$ (Applying De Morgan's Law)

$\exists x\,(P(x) \wedge \neg P(x))$ (Drop universal identifiers)

$P(x) \wedge \neg P(x)$.



∵ $\bar{S}$ is a contradiction

∴ S is valid.

(b) $S: (\neg \exists x\, P(x)) \Rightarrow (\forall x\, \neg P(x))$

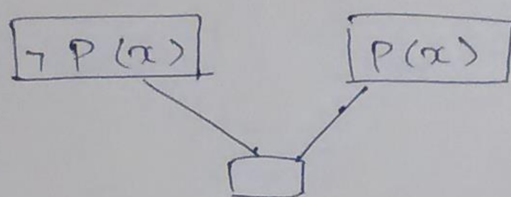
$\bar{S}: \neg((\neg \exists x\, P(x)) \Rightarrow (\forall x\, \neg P(x)))$ (Remove Biconditionals)

$\neg(\exists x\, P(x) \vee \forall x\, \neg P(x))$ (Applying De Morgan's)

$\neg \exists x\, P(x) \wedge \neg \forall x\, \neg P(x)$

$\forall x\, \neg P(x) \wedge \exists x\, P(x)$ (Drop universal quantifiers)

$\neg P(x) \wedge P(x)$



∴ $\bar{S}$ is a contradiction.

∴ S is valid.

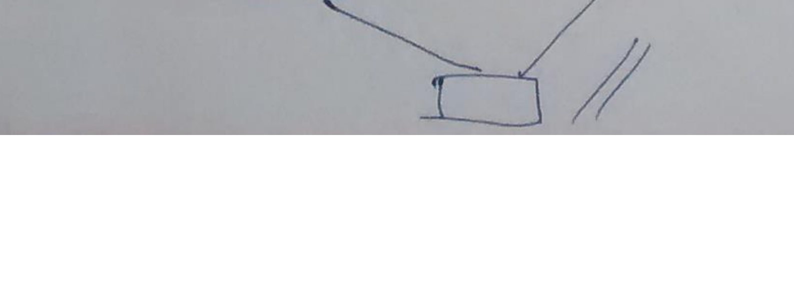
(c) $S: (\forall x(P(x) \vee Q(x)))) \Rightarrow ((\forall x\, P(x)) \vee (\exists x\, Q(x)))$

$\bar{S}: \neg((\forall x(P(x) \vee Q(x))) \Rightarrow ((\forall x\, P(x)) \vee (\exists x\, Q(x))))$

$\neg(\neg(\forall x(P(x) \vee Q(x))) \vee ((\forall x\, P(x)) \vee (\exists x\, Q(x))))$

$\forall x\, (P(x) \vee Q(x)) \wedge (\exists x\, \neg P(x)) \wedge (\forall x\, \neg Q(x))$

$(P(x) \vee Q(x)) \wedge (\neg P(x)) \wedge (\neg Q(x))$



$\because$ $\bar{S}$ is a contradiction.

$\therefore$ S is valid