

13/9/2023

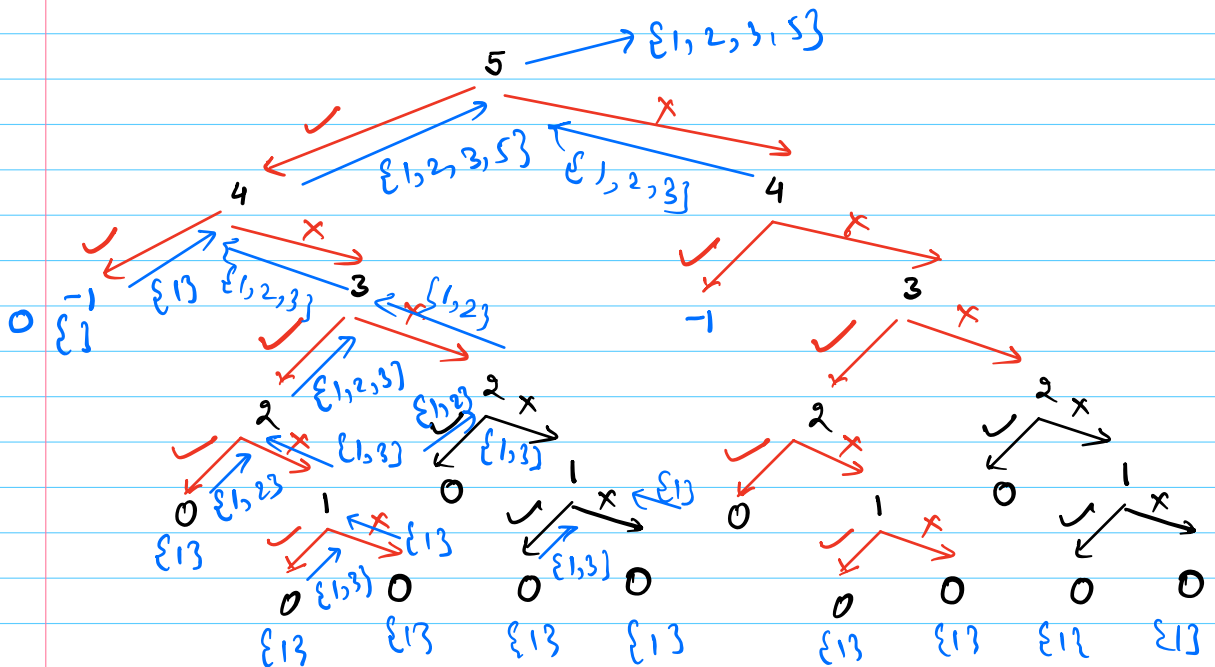
DP-6

Q Find any longest increasing subsequence (LIS) of the given array A.

A = [1, 3, 2, 3, 1, 5] Ans = [1, 2, 3, 5]

A = [1, 5, 8, 2, 1, 10] Ans = [1, 5, 8, 10]

$$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ [1 & 3 & 2 & 3 & 1 & 5] \\ L & \leftarrow & R \end{matrix}$$



$$\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ [1 & 3 & 2 & 3 & 1 & 5 & 2] \\ \text{len} \rightarrow & 1 & 2 & 2 & 3 & 1 & 4 & 2 \\ \text{LIS} \rightarrow & \{1\} & \downarrow & \{1, 2\} & \downarrow & \{1\} & \{1, 2, 3, 5\} & \downarrow \\ & \{1, 3\} & & \{1, 2, 3\} & & \{1, 2\} & & \end{matrix}$$

1 \rightarrow {1, 5}
 3 \rightarrow {1, 3, 5}
 2 \rightarrow {1, 2, 5}
 3 \rightarrow {1, 2, 3, 5}
 1 \rightarrow {1, 5}

So: $O(N^2)$

$len[i] \rightarrow$ longest increasing subsequence ending at 'i'

	0	1	2	3	4	5	6
	[1	3	2	3	1	5	2]
len \rightarrow	1	2	2	3	1	4	2
Prev \rightarrow	-1	0	0	2	-1	3	0/4

```

for i  $\rightarrow$  0 to N-1 {
    len[i] = 1, Prev[i] = -1
    for j  $\rightarrow$  0 to (i-1) {
        if (A[j] < A[i] && len[j] >= len[i]) {
            len[i] = len[j] + 1;
            prev[i] = j;
        }
    }
}
    
```

	0	1	2	3	4
A = [8	5	7	10	7]
len \rightarrow [1	1	2	3	2
prev \rightarrow [-1	-1	1	2	1

5, 7, 10

```

mid = 0
for i  $\rightarrow$  0 to (N-1) {
    if (len[i] > len[mid]) {
        mid = i;
    }
}
    
```

```

while (mid != -1) {
    st.push(A[mid])
    mid = prev[mid]
}
    
```

5
7
10

```

while (!st.empty()) {
    print(st.pop());
}
    
```

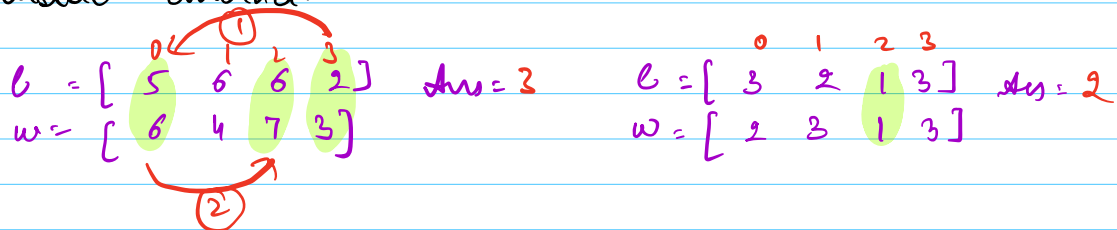
Tc: $O(N^2)$
 Sc: $O(N)$

Q Find length of LIS \rightarrow Binary search

Q2 Given N envelopes with their length & width.
An envelope i can fit in envelope j if

$$l[i] \leq l[j] \text{ \& } w[i] \leq w[j]$$

Find the max count of envelopes that can fit one inside another.



$l = [5, 6, 6, 2]$ $w = [6, 4, 7, 3]$ $Ans = 3$
 $l = [3, 2, 1, 3]$ $w = [2, 3, 1, 3]$ $Ans = 2$

Observations:

- 1) We can change the order of the elements.
- 2) Sort w.r.t $l[i] / w[i]$

$Ans \Rightarrow$ LIS w.r.t $w[i]$ s.t. equal length is not included.

$$l \rightarrow [2, 5, 6, 6]$$

$$w \rightarrow [3, 6, 4, 7]$$

$$Tc: O(N \log N + N^2) \approx O(N^2)$$

$$Sc: O(N)$$

$$len \rightarrow [1, 2, 2, 2, 3]$$

Meet at 8:28 am IST

Q

Given a string, check if substrings whether the substring is a palindrome. (Ans \rightarrow 2D array)

$s = a b c d$
0 1 2 3

	0	1	2	3
0	T	F	F	F
1	—	T	F	F
2	—	—	T	F
3	—	—	—	T

\leftarrow Ans

Bruteforce \rightarrow if substring, check if it is palindrome

\downarrow N^2 \downarrow N
 $\approx O(N^3)$, sc: $O(1)$

Observation :-

$a \text{ --- } a$ is palindrome.
 $i \quad i+1 \quad j-1 \quad j$

Base case \rightarrow

length

Ans

1

true

2

$s[i] == s[i+1]$

```

for l  $\rightarrow$  1 to N { // len
    for i  $\rightarrow$  0 to (N-1) { // start
        j = l+i-1 // end
        if (j > N) break;
        if (l == 1) isP[l][j] = true;
        else if (l == 2) isP[l][j] = (s[i] == s[j]);
        else {
            isP[l][j] = (s[i] == s[j] && isP[l+1][j-1] == true);
        }
    }
}

```

Tc: $O(N^2)$
Sc: $O(1)$

S = abab

	0	1	2	3
0	T	F	T	F
1	—	T	F	T
2	—	—	T	F
3	—	—	—	T

← Ans

Q find **mini cuts** to partition the string s.t. all the partitions are palindromic.

Eg \rightarrow $xxyy$, Ans = 1

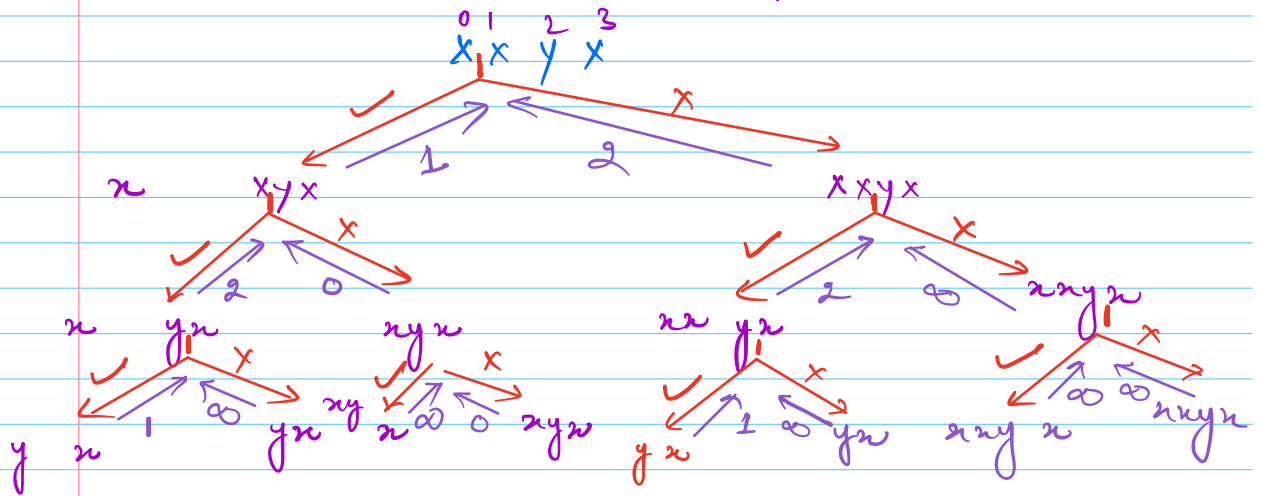
$a|bcb|b$, Ans = 2

$x|a|b|a|a|b|p$

Ans = 3

$a|b|c|b$
 $\checkmark \checkmark \checkmark \checkmark$

2^{N-1} # ways



$cuts[i] = \text{minimum cuts for string } [0 \text{ to } i]$

$\forall i, \text{cuts}[i] = \text{INT_MAX}$

for $i \rightarrow 0$ to $(N-1)$ {

if ($\text{isP}[0][i]$) { $\text{cuts}[i] = 0;$ }

else {

for $j \rightarrow 0$ to $(i-1)$ {

if ($\text{isP}[j+1][i]$) {

$\text{cuts}[i] = \min(\text{cuts}[i], \text{cuts}[j] + 1);$

}

}

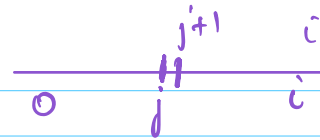
}

return $\text{cuts}[N-1];$

}

Tc: $O(N^2)$

Sc: $O(N^2)$



abab \rightarrow a bab
 \rightarrow aba b

S = abab

	0	1	2	3
0	T	F	T	F
1	—	T	F	T
2	—	—	T	F
3	—	—	—	T

$\leftarrow \text{Ans}$

cuts \rightarrow $\begin{bmatrix} \text{MAX} & \text{MAX} & \text{MAX} & \text{MAX} \\ 0 & 1 & 0 & 1 \end{bmatrix}$