Regression

Types of Regression Models

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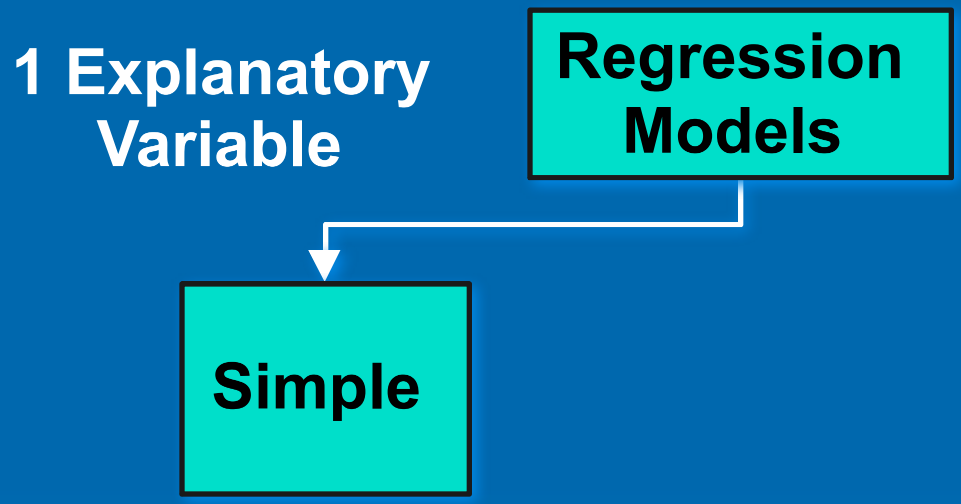

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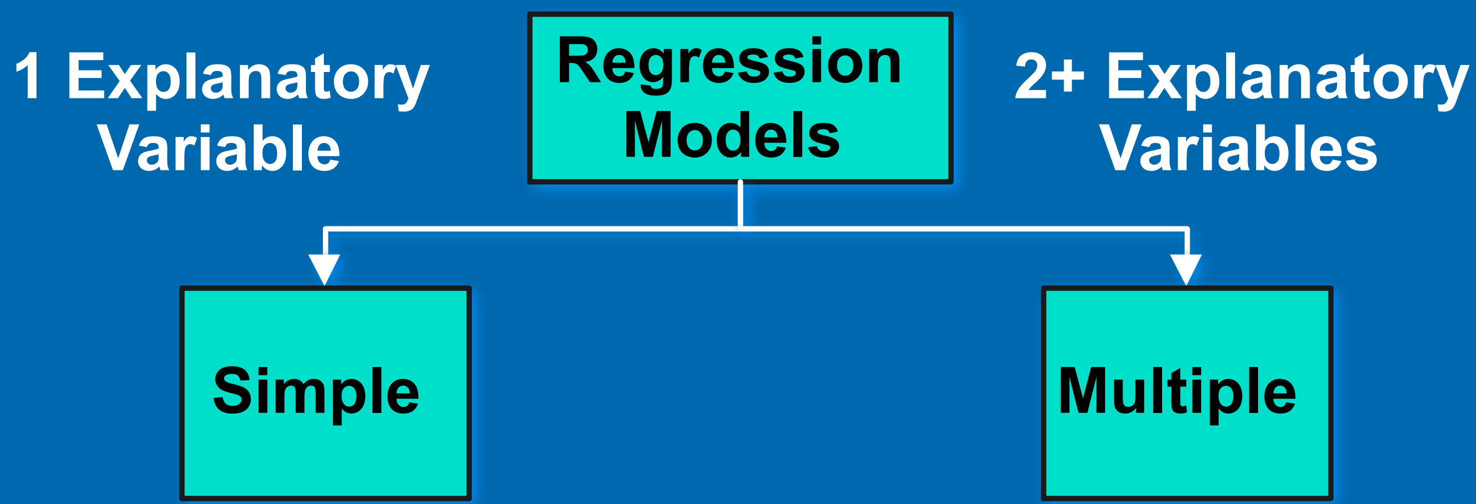

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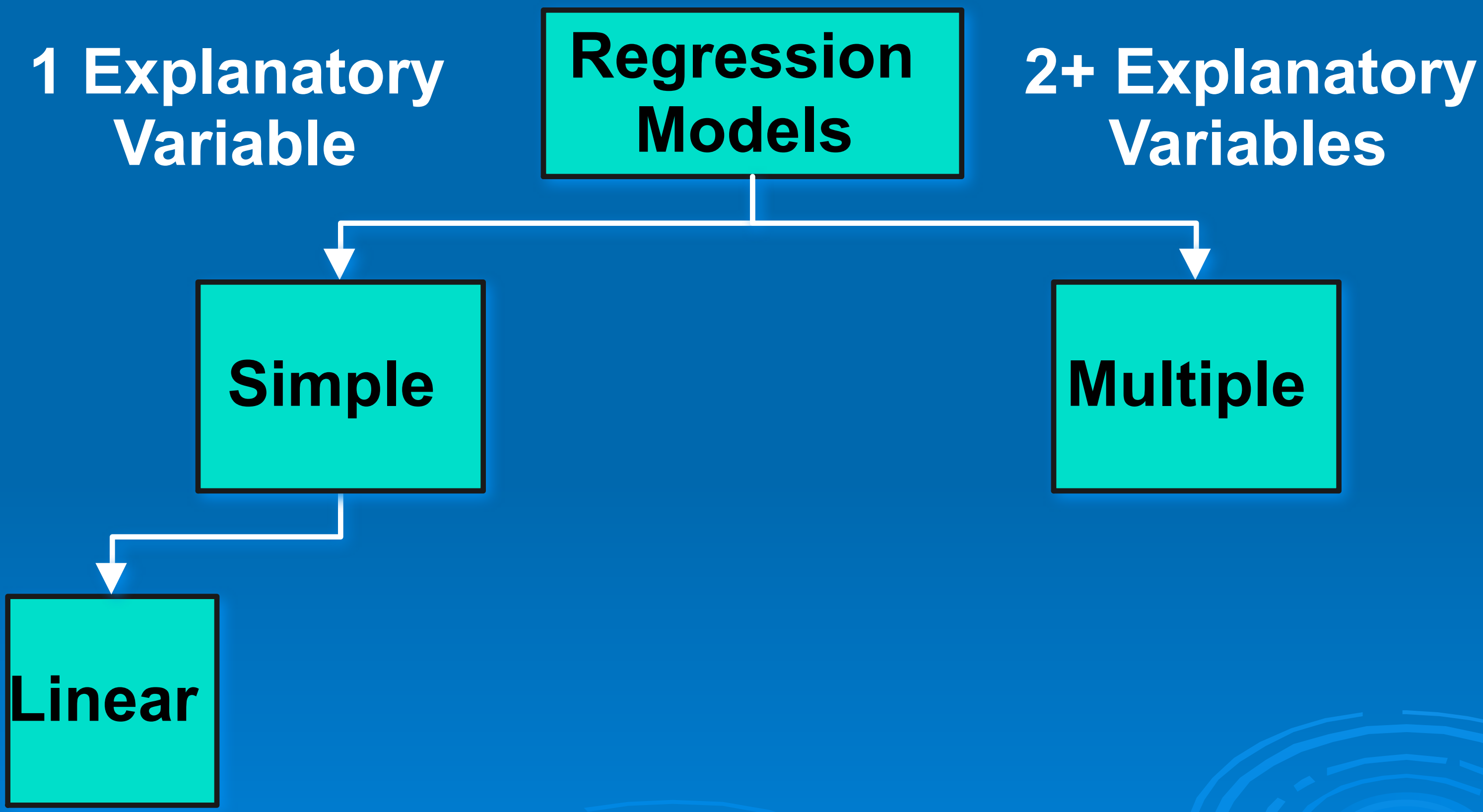

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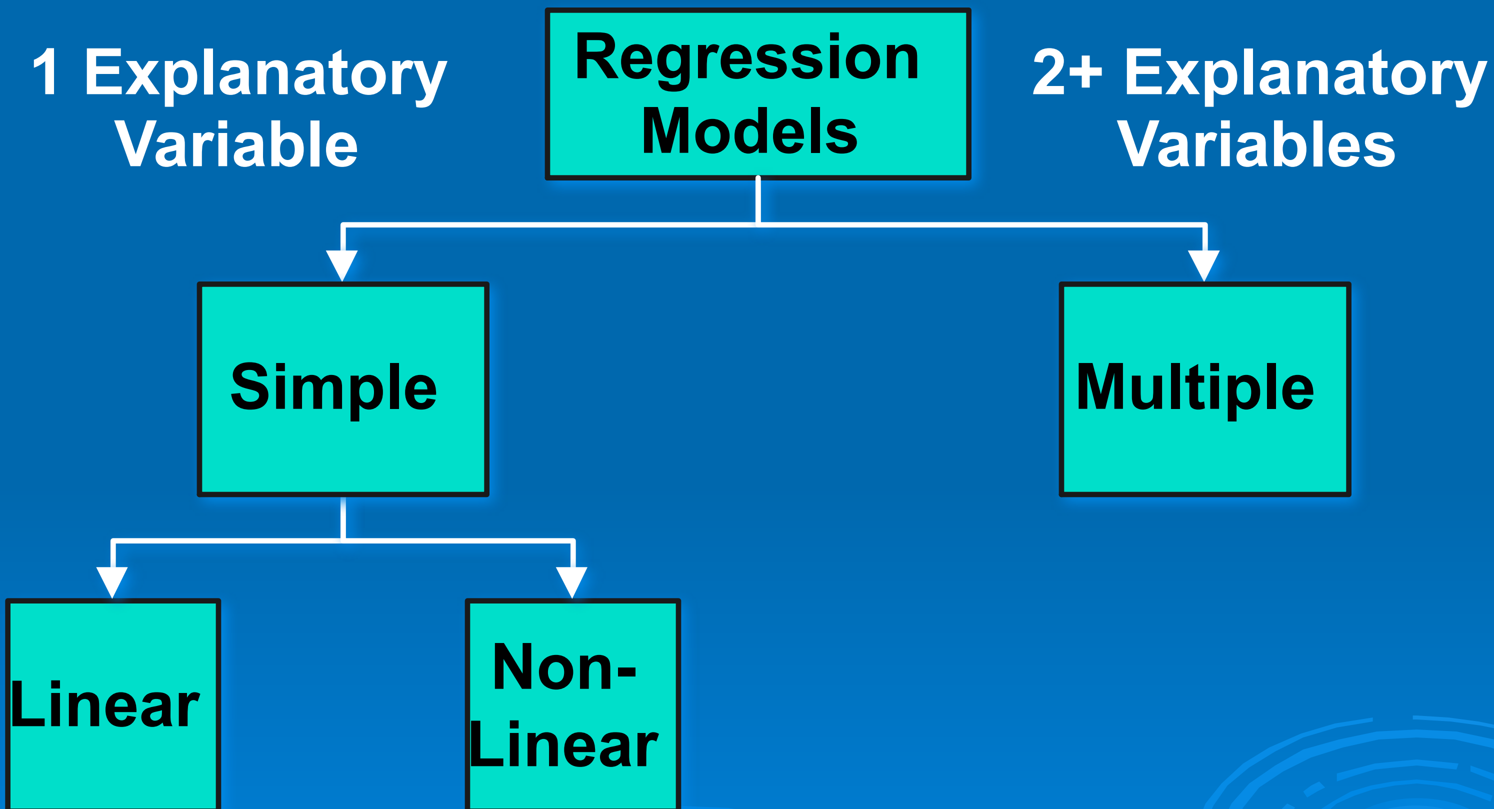

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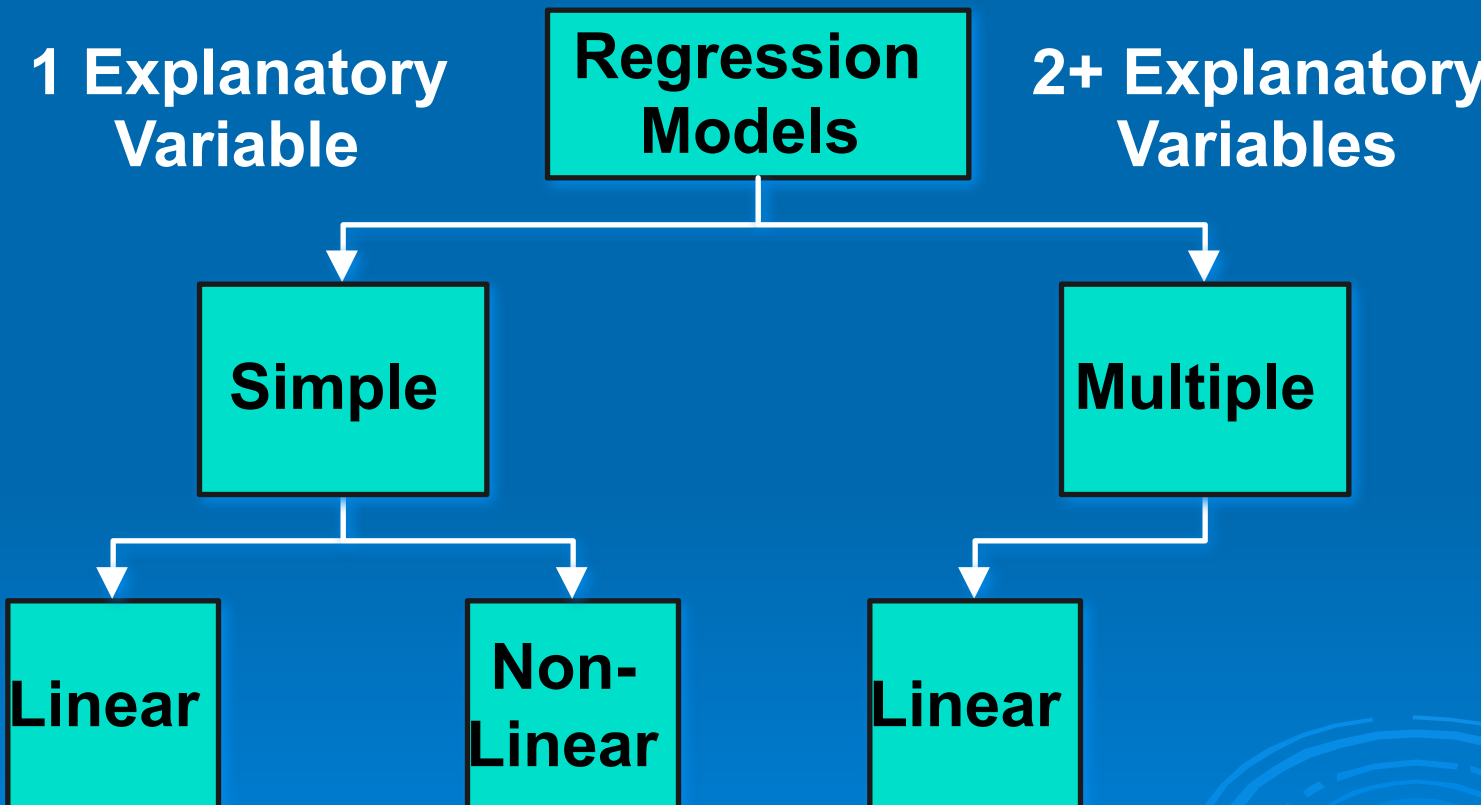

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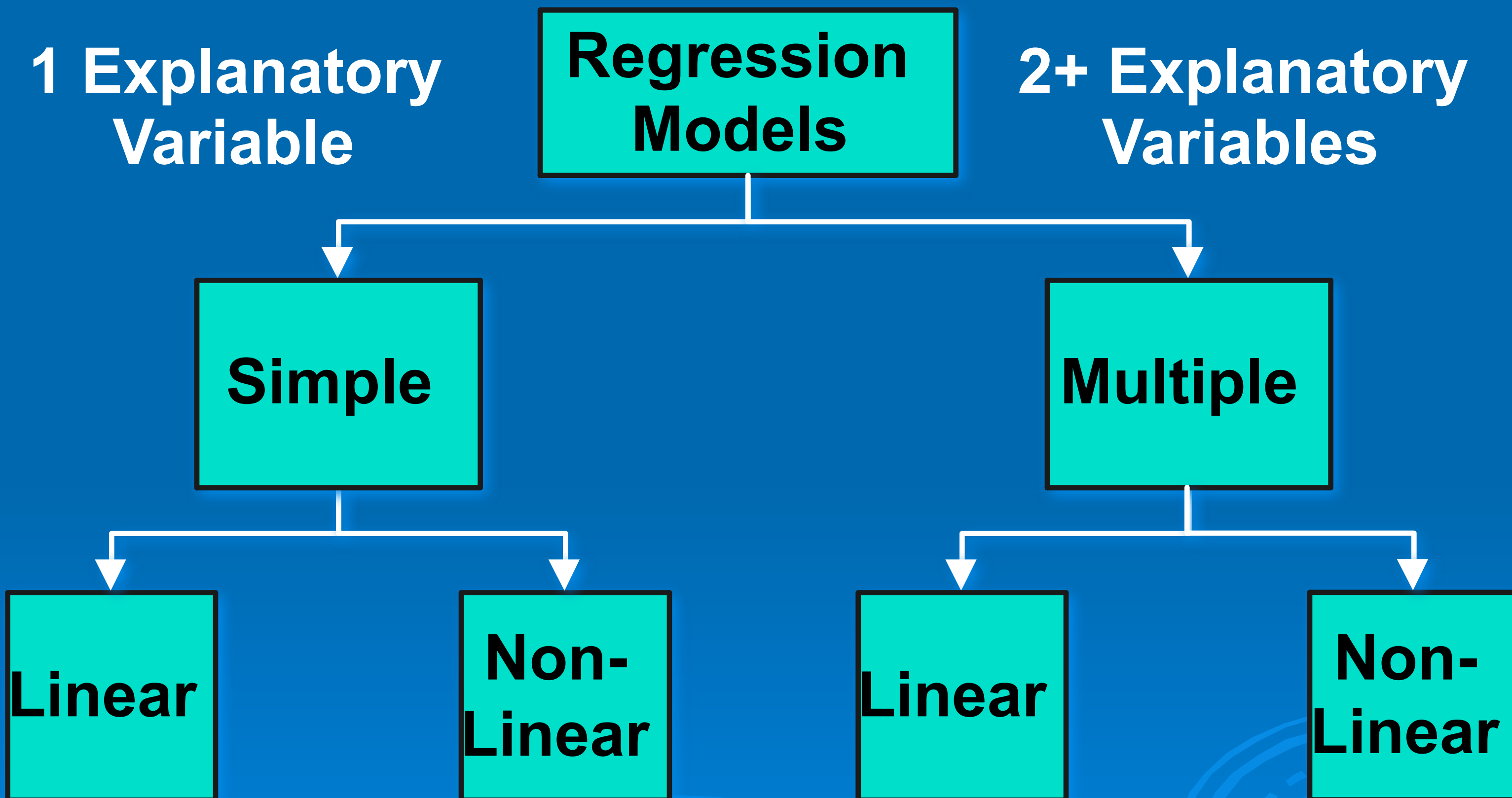
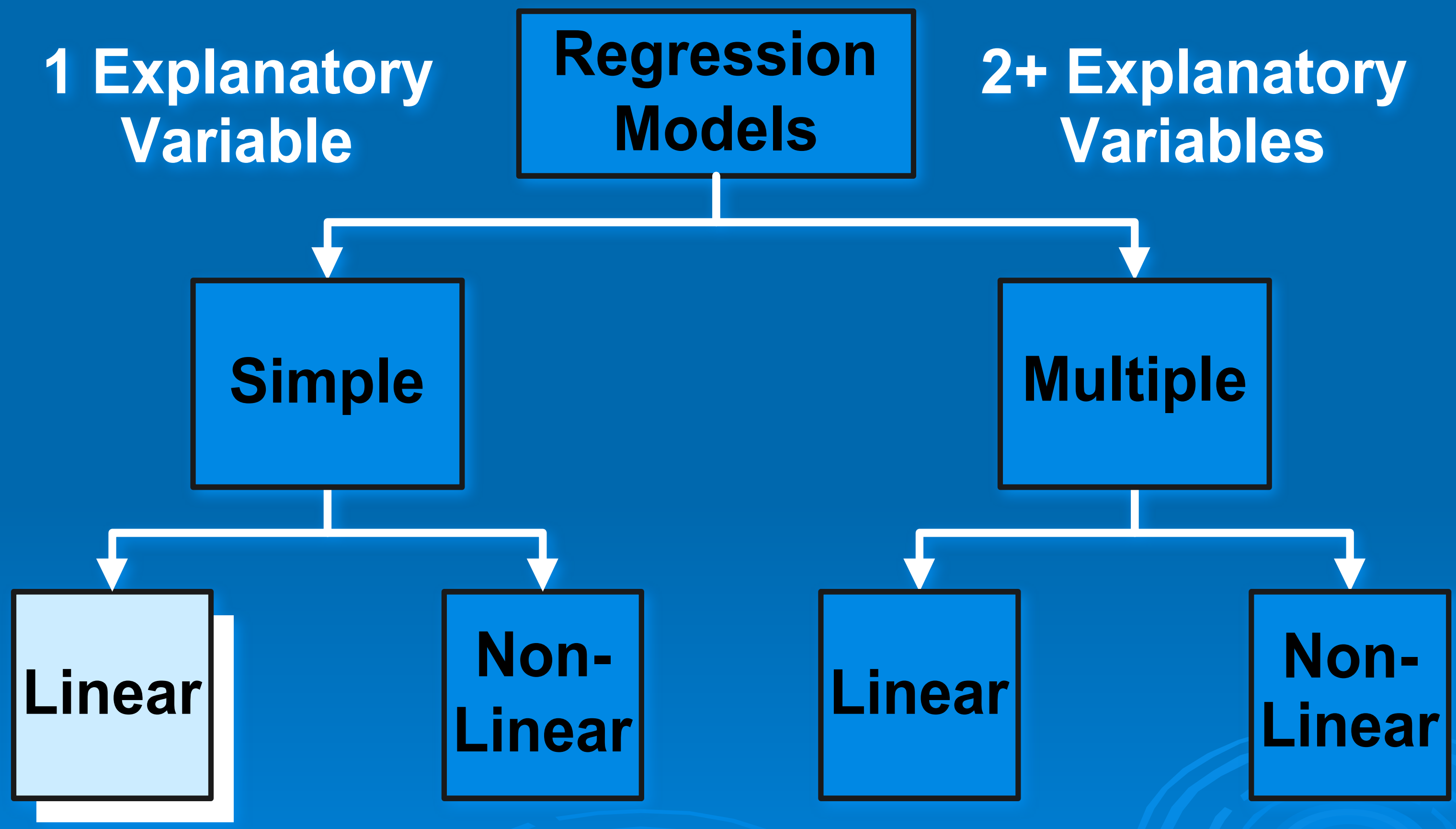
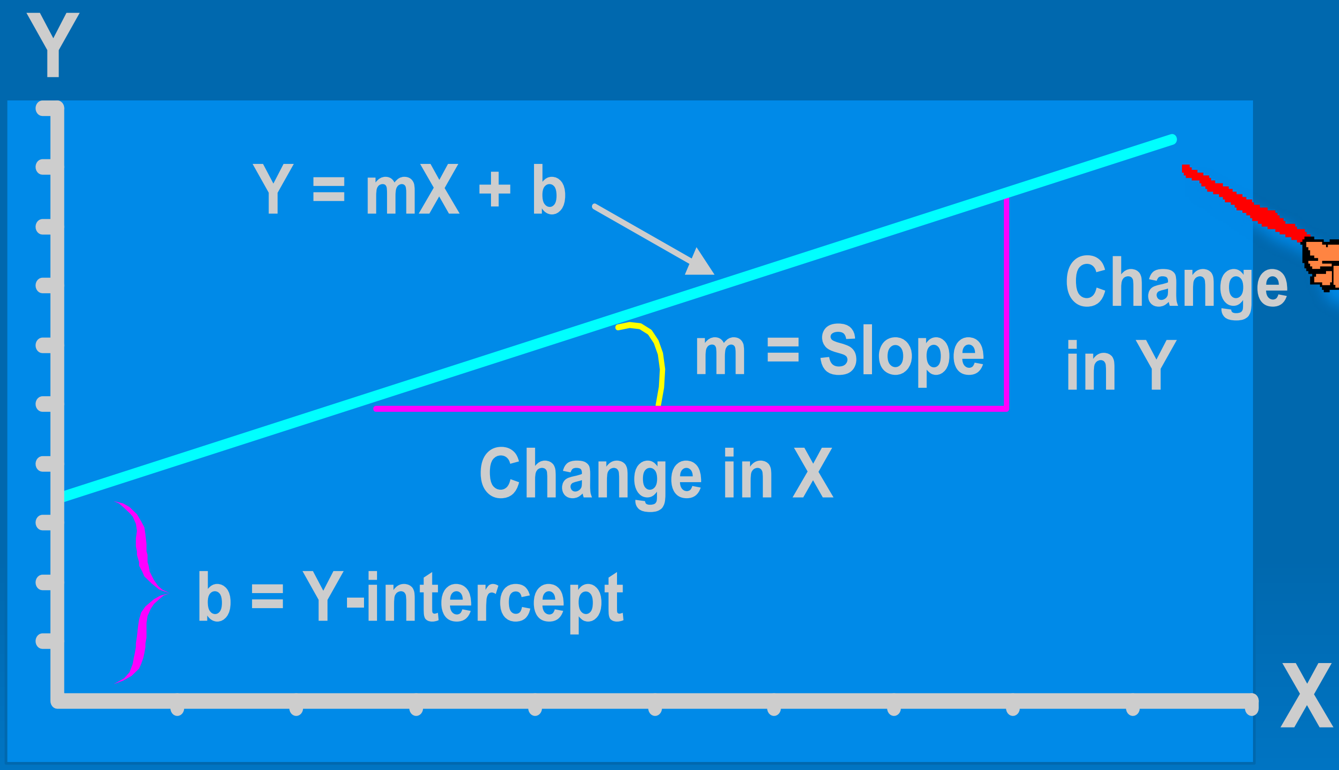


Linear Regression Model

Types of Regression Models



Linear Equations



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Linear Regression Model

- 1. Relationship Between Variables Is a Linear Function

Population Y-Intercept

Population Slope

Random Error

$$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$$

Dependent (Response) Variable (e.g., CD+ c.)

Independent (Explanatory) Variable (e.g., Years s. serocon.)

Population & Sample Regression Models

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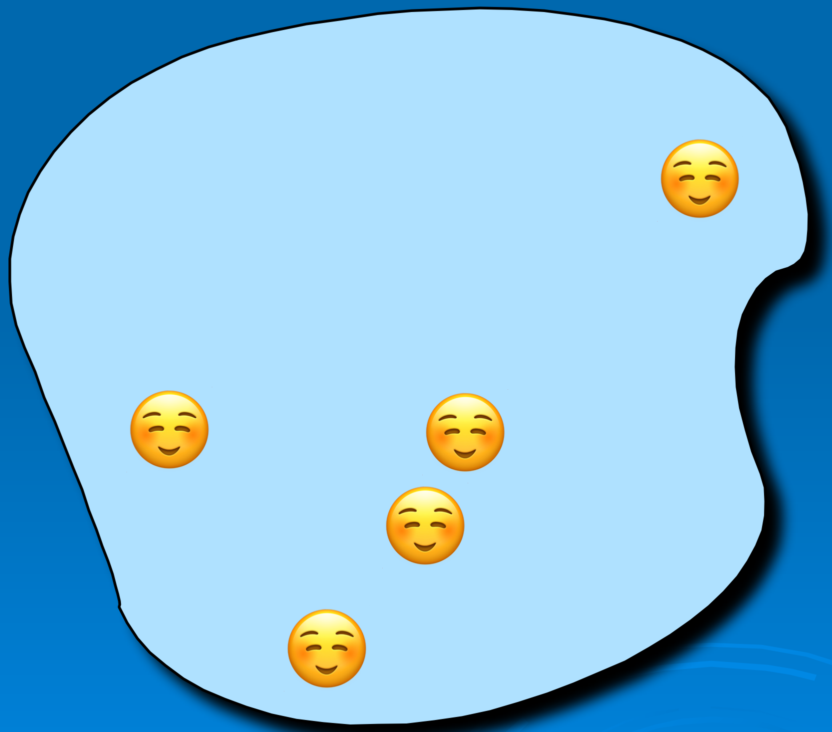
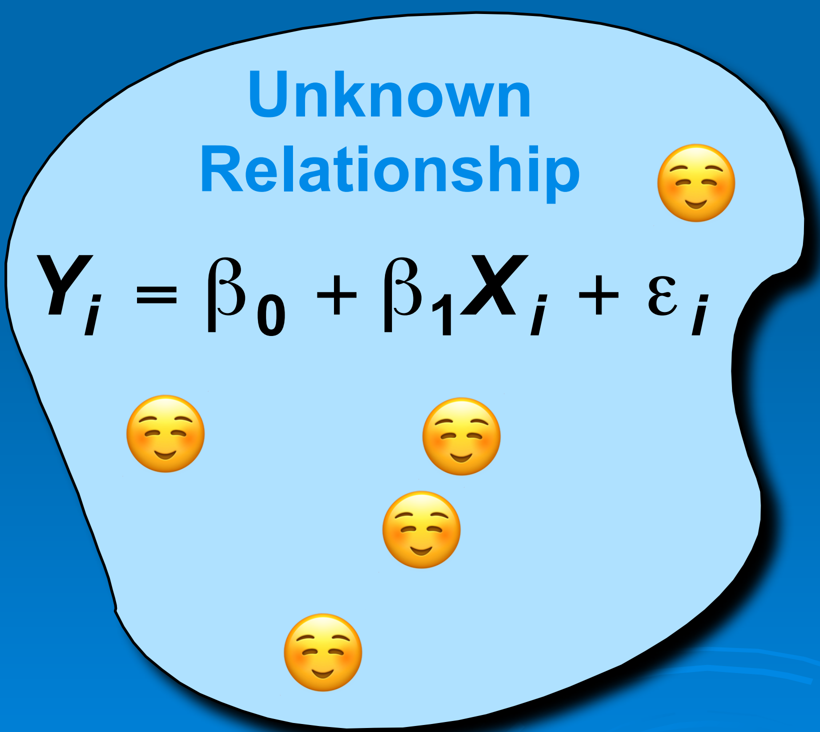
Population



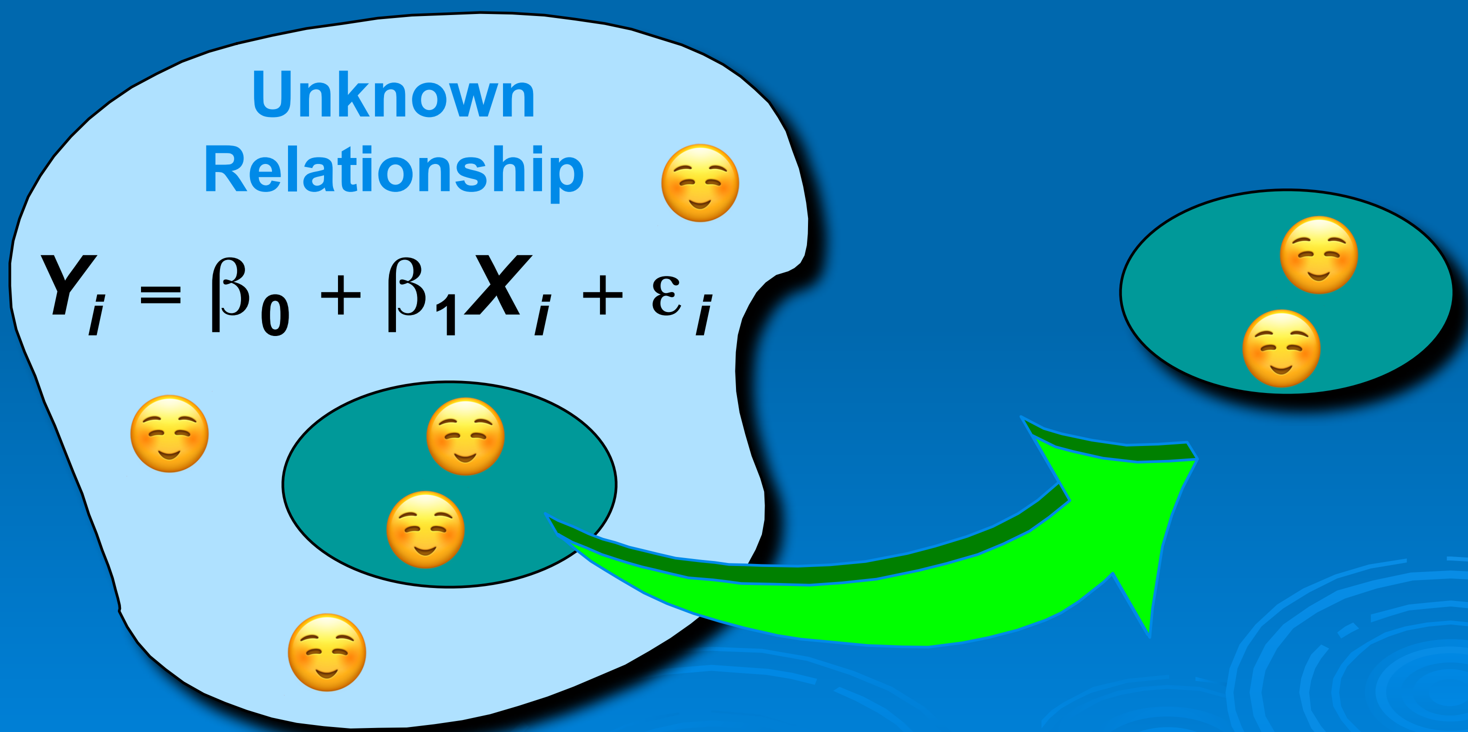
Population & Sample Regression Models

Population



Population & Sample Regression Models

Population **Random Sample**



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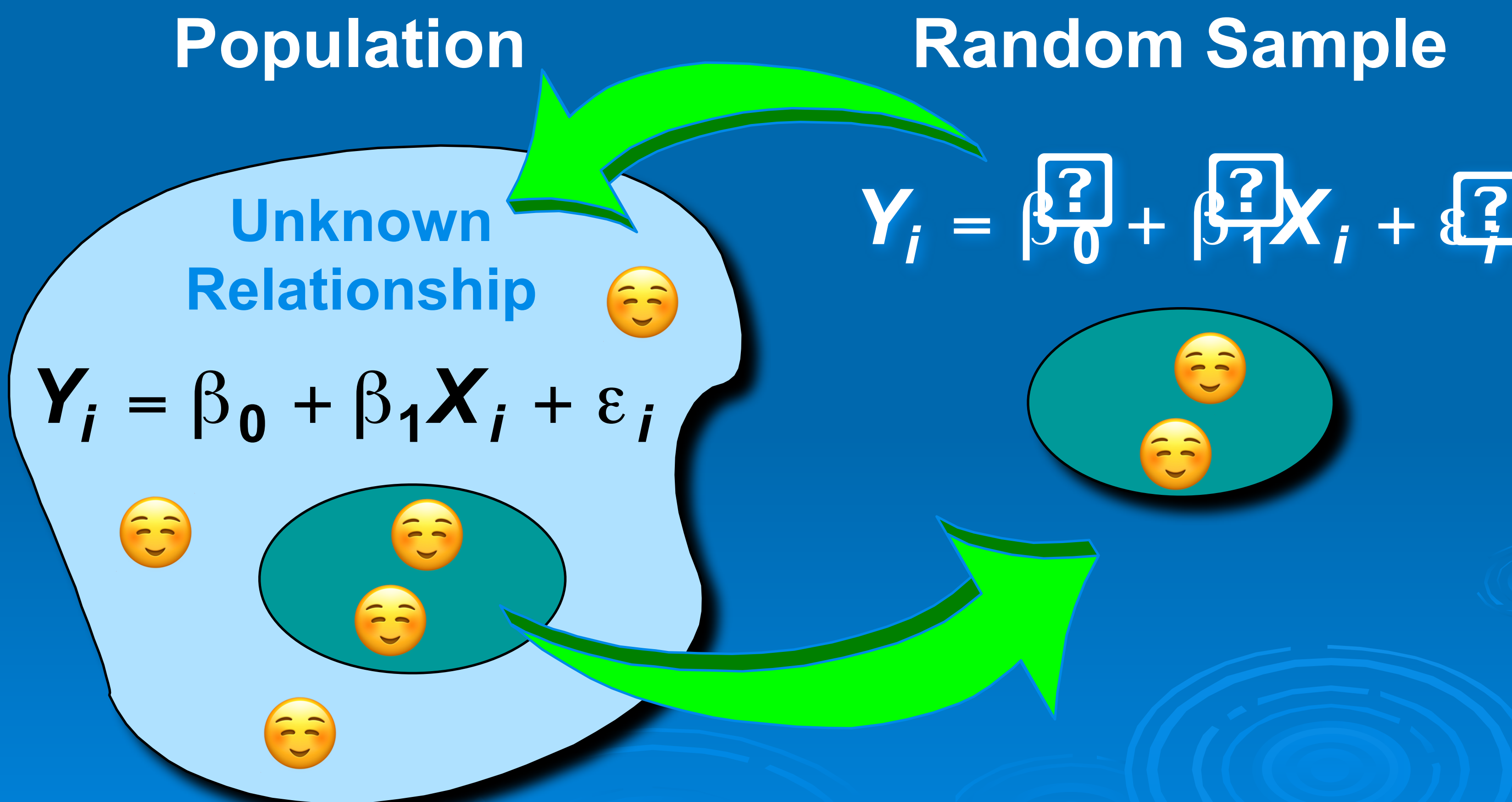
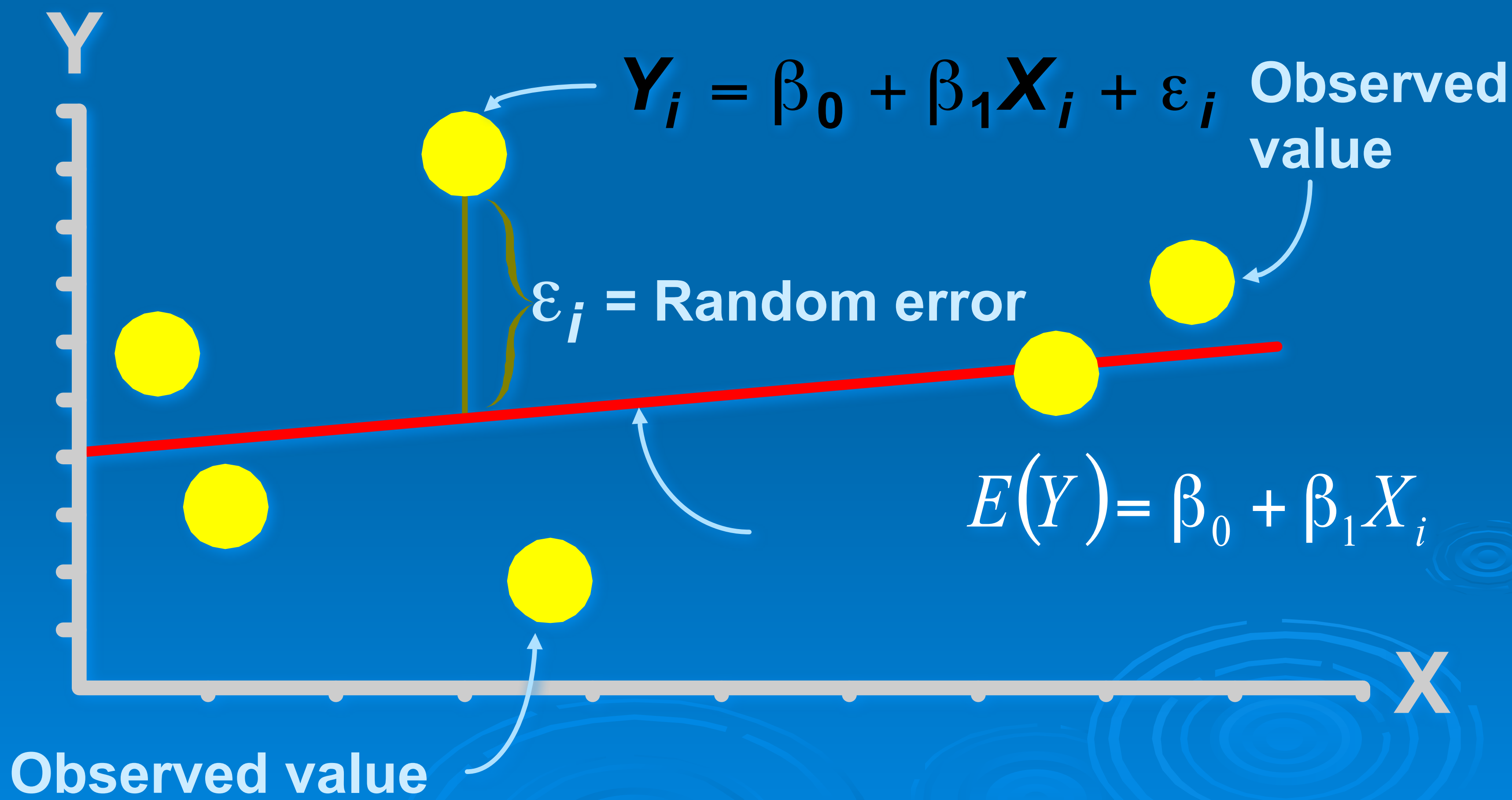
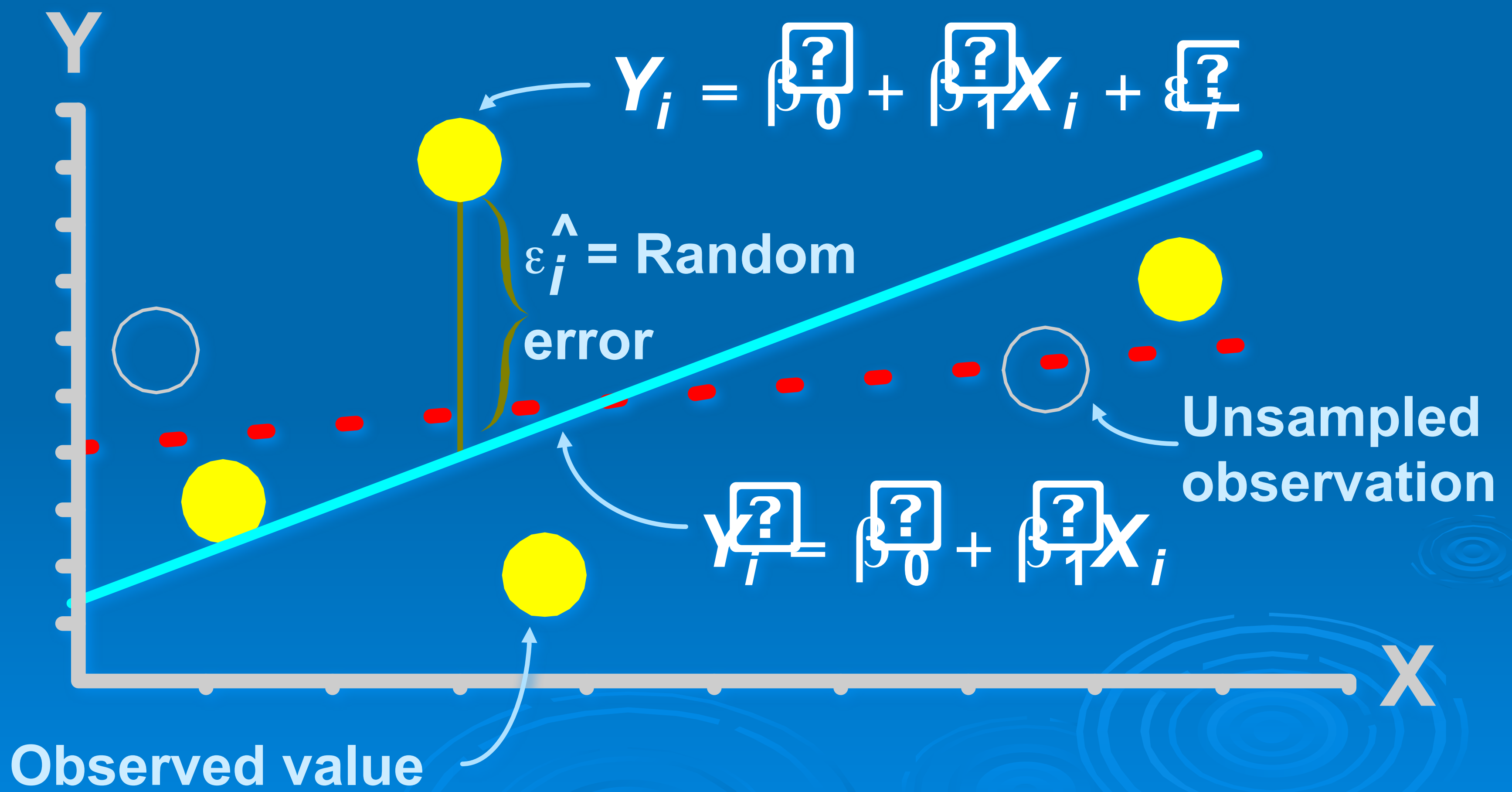


Population Linear Regression Model



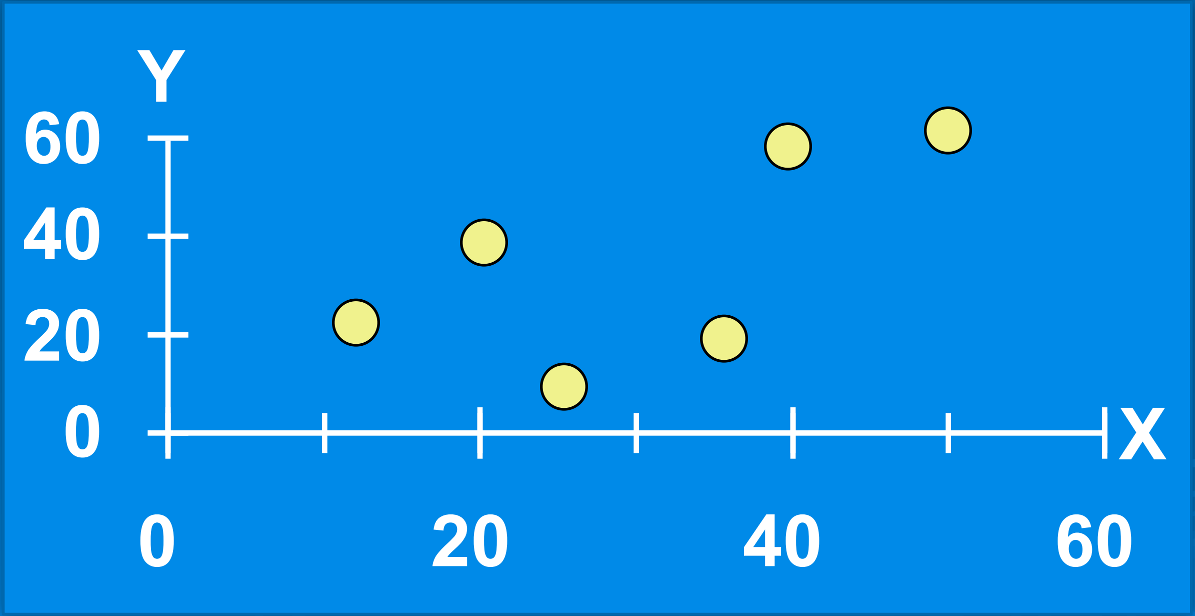
Sample Linear Regression Model



Estimating Parameters: Least Squares Method

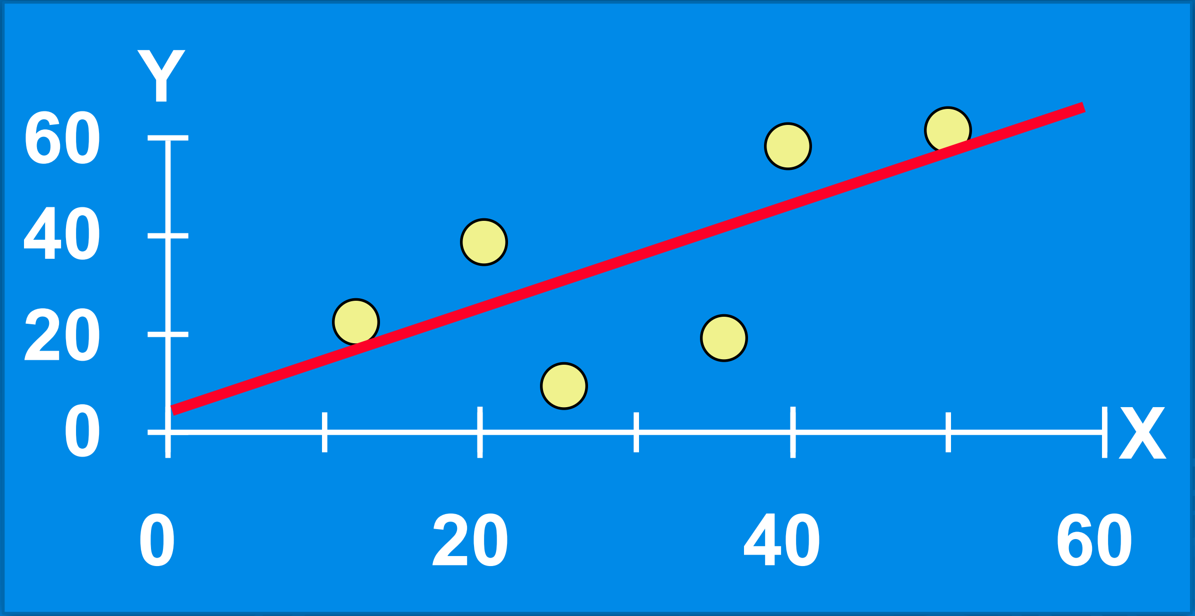
Scatter plot

- 1. Plot of All (X_i, Y_i) Pairs
- 2. Suggests How Well Model Will Fit



Thinking Challenge

How would you draw a line through the points? How do you determine which line 'fits best'?



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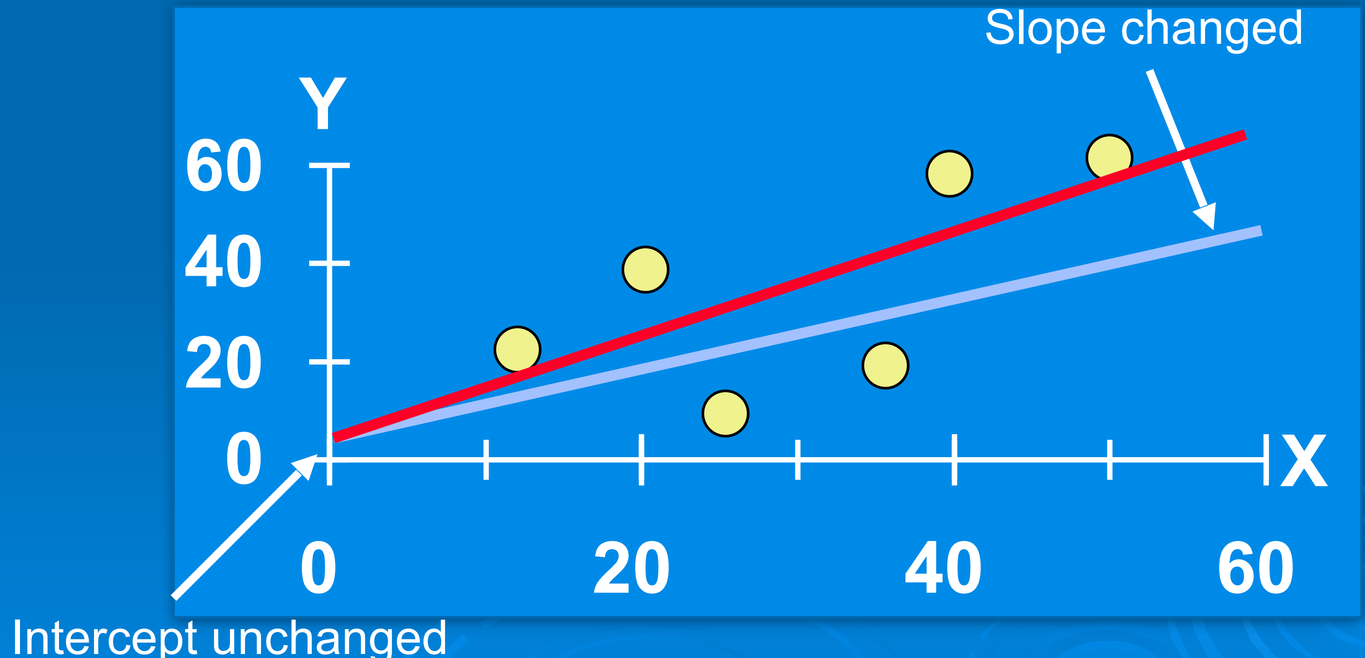

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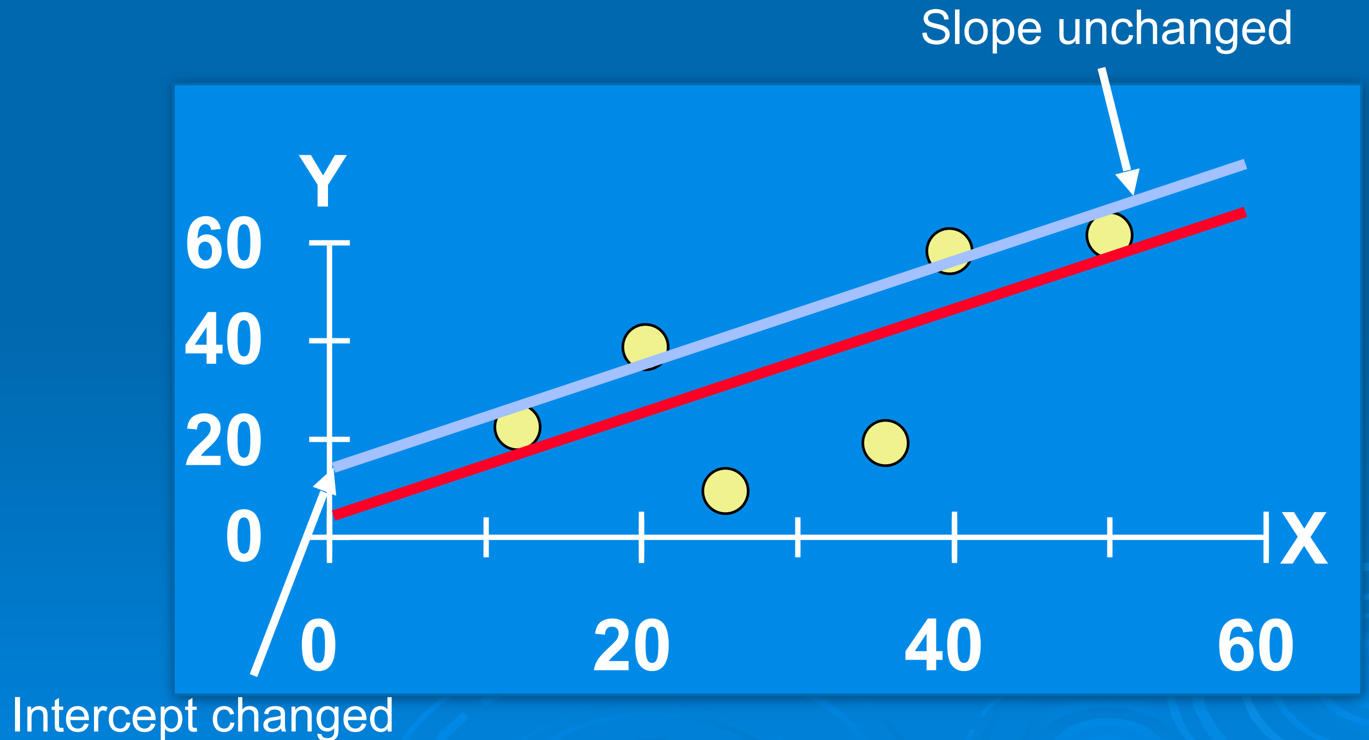

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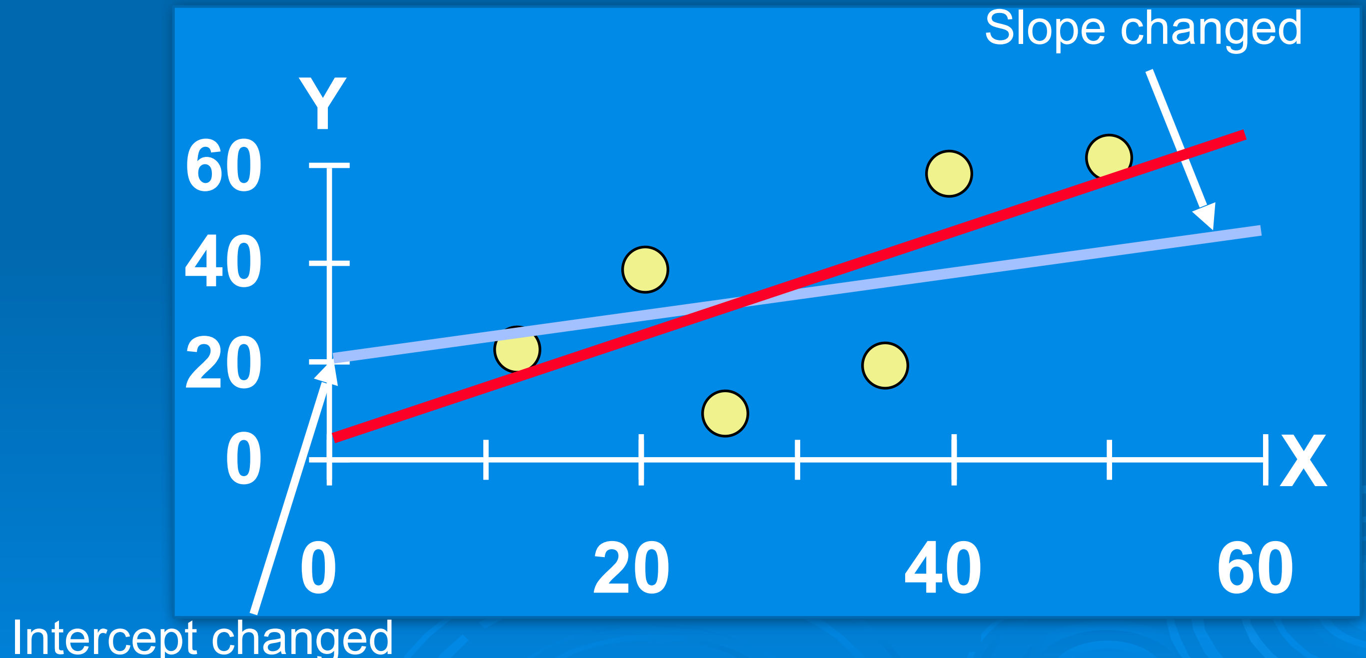

Least Squares

- 1. ‘Best Fit’ Means Difference Between Actual Y Values & Predicted Y Values Are a Minimum. *But* Positive Differences Off-Set Negative ones

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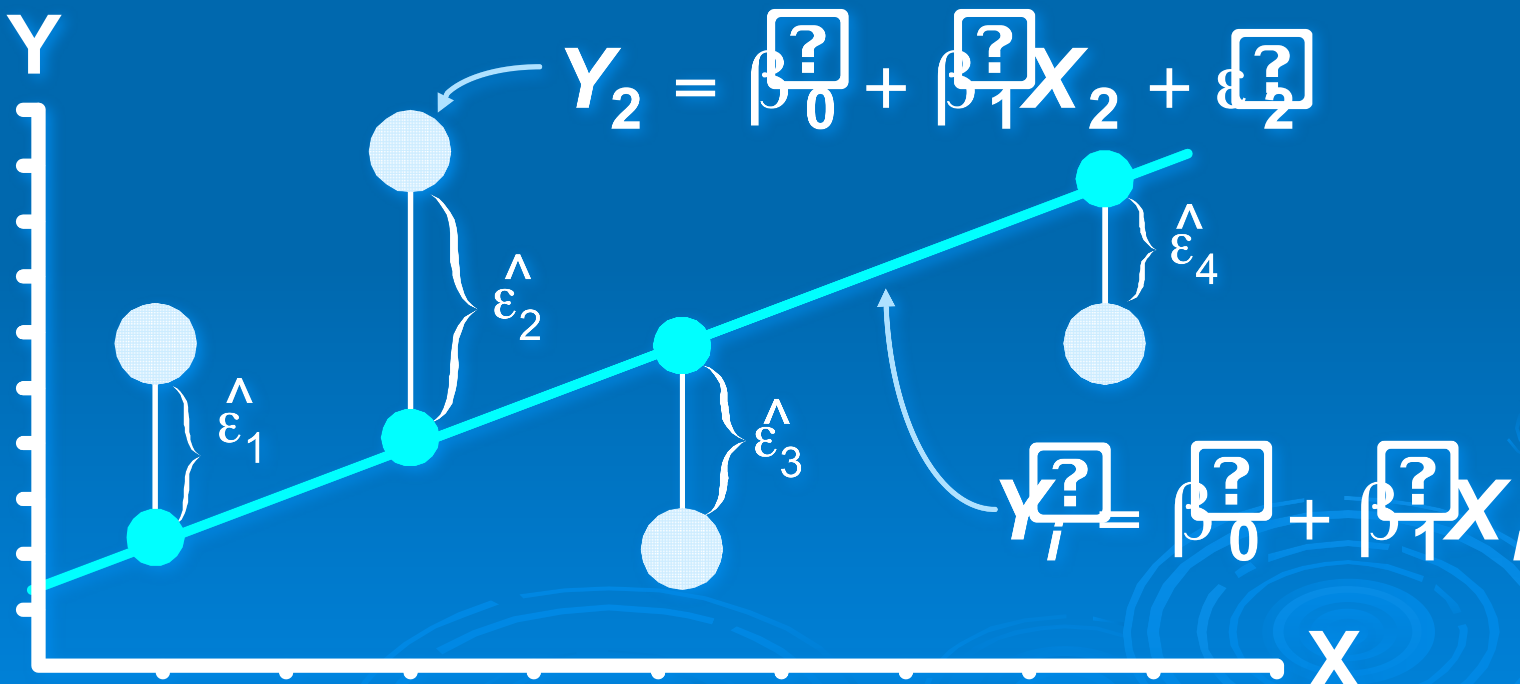
$$\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n}\hat{\varepsilon}_i^2$$

- 2. LS Minimizes the Sum of the Squared Differences (errors) (SSE)

Least Squares Graphically

LS minimizes $\sum_{i=1}^{n} \hat{\varepsilon}_i^2 = \hat{\varepsilon}_1^2 + \hat{\varepsilon}_2^2 + \hat{\varepsilon}_3^2 + \hat{\varepsilon}_4^2$



Coefficient Equations

Prediction equation

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Sample slope

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

Sample Y - intercept

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Interpretation of Coefficients

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- 1. Slope ($\hat{\beta}_1$)
  - Estimated *Y* Changes by $\hat{\beta}_1$ for Each 1 Unit Increase in *X*
    - If $\hat{\beta}_1 = 2$, then *Y* Is Expected to Increase by 2 for Each 1 Unit Increase in *X*

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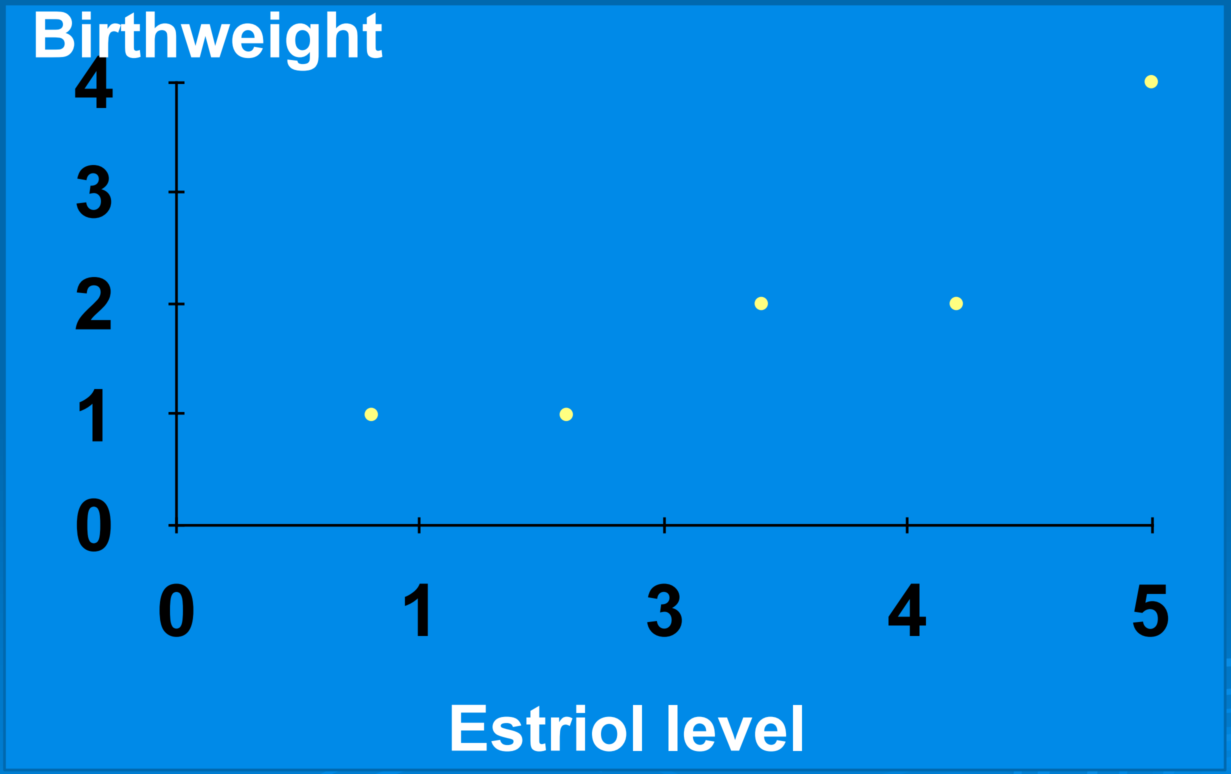
    - If $\hat{\beta}_1 = 2$, then *Y* Is Expected to Increase by 2 for Each 1 Unit Increase in *X*
- 2. Y-Intercept ($\hat{\beta}_0$)
  - Average Value of *Y* When *X* = 0
    - If $\hat{\beta}_0 = 4$, then Average *Y* Is Expected to Be 4 When *X* Is 0

Parameter Estimation Example

- Obstetrics: What is the **relationship** between Mother's Estriol level & Birthweight using the following data?

Estriol (mg/24h)	Birthweight (g/1000)
1	1
2	1
3	2
4	2
5	4

Scatterplot
Birthweight vs. Estriol level



Parameter Estimation Solution Table

X_i	Y_i	X_i^2	Y_i^2	X_iY_i
1	1	1	1	1
2	1	4	1	2
3	2	9	4	6
4	2	16	4	8
5	4	25	16	20
15	10	55	26	37

Parameter Estimation Solution

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{n}}{\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}} = \frac{37 - \frac{(15)(10)}{5}}{55 - \frac{(15)^2}{5}} = 0.70$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X} = 2 - (0.70)(3) = -0.10$$

Coefficient Interpretation Solution

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- 1. Slope ($\hat{\beta}_1$)
  - Birthweight (*Y*) Is Expected to Increase by .7 Units for Each 1 unit Increase in Estriol (*X*)

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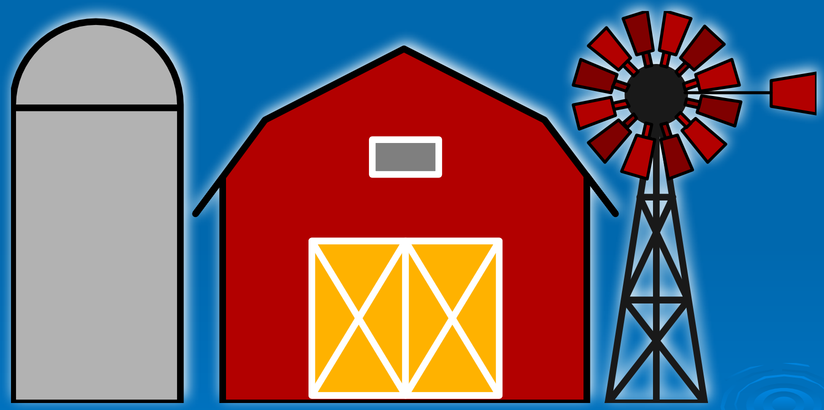
  - Birthweight (*Y*) Is Expected to Increase by .7 Units for Each 1 unit Increase in Estriol (*X*)
- 2. Intercept ($\hat{\beta}_0$)
  - Average Birthweight (*Y*) Is -.10 Units When Estriol level (*X*) Is 0
    - Difficult to explain
    - The birthweight should always be positive

Parameter Estimation Thinking Challenge

- You're a Vet epidemiologist for the county cooperative. You gather the following data:
- Food (lb.) Milk yield (lb.)

Food (lb.)	Milk yield (lb.)
4	3.0
6	5.5
10	6.5
12	9.0



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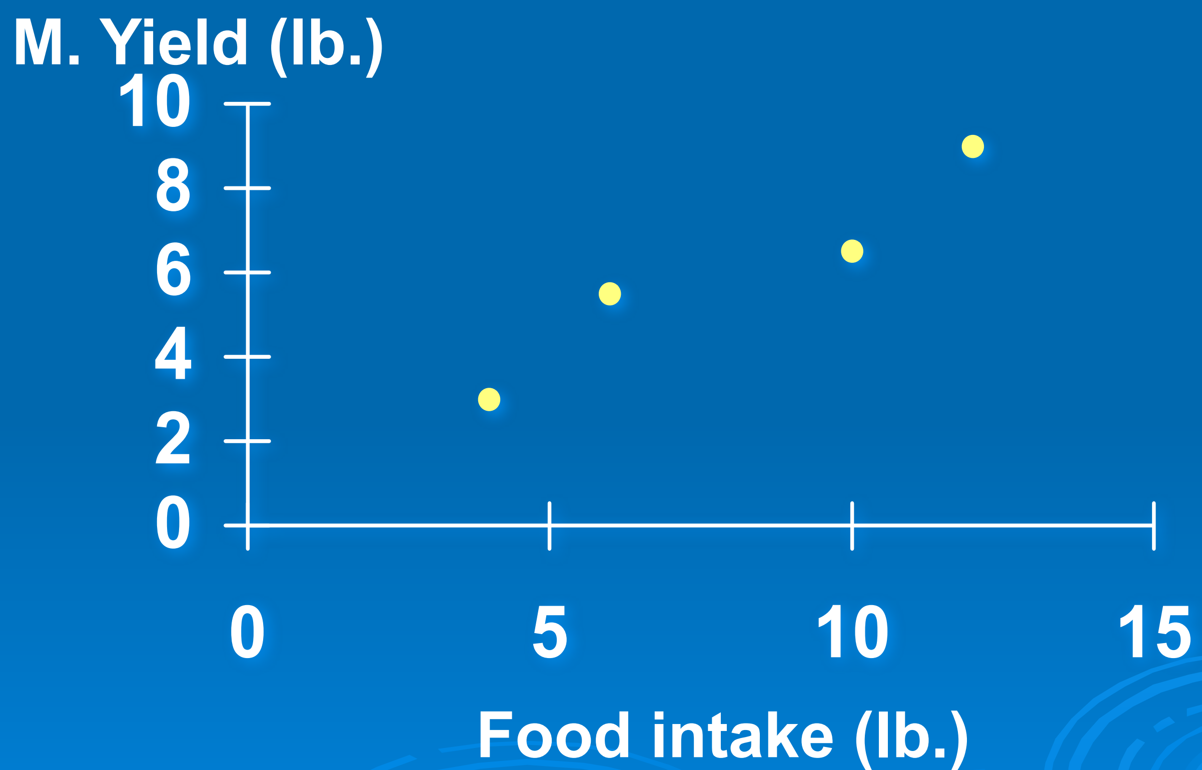
- What is the **relationship** between cows' food intake and milk yield?

Scattergram
Milk Yield vs. Food intake*



Parameter Estimation Solution Table*

X_i	Y_i	X_i^2	Y_i^2	X_iY_i
4	3.0	16	9.00	12
6	5.5	36	30.25	33
10	6.5	100	42.25	65
12	9.0	144	81.00	108
32	24.0	296	162.50	218

Parameter Estimation Solution*

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} X_i Y_i - \frac{\left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right)}{n}}{\sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n}} = \frac{218 - \frac{(32)(24)}{4}}{296 - \frac{(32)^2}{4}} = 0.65$$

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X} = 6 - (0.65)(8) = 0.80$$

Coefficient Interpretation Solution*

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- 1. Slope ($\hat{\beta}_1$)
  - Milk Yield (Y) Is Expected to Increase by .65 lb. for Each 1 lb. Increase in Food intake (X)
- 2. Y-Intercept ($\hat{\beta}_0$)
  - Average Milk yield (Y) Is Expected to Be 0.8 lb. When Food intake (X) Is 0

Coefficient of determination

– To measure the strength of the linear relationship we use the coefficient of determination.

$$R^2 = \frac{\left[\sum (x_i - \bar{x})(y_i - \bar{y})\right]^2}{s_x^2 s_y^2}$$

$$or \quad R^2 = 1 - \frac{SSE}{\sum (y_i - \bar{y})^2}$$

Note that the coefficient of determination is r^2

Coefficient of determination

- R^2 measures the proportion of the variation in y that is explained by the variation in x.

$$R^2 = 1 - \frac{SSE}{\sum (y_i - \bar{y})^2} = \frac{\sum (y_i - \bar{y})^2 - SSE}{\sum (y_i - \bar{y})^2} = \frac{SSR}{\sum (y_i - \bar{y})^2}$$

- R^2 takes on any value between zero and one.
  $R^2 = 1$: Perfect match between the line and the data points.
  $R^2 = 0$: There are no linear relationship between x and y.