

8/9/2023

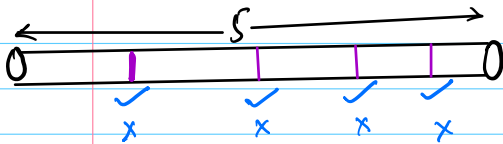
DP-4

Q1) Given a rod of length N & an array A of length N , $A[i] \rightarrow$ price of i length rod (1 based index)

Find max value that can be obtained by cutting the rod in some pieces & selling them.
profit

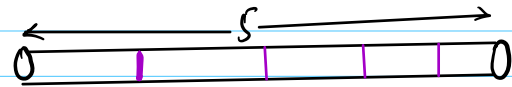
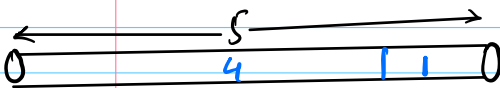
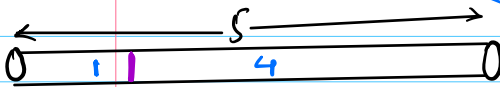
$N = 5$

$A = [1, 4, 2, 5, 6]$
1 2 3 4 5



#ways = $2^4 = 16$

$N \rightarrow 2^{N-1}$



Sold length

Total Value

5

6

4+1

5+1=6

3+2

2+4=6

3+1+1

2+1+1=4

2+2+1

4+4+1=9 (ans)

2+1+1+1

4+1+1+1=7

1+1+1+1+1

1+1+1+1+1=5

→ optimal substructure

→ overlapping subproblems.

capacity \rightarrow length of given rod

one part of rod

$\rightarrow i$ is the length

$\rightarrow A[i]$ is received.

Σ length of each part $\leq N$

unbounded
knapsack
0-N

$dp[i] \rightarrow$ max value that can be received by a rod of length i .

$dp[0] = 0$

$\forall i, dp[i] = 0$

for $i \rightarrow 1$ to N { // length of rod to sell

for $j \rightarrow 1$ to i {

$dp[i] = \max(dp[i], A[j] + dp[i-j])$

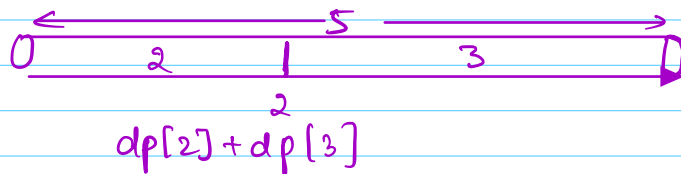
}

}

return $dp[N]$

Tc: $O(N^2)$

Sc: $O(N)$



$A = [1, 4, 2, 5, 6]$
1 2 3 4 5

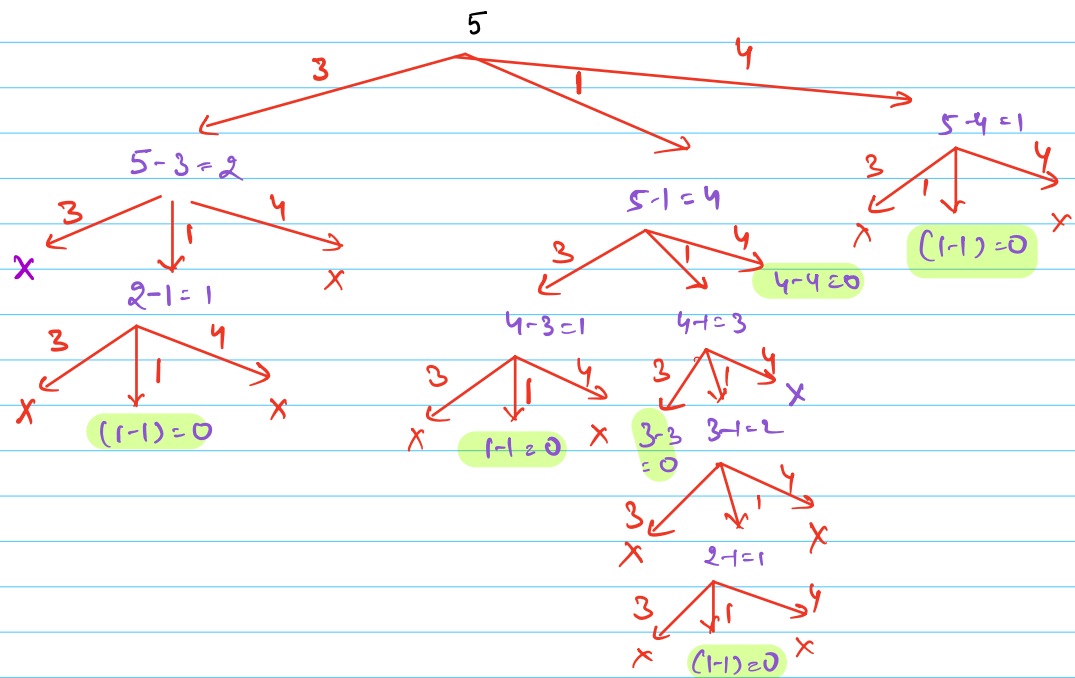
$dp[0, 1, 2, 3, 4, 5]$

Q2 In how many ways can sum be equal to N by using coins given in the array.

One coin can be used multiple times

$A \rightarrow$ Ordered selection. $(x, y) \neq (y, x)$

$N = 5$ $\{1, 4\}$ $\{3, 1, 1\}$, $\{1, 1, 3\}$
 $A = [3, 1, 4]$ $\{4, 1\}$ $\{1, 3, 1\}$, $\{1, 1, 1, 1, 1\}$ $ans = 6$



Unbounded knapsack

$\sum \text{selected } A[i] \leq N$

one coin can be selected multiple times

$\text{ways}(0) = 1$

$\forall i, dp[i] = 0$

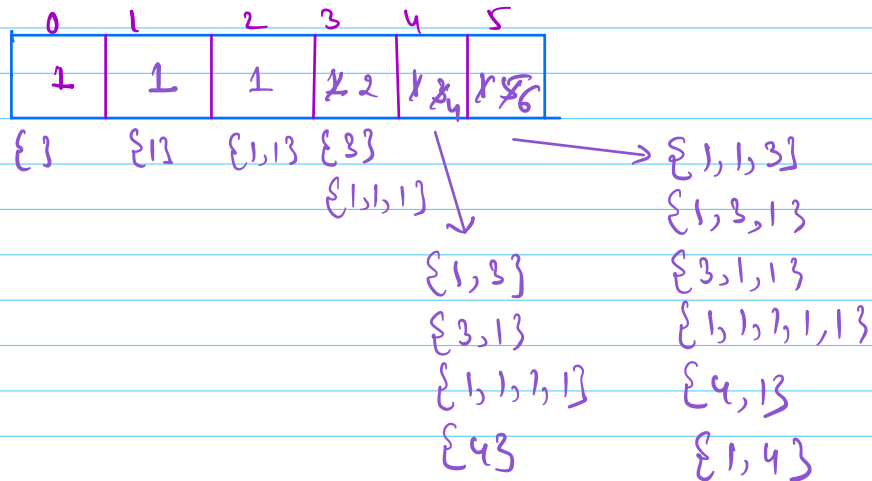
$dp[0] = 1$

```
for i → 1 to N {  
  for j → 0 to (A.length - 1) {  
    if (A[j] ≤ i) {  
      dp[i] = dp[i] + dp[i - A[j]]  
    }  
  }  
}
```

Tc: $O(N * A.length)$
Sc: $O(N)$

$N = 5$

$A = [3, 1, 4]$

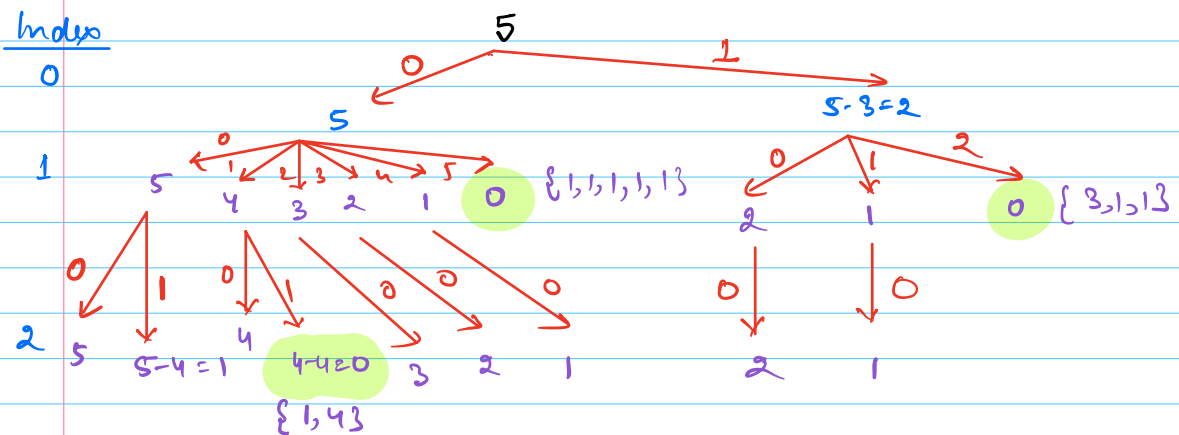


Met at 8:33 am IST

Q unordered selection $(x, y) = (y, x)$

$N = 5$ $\{1, 4\}$ $\{3, 1, 1\}$, $\{1, 1, 3\}$ $Ans = 3$
 $A = [3, 1, 4]$ $\{4, 1\}$ $\{1, 3, 1\}$, $\{1, 1, 1, 1, 1\}$
 $L \rightarrow R$

decide one order that will make repetitions as invalid.



$dp[i] = \# \text{ ways to get sum} = i \text{ by selecting coins from } L \text{ to } R \text{ in the array.}$

$\forall i, dp[i] = 0$

$dp[0] = 1$

TC: $O(N * A.length)$
 SC: $O(N)$

```

for j = 0 to (A.length-1) { // coins
    for i = 1 to N { // sum.
        if (A[j] <= i) {
            dp[i] = dp[i] + dp[i-A[j]]
        }
    }
}
return dp[N]
    
```

Q 0-1 Knapsack 2 (object cannot be divided)

Given N toys with their happiness & weight.
Find max total happiness that can be kept in a bag with capacity = W (toys cannot be divided)

$$1 \leq N \leq 500$$

$$1 \leq h[i] \leq 50$$

$$1 \leq wt[i] \leq 10^9$$

$$1 \leq W \leq 10^9$$

$$TC = O(N * W) \Rightarrow 500 * 10^9 = 5 \times 10^{11}$$

(TLE)

$dp[N][W] \rightarrow$ max happiness



$dp[N][H] \rightarrow$ min weight required to achieve H

$$500 * (500 * 50)$$

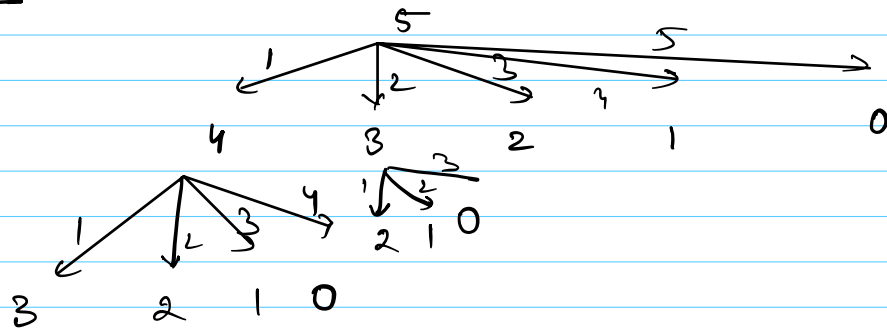
$$500 * (25000)$$

$$1.25 \times 10^7$$

for $i \rightarrow H$ to 0 { \rightarrow min weight to get happiness i
if ($dp[N][i] \leq W$)
return i ;
}



Solution



$N = 5$
 $A = [1, 4, 2, 5, 6]$
 1 2 3 4 5

for $i \rightarrow 1$ to N { // length of rod to sell

for $j \rightarrow 1$ to i {

$dp[i] = \max(dp[i], A[j] + dp[i-j])$

}

}
return $dp[N]$

$\{1, 2, 2\} = 9$

$\{2, 1, 2\} \quad 4+5$

$\{3, 2\} \rightarrow 2+4=6$

$\{4, 1\} \rightarrow 5+1$

$\{1\} \quad \{2\} \quad \{1, 2\} \quad \{2, 2\} \quad \{1, 2, 2\} \quad \{5\} \rightarrow 6$

dp \rightarrow

0	1	2	3	4	5
0	1	4	5	8	9

