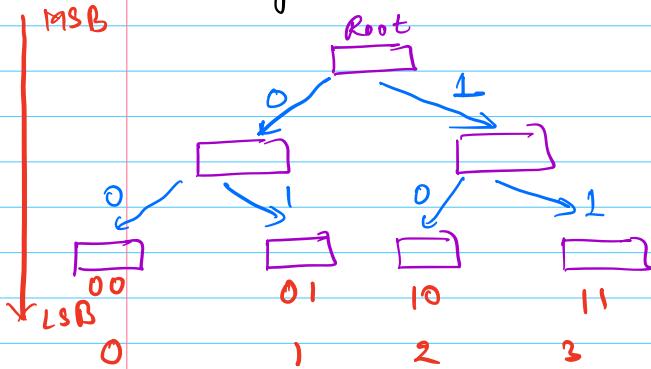


14th August 23

Trees - 2

Tree of Bits



Numbers \rightarrow 0 to 15

Height \rightarrow 4 ($0 - 2^4 - 1$)

Numbers \rightarrow 0 to 31

Height \rightarrow 5 ($0 - 2^5 - 1$)

Q1 Given an integer array A, find the max. value of $A[i] \wedge A[j]$ for (i, j) pairs.

$$A = \begin{bmatrix} 0 \\ 3 \\ 5 \\ 2 \end{bmatrix}$$

$$3^4 = 011$$

$$\begin{array}{r} 101 \\ \hline 110 \Rightarrow 6 \end{array}$$

$$3^2 = 011$$

$$\begin{array}{r} 010 \\ \hline 001 \Rightarrow 1 \end{array}$$

$$5^2 \Rightarrow \begin{array}{r} 101 \\ 010 \\ \hline 111 \Rightarrow 7 \text{ (Ans)} \end{array}$$

$$A = \begin{bmatrix} 0 \\ 9 \\ 8 \\ 10 \\ 7 \end{bmatrix}$$

$$8^4 = 1000$$

$$\begin{array}{r} 0111 \\ \hline 1111 \Rightarrow 15 \text{ (Ans)} \end{array}$$

Brute force \Rightarrow i, j check
 $A[i] \wedge A[j]$ & calculate max.

TC: $O(N^2)$

SC: $O(1)$

$$\begin{array}{r}
 15 \rightarrow 1 \ 1 \ 1 \ 1 \\
 7 \quad 0 \ 1 \ 1 \ 1 \\
 \hline
 10 \ 0 \ 0 \ 28
 \end{array}
 \quad
 \begin{array}{r}
 15 \rightarrow 1 \ 1 \ 1 \ 1 \\
 11 \quad 1 \ 0 \ 1 \ 1 \\
 \hline
 0 \ 1 \ 0 \ 0 \ 29
 \end{array}$$

? To maximize the number MSB (leftmost) should be set(1)

Travelling bits from MSB to LSB \rightarrow Trie of exits.

$A[i] \leq 10^9 \rightarrow$ Height of trie = 30

$$(2^{30} - 1) \geq 10^9$$

$$A = [20 \ 30 \ 15 \ 25 \ 10 \ 5]$$

$$4 \ 3 \ 2 \ 1 \ 0$$

$$20 \rightarrow 1 \ 0 \ 1 \ 0 \ 0$$

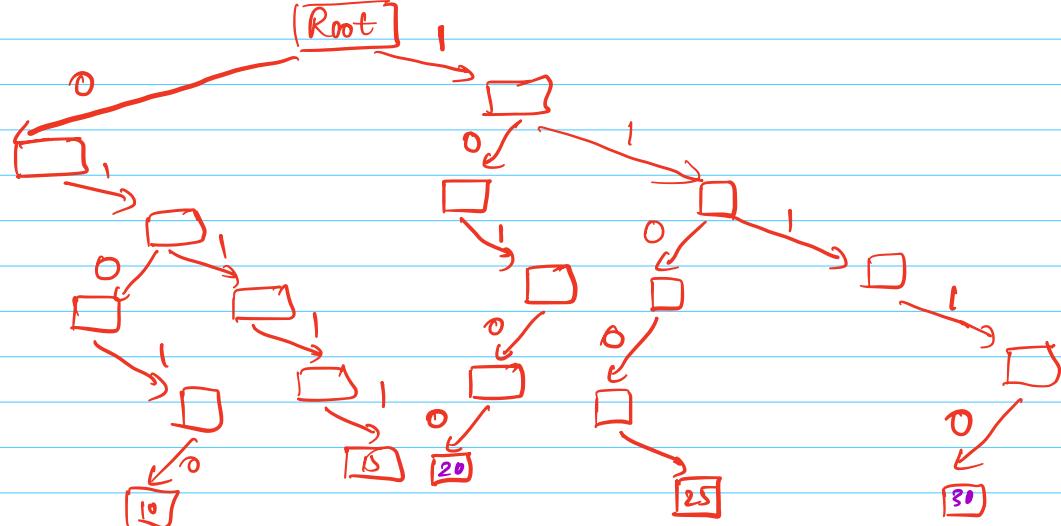
$$30 \rightarrow 1 \ 1 \ 1 \ 1 \ 0 \ ^1 20 = 01010 \Rightarrow 10$$

$$15 \rightarrow 0 \ 1 \ 1 \ 1 \ 1 \ ^1 20 = 11011 \Rightarrow 27$$

$$25 \rightarrow 1 \ 1 \ 0 \ 0 \ 1 \ ^1 15 = 10110 \Rightarrow 22$$

$$10 \rightarrow 0 \ 1 \ 0 \ 1 \ 0 \ ^1 20 = 11110 \Rightarrow 30$$

$$5 \rightarrow 0 \ 0 \ 1 \ 0 \ 1 \ ^1 25 = 11100 \Rightarrow 28$$



Steps > Find max xor for $A[i]$
> Insert $A[i]$

with find (root, x) {

temp = root;

for $i \rightarrow 29 \rightarrow 0$ {

$b = (x \gg i) \& 1$ // ith bit of x.

$t = 1 - b$ // 0 $\rightarrow 1$, 1 $\rightarrow 0$ { $1 \wedge b$ }

if (temp.child[t]) {

temp = temp.child[t];

} else {

temp = temp.child[b];

}

return temp.data $\wedge x$;

}

Tc: $O(30) \approx O(1)$

Overall Tc: $O(N)$, Sc: $O(N * 30) \approx O(N)$

main () {

for \forall res {

cur = find (root, num);

ans = max (ans, cur);

insert (root, num);

}

Q2

Given an integer array A, find subarray with max XOR value.

$$A = [\begin{smallmatrix} 0 & 1 & 2 \\ 4 & 6 & 1 \end{smallmatrix}]$$

$$\begin{array}{rcl} 4 & \rightarrow & 4 \\ 4 \wedge 6 & \rightarrow & 2 \\ 4 \wedge 6 \wedge 1 & \rightarrow & 3 \end{array}$$

Bruteforce \rightarrow # subarrays

$$6 \rightarrow 6$$

$$6 \wedge 1 \rightarrow 1$$

$$6 \wedge 1 \rightarrow 7 \text{ (Ans)}$$

calculate XOR value & store max as answer.

max as answer.

$$TC: O(N^3) \rightarrow O(N^2), SC: O(1)$$

$$a \wedge a = 0$$

$$a \wedge b \wedge c = b$$

$$\text{subarray XOR} \rightarrow i-j = A[i] \wedge A[i+1] \wedge A[i+2] \wedge \dots \wedge A[j-1] \wedge A[j]$$

$$P[j] = A[0] \wedge A[1] \wedge A[2] \wedge \dots \wedge A[i-1] \wedge A[i] \wedge A[j]$$

$$P[i-1] = A[0] \wedge A[1] \wedge \dots \wedge A[i-2] \wedge A[i-1].$$

$$A[i] \wedge A[i+1] \wedge \dots \wedge A[j]$$

$$\text{Subarray XOR } i-j = P[j] \wedge P[i-1].$$

$$P[i] = P[i-1] \wedge A[i]$$

$$A = [\begin{smallmatrix} 0 & 1 & 2 & 3 \\ 4 & 6 & 1 \end{smallmatrix}]$$

$$P = [\begin{smallmatrix} 0 & 1 & 2 & 3 \\ 1 & 5 & 7 & 1 \end{smallmatrix}] \rightarrow \text{max XOR pair} \rightarrow \text{answer.}$$

$$\text{Subarray XOR } i-j = P[j] \wedge P[i-1], i > 0$$

$$P[j], i = 0$$

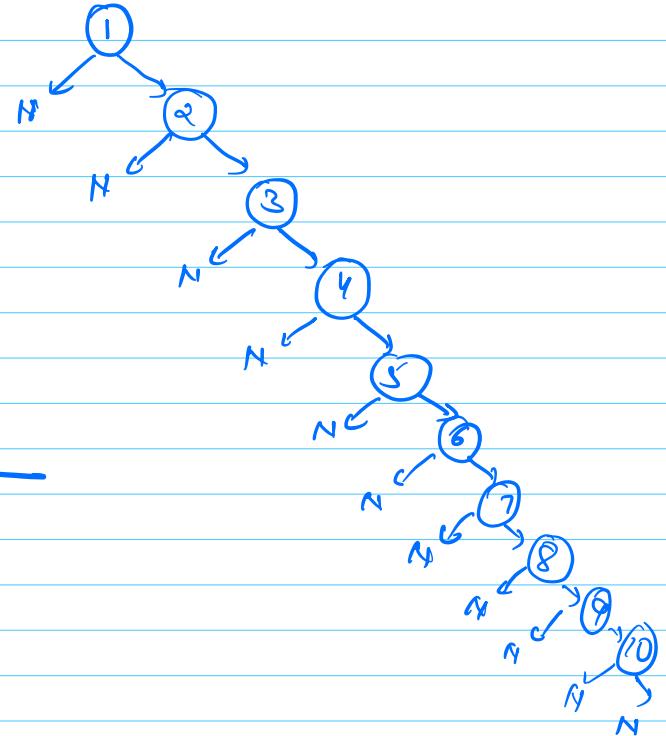
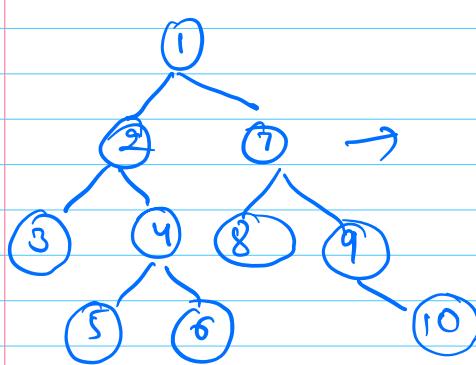
Ans $\Rightarrow \max(\max_{i \in P[i]}, \max_{i \in P[i]} \text{prefix XOR})$

prefix sum $\Rightarrow P[i] = P[i-1] + A[i]$
 prefix XOR $\Rightarrow P[i] = P[i-1] \wedge A[i]$

TC: $O(N + N) \approx O(N)$
 SC: $O(N + N) \approx O(N)$

Q: Flatten the given binary tree to linked list in pre-order manner, such that right child becomes next & left child & nodes should be null.

Node
left
right



Meet at 8:45 am IST

→ {Head, tail)

pair flatten (root) {

if (root == null) return {null, null};

L = flatten (root.left);

R = flatten (root.right);

root.left = null;

if (L.head == null & R.head == null) {

return {root, root};

}

else if (L.head == null) {

root.right = R.head;

return {root, R.tail};

}

else if (R.head == null) {

root.right = L.head;

return {root, L.tail};

}

else {

root.right = L.head;

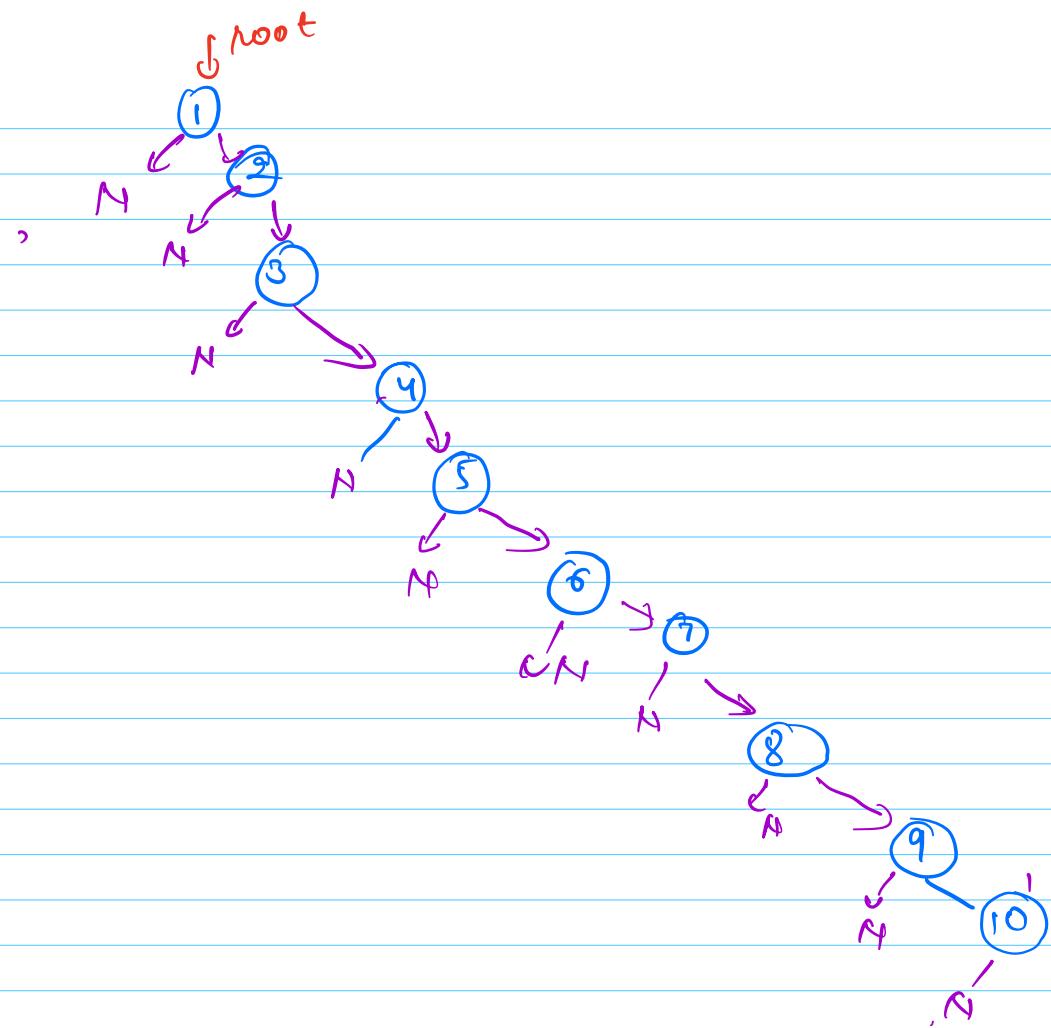
L.tail.right = R.head;

return {root, R.tail};

}

TC: O(N)

SC: O(H)



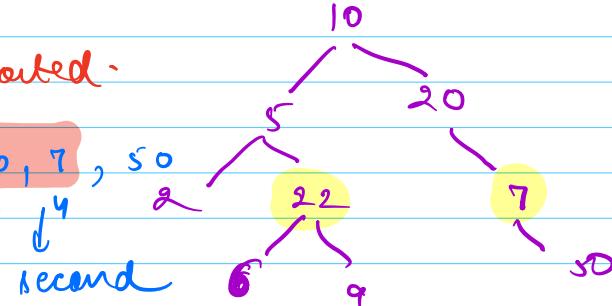
Q Given a BST where exactly 2 nodes are swapped, find the 2 nodes (distinct nodes)

inorder \rightarrow LNR

inorder traversal is sorted

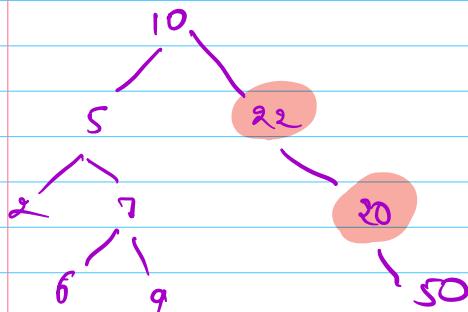
2, 5, 6, 22, 9
1 2

element of first pair



element of 2nd pair

2, 5, 6, 7, 9, 10, 22, 20, 50
ans



Tc: $O(N)$, sc: $O(H) \rightarrow O(1)$