

8/9/2023

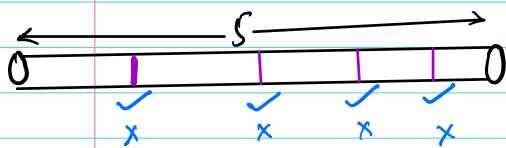
DP-4

Q1 Given a rod of length N & an array A of length N , $A[i] \rightarrow$ price of i length rod (1-based index)

Find max value that can be obtained by cutting the rod in some pieces & selling them.
profit

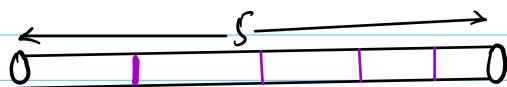
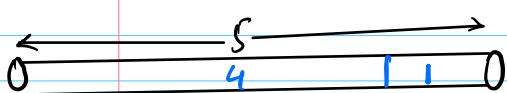
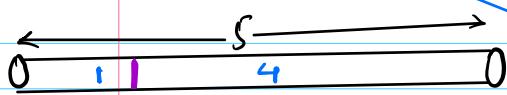
$$N = 5$$

$$A = [1 \ 4 \ 2 \ 5 \ 6]$$



$$\# \text{ways} = 2^4 = 16$$

$$N \rightarrow 2^{N-1}$$



Sold Length

5

4+1

3+2

3+1+1

2+2+1

2+1+1+1

1+1+1+1+1

Total Value

6

5+1=6

2+4=6

2+1+1=4

4+4+1=9 (Ans)

4+1+1+1=7

1+1+1+1+1=5

optional substructure

overlapping subproblems.

Capacity \rightarrow length of given rod

one part of rod \rightarrow i is the length

$\rightarrow A[i]$ is received.

\sum length of each part $\leq N$

unbounded knapsack O-N

$dp[i] \rightarrow$ max value that can be received by a rod of length i .

$$dp[0] = 0$$

$$\forall i, dp[i] = 0$$

for $i \rightarrow 1 \text{ to } N \{ // \text{length of rod to sell}$

for $j \rightarrow 1 \text{ to } i \{$

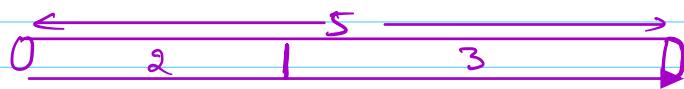
$$dp[i] = \max(dp[i], A[j] + dp[i-j])$$

}

return $dp[N]$

TC: $O(N^2)$

SC: $O(N)$



$$dp[2] + dp[3]$$

$$A = \begin{bmatrix} 1 & 4 & 2 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

$$dp[0 \ 1 \ 2 \ 3 \ 4 \ 5]$$

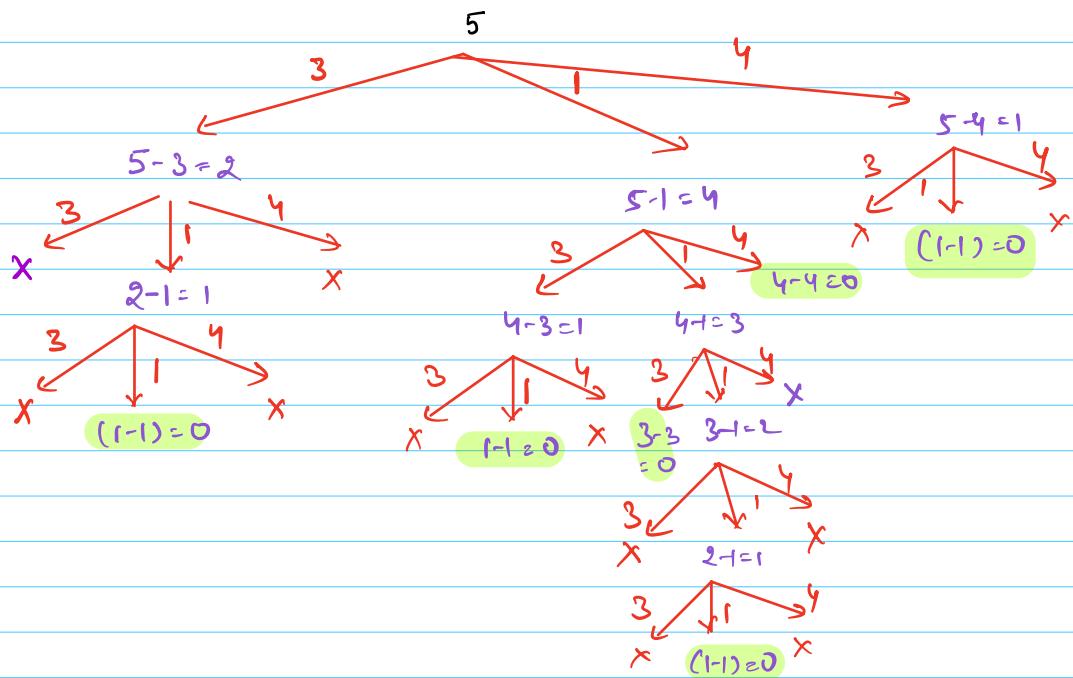
Q2 In how many ways can sum be equal to N by using coins given in the array.

One comb can be used multiple times

$A \rightarrow$ Ordered selection. $(x,y) \neq (y,x)$

$$N=5 \quad \{1, 4\} \quad \{3, 1, 1\}, \{1, 1, 3\}$$

$$A = [3 \ 1 \ 4] \quad \{4, 1\} \quad \{1, 3, 1\}, \{1, 1, 1, 1, 1\} \quad \text{dim}=6$$



Unbounded knapsack

$\sum \text{selected } A[i] < N$

one card can be selected multiple times

ways (0) = 1

$\forall i, dp[i] = 0$

$dp[0] = 1$

```
for i → 1 to N {  
    for j → 0 to (A.length - 1) {  
        if (A[j] <= i) {  
            dp[i] = dp[i] + dp[i - A[j]]  
        }  
    }  
}
```

TC: $O(N * A.length)$
SC: $O(N)$

$N = 5$

$A = [3 \ 1 \ 4]$

0	1	2	3	4	5	
1	1	1	2	1	1	6

{ } {1} {1, 1} {1, 1, 1} {1, 1, 1, 1} {1, 1, 1, 1, 1} {1, 1, 1, 1, 1, 1}

{ } {1, 3} {1, 1, 3} {1, 1, 1, 3} {1, 1, 1, 1, 3} {1, 1, 1, 1, 1, 3} {1, 1, 1, 1, 1, 1, 3}

{ } {3} {3, 1} {3, 1, 1} {3, 1, 1, 1} {3, 1, 1, 1, 1} {3, 1, 1, 1, 1, 1}

{ } {4} {4, 1} {4, 1, 1} {4, 1, 1, 1} {4, 1, 1, 1, 1} {4, 1, 1, 1, 1, 1}

{ } {1, 4} {1, 4, 1} {1, 4, 1, 1} {1, 4, 1, 1, 1} {1, 4, 1, 1, 1, 1} {1, 4, 1, 1, 1, 1, 1}

Meet at 8:33 am IST

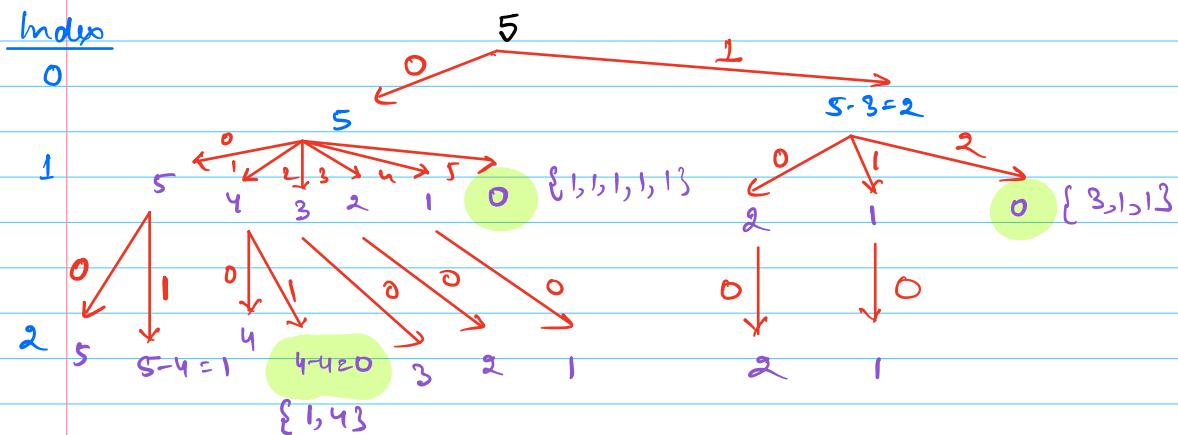
Q Unordered selection $(x, y) = (y, x)$

$$N = 5 \quad \{1, 4\} \quad \{3, 1, 1\}, \{1, 1, 3\} \quad Ans = 3$$

$$A = [3 \ 1 \ 4] \quad \{4, 1\} \quad \{1, 3, 1\}, \{1, 1, 1, 1, 1\}$$

$$L \rightarrow R$$

decide one order that will make repetitions as invalid.



$dp[i] = \# \text{ ways to get sum } = i \text{ by selecting coins from L to R in the array.}$

If i , $dp[i] = 0$

$dp[0] = 1$

TC: $O(N * A.length)$
SC: $O(N)$

```

for j > 0 to (A.length - 1) {
    for i > 1 to N {
        if (A[j] <= i) {
            dp[i] = dp[i] + dp[i - A[j]]
        }
    }
}
return dp[N]
    
```

Q

0-1 Knapsack 2 (object cannot be divided)

Given N toys with their happiness & weight.
Find max total happiness that can be kept in a bag with capacity = w (Toys cannot be divided)

$$1 \leq N \leq 500$$

$$1 \leq h[i] \leq 50$$

$$1 \leq wt[i] \leq 10^9$$

$$1 \leq w \leq 10^9$$

$$TC = O(N \times w) \Rightarrow 500 \times 10^9 = 5 \times 10^{11}$$

(TLE)

$dp[N][w] \rightarrow \max \text{ happiness}$

$dp[N][H] \rightarrow \min \text{ weight required to achieve } H$

$$500 * (500 * 50)$$

$$500 * (25000)$$

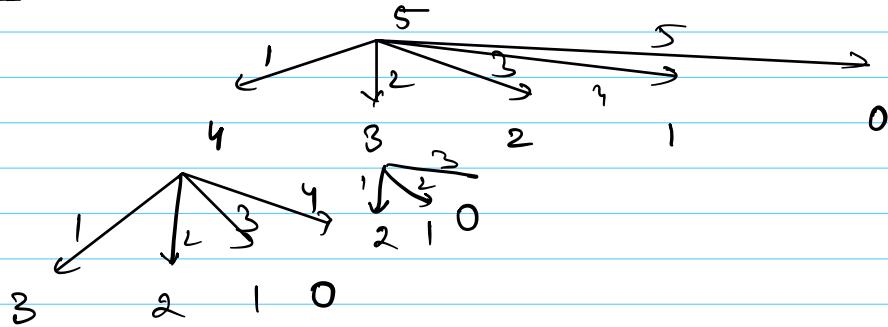
$$1.25 \times 10^7$$

for $0 \rightarrow H$ to $0 \{$
if ($dp[N][i] \leq w$)
return i

3



Solutions



$$N = 5$$

$$A = \begin{bmatrix} 1 & 4 & 2 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

for $i \rightarrow 1 \text{ to } N$ { // length of rod to sell }

for $j \rightarrow 1 \text{ to } i$ {

$$dp[i] = \max(dp[i], A[j] + dp[i-j])$$

}
return $dp[N]$

$$\begin{aligned} \{1, 2, 2\} &= 9 \\ \{2, 1, 2\} &= 4+5 \\ \{3, 2\} &= 2+4=6 \end{aligned}$$

$dp \rightarrow$	0	1	2	3	4	5
	0	1	4	5	8	9
	{1}	{2}	{1, 2}	{2, 2}	{1, 2, 2}	{5}

$$\{1, 2, 2\} \rightarrow 6$$

0 1 2 4 2 0 0 2 1 1 2 0