3. Using Master Method

a) $T(N) = 2T(N/2) + N^4$

$a = 2 \quad b = 2 \quad c = 4$

$\log_2 2 = 1 \quad c = 4 \quad c > \log_b a$

$\therefore$ Using 3 $T(N) = \underline{\underline{\theta(N^4)}}$

b) $T(N) = T(9N/10) + N$

$a = 1 \quad b = 10 \quad c = 1$

$\log_{10} 1 = 0 \quad c = 1 \quad c > \log_b a$

using 3 we have $T(N) = \theta(f(n))$

$\underline{\underline{T(N) = \theta(N)}}$

c) $T(N) = 16T(N/4) + N^2$

$a = 16 \quad b = 4 \quad c = 2$

$\log_b a = \log_4 16 = 2 \quad c = 2$

$c = \log_b a$ using rule 2

$T(N) = \underline{\underline{\theta(n^2 \log n)}}$

d) $T(N) = 2T\left(\frac{N}{4}\right) + \sqrt{N}$

$n^{\log_b a} = n$

$\therefore n^{\log_4 2 - 0.5} = n^{0.5 - 0.5} = n^0$

$f(n) = \sqrt{N} \in O(\sqrt{N} = O(N^{0.5})$

Now T(N) becomes

$T(N) = 2T\left(\frac{N}{4}\right) + O(N^{0.5})$

$a = 2 \quad b = 4 \quad c = 0.5$

$\log_b a \Rightarrow \log_4 2 = \frac{\log 2}{\log 4} = 0.5$

$c = \log_b a$ so using rule 2

$n^{0.5} \log n \Rightarrow \theta(\sqrt{N} \log N)$

e) $T(N) = T(N-1) + N$

$T(N-1) = T(N-2) + N - 1$

$T(N-2) = T(N-3) + N - 2$

$T(N-3) = T(N-4) + N - 3$

$T(N) = T(N-K) + KN - K(K-1)/2$

Substituting $T(1)$

$N - K = 1 \Rightarrow K = N - 1$

$T(N) = T(1) + (N-1)N - (N-1)(N-2)/2$

$T(N) = O(N^2)$

b) $T(N) = T(\sqrt{N}) + 1$

Considering $\sqrt{N}$ as $\lfloor \sqrt{N} \rfloor$

$T(N) = T(\lfloor \sqrt{N} \rfloor) + 1$

Assuming N as very big number 2^{2^k}

$T(2^{2^k}) = T(\lfloor \sqrt{2^{2^k}} \rfloor) + 1$

$= T(2^{2^{k-1}}) + 1$

$= T(2^{2^{k-2}}) + 1 + 1 = T(2^{2^{k-2}}) + 2$

Similarly

$= T(2^{2^{k-3}}) + 3$

$= T(2) + k$ — ①

Substituting for $T(2)$

$T(2) = T(\lfloor \sqrt{2} \rfloor) + 1 = T(1) + 1 = 1 + 1 = 2$ — ②

Using ① & ②

$T(2^{2^k}) = k + 2$ also, $T(2^{2^{k+1}} - 1) = T(2^{2^k})$

$\therefore$ for all $N \leq M$ $T(N) < T(M)$

$\therefore$ $T(N) = \log(\log N) + 2$ if $N > 1$

$\Rightarrow$ $\underline{\theta(\log(\log N))}$