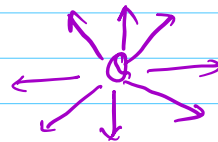
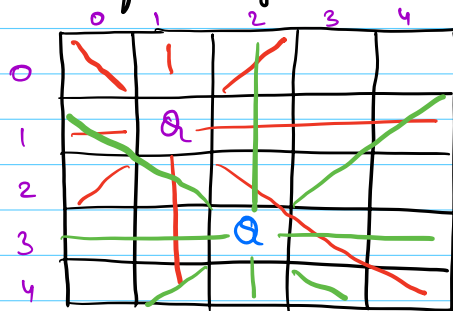


25/8/23

## Backtracking - 2

Q Given  $N \times N$  chessboard & location of 2 queens. Check if they can attack each other.



i/p  $\Rightarrow (1,1), (3,2) \rightarrow \text{false}$

i/p  $\Rightarrow (1,1), (3,1) \rightarrow \text{true}$

$(r_1, c_1) \rightarrow Q_1$   
 $(r_2, c_2) \rightarrow Q_2$

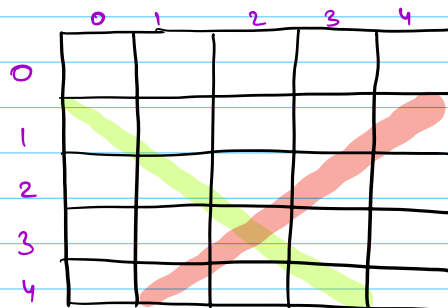
### Directions

1)  $\longleftrightarrow$  same row ( $r_1 == r_2$ )

2)  $\updownarrow$  same col ( $c_1 == c_2$ )

3)  $\swarrow \searrow$   
 $(r_1 - r_2) == (c_1 - c_2)$

4)  $\swarrow \searrow$   
 $(r_1 + c_1) == (r_2 + c_2)$

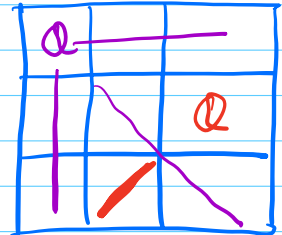
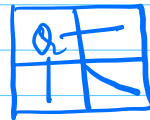


$(1,0)$        $(1,4)$   
 $(2,1)$        $(2,3)$   
 $(3,2)$        $(3,2)$   
 $(4,3)$        $(4,1)$

Q. Given an integer  $N$ , check if it is possible to place  $N$  queens on an  $N \times N$  chessboard s.t. no queen attacks each other.

$N$

Ans



1

true

2

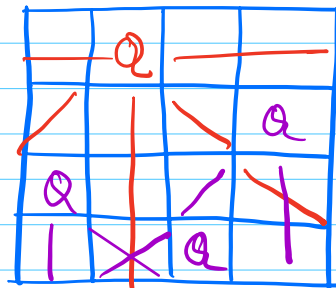
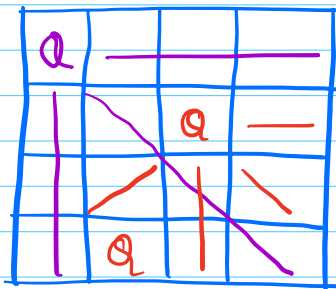
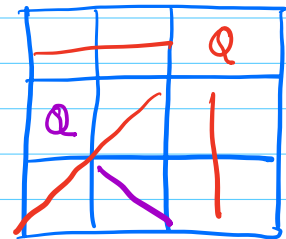
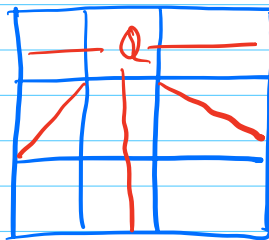
false

3

false

4

true.



$N$  Queens &  $N \times N$  chessboard.

- 1) Every row should have exactly 1 queen.
- 2) Every col should have exactly 1 queen.

- 1) Place the queens row by row.
- 2) Place the queens col by col.

Do we really need  $N \times N$  extra space to keep track of placed queens?

	0	1	2	3
0		Q		
1				Q
2	Q			
3			Q	

	0	1	2	3
col	1	3	0	2

$(i, \text{col}[i]) \rightarrow \text{loc}^{\text{th}}$  of  $i^{\text{th}}$  queen.

	0	1	2	3
row	2	0	3	1

SC:  $O(N)$

boolean  $\text{nqueen}(r, \text{col}) \{ \quad i/p \rightarrow N$

if  $(r == N)$  return true; // Base case.

for  $c \rightarrow 0$  to  $(N-1) \{ \quad // \text{all possibilities}$

if  $(\text{isValid}(\text{col}, r, c)) \{ \quad // \text{valid possibility}$

$\text{col}[r] = c \quad // \text{DO}$

if  $(\text{nqueen}(r+1, \text{col})) \{ \quad // \text{Recursion}$   
return true;  
}

$\text{col}[r] = -1 \quad // \text{undo (optional)}$

return false;

}

```

boolean isValid (col[], r, c) {
    for i = 0 to (r-1) {
        j = col[i] (i, j)
        if (j == c || (i-r) == (j-c) || (i+j) == (r+c)) {
            return false;
        }
    }
    return true;
}

```

	0	1	2	3
0	Q			
1			Q	Q
2		Q		
3				

0	1	2	3
10	13	1	-1

$fn(0)$   
 $\swarrow$   
 $fn(1)$   
 $\downarrow$   
 $fn(2)$

$TC < O(N! * N)$   
 sc:  $O(N + N) \approx O(N)$

	N
	N-1
	N+2
	...
	1

} N!

Meet at 8:40 am IST

Q Solve the given incomplete Sudoku.

Sudoku is a  $N \times N$  grid where  $N$  is a perfect square & every row, column or block has unique elements.

$N=4$

	0	1	2	3
0				
1				
2				
3				

	0	1	2	3
0	2	4	1	3
1	1	3	2	4
2	3	2	4	1
3	4	1	3	2



$N=9$

	0	1	2	3	4	5	6	7	8
0									
1									
2									
3									
4									
5									
6									
7									
8									

i/p  $\Rightarrow$

	0	1	2	3
0	2	4	1	3
1	1	3	2	4
2	3	2	4	1
3	4	1	3	2

0 1 2 3 4 5 6 7 8

$(7, 5) \rightarrow (6, 3)$   
 $(8, 7) \rightarrow (6, 6)$

$$\left(\frac{x}{3}\right) \times 3$$

$$x - (x \% 3)$$

```

boolean sudoku (A[][], N, r, c) {
    if (c == N) {
        r += 1; c = 0;
    }
    if (r == N) { return true; } // Base case.

    if (A[r][c] > 0) { // Already filled.
        return sudoku (A, N, r, c+1);
    }

    for (i = 1 to N) { // All possibilities.
        A[r][c] = i; // Do
        if (check (A, N, r, c) && sudoku (A, N, r, c+1)) {
            return true;
        }
        A[r][c] = 0;
    }

    return false.
}

```

Valid possibility of recursion.

$N \times N \times N$   $\cdot N^2$  times  
 $TC < O(N^{N^2})$   
 $SC: O(N^2)$

```
boolean check(A[][] A, N, x, c) {
```

```
    for i = 0 to (N-1) {  
        if (i != c && A[x][c] == A[x][i]) ←  
            return false;  
  
        if (i != x && A[i][c] == A[x][c]) ↑  
            return false;  
    }
```

```
    sq = sqrt(N)
```

```
    u = x - x / sq
```

```
    v = c - c / sq
```

```
    for i = 0 to (sq-1) {  
        for j = 0 to (sq-1) {
```

```
            x = u + i
```

```
            y = v + j
```

```
            if ((x != x || y != c) && A[x][y] == A[x][c]) {  
                return false;  
            }
```

```
        }  
    }  
    return true;
```

```
}
```

i/p →

	0	1	2	3
0	2	4	1	3
1	1	3	2	4
2	3	2	4	1
3	4	1	3	2

	1	2	3	4
1	0	0	0	0
2		1		1
3				
4				

~~N \* 4~~

~~N \* 4~~

~~N \* 4~~

~~N \* 4~~



```

void permutation (A[], vst[], ans[], mid) { N → A.length
    if (mid == N) { // Base case
        print array (ans);
        return;
    }

    for i = 0 to (N-1) { // All possibilities.
        if (!vst[i]) { // Valid possibility
            vst[i] = true; // do
            ans[mid] = A[i];

            permutation (A, vst, ans, mid+1) // Recursion.

            vst[i] = false; // Undo
        }
    }
}

```

mid = 0

		0	1	2	
[a, b, c]	ans	<del>b</del>	a	c	}
	vst	T	<del>T</del>	<del>F</del>	}