EE5600 Assignment 1

Perabhattula Venkatesh
AI20MTECH01004

***Abstract*—This document contains the solution to a Area of triangle problem.**

Download all python codes from

https://github.com/venky−p/EE5600/Assignment_1

1 Problem

Problem Set: Vector2, Example II, Problem 5

1.1. Find the area of the triangle formed by the points (a, c+a), (a, c) and (-a, c-a)

2 Solution

We are going to solve this problem using vectors

$$\mathbf{P} = \begin{pmatrix} a \\ c+a \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} a \\ c \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -a \\ c-a \end{pmatrix} \tag{2.1.1}$$

Rewriting P, Q and R as product of a matrix and a vector

$$\mathbf{P} = \mathbf{A}\mathbf{u} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \tag{2.1.2}$$

$$\mathbf{Q} = \mathbf{B}\mathbf{u} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \tag{2.1.3}$$

$$\mathbf{R} = \mathbf{C}\mathbf{u} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} \tag{2.1.4}$$

$$Area\, of\ given\ \triangle le \ = \ \frac{1}{2} \left\| \mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} \right\| \tag{2.1.5}$$

$$\mathbf{P} - \mathbf{Q} = (\mathbf{A} - \mathbf{B})\mathbf{u} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ a \end{pmatrix} \tag{2.1.6}$$

$$\mathbf{P} - \mathbf{R} = (\mathbf{A} - \mathbf{C})\mathbf{u} = \begin{pmatrix} 2 & 0 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 2a \\ 2a \end{pmatrix} \tag{2.1.7}$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = (\mathbf{A} - \mathbf{B})\mathbf{u} \times (\mathbf{A} - \mathbf{C})\mathbf{u} \tag{2.1.8}$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \begin{pmatrix} 0 \\ a \end{pmatrix} \times \begin{pmatrix} 2a \\ 2a \end{pmatrix} \tag{2.1.9}$$

$$\mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ -a & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 2a \\ 2a \\ 0 \end{pmatrix} \tag{2.1.10}$$

$$\implies \mathbf{P} - \mathbf{Q} \times \mathbf{P} - \mathbf{R} = \begin{pmatrix} 0 \\ 0 \\ -2a^2 \end{pmatrix} \tag{2.1.11}$$

By substituting (2.1.11) in (2.1.5), we get

$$Area\, of\ given\ \triangle le \ = \ \frac{1}{2} \left\| \begin{matrix} 0 \\ 0 \\ -2a^2 \end{matrix} \right\| \tag{2.1.12}$$

$$\implies \frac{1}{2} \sqrt{0^2 + 0^2 + (-2a^2)^2} \tag{2.1.13}$$

$$\implies \frac{1}{2} 2a^2 \tag{2.1.14}$$

$$\implies a^2 \tag{2.1.15}$$

$$\boxed{\therefore Area\, of\ given\ \triangle le \ = \ a^2 units^2} \tag{2.1.16}$$

3 Numerical Example

Let,

$$\mathbf{u} = \begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{3.1.17}$$

Solution: By substituting the given values in (2.1.1), We get

$$\mathbf{P} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \tag{3.1.18}$$

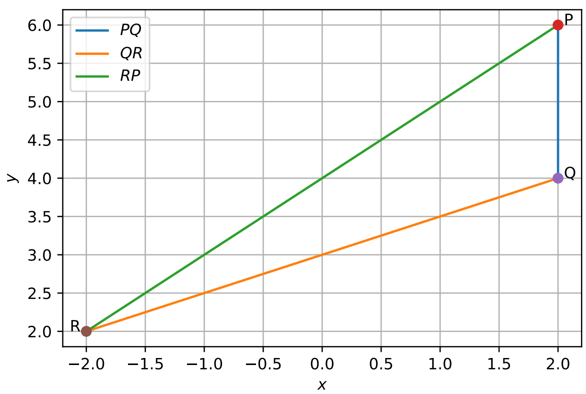

Fig. 3.1: Plot obtained from Python code

Using (2.1.16),

$$\begin{aligned} Area\,of\,\triangle le &= a^2 && (3.1.19)\\ &= 2^2 && (3.1.20)\\ &= 4\,units^2 && (3.1.21)\end{aligned}$$