EE5600 Assignment 1

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***Abstract*—This document contains the solution to a Area of triangle problem.**

Download all python and latex codes from

https://github.com/venky−p/EE5600/Assignment_1

1 Problem

Problem Set: Vector2, Example II, Problem 5

1.1. Find the area of the triangle formed by the points (a, c+a), (a, c) and (-a, c-a)

2 Solution

We are going to solve this problem using vectors

$$\mathbf{P} = \begin{pmatrix} a \\ c+a \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} a \\ c \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -a \\ c-a \end{pmatrix} \tag{2.1.1}$$

Rewriting P, Q and R as product of a matrix and a vector

$$\mathbf{P} = \mathbf{Au} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \tag{2.1.2}$$

$$\mathbf{Q} = \mathbf{Bu} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \tag{2.1.3}$$

$$\mathbf{R} = \mathbf{Cu} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \tag{2.1.4}$$

$$\text{Area of given Triangle } = \frac{1}{2} \left\| \mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} \right\| \tag{2.1.5}$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = (\mathbf{Au} - \mathbf{Bu}) \times (\mathbf{Au} - \mathbf{Cu}) \tag{2.1.6}$$

$$= \mathbf{Au} \times \mathbf{Au} - \mathbf{Au} \times \mathbf{Cu} - \mathbf{Bu} \times \mathbf{Au} \tag{2.1.7}$$

$$+\mathbf{Bu} \times \mathbf{Cu} \tag{2.1.8}$$

$$= \mathbf{Au} \times \mathbf{Bu} + \mathbf{Bu} \times \mathbf{Cu} + \mathbf{Cu} \times \mathbf{Au} \tag{2.1.9}$$

$$= \begin{pmatrix} a \\ c+a \end{pmatrix} \times \begin{pmatrix} a \\ c \end{pmatrix} + \begin{pmatrix} a \\ c \end{pmatrix} \times \begin{pmatrix} -a \\ c-a \end{pmatrix} + \begin{pmatrix} -a \\ c-a \end{pmatrix} \times \begin{pmatrix} a \\ c+a \end{pmatrix} \tag{2.1.10}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -a^2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -a^2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.1.11}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2a^2 \end{pmatrix} \tag{2.1.12}$$

By Substituting (2.1.12) in (2.1.5), We get

$$\text{Area of given Triangle } = \frac{1}{2} \left\| \begin{matrix} 0 \\ 0 \\ -2a^2 \end{matrix} \right\| \tag{2.1.13}$$

$$= \frac{1}{2} \sqrt{0^2 + 0^2 + (-2a^2)^2} \tag{2.1.14}$$

$$= \frac{1}{2} 2a^2 \tag{2.1.15}$$

$$= a^2 \tag{2.1.16}$$

$$\boxed{\therefore \text{ Area of given Triangle } = a^2 units^2} \tag{2.1.17}$$

3 Numerical Example

Let,

$$\mathbf{u} = \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{3.1.18}$$

Solution: By substituting the given values in (2.1.1), We get

$$\mathbf{P} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{3.1.19}$$

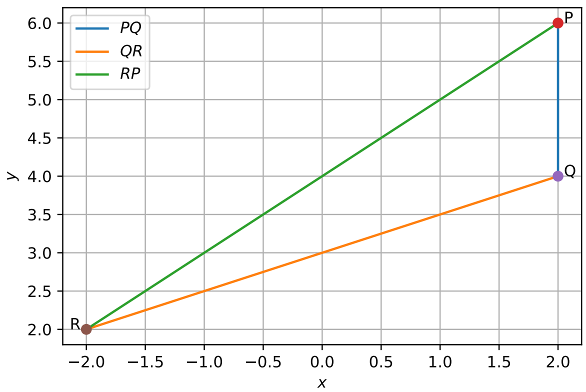

Fig. 3.1: Plot obtained from Python code

Using (2.1.17),

$$\begin{aligned} \text{Area of given Triangle} &= a^2 &(3.1.20)\\ &= 2^2 &(3.1.21)\\ &= 4units^2 &(3.1.22)\end{aligned}$$