1. If f_1/n $O(g_1/n)$ and $f_2(n)$ to/g^2 then $f_1(n)+f_2(n)$ to O (max {$g_1(m)$, $g_2(n)$}. prove the assertions.

Sol By definition, there exist constant c_1, n such that for all $n \geq n$; $f(n) < c_1 g_1(n)$

similary there exist constant $c_1 n_1$ such that for all $n \geq n_2$; $f_2(n) \leq c_2 \cdot g_2(n)$

Let $n_0 = \max(n_1 n_2)$ and $c = c_1 + c_2$ for all $n \geq n_0$

$$f(n) + f_2(n) \leq c_1 \cdot g_1(n) + c_2 \cdot g_2(n)$$

By definition of maximum

$$g_1(n) \leq \max \{g_1(n), g_2(n)\}$$

$$g_2(n) \leq \max \{g_1(n), g_2(n)\}$$

Thus

$$f_1(n) + f_2(n) \leq c_1$$

$$\max \{g_1(n), g_2(n)\} + c_2$$

$$\max \{g_1(n), g_2(n)\}$$

$$f_1(n) + f_2(n) \leq (c_2 + c_1)$$

$$\max (g_1(n), g_2(n)\}$$

Hence

$$f_1(n) + f_2(n) \in O\{\max\{g_1(n), g_2(n)\}\}$$

2. find the time complexity of the recurrence equation.

Sol Let us consider such that recurrance for merge sort

$$T(n) = 2T(n/2) + n$$

By using master theorem

$$T(n) = aT(n/b) + f(n)$$