Definite Integrals and Applications of Integrals: JEE Maths

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1. $$f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$$
   Then $\int_0^{\pi/2} f(x)dx$=......
2. The integral $\int_0^{1.5}[x^2]dx$, Where [ ] denotes the greatest integer function, equals.......
3. The value of
   $$\int_{-2}^{2} |1 - x^2| dx =$$
4. The value of
   $$\int_{\pi/4}^{3\pi/4} \frac{\phi}{1 + \sin\phi} d\phi =$$
5. The value of
   $$\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx =$$
6. If for non-zero x,
   $$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$$
   where $a \neq b$, then $\int_1^2 f(x)dx =$ ....
7. For $n > 0$,
   $$\int_0^{2\pi} \frac{x \sin^{2n} x}{\sin^{2n} x + \cos^{2n} x} dx =$$
8. The value of
   $$\int_1^{e^{37}} \frac{\pi \sin(\pi ln x)}{x} dx =$$
9. Let $\frac{d}{dx}F(x) = \frac{e^{\sin x}}{x}$, $x > 0$. If
   $$\int_1^4 \frac{2e^{\sin x^2}}{x} = F(k) - F(1)$$
   then one of the possible values of k is..........

True/False

10. The value of the integral
   $$\int_0^{2a} \frac{f(x)}{f(x) + f(2a - x)} dx$$
   is equal to a.

MCQs with One Correct Answer

11. The value of the definite integral $\int_0^1 (1 + e^{-x^2}) dx$ is
    a) -1
    b) 2
    c) $1 + e^{-1}$
    d) None of these
12. Let a, b, c be non-zero real numbers such that
    $$\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx$$
    Then the quadratic equation $ax^2 + bx + c = 0$ has
    a) no root in (0, 2)
    b) at least one root in (0, 2)
    c) a double root in (0, 2)
    d) two imaginary roots
13. The area bounded by the curves $y = f(x)$, the x-axis and the ordinates x = 1 and x = b is $(b - 1)\sin(3b + 4)$. Then $f(x)$ is
    a) $(x - 1)\cos(3x + 4)$
    b) $\sin(3x + 4)$
    c) $\sin(3x + 4) + 3(x - 1)\cos(3x + 4)$
    d) None of these

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14. The value of the integral

$$\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx =$$

a) $\pi/4$
b) $\pi/2$
c) π
d) None of these

15. For any integer n the integral

$$\int_0^{\pi} e^{\cos^2 x} \cos^3(2n+1) dx$$

has the value

a) π
b) 1
c) 0
d) None of these

16. Let $f : R \to R$ and $g : R \to R$ be continuous functions. Then the value of the integral

$$\int_{-\pi/2}^{\pi/2} [f(x) + f(-x)][g(x) - g(-x)] dx =$$

a) π
b) 1
c) -1
d) 0

17. The value of

$$\int_0^{\pi/2} \frac{dx}{1 + \tan^3 x} =$$

a) 0
b) 1
c) $\pi/2$
d) $\pi/4$

18. If $f(x) = A\sin(\frac{\pi x}{2}) + B$, $f'(\frac{1}{2}) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2A}{\pi}$, then constants A and B are

a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$
b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$
c) 0 and $\frac{-4}{\pi}$
d) $\frac{4}{\pi}$ and 0

19. The value of $\int_{\pi}^{2\pi} [2\sin x] dx$ where [ ] represents the greatest integer funcion is

a) $\frac{-5\pi}{3}$
b) $-\pi$
c) $\frac{5\pi}{3}$
d) -2π

20. If

$$g(x) = \int_0^x \cos^4 t dt$$

then $g(x + \pi)$ equals

a) $g(x) + g(\pi)$
b) $g(x) - g(\pi)$
c) $g(x)g(\pi)$
d) $\frac{g(x)}{g(\pi)}$

21.

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x} =$$

a) 2
b) -2
c) 1/2
d) -1/2

22. If for a real number y, [y] is the greatest integer less than or equal to y, then the value of the integral

$$\int_{\pi/2}^{3\pi/2} [2\sin x] dx =$$

a) -π
b) 0
c) $\pi/2$
d) $\pi/2$

23. Let

$$g(x) = \int_0^x f(t) dt$$

where f is such that $\frac{1}{2} \le f(t) \le 1$ for $t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$, for $t \in [1,2]$. Then g(2) satisfies the inequality

a) $\frac{3}{2} \le g(2) < \frac{1}{2}$
b) $0 \le g(2) < 2$
c) $\frac{3}{2} < g(2) \le \frac{5}{2}$
d) $2 < g(2) < 4$

24. If

$$f(x) = \left\{ \begin{array}{ll} e^{\cos x} \sin x & for |x| \le 2 \\ 2 & otherwise \end{array} \right\}$$

then $\int_{-2}^{3} f(x) dx =$

a) 0
b) 1
c) 2
d) 3

25. The value of the integral
$$\int_{e^{-1}}^{e^2} \left|\frac{log_e x}{x}\right| dx =$$
   a) 3/2
   b) 5/2
   c) 3
   d) 5
26. The value of
$$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx =$$
   a) π
   b) $a\pi$
   c) $\pi/2$
   d) 2π
27. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is
   a) 1
   b) 2
   c) $2\sqrt{2}$
   d) 4
28. Let
$$f(x) = \int_1^x \sqrt{2-t^2} dt$$
   Then the real roots of the equation $x^2 - f'(x) = 0$ are
   a) ± 1
   b) $\pm\frac{1}{\sqrt{2}}$
   c) $\pm\frac{1}{2}$
   d) 0 and 1
29. Let $T > 0$ be a fixed real number. Suppose f is a continuous function such that for all $x \in R$, $f(x+T) = f(x)$. If $I = \int_0^T f(x)dx$ then the value of $\int_3^{3+3T} f(2x)dx$ is
   a) 3/2I
   b) 2I
   c) 3I
   d) 6I
30. The integral
$$\int_{-1/2}^{1/2} ([x] + ln(\frac{1+x}{1-x}))dx =$$
   a) $\frac{-1}{2}$
   b) 0
   c) 1
   d) $2ln(\frac{1}{2})$
31. If $l(m,n) = \int_0^1 t^m(1+t)^n dt$ then the expression for l(m, n) in terms of l(m + 1, n - 1) is
   a) $\frac{2^n}{m+1} - \frac{n}{m+1} l(m+1, n-1)$
   b) $\frac{n}{m+1} l(m+1, n-1)$
   c) $\frac{2^n}{m+1} + \frac{n}{m+1} l(m+1, n-1)$
   d) $\frac{m}{m+1} l(m+1, n-1)$
32. If
$$f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$$
   then, f(x) increases in
   a) (-2, 2)
   b) No value of x
   c) $(0, \infty)$
   d) $(-\infty, 0)$
33. The area bounded by the curves $y = \sqrt{x}$, 2y + 3 = x and x-axis in the 1^{st} quadrant is
   a) 9
   b) 27/4
   c) 36
   d) 18
34. If f(x) is differentiable and
$$\int_0^{t^2} xf(x)dx = \frac{2}{5}t^5$$
   then $f(\frac{4}{25})$ equals
   a) 2/5
   b) -5/2
   c) 1
   d) 5/2
35. The value of the integral
$$\int_0^1 \sqrt{\frac{1-x}{1+x}} dx =$$
   a) $\frac{\pi}{2} + 1$
   b) $\frac{\pi}{2} - 1$
   c) -1
   d) 1
36. The area enclosed between the curves $y = ax^2$ and $x = ay^2 (a > 0)$ is 1 sq. unit, then the value of a is
   a) $1/\sqrt{3}$
   b) 1/2
   c) 1
   d) 1/3

37.
$$\int_{-2}^{0}\{x^3+3x^2+3x+3+(x+1)\cos(x+1)\}dx=$$
a) -4
b) 0
c) 4
d) 6

38. The area bounded by the parabolas $y=(x+1)^2$ and $y=(x-1)^2$ and the line $y=1/4$ is
a) 4 sq.units
b) 1/6 sq.units
c) 4/3 sq.units
d) 1/3 sq.units

39. The area of the region between the curves $y=\sqrt{\frac{1+\sin x}{\cos x}}$ and $y=\sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines x = 0 and $x=\frac{\pi}{4}$ is
a) $\int_0^{\sqrt{2}-1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$
b) $\int_0^{\sqrt{2}-1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$
c) $\int_0^{\sqrt{2}+1}\frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$
d) $\int_0^{\sqrt{2}+1}\frac{t}{(1+t^2)\sqrt{1-t^2}}dt$

40. Let f be a non-negative function defined on the interval [0, 1]. If
$$\int_0^x\sqrt{1-(f'(t))^2}dt=\int_0^x f(t)dt,$$
$0\le x\le 1$, and f(0) = 0, then
a) $f(\frac{1}{2})<\frac{1}{2}$ and $f(\frac{1}{3})>\frac{1}{3}$
b) $f(\frac{1}{2})>\frac{1}{2}$ and $f(\frac{1}{3})>\frac{1}{3}$
c) $f(\frac{1}{2})<\frac{1}{2}$ and $f(\frac{1}{3})<\frac{1}{3}$
d) $f(\frac{1}{2})>\frac{1}{2}$ and $f(\frac{1}{3})<\frac{1}{3}$

41. The value of
$$\lim_{x\to 0}\frac{1}{x^3}\int_0^x\frac{tln(1+t)}{t^4+4}dt$$
a) 0
b) $\frac{1}{12}$
c) $\frac{1}{24}$
d) $\frac{1}{64}$

42. Let f be a real-valued function defined on the interval (-1, 1) such that
$$e^{-x}f(x)=2+\int_0^x\sqrt{t^4+1}dt$$
for all $x\in(-1,1)$, and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to
a) 1
b) 1/3
c) 1/2
d) 1/e

43. The value of
$$\int_{\sqrt{\ln 2}}^{\sqrt{ln3}}\frac{x\sin x^2}{\sin x^2+\sin(ln6-x^2)}dx=$$
a) $\frac{1}{4}ln\frac{3}{2}$
b) $\frac{1}{2}ln\frac{3}{2}$
c) $ln\frac{3}{2}$
d) $\frac{1}{6}ln\frac{3}{2}$

44. Let the straight line x = b divide the area enclosed by $y=(1-x)^2$, y = 0 and x = 0 into two parts $R_1(0\le x\le b)$ and $R_2(b\le x\le 1)$ such that $R_1-R_2=\frac{1}{4}$. Then b equlas
a) 3/4
b) 1/2
c) 1/3
d) 1/4

45. Let $f:[-1,2]\to[0,\infty)$ be a continuous function such that $f(x)=f(1-x)$ for all $x\in[-1,2]$. Let $R_1=\int_{-1}^{2}xf(x)dx$, x = -1, x = 2 and the x-axis. Then
a) $R_1=2R_2$
b) $R_1=3R_2$
c) $2R_1=R_2$
d) $3R_1=R_2$

46. The value of the integral
$$\int_{-\pi/2}^{\pi/2}(x^2+ln\frac{\pi+x}{\pi-x})\cos xdx$$
a) 0
b) $\frac{\pi^2}{2}-4$
c) $\frac{\pi^2}{4}+4$
d) $\frac{\pi^2}{2}$

47. The area enclosed by the curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ over the interval $[0,\frac{\pi}{2}]$ is
a) $4(\sqrt{2}-1)$
b) $2\sqrt{2}(\sqrt{2}-1)$
c) $2(\sqrt{2}+1)$
d) $2\sqrt{2}(\sqrt{2}+1)$

48. Let $f:[\frac{1}{2},1]\to R$(the set of all real number) be a positive, non-constant and diffrentiable function such that $f'(x)<2f(x)$ and $f(\frac{1}{2})=1$. Then the value of $\int_{1/2}^{1}f(x)dx$ lies in the interval

a) (2e-1, 2e)
b) (e - 1, 2e - 1)
c) $(\frac{e-1}{2}, e-1)$
d) $(0, \frac{e-1}{2})$

49. The following integral

$$\int_{\pi/4}^{\pi/2} (2\operatorname{cosec} x)^{17} dx$$

is equal to

a) $\int_0^{log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} dx$
b) $\int_0^{log(1+\sqrt{2})} (e^u + e^{-u})^{17} dx$
c) $\int_0^{log(1+\sqrt{2})} 2(e^u - e^{-u})^{17} dx$
d) $\int_0^{log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} dx$

50. The value of

$$\int_{\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx$$

is equal to

a) $\frac{\pi^2}{4} - 2$
b) $\frac{\pi^2}{4} + 2$
c) $\pi^2 - e^{\frac{\pi}{2}}$
d) $\pi^2 + e^{\frac{\pi}{2}}$

51. Area of the region

$$\{(x, y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq x + 9 \leq 15\}$$

is equal to

a) $\frac{1}{6}$
b) $\frac{4}{3}$
c) $\frac{3}{2}$
d) $\frac{5}{3}$

52. The area of the region

$$\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\} =$$

a) $8log_e 2 - \frac{14}{3}$
b) $16log_e 2 - \frac{14}{3}$
c) $8log_e 2 - \frac{7}{3}$
d) $16log_e 2 - 6$

MCQs with One or More than One Correct Answer

53. If

$$\int_0^x f(t)dt = x + \int_x^t tf(t)dt$$

then the value of f(1) is

a) 1/2
b) 0
c) 1
d) -1/2

54. Let $f(x) = x - [x]$, for every real number x, where [x] is the integral part of x. Then

$$\int_{-1}^{1} f(x)dx =$$

a) 1
b) 2
c) 0
d) 1/2

55. For which of the following values of m, is the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 9/2?

a) -4
b) -2
c) 2
d) 4

56. Let $f(x)$ be a non-constant twice diffrentiable function defined on $(-\infty, \infty)$ such that $f(x) = f(1-x)$ and $f'(\frac{1}{4}) = 0$. Then,

a) $f''(x)$ vanishes at least twice on [0, 1]
b) $f'(\frac{1}{2}) = 0$
c) $\int_{-1/2}^{1/2} f(x + \frac{1}{2}) \sin x dx = 0$
d) $\int_0^{1/2} f(t)e^{\sin \pi t} dt = \int_{1/2}^{1} f(1-t)e^{\sin \pi t} dt$

57. Area of the region bounded by the curve $y = e^x$ and lines x = 0 and y = e is

a) e - 1
b) $\int_1^e ln(e + 1 - y)dy$
c) $e - \int_0^1 e^x dx$
d) $\int_1^e lny dy$

58. If

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$$

where n = 0, 1 , 2..... then

a) $I_n = I_{n+2}$
b) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$
c) $\sum_{m=1}^{10} I_{2m} = 0$
d) $I_n = I_{n+1}$

59. The value(s) of

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx =$$

a) $\frac{22}{7} - \pi$
b) $\frac{2}{105}$
c) 0

d) $\frac{71}{15} - \frac{3\pi}{2}$

60. Let f be a real-valued function defined on the interval $(0, \infty)$ by

$$f(x) = lnx + \int_0^x \sqrt{1 + \sin t} dt$$

Then which of the following statement(s) is(are) true?

a) $f''(x)$ exists for all $x \in (0, \infty)$
b) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$
c) There exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$
d) There exists $\beta > 1$ such that $|f'(x)| + |f(x)| \leq \beta$ for all $x \in (0, \infty)$

61. Let S be the area of the region enclosed by $y = e^{x^2}$, y = 0, x = 0 and x = 1: then

a) $S \geq \frac{1}{e}$
b) $S \geq 1 - \frac{1}{e}$
c) $S \leq \frac{1}{4}(1 + \frac{1}{\sqrt{e}})$
d) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}(1 - \frac{1}{\sqrt{2}})$

62. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)dt} = L?$$

a) a = 2, $L = \frac{e^{4\pi}-1}{e^{\pi}-1}$
b) a = 2, $L = \frac{e^{4\pi}+1}{e^{\pi}+1}$
c) a = 4, $L = \frac{e^{4\pi}-1}{e^{\pi}-1}$
d) a = 4, $L = \frac{e^{4\pi}+1}{e^{\pi}+1}$

63. Let

$$f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$$

for all $x \in (\frac{\pi}{2}, \frac{\pi}{2})$. Then the correct expression(s) is(are)

a) $\int_0^{\pi/4} xf(x)dx = \frac{1}{12}$
b) $\int_0^{\pi/4} f(x)dx = 0$
c) $\int_0^{\pi/4} xf(x)dx = \frac{1}{6}$
d) $\int_0^{\pi/4} f(x)dx = 1$

64. Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f(\frac{1}{2}) = 0$. If

$$m <\leq \int_{1/2}^1 f(x)dx \leq M$$

then the possible values of m and M are

a) m = 13, M = 24
b) m = 0.25, M = 0.5
c) m = -11, M = 0
d) m = 1, M = 12

65. Let

$$f(x) = \lim_{x\to\infty}\left(\frac{n^n(x+n)(x+\frac{n}{2}).....(x+\frac{n}{n})}{n!(x^2+n^2)(x^2+\frac{n^2}{4}).......(x^2+\frac{n^2}{n^2})}\right)^{\frac{x}{n}}$$

for all $x > 0$. Then

a) $f(\frac{1}{2}) \geq f(1)$
b) $f(\frac{1}{3}) \leq f(\frac{2}{3})$
c) $f'(2) \leq 0$
d) $\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$

66. Let $f : R \to (0, 1)$ be a continuous function. Then, which of the following function(s) has(have) the value zero at some point in the interval (0, 1)?

a) $x^9 - f(x)$
b) $x - \int_0^{\frac{\pi}{2}-x} f(t)\cos t dt$
c) $e^x - \int_0^x f(t)\sin t dt$
d) $f(x) + \int_0^{\pi/2} f(t)\sin t dt$

67. If

$$g(x) = \int \sin x^{\sin(2x)} \sin^{-1}(t)dt$$

then,

a) $g'(\frac{\pi}{2}) = -2\pi$
b) $g'(\frac{-\pi}{2}) = 2\pi$
c) $g'(\frac{\pi}{2}) = 2\pi$
d) $g'(\frac{-\pi}{2}) = -2\pi$

68. If the line $sx = \alpha$ divides the area of the region

$$R = \{(x, y) \in R^2 : x^3 \leq y \leq x, 0 \leq x \leq 1\}$$

into two equal parts, then

a) $0 < \alpha \leq \frac{1}{2}$
b) $\frac{1}{2} < \alpha < 1$
c) $2\alpha^4 - 4\alpha^2 + 1 = 0$
d) $\alpha^4 + 4\alpha^2 - 1 = 0$

69. If

$$I = \sum_{k=1}^{98} \int_k^{k+1} \frac{k+1}{x(x+1)}dx, then$$

a) $1 > log_e 99$
b) $1 < log_e 99$
c) $1 < \frac{49}{50}$
d) $1 > \frac{49}{50}$

70. For, $a \in R$, $|a| > 1$, let

$$\lim_{x\to\infty} \frac{1 + 2^{3/2} + ... + n^{3/2}}{n^{7/3}(\frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + ... + \frac{1}{(an+n)^2})} = 54$$

Then the possible value(s) of a is/are

a) -9
b) 7
c) 6
d) 8

Subjective Problems

71. Find the area bounded by the curve $x^2 = 4y$ and the straight line x = 4y - 2.

72. Show that:

$$\lim_{n\to\infty}(\frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{6n}) = log6$$

73. Show that

$$\int_0^{\pi} xf(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$$

74. Find the value of

$$\int_{-1}^{3/2} |x \sin \pi x| dx$$

75. For any real t, $x = \frac{e^t+e^{-t}}{2}$, $y = \frac{e^t-e^{-t}}{2}$ is a point on the hyperbola $x^2 - y^2 = 1$. Show that the area bounded by the this hyperbola and the line joining its centre to the points corresponding to t_1 and $-t_1$ is t_1.

76. Evaluate

$$\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$$

77. Find the area bounded by the x-axis, part of the curve $y = (1 + \frac{8}{x^2})$ and the ordinates at x = 2 and x = 4. If the ordinate at x = a divides the area into two equal parts, find a.

78. Evaluate the follwing

$$\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx.$$

79. Find the area of the region bounded by the x-axis and the curves defined by

$$y = \tan x, \frac{-\pi}{3} \le x \le \frac{\pi}{3}$$

$$y = \cot x, \frac{\pi}{6} \le x \le \frac{3\pi}{2}$$

80. Given a function $f(x)$ such that

a) it is integratable over every interval on the real line and
b) $f(t + x) = f(x)$, for every x and a real t, then Show that the integral $\int_a^{a+1} f(x)dx$ is independent of a.

81. Evaluate the following:

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx$$

82. Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and find its area.

83. Evaluate

$$\int_0^{\pi} \frac{xdx}{1 + \cos\alpha \sin x}, 0 < \alpha < \pi$$

84. Find the area bounded by the curves

$$x^3 + y^2 = 25, \tag{84.1}$$

$4y = |4 - x^2|$ and x = 0 above the x-axis.

85. Find the area of the region bounded by the curves $C : y = \tan x$, tangent drawn to C at $x = \frac{\pi}{4}$ and the x-axis.

86. Evaluate

$$\int_0^1 log[\sqrt{1 - x} + \sqrt{1 + x}]dx$$

87. If f and g are continuous function on [0, a] satisfying $f(x) = f(a-x)$ and $g(x)+g(a-x) = 2$, then Show that

$$\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$$

88. Show that

$$\int_0^{\pi/2} f(\sin 2x)\sin x dx = \sqrt{2}\int_0^{\pi/4} f(\cos 2x)\cos x dx.$$

89. Prove that for any positive integer k,

$$\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + ... + \cos(2k - 1)x]$$

Hence prove that

$$\int_0^{\pi/2} \sin 2kx \cot x dx = \frac{\pi}{2}$$

90. Compute the area of the region bounded by the curves $y = exlnx$ and $y = \frac{lnx}{ex}$ where $lne = 1$.

91. Sketch the curves and identify the region bounded by the $x = \frac{1}{2}$, x = 2, $y = lnx$ and $y = 2^x$. Find the area of the region.

92. Evaluate

$$\int_0^{\pi} \frac{x\sin 2x \sin(\frac{\pi}{2}\cos x)}{2x-\pi}dx$$

93. Sketch the region bounded by the curves $y = x^2$ and $y = \frac{2}{1+x^2}$. Find the area.

94. Determine a positive integer $n \leq 5$, such that

$$\int_0^1 e^x(x-1)^n dx = 16 - 6e$$

95. Evaluate

$$\int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)}dx$$

96. Show that

$$\int_0^{n\pi+v} |\sin x| dx = 2n + 1 - \cos v$$

where n is a positive integer and $0 \leq v < \pi$.

97. In what ratio does the x-axis divide the area of the region bounded by the parabolas $y = 4x - x^2$ and $y = x^2 - x$?

98. Let

$$I_m = \int_0^{\pi} \frac{1-\cos mx}{1-\cos x}dx$$

Use the mathematical induction to prove that $I_m = m\pi$, m = 0, 1, 2........

99. Evaluate the definite integral

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \left(\frac{x^4}{1-x^4}\right)\cos^{-1}\left(\frac{2x}{1+x^2}\right)dx$$

100. Consider a square with vertices at (1, 1), (-1, 1), (-1, -1) and (1, -1). Let S be the region consisting of all points inside the square which are near to the origin than to any edge. Sketch the region S and find its area.

101. Let A_n be the area bounded by the curve $y = (\tan x)^n$ and the lines x = 0, y = 0 and $x = \frac{\pi}{4}$. Prove that $n > 2$,

$$A_n + A_{n-2} = \frac{1}{n-1}$$

and deduce

$$\frac{1}{2n+2} < A_n < \frac{1}{2n-2}$$

102. Determine the value of

$$\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx.$$

103. Let

$$f(x) = Maximum\{x^2, (1-x^2), 2x(1-x)\}$$

where $0 \leq x \leq 1$. Determine the area of the region bounded by the curves $y = f(x)$ x-axis x = 0 and x = 1.

104. Prove that

$$\int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = 2\int_0^1 \tan^{-1} x dx$$

Hence or otherwise, evaluate the integral

$$\int_0^1 \tan^{-1}\left(1-x+x^2\right)dx$$

105. Integrate

$$\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}}dx.$$

106. Let $f(x)$ be a continuous function given by

$$f(x) = \left\{ \begin{array}{ll} 2x & |x| \leq 1 \\ x^2 + ax + b & |x| > 1 \end{array} \right\}$$

Find the area of the region in the third quadrant bounded by the curves $x = -2y^2$ and $y = f(x)$ lying on the left of the line $8x + 1 = 0$.

107. For $x > 0$, let

$$f(x) = \int_e^x \frac{lnt}{1+t}dt$$

Find the function $f(x) + f(\frac{1}{x})$ and show that $f(e) + f(\frac{1}{e}) = \frac{1}{2}$.

108. Let $b \neq 0$ and for j = 0, 1, 2, ...n, let S_j be the area of the region bounded by the y-axis and the curve $xe^{ay} = \sin by$, $\frac{jr}{b} \leq y \leq \frac{(j+1)\pi}{b}$. Show that S_0, S_1, S_2,.......S_n are in geometric progression. Also, find their sum for a = -1 and $b = \pi$.

109. Find the area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and y = 2, which lies to the right of the line x = 1.

110. If f is an even function then prove that

$$\int_0^{\pi/2} f(\cos 2x)\cos x dx = \sqrt{2}\int_0^{\pi/4} f(\sin 2x)\cos x dx$$

111. Find the value of

$$\int_{-\pi/3}^{\pi/3} \frac{\pi + 4x^3}{2 - \cos\left(|x| + \frac{\pi}{3}\right)}dx$$

112. If

$$y(x) = \int_{\pi^2/16}^{x^2} \frac{\cos x \cos \sqrt{\theta}}{1 + \sin^2 \sqrt{\theta}} d\theta$$

find $\frac{dy}{dx}$ at $x = \pi$

113. Evaluate

$$\int_0^{\pi} e^{|\cos x|} \left(2 \sin\left(\frac{1}{2}\cos x\right) + 3\cos\left(\frac{1}{2}\cos x\right)\right) \sin x dx$$

114. Find the area bounded by the curves $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$.

115. $f(x)$ is a differentiable function and $g(x)$ is a double differentiable function such that $|f(x)| \leq 1$ and $f'(x) = g(x)$. If $f^2(0) + g^2(0) = 9$. Prove that there exists some $c \in (-3, 3)$ such that $g(c).g''(c) < 0$.

116. If

$$\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix} \begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2 + 3a \\ 3b^2 + 3b \\ 3c^2 + 3c \end{bmatrix}$$

$f(x)$ is a quadrant function and its maximum value occurs at a point V. A is a point of intersection of $y = f(x)$ with x-axis and point B is such that chord AB subtends a right angle at V. Find the area enclosed by $f(x)$ and chord AB.

117. Find the value of

$$5050 \frac{\int_0^1 (1 - x^{50})^{100} dx}{\int_0^1 (1 - x^{50})^{101} dx}$$

118. Let $f : R \to R$ be a function defined by

$$f(x) = \left\{ \begin{array}{ll} [x] & x \leq 2 \\ 0 & x > 2 \end{array} \right\}$$

where [x] is the greatest integer less than or equal to x, if

$$I = \int_{-1}^{2} \frac{x f(x^2)}{2 + f(x+1)} dx$$

then the value of (4I - 1) is

119. Let

$$F(x) = \int_x^{x^2 + \frac{\pi}{6}} 2\cos^2 t dt$$

for all $x \in R$ and $f : [0, 0.5]$, if $F'(a) + 2$ is the area of the region bounded by x = 0, y = 0, y = f(x) and x = a, then f(0) is

120. If

$$\alpha = \int_0^1 (e^{9x + 3\tan^{-1} x}) \left(\frac{12 + 9x^2}{1 + x^2}\right) dx$$

where $\tan^{-1} x$ takes only polynomial values, then the value of $\left(log_e|1 + \alpha| - \frac{3\pi}{4}\right)$ is

121. Let $f : R \to R$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose

$$F(x) = \int_{-1}^{x} f(t) dt$$

for all $x \in [-1, 2]$ and

$$G(x) = \int_{-1}^{x} t|f(f(t))| dt$$

for all $x \in [-1, 2]$. If $\lim_{x \to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f(\frac{1}{2})$ is

122. The total number of distinct $x \in [0, 1]$ for which

$$\int_{t^2}^{1+t^4} dt = 2x - 1$$

is

123. Let $f : R \to R$ be a differentiable function such that f(0)=0, $f(\frac{\pi}{2}) = 3$ and $f'(0) = 1$. If

$$g(x) = \int_x^{\pi/2} [f'(t) \operatorname{cosec} t - \cot t \operatorname{cosec} t f(t)] dt$$

for $x \in (0, \frac{\pi}{2}]$, then $\lim_{x \to 0} g(x) =$

124. For positive integer n, let

$$y_n = \frac{1}{n}(n+1)(n+2).....(n+n)^{\frac{1}{n}}$$

For $x \in R$, let [x] be the greatest integer less than or equal to x. If $\lim_{n \to \infty} y_n = L$, then the value of $f(L) =$

125. A farmer F_1 has a land in the shape of triangle with vertices at P(0, 0), Q(1, 1) and R(2, 0). From this land, a neighbouring farmer F_2 takes away the region which lies between the side PQ and a curve of the form $y = x^n (n > 1)$. If the area of the region taken away by the farmer F_2 is exactly 30 percentage of the area of ΔPQR, then the value of n is...........

Match the Following

126. Match the following

Column I	**Column II**
(A) $\int_0^{\pi/2}(\sin x)^{\cos x}\left(\cos x\cot x - log(\sin x)^{\sin x}\right)dx$	(p) 1
(B) Area bounded by $-4y^2 = x$ and $x - 1 = -5y^2$	(q) 0
(C) Cosine of angle of intersection of curves $y = 3^{x-1}logx$ and $y = x^x - 1$ is	(r) $6ln2$
(D) Let $\frac{dy}{dx} = \frac{6}{x+y}$ where y(0) = 0 then value of y when x + y = 6 is	(s) $\frac{4}{3}$

127. Match the following

Column I	**Column II**
(A) $\int_{-1}^{1}\frac{dx}{1+x^2}$	(p) $\frac{1}{2}log\left(\frac{2}{3}\right)$
(B) $\int_0^1\frac{dx}{\sqrt{1-x^2}}$	(q) $2log\left(\frac{2}{3}\right)$
(C) $\int_2^3\frac{dx}{1-x^2}$	(r) $\frac{\pi}{3}$
(D) $\int_1^2\frac{dx}{x\sqrt{x^2-1}}$	(s) $\frac{\pi}{2}$

128. Match the following

Column I	**Column II**
(A) The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying f(0) = 0 and $\int_0^1 f(x)dx = 1$, is	(p) 8
(B) The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ of the maximum value is	(q) 2
(C) $\int_{-2}^{2}\frac{3x^2}{(1+e^x)}dx$ equals	(r) 4
(D) $\dfrac{\int_{-1/2}^{1/2}\cos 2x log\left(\frac{1+x}{1-x}\right)dx}{\int_0^{1/2}\cos 2x log\left(\frac{1+x}{1-x}\right)dx}$	(s) 0

codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	2	3	4	1
(c)	3	2	1	4
(d)	2	3	1	4

Comprehension Based Questions

PASSAGE-1

Let the definite integral be defined by the fomula

$$\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$$

For more acurate results for $c \in (a,b)$ we can use

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx = F(c)$$

so that for $c = \frac{a+b}{2}$, we get

$$\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c)).$$

129. $\int_0^{\pi/2} \sin x dx =$
   a) $\frac{\pi}{8}(1+\sqrt{2})$
   b) $\frac{\pi}{4}(1+\sqrt{2})$
   c) $\frac{\pi}{8\sqrt{2}}$
   d) $\frac{\pi}{4\sqrt{2}}$
130. If

$$\lim_{x\to a} \frac{\int_a^x f(x)dx - \left(\frac{x-a}{2}\right)(f(x)+f(a))}{(x-a)^3} = 0$$

then $f(x)$ is of maximum degree
   a) 4
   b) 3
   c) 2
   d) 1
131. If $f''(x) < 0$ for all $x \in (a,b)$ and c is a point such that $a < c < b$ and $(c, f(c))$ is the point lying on the curve for which $F(c)$ is maximum, then $f'(c)$ is equal to
   a) $\frac{f(b)-f(a)}{b-a}$
   b) $2\frac{f(b)-f(a)}{b-a}$
   c) $2\frac{f(b)-f(a)}{2b-a}$
   d) 0

PASSAGE-2

Consider the functions defined implicity by the equation

$$y^3 - 3y + x = 0 \qquad (131.1)$$

on various intervals in the real line. If $x \in (-\infty,-2) \cup (2,\infty)$, the equation implicity defines a unique real valued differentiable function $y = f(x)$. If $x \in (-2,2)$, the equation implicity defines a unique real valued differentiable function $y = g(x)$ satisfying $g(0) = 0$.

132. If $f(-10\sqrt{2}) = 2\sqrt{2}$, then $f''(-10\sqrt{2}) =$
   a) $\frac{4\sqrt{2}}{7^3 3^2}$
   b) $-\frac{4\sqrt{2}}{7^3 3^2}$
   c) $\frac{4\sqrt{2}}{7^3 3}$
   d) $-\frac{4\sqrt{2}}{7^3 3}$
133. The area of the region bounded by the curve $y = f(x)$, the x-axis and the lines x = a and x = b, where $-\infty < a < b < -2$ is
   a) $\int_a^b \frac{x}{3(f(x))^2-1}dx + bf(b) - af(a)$
   b) $-\int_a^b \frac{x}{3(f(x))^2-1}dx + bf(b) - af(a)$
   c) $\int_a^b \frac{x}{3(f(x))^2-1}dx - bf(b) + af(a)$
   d) $-\int_a^b \frac{x}{3(f(x))^2-1}dx - bf(b) + af(a)$
134.

$$\int_{-1}^{1} g'(x)dx =$$

   a) $2g(-1)$
   b) 0
   c) $-2g(1)$
   d) $2g(1)$

PASAAGE-3

Consider the function $f : (-\infty,\infty) \to (-\infty,\infty)$ defined by

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, 0 < a < 2.$$

135. Which of the following is True?
   a) $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$
   b) $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$
   c) $f'(1)f'(-1) = (2-a)^2$
   d) $f'(1)f'(-1) = -(2-a)^2$
136. Which of the following is True?
   a) $f(x)$ is decreasing on (-1, 1) and has a local minimum at x = 1
   b) $f(x)$ is increasing on (-1, 1) and has a local minimum at x = 1
   c) $f(x)$ is increasing on (-1, 1) but has a neither local maximum nor local minimum at x = 1
   d) $f(x)$ is decreasing on (-1, 1) but has a neither local maximum nor local minimum at x = 1
137. Let

$$g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2}dt$$

Which of the following is True?

a) $g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$
b) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$
c) $g'(x)$ changes sign on both $(-\infty, 0)$ and $(0, \infty)$
d) $g'(x)$ does not change sign on $(-\infty, \infty)$

PASSAGE-4

Consider the polynomial

$$f(x) = 1 + 2x + 3x^2 + 4x^3 \tag{137.1}$$

Let s be the sum of all distinct real roots of $f(x)$ and let $t = |s|$

138. The real numbers lies in the interval
a) $\left(-\frac{1}{4}, 0\right)$
b) $\left(-11, -\frac{3}{4}\right)$
c) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$
d) $\left(0, \frac{1}{4}\right)$

139. The area bounded by the curve $y = f(x)$ and the lines x = 0, y = 0 and x = t lies in the interval
a) $\left(\frac{3}{4}, 3\right)$
b) $\left(\frac{21}{64}, \frac{11}{16}\right)$
c) (9, 10)
d) $\left(0, \frac{21}{64}\right)$

140. The function $f'(x)$ is
a) increasing in $\left(-t, -\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$
b) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$
c) increasing in (-t, t)
d) decreasing in (-t, t)

PASSAGE-5

Given that for each $a \in (0, 1)$,

$$\lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}dt$$

exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on (0, 1).

141. The value of $g\left(\frac{1}{2}\right)$ is
a) π
b) 2π
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4}$

142. The value of $g'\left(\frac{1}{2}\right)$ is
a) $\frac{\pi}{2}$
b) π
c) $-\frac{\pi}{2}$
d) 0

PASSAGE-6

Let $F : R \to R$ be a thrice differentiable function. Suppose that F(1) = 0, F(3) = -4 and $F(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in R$.

143. The correct statement(s) is(are)
a) $f'(1) < 0$
b) $f(2) < 0$
c) $f'(x) \neq 0$ for any $x \in (1, 3)$
d) $f'(x) = 0$ for any $x \in (1, 3)$

144. If

$$\int_1^3 x^2F'(x)dx = -12$$

and

$$\int_1^3 x^3F''(x)dx = 40$$

then the correct expression(s) is(are)
a) $9f'(3) + f'(1) - 32 = 0$
b) $\int_1^3 f(x)dx = 12$
c) $9f'(3) - f'(1) + 32 = 0$
d) $\int_1^3 f(x)dx = -12$

Integer Value Correct Type

145. Let $f : R \to R$ be a continuous function which satisfies

$$f(x) = \int_0^x f(t)dt$$

Then the value of $f(ln5)$ is

146. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [-10, 10] by

$$f(x) = \begin{Bmatrix} x - [x] & if[x]isodd \\ 1 + [x] - x & if[x]iseven \end{Bmatrix}$$

Then the value of

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x dx =$$

147. The value of

$$\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx =$$

148. The value of the integral

$$\int_0^{1/2} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{1/4}}dx$$

is

149. If

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)}$$

then $27I^2$ equals........

150. The value of the integral

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5}d\theta$$

equals............

Section - B

151. $\int_0^{10\pi} |\sin x|dx$ is
   a) 20
   b) 8
   c) 10
   d) 18

152. $I_n = \int_0^{\pi/4} \tan^n x dx$ then $\lim_{n\to\infty} n[I_n + I_{n+2}]$ equals
   a) 1/2
   b) 1
   c) ∞
   d) 0

153. $\int_0^2 [x^2]dx$ is
   a) $2-\sqrt{2}$
   b) $2+\sqrt{2}$
   c) $\sqrt{2}-1$
   d) $-\sqrt{2}-\sqrt{3}+5$

154. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is
   a) $\frac{\pi^2}{4}$
   b) π^2
   c) 0
   d) $\frac{\pi}{2}$

155. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is
   a) 3/2
   b) 1
   c) 5/4
   d) -3/4

156. The area of bounded by the curves $y = lnx$, $y = ln|x|$, $y = |lnx|$ and $y = |ln|x||$ is
   a) 4 sq.units
   b) 6 sq.units
   c) 10 sq.units
   d) None of these

157. If $f(a+b+-x) = f(x)$, then

$$\int_a^b xf(x)dx$$

is equal to
   a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$
   b) $\frac{a+b}{2}\int_a^b f(b-x)dx$
   c) $\frac{a+b}{2}\int_a^b f(x)dx$
   d) $\frac{b-a}{2}\int_a^b f(x)dx$

158. The area of the region bounded by the curves $y = |x-1|$ and $y = 3-|x|$ is
   a) 6 sq.units
   b) 2 sq.units
   c) 3 sq.units
   d) 4 sq.units

159. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral

$$\int_0^1 f(x)g(x)dx =$$

   a) $e + \frac{e^2}{2} + \frac{5}{2}$
   b) $e - \frac{e^2}{2} - \frac{5}{2}$
   c) $e + \frac{e^2}{2} - \frac{3}{2}$
   d) $e - \frac{e^2}{2} - \frac{5}{2}$

160. The value of the integral $I = \int_0^1 x(1-x)^n dx$
   a) $\frac{1}{n+1} + \frac{1}{n+2}$
   b) $\frac{1}{n+1}$
   c) $\frac{1}{n+2}$
   d) $\frac{1}{n+1} - \frac{1}{n+2}$

161. $\lim_{n\to\infty} \sum_{r=1}^{n} e^{\frac{r}{n}} =$
   a) e + 1
   b) e - 1
   c) 1 - e
   d) e

162. The value of

$$\int_{-2}^{3} |1-x^2|dx =$$

   a) $\frac{1}{3}$
   b) $\frac{14}{3}$

c) $\frac{7}{3}$
d) $\frac{28}{3}$

163. The value of

$$I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx =$$

a) 3
b) 1
c) 2
d) 0

164. If $f(x) = \frac{e^x}{1+e^x}$,

$$I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$$

and

$$I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$$

then the value of $\frac{I_2}{I_1}$ is
a) 1
b) -3
c) -1
d) 2

165. The area of the region bounded by the curves $y = |x - 2|$, x = 1, x = 3 and the x-axis is
a) 4
b) 2
c) 3
d) 1

166. If

$$I_1 = \int_0^1 2^{x^2} dx, I_2 = \int_0^1 2^{x^3} dx$$

$$I_3 = \int_1^2 2^{x^2} dx, I_4 = \int_1^2 2^{x^3} dx$$

then
a) $I_2 > I_1$
b) $I_2 < I_1$
c) $I_3 = I_4$
d) $I_3 > I_4$

167. The area enclosed between the curve $y = log_e(x + e)$ and the coordinate axes is
a) 1
b) 2
c) 3
d) 4

168. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines x = 4, y = 4 and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is
a) 1:2:1
b) 1:2:3
c) 2:1:2
d) 1:1:1

169. Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f(\frac{\pi}{2})$ is
a) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$
b) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$
c) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$
d) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

170. The value of

$$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx =$$

a) $a\pi$
b) $\frac{\pi}{2}$
c) $\frac{\pi}{a}$
d) 2π

171. The value of integral

$$\int_3^6 \frac{\sqrt{x}}{\sqrt{9 - x} + \sqrt{x}} dx =$$

a) 1/2
b) 3/2
c) 2
d) 1

172. $\int_0^{\pi} xf(\sin x)dx =$
a) $\pi \int_0^{\pi} f(\cos x)dx$
b) $\pi \int_0^{\pi} f(\sin x)dx$
c) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x)dx$
d) $\pi \int_0^{\pi/2} f(\cos x)dx$

173.

$$\int_{-3\pi/2}^{-\pi/2} [(x + \pi^3) + \cos^2(x + 3\pi)]dx =$$

a) $\frac{\pi^4}{32}$
b) $\frac{\pi^4}{32} + \frac{\pi}{2}$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4} - 1$

174. The value of

$$\int_1^a [x]f'(x)dx$$

$a > 1$ where [x] denotes the greatest integer not exceeding x is

a) $af(a) - \{f(1) + f(2) + ...f([a])\}$
b) $[a]f(a) - \{f(1) + f(2) + ...f([a])\}$
c) $[a]f([a]) - \{f(1) + f(2) + ...f(a)\}$
d) $af([a]) - \{f(1) + f(2) + ...f(a)\}$

175. Let $F(x) = f(x) + f(\frac{1}{x})$, where

$$f(x) = \int_l^t \frac{logt}{1+t}dt$$

Then $F(e)$ equals

a) 1
b) 2
c) 1/2
d) 0

176. The solution for x of the equation

$$\int_{\sqrt{2}}^x \frac{dt}{\sqrt{t^2-1}} = \frac{\pi}{2} =$$

a) $\frac{\sqrt{3}}{2}$
b) $2\sqrt{2}$
c) 2
d) None

177. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is

a) 1/6
b) 1/3
c) 2/3
d) 1

178. Let

$$I = \int_0^1 \frac{\sin x}{\sqrt{x}}dx, J = \int_0^1 \frac{\cos x}{\sqrt{x}}dx$$

Then which one of the following is True?

a) $I > \frac{2}{3}$ and $J > 2$
b) $I < \frac{2}{3}$ and $J < 2$
c) $I < \frac{2}{3}$ and $J > 2$
d) $I > \frac{2}{3}$ and $J < 2$

179. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent of the parabola at the point (2, 3) and the x-axis is

a) 6
b) 9
c) 12
d) 3

180. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is eqaul to

a) 5/3
b) 1/3
c) 2/3
d) 4/3

181. $\int_0^\pi [\cot x]$, where [ ] denotes the greatest integer function is equal to

a) 1
b) -1
c) $\frac{\pi}{2}$
d) $\frac{\pi}{2}$

182. The area of bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

a) $4\sqrt{2} + 2$
b) $4\sqrt{2} - 1$
c) $4\sqrt{2} + 1$
d) $4\sqrt{2} - 2$

183. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, p(0) = 1 and p(1) = 41. Then

$$\int_0^1 p(x)dx =$$

a) 21
b) 41
c) 42
d) $\sqrt{41}$

184. The value of

$$\int_0^1 \frac{8log(1+x)}{1+x^2}dx =$$

a) $\frac{\pi}{8}log2$
b) $\frac{\pi}{2}log2$
c) $log2$
d) $\pi log2$

185. The area of the region enclosed by the curves y = x, x = e, $y = \frac{1}{x}$ and the positive x-axis is

a) 1 sq.units
b) $\frac{3}{3}$ sq.units
c) $\frac{5}{3}$ sq.units
d) $\frac{1}{2}$ sq.units

186. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is

a) $20\sqrt{2}$
b) $\frac{10\sqrt{2}}{3}$

c) $\frac{20\sqrt{2}}{2}$
d) $10\sqrt{2}$

187. If

$$g(x) = \int_0^x \cos 4t dt$$

then $g(x+\pi)$ is equal to

a) $\frac{g(x)}{g(\pi)}$
b) $g(x) + g(\pi)$
c) $g(x) - g(\pi)$
d) $g(x).g(\pi)$

188. **Statement-1:** The value of the integral

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$$

is equal to $\pi/6$

Statement-2:

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-2
b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-2
c) Statement-1 is true, Statement-2 is false
d) Statement-1 is false, Statement-2 is true

189. The area(in square units) bounded by the curves $y = \sqrt{x}$, 2y - x + 3 = 0, x-axis and lying in the first quadrant is

a) 9
b) 36
c) 18
d) 27/4

190. The integral

$$\int_0^\pi \sqrt{1 + 4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}}dx =$$

a) $4\sqrt{3} - 4$
b) $4\sqrt{3} - 4 - \frac{\pi}{3}$
c) $\pi - 4$
d) $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

191. The area of the region bounded by

$$\{(x,y) : y^2 \le 2x, y \ge 4x - 1\} =$$

a) $\frac{15}{64}$
b) $\frac{9}{32}$
c) $\frac{7}{32}$
d) $\frac{5}{64}$

192. The area of the region bounded by

$$A = \{(x,y) : x^2 + y^2 \le 1, y^2 \le 1 - x\} =$$

a) $\frac{\pi}{2} - \frac{2}{3}$
b) $\frac{\pi}{2} + \frac{2}{3}$
c) $\frac{\pi}{2} + \frac{4}{3}$
d) $\frac{\pi}{2} - \frac{4}{3}$

193. The integral

$$\int_2^4 \frac{logx^2}{logx^2 + log(36 - 12x + x^2)}dx$$

is equal to

a) 1
b) 6
c) 2
d) 4

194. The area(in square units) of the region

$$\{(x,y) : y^2 \ge 2x, x^2 + y^2 \le 4x, x \ge 0, y \ge 0\} =$$

a) $\pi - \frac{4\sqrt{2}}{¢3}$
b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
c) $\pi - \frac{4}{3}$
d) $\pi - \frac{8}{3}$

195. The area(in square units) of the region

$$\{(x,y) : x \ge 0, x + y \le 3, x^2 \le 4y, y \ge 1 + \sqrt{x}\} =$$

a) 5/2
b) 59/12
c) 3/2
d) 7/3

196. The integral

$$\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x} =$$

a) -1
b) -2
c) 2
d) 4

197. Let $g(x) = \cos^2 x$, $f(x) = \sqrt{x}$ and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation

$$18x^2 - 9\pi x + \pi^2 = 0 \qquad (197.1)$$

Then the area(in sq.units) bounded by the curve $y = (gof)(x)$ and he lines $x = \alpha$, $x = \beta$ and y = 0 is

a) $\frac{1}{2}(\sqrt{3}+1)$
b) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$
c) $\frac{1}{2}(\sqrt{2}-1)$
d) $\frac{1}{2}(\sqrt{3}-1)$

198. The value of

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx =$$

a) $\frac{\pi}{2}$
b) 4π
c) $\frac{\pi}{4}$
d) $\frac{\pi}{8}$

199. The value of

$$\int_{0}^{\pi} |\cos x| dx =$$

a) 0
b) 4/3
c) 2/3
d) -4/3

200. The area(in sq.units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y-axis is

a) 8/3
b) 32/3
c) 56/3
d) 14/3

201. The value of

$$\int_{0}^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx =$$

a) $\frac{\pi-2}{8}$
b) $\frac{\pi-1}{4}$
c) $\frac{\pi-2}{4}$
d) $\frac{\pi-1}{2}$

202. The area(in sq.units) of the region

$$A = \{(x, y) : x^2 \le y \le x + 2\} =$$

a) 10/3
b) 9/2
c) 31/6
d) 13/6