solve the following recurrence relation

a. $x(n) = x(n-1)+5$ for $n>1$ with $x(1)=0$

1. write down the first two terms to identify the pattern

$x(1)=0$

$x(2)=x(1)+5=5$

$x(3)=x(2)+5=10$

$x(4)=x(3)+5=15$

2. Identify the pattern or the general term

→ The first term $x(1)=0$

The common difference $d=5$

The general formula for the nth term of an A.P is

$x(n)=x(1)+(n-1)d$

substituting the given values

$x(n)=0+(n-1)\cdot 5=5(n-1)$

The solution is $x(n)=5(n-1)$

b. $x(n)=3x(n-1)$ for $n>1$ with $x(1)=4$ the pattern

1. write down the first two terms to identify the pattern

$x(1)=4$

$x(2)=3x(1)=3.4=12$

$x(3)=3x(2)=36$

$x(4)=3x(3)=108$

2. Identify the general term

→ The first term $x(1)=4$

→ The common ratio $r=3$

The general formula for the nth term of gp is

$x(n)=x(1)\cdot r^{n-1}$

substituting the given values

$$x(n) = 4 \cdot 3^{n-1}$$

The solution is $x(n) = 4 \cdot 3^{n-1}$

e. $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^k$)

for 2^k, we can write recorrence terms of k

1. Substitute $n = 2^k$ in the recorrence

$$x(2^k) = x(2^{k-1}) + 2^k$$

2. write down the 1st few time to identify the pattern. $x(1) = 1$

$$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$$

$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$

$$x(8) = x(2^3) = x(4) + 8 = 7 + 8 = 15$$

3. Identify the general term by finding the pattern

we observation that

$$x(2^k) = x(2^{k-1}) + 2^k$$

we sum the series

$$x(2^k)\ x(1) = 1?$$

the geometric series with the term $a=2$ and the last term 2^k except for the additional term

The sum of a geometric series S with ratio $r=2$ is given by $S = \frac{ar^{n-1}}{r-r}$

Here $a=2$, $r=2$ and $n=k$

$S = 2 \quad \frac{2^{k-1}}{2-1} = 2(2^{k-1}) = 2^{k+1}$

Adding the +1 term

$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$

solution is

$x(2^k) = 2^{k+1} - 1$

d $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve for $n = 3^k$)

for $n = 3^k$, we can write recurrence in terms of k

1. Substitute $n = 3^k$ in the recurrence

$x(3^k) = x(3^{k-1}) + 1$

2. write down the first few terms to identify the

$x(1) = 1$

$x(3) = x(3^1) = x(1) + 1 = 1 + 1 = 2$

$x(9) = x(3^2) = x(3) + 1 = 2 + 1 = 3$

$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4$

3. Identify the general term

we observe that

$x(3^k) = x(3^{k-1}) + 1$

Summing up the series

$x(3^k) = 1 + 1 + 1 + 1 + \dots + 1$

$x(3^k) = k + 1$

The solution is $x(3^k) = k + 1$

2 Evaluate the following recorrences complexity.

(i) $T(n) = T(n/2)+1$, where $n=2^k$ for all $k \geq 0$.

The recorrence relation can be sloved using iteration method

(i) substitute $n=2^k$ in the recurrance

(ii) Iterate the recorrence

for $k=0$: $T(2^0)=T(1)=T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$: $T(2^2) = T(4) = T(n)+1 = T(1)+1)+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = T(1)+2)+1 = T(1)+3$

3 Generate the pattern

$T(2^k) = T(1)+k$

since $n=2^k$, $k=\log_2 n$

$T(n) = T(2^k) = T(1)+\log_2 n$

4. Assume $T(1)$ is a constant c

$T(n) = c+\log_2 n$

The solution is $T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3)+n$ where c is constant and n is input site

The reconence can be sloved using the method theorem for divide and conquer recoirance of form

$T(n) = aT(n/b)+f(n)$

$a=2$, $b=3$ and $f(n)=n$

lets determine the value of $\log_b a$

$\log_b a = \log_3 2$

using the properties of logarithms

$$\log_3 2 = \frac{\log_2}{\log_3}$$

Now, we compare $f(n) = n$ with $n^{\log_3 2}$

$f(n) = O(n)$

$n = n^1$

since, $\log_3 2$ we are in the third case of master theorem.

$f(n) = O(n^c)$ with $c > \log_b a$

The solution is $T(n) = O f n = O(n) = O(n)$

consider the following recurrence algorithm

min [A[0, ... n-2] if n=1 return A[0]

else temp=min [A[0 ... n-2] if temp <= A[n-1]

else return A(n-1)

a. what does this algorithm compute

The given algorithm, min [A[0 ... n-1] computes minimum value in the array 'A' from index '0' for 'n-1' it does this by recursively finding the minimum value in the subarray A[0 ... n-2] and then comparing it it with the last element 'A[n-1] to determine the overall maximum value

b. setup a recurrence relation for the algorithm basic operation count and solve it

To determine the reconrence relation for the algorithm basic operation count, let's analyse the steps involved in the algorithm the basic operation are the comparison and function calls

Recurrence relation setup

1. Base case when $n=1$ the algorithm performs a single operation to return $A[0]$

2. Recursive case, when $n>1$ the algorithm makes a recursive call to $min(A[0, \cdots n-2])$ perform a comparison b/w temp and $A(n-1)$

1. Base case:

$$T(1)=1$$

2. Recursive case:

$$T(n)=T(n-1)+1$$

Here $T(n-1)$ accounts for the operations performed by the recursive call to $min[A[0 \cdots n-2)$ and the $+1$ account for the comparision b/w temp and $A(n)$

To slove this reconrence relation we can use iteration method.

$$T(n)=T(n-1)+1$$

$$=T(n-2)+1+1$$

$$=T(n-3)+1)+1+1)$$

$$=T+(n-1)$$

$$=n$$

The solution is

$$T(n) = n$$

This means the algorithm perform n basic operations for an input array of size n

1. Analyse the order of growth

(i) $f(n) = 2n^2+5$ and $g(n)$ use the $\Omega\, g(n)$ notation

To analyze the order of growth and use the Ω notation, we need to compare the given function $f(n)$ and $g(n)$

given functions:

$$f(n) = 2n^2+5$$

$$g(n) = 7n$$

Order of growth using $\Omega\, g(n)$ notation

The notation $\Omega\, g(n)$ describes a lower bound as the growth rate that for sufficiently large n, $f(n)$, $g(n)$

$$f(n) \geq c \cdot g(n)$$

let analyze $f(n) = 2n^2+5$ with respect to $g(n) = 7$

1. Identify dominant terms:

→ The dominant terms in $f(n)$ is $2n^2$ since it grows faster then the constant terms in $g(n)$ is $7n$

→ The dominant term in $g(n)$ is $7n$.

2. Establish the inequality

→ we want to find constants c and n_0 such that

$$2n^2+5 \geq c\, 7n \text{ for all } n \geq n_0$$

3. simplify the inequality:

→ Ignore the lower order term 5 for larger

$2n^2 \geq 7cn$

→ Divide both sides by n

$2n \geq 7c$

→ solve from n

$n \geq 7c/2$

c. choose constants

let $c = 1$

$n \geq \frac{7.1}{2} = 3.5$

for $n \geq n$ the inequality holds

$2n^2+5 \geq 7n$ for all $n \geq n$

we use shown the there exist constants $c \geq 1$ and $n_0 \geq n$

such that for all $n \geq n_0$

$2n^2+5 \geq 7n$

thus, we can conclude that

$n(n) = 2n^2+5 = \Omega\, 7(n)$

However for the specific comparison asked

$f(n) = \Omega\, 7(n)$ is also correct

showing that $f(n)$ grows at least as fast