2) Solve the following recurrence relations

a) $T(N) = T(N-1) + 5$ if $n > 1$
$T(1) = 0$

$$T(N) = T(N-1) + 5$$
$$= T(N-2) + 10$$
$$= T(N-3) + 15$$
$$= T(N-k) + 5k$$

To substitute $T(1)$

$$N - k = 0 \quad N = k$$

$$T(N) = 5N + 1 \approx \underline{\underline{O(N)}}$$

b) $T(N) = 3T(N-1)$ if $N > 1$
$T(1) = 4$ if $N = 1$

$$T(N) = 3T(N-1)$$
$$= 3(3T(N-2))$$
$$= 9T(N-2)$$

$$= 9(3T(N-3))$$
$$= 27T(N-3)$$

$$= 27(3T(N-4))$$
$$= 81T(N-4)$$

$= 3^k T(N-k)$

using $T(1)$ we have

$N-k = 4 \quad \therefore N = k+4$

$\& \; k = n-4$

$\therefore 3^{n-4} \simeq \underline{\underline{O(3^n)}}$

c) $T(N) = T(\frac{N}{3}) + 1 \quad$ if $n > 1$

$T(1) = 1 \quad$ if $n = 1$

$T(N) = T(\frac{N}{3}) + 1$

$= T(\frac{N}{9}) + 2$

$= T(\frac{N}{27}) + 3$

$= T(\frac{N}{81}) + 4$

$= T(\frac{N}{3^k}) + k$

using $T(1)$ we have $\frac{N}{3^k} = 1$

$\therefore k = \log_3 n$

$\underline{\underline{O(\log n)}}$