Quick Summary of Robot Motion

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Basic Linear Algebra

- **2D Translation**
- **Definition:**
  Translation in 2D refers to shifting an object from one position to another in a two-dimensional plane without changing its orientation, shape, or size.
- **Mathematical Representation:**
  If a point $P(x, y)$ is translated by $\left(t_x, t_y\right)$, the new point $P'(x', y')$ is:

$$x' = x + t_x, y' = y + t_y$$

- **Matrix Form (Homogeneous Coordinates):**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Basic Linear Algebra

- **2D Transformation**
- **Definition:**
  A transformation in 2D includes operations like **translation, rotation, scaling, reflection, and shearing** applied to objects in a 2D plane.
- **General Transformation Matrix:**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- where $a_{11}, a_{12}, a_{21}, a_{22}$ define rotation, scaling, and shear, and t_x, t_y define translation.
- For example, for a pure rotation by angle θ:

$$a_{11} = \cos\theta\,,\ a_{12} = -\sin\theta\,,\ a_{21} = \sin\theta\,,\ a_{22} = \cos\theta$$

ROS IN MOTION

Basic Linear Algebra

- **3D Translation:**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- **3D Transformation:** Combines rotation and translation:

$$T = \begin{bmatrix} R_{3\times3} & t_{3\times1} \\ 0 & 1 \end{bmatrix}$$

- **3D rotation matrix** R is:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

- Each r_{ij} is an element of the matrix, and its value depends on the chosen rotation representation (Euler angles, axis-angle, etc.).

Basic Linear Algebra

- **For Euler Angles (Yaw-Pitch-Roll, Z-Y-X):**
- If:
  - ψ= yaw (rotation about Z)
  - θ= pitch (rotation about Y)
  - ϕ= roll (rotation about X)
- Then:

$$R = \begin{bmatrix} \cos\psi\cos\theta & \cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\cos\theta & \sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi & \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Differential Drive Robot Basics

- A **differential drive robot** has:
  - Two independently driven wheels (left and right).
  - A fixed axle length L between the wheels.
  - Wheel radius r.
- The robot's configuration in the plane is:

$$q = [x, y, \theta]^T$$

- where:
  - (x, y) = position of the robot's center in the global frame.
  - θ= heading angle.

Euler's Equations for Motion

- **Kinematic Model**
- The linear and angular velocities:

$$v = \frac{r}{2}(\omega_R + \omega_L), \omega = \frac{r}{L}(\omega_R - \omega_L)$$

- The motion equations:

$$\dot{x} = v\cos\theta\,, \dot{y} = v\sin\theta\,, \dot{\theta} = \omega$$

- For a small time step Δt:

$$x_{k+1} = x_k + \dot{x}_k\Delta t = x_k + v_k\cos\theta_k\,\Delta t$$
$$y_{k+1} = y_k + \dot{y}_k\Delta t = y_k + v_k\sin\theta_k\,\Delta t$$
$$\theta_{k+1} = \theta_k + \omega_k\Delta t$$

- Assume
- $r = 0.05\ m, L = 0.3\ m$
- $\omega_R = 10\ rad/s, \omega_L = 8\ rad/s$
- Initial pose: $(x_0, y_0, \theta_0) = (0, 0, 0)$
- $\Delta t = 0.1\ s$
- Compute:
- $v = \frac{0.05}{2}(10 + 8) = 0.45\ m/s,\ \ \omega = \frac{0.05}{0.3}(10 - 8)$
  $= 0.333\ rad/s$
- Update:

$$x1=0+0.45\cos(0)\cdot 0.1=0.045,$$

$$y1=0+0.45\sin(0)\cdot 0.1=0,$$

$$\theta 1=0+0.333\cdot 0.1=0.0333\text{rad}$$

Euler (dt = 0.1 s):

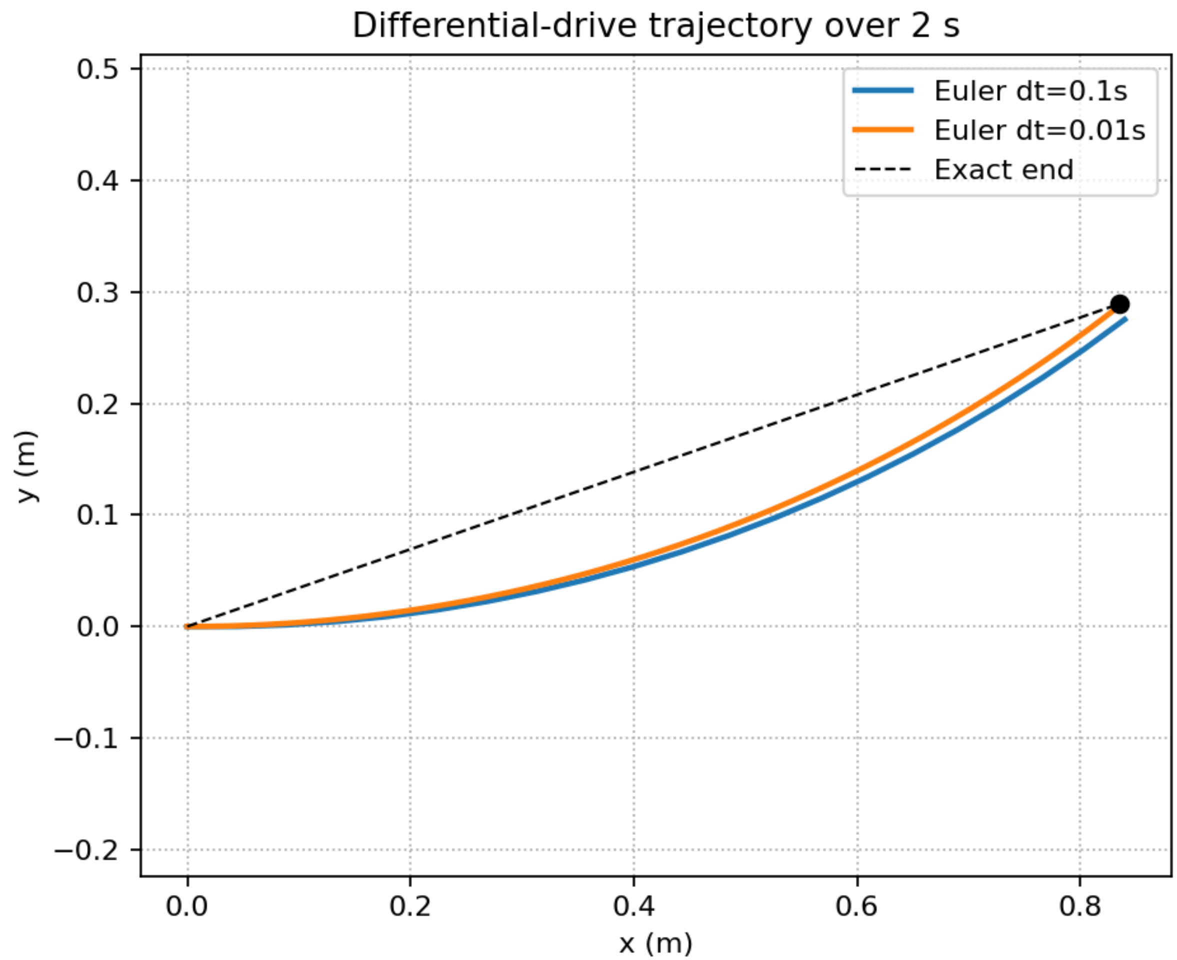
- $x = 0.839539$ m
- $y = 0.275112$ m
- $\theta = 0.666667$ rad $\approx 38.2^\circ$

Euler (dt = 0.01 s):

- $x = 0.835280$ m
- $y = 0.287661$ m
- $\theta = 0.666667$ rad

Exact (closed-form):

- $x = 0.834799$ m
- $y = 0.289052$ m
- $\theta = 0.666667$ rad



Euler $dt = 0.1$ s:

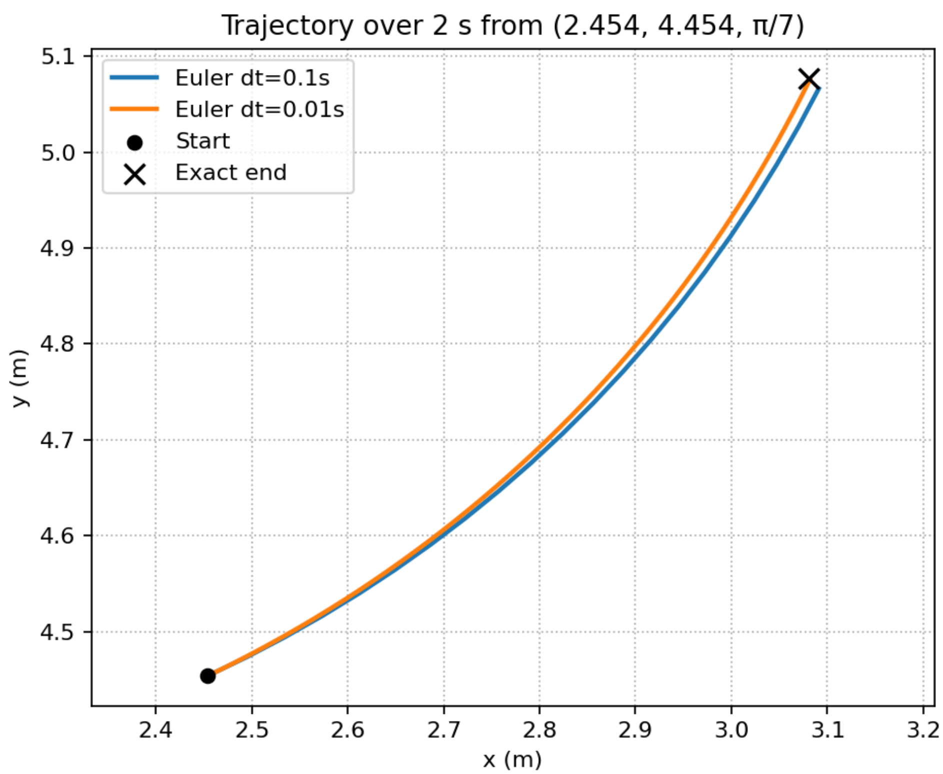
$x = 3.091032$ m, $y = 5.066130$ m, $\theta = 1.115466$ rad

Euler $dt = 0.01$ s:

$x = 3.081750$ m, $y = 5.075588$ m, $\theta = 1.115466$ rad

Exact (closed-form):

$$\boxed{x = 3.080713 \text{ m}, \quad y = 5.076633 \text{ m}, \quad \theta = 1.115466 \text{ rad } (\approx 63.9^\circ)}$$



ICR Equations for Motion

- **Instantaneous Center of Rotation** (also called ICC: Instantaneous Center of Curvature) is the point about which the robot is rotating at any given instant.
- For a differential drive robot, the two wheels usually have different speeds, so the robot follows a circular arc. The center of that circle is the ICR.
- Motion:
  - If both wheels move at the same speed → ICR is at infinity → robot moves straight.
  - If one wheel is stationary → ICR is at the stationary wheel → robot pivots around that wheel.
  - If wheels move in opposite directions → ICR is between the wheels → robot spins in place.

ICR Equations

- Let:
- v_L, v_R= linear velocities of left and right wheels
- L= distance between wheels
- v= linear velocity of the robot’s midpoint
- ω= angular velocity of the robot
- R= distance from the robot’s midpoint to the ICR
- Then:

$$v = \frac{v_R + v_L}{2}, \omega = \frac{v_R - v_L}{L}$$

- The ICR radius:

$$R = \frac{v}{\omega} = \frac{\frac{v_R + v_L}{2}}{\frac{v_R - v_L}{L}} = \frac{L}{2} \cdot \frac{v_R + v_L}{v_R - v_L}$$

Pose Update Using ICR

- If $\omega \neq 0$:

$$x_{new} = x + R[\sin(\theta + \omega\,\Delta t) - \sin\theta]$$
$$y_{new} = y - R[\cos(\theta + \omega\,\Delta t) - \cos\theta]$$
$$\theta_{new} = \theta + \omega\,\Delta t$$

- If $\omega = 0$ (straight motion):

$$x_{new} = x + v\cos\theta\,\Delta t, y_{new} = y + v\sin\theta\,\Delta t$$

- **Example**

$$L = 0.3\,m, r = 0.05\,m$$

$$\omega_R = 10\,rad/s, \omega_L = 8\,rad/s$$

- So:

$$v_R = r\omega_R = 0.5\,m/s,\ \ v_L = r\omega_L = 0.4\,m/s$$

$$v = 0.45\,m/s,\ \ \omega = \frac{0.5 - 0.4}{0.3} = 0.333\,rad/s$$

$$R = \frac{0.45}{0.333} \approx 1.35\,m$$

- **Final Pose ICR Equations**:

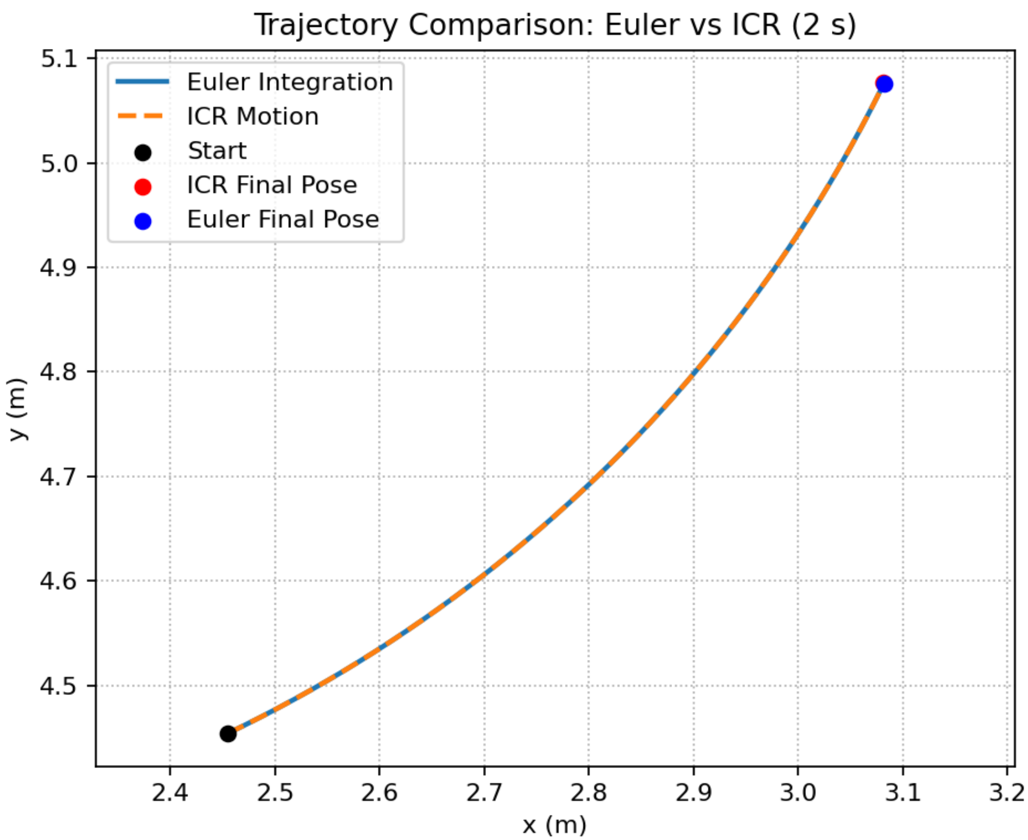
$$x = 3.0807\ m,\ \ y = 5.0766\ m,\ \ \theta = 1.1155\ rad$$

Euler Integration:

$$x = 3.0818\text{ m},\ y = 5.0756\text{ m},\ \theta = 1.1155\text{ rad } (\approx 63.9^\circ)$$

ICR Equations:

$$x = 3.0807\text{ m},\ y = 5.0766\text{ m},\ \theta = 1.1155\text{ rad}$$



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