

13/9/2023

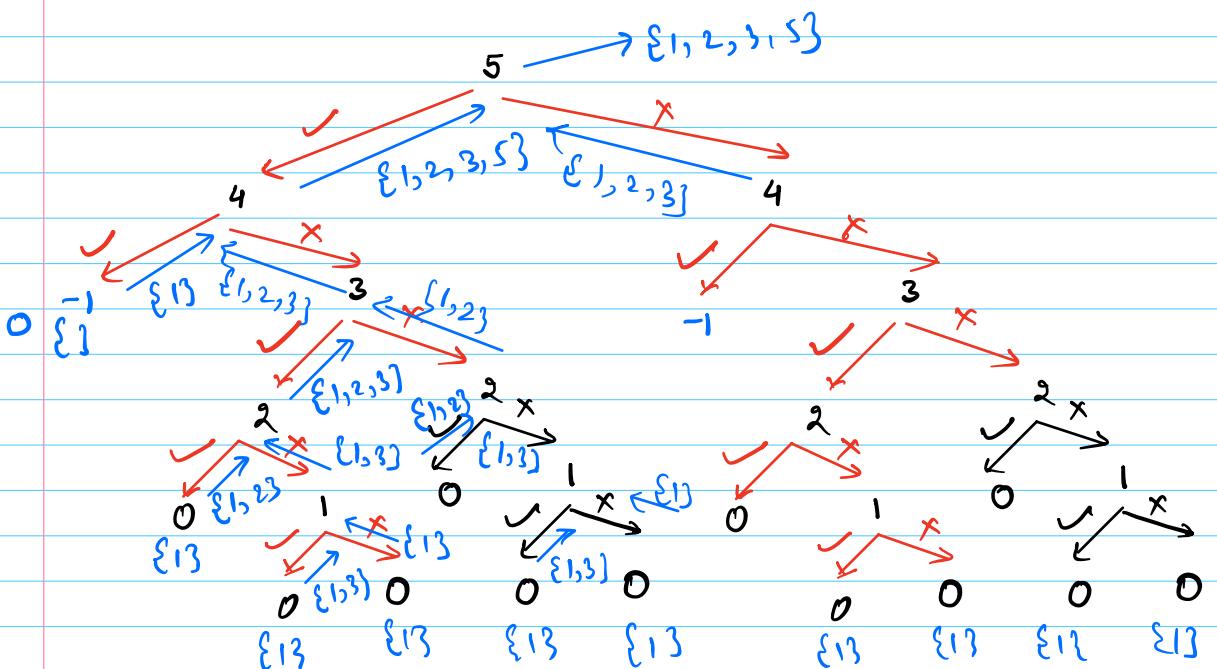
DP-6

Q Find any longest increasing subsequence (LIS) of the given array A.

$$A = [1, 3, 2, 3, 1, 5] \quad \text{Ans} = [1, 2, 3, 5]$$

$$A = [1, 5, 8, 2, 1, 10] \quad \text{Ans} = [1, 5, 8, 10]$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 3 & 1 & 5 \end{bmatrix} \quad L \leftarrow R$$



$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 2 & 3 & 1 & 5 & 2 \end{bmatrix}$$

lcm \rightarrow 1 2 2 3 1 4 2

LIS \rightarrow {1, 3} | {1, 2, 3} | {1, 3} {1, 2, 3, 5} | {1, 2, 3} {1, 2, 3, 5}

sc: $O(N^2)$

1 \rightarrow {1, 5} 3 \rightarrow {1, 3, 5} 2 \rightarrow {1, 2, 5} 3 \rightarrow {1, 2, 3, 5} 1 \rightarrow {1, 5}

$lis[i] \rightarrow$ longest increasing subsequence ending at 'i'

$A = [1, 3, 2, 3, 4, 5, 2]$
len \rightarrow 1 2 2 3 1 4 2
prev \rightarrow -1 0 0 2 -1 3 0/4

for $i \rightarrow 0$ to $N-1$ {

len[i] = 1, prev[i] = -1

for $j \rightarrow 0$ to $(i-1)$ {

if ($A[j] < A[i]$ & & len[j] \geq len[i]) {

len[i] = len[j] + 1;

prev[i] = j;

$A = [8, 5, 7, 10, 7]$
len \rightarrow 1 1 2 3 2
prev \rightarrow -1 -1 1 2 1
5, 7, 10

mid = 0

for $i \rightarrow 0$ to $(N-1)$ {

if (len[i] $>$ len[mid]) {

mid = i;

}

}

while ($ind \neq -1$) {

st.push(A[ind])

mid = prev[mid]

}

while (!st.empty()) {

print(st.pop());

}

s
7
10

Tc: $O(N^2)$

Sc: $O(N)$

Q

Find length of LIS \rightarrow Binary search

Q2

Given N envelopes with their length & width.
An envelope i can fit in envelope j if

$$l[i] \leq l[j] \wedge w[i] \leq w[j]$$

Find the max count of envelopes that can fit one inside another.

$$\begin{aligned} l &= [5, 6, 6, 2] \quad \text{Ans: 3} \\ w &= [6, 4, 7, 3] \end{aligned}$$

Diagram showing envelopes 1 and 2 fitting inside each other. Envelope 1 (l=6, w=7) contains envelope 2 (l=4, w=3).

$$\begin{aligned} l &= [3, 2, 1, 3] \quad \text{Ans: 2} \\ w &= [2, 3, 1, 3] \end{aligned}$$

Observations:

- 1) We can change the order of the elements.
- 2) Sort w.r.t $l[i] / w[i]$

Ans \Rightarrow LIS w.r.t $w[i]$ s.t. equal length is not included.

$$\begin{aligned} l &\rightarrow [2, 5, 6, 6] \\ w &\rightarrow [3, 6, 4, 7] \end{aligned}$$

$$\begin{aligned} Tc &: O(N \log N + N^2) \approx O(N^2) \\ Sc &: O(N) \end{aligned}$$

$$\text{Ans} \Rightarrow [L \ 2 \ 2 \ 2 \ 3]$$

Meet at 8:28 am IST

Q

Given a string, check if substrings whether the substring is a palindrome. (Ans \rightarrow 2D array)

$s = a b c d$
0 1 2 3

	0	1	2	3
0	T	F	F	F
1		T	F	F
2			T	F
3				T

Ans

Bruteforce \rightarrow if substring, check if it is palindrome

$\downarrow N^2$ $\downarrow N$
 $\approx O(N^3)$, sc: $O(1)$

Observation:-

$a - - - - a \quad \{$ is palindrome.

Base case \rightarrow length Ans

1 true
2 $s[i] == s[i+1]$

```

for l = 1 to N { // len [i, j]  $\rightarrow$  j - i + 1 = l
    for i = 0 to (N-1) { // start j = l + i - 1
        j = l + i - 1 // end
        if (j >= N) break;
        if (l == 1) isP[i][j] = true;
        else if (l == 2) isP[i][j] = (s[i] == s[j]);
        else {
            isP[i][j] = (s[i] == s[j] && isP[i+1][j-1] == true)
        }
    }
}
  
```

Tc: $O(N^2)$
Sc: $O(1)$

$s = abab$

	0	1	2	3
0	T	F	T	F
1		T	F	T
2			T	F
3				T

← Ans

Q find min cuts to partition the string s.t. all the partitions are palindrome.

Eg \Rightarrow justify , Ans = 1

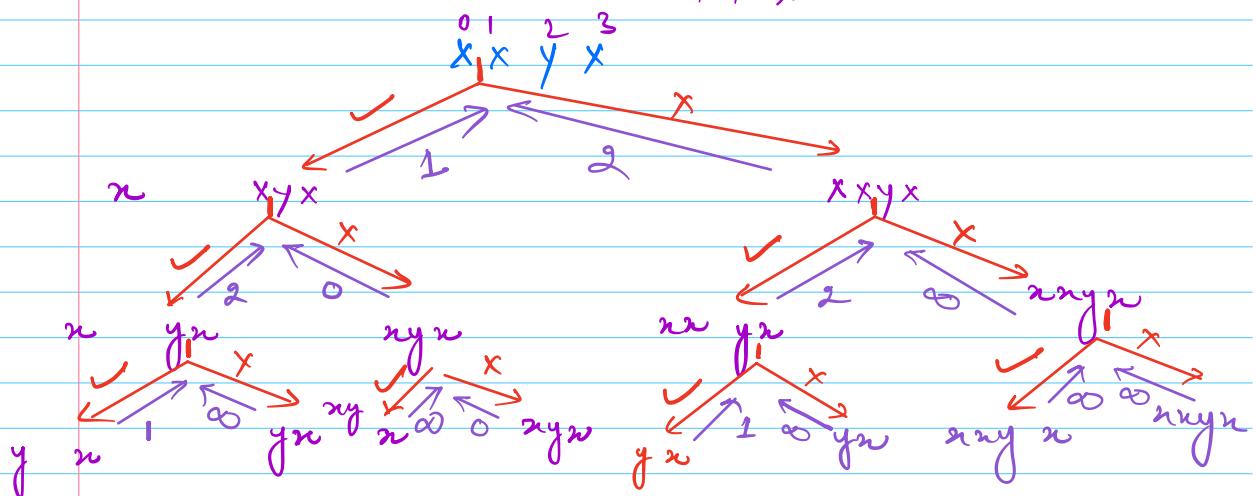
$$a \mid b c b \mid b, \text{ thus } 2$$

2| a b a a b | p

$$d\omega = 3$$

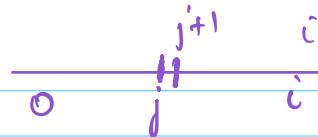
$$\begin{array}{c|c|c|c|c} a & b & c & b \\ \checkmark & \checkmark & \checkmark & \checkmark \end{array}$$

$$2^{N-1} \text{ # ways}$$



`cuts[i]` = minimum cuts for string [0 to i]

$\forall i, \text{cuts}[i] = \text{INT_MAX}$
 $\text{for } i \rightarrow 0 \text{ to } (N-1) \{$
 if ($\text{isp}[0][i]$) { $\text{cuts}[i] = 0;$ }



close {

$\text{for } j \rightarrow 0 \text{ to } (i-1) \{$
 if ($\text{isp}[j+1][i]$) {
 $\text{cuts}[0] = \min(\text{cuts}[i], \text{cuts}[j] + 1);$

}
3

return $\text{cuts}[N-1];$

3

TC: $O(N^2)$

SC: $O(N^2)$

$\text{abab} \rightarrow \text{a } \text{bab}$
 $\text{abab} \rightarrow \text{aba } \text{b}$

$s = \text{abab}$

	0	1	2	3
0	T	F	T	F
1		T	F	T
2			T	F
3				T

Ans

$\text{cuts} \rightarrow \begin{bmatrix} \text{MAX} & \text{MAX} & \text{MAX} & \text{MAX} \\ 0 & 1 & 0 & 1 \end{bmatrix}$