

Aim: To find the solution of given equation by Gauss-Seidel method.

Apparatus Required: Laptop - lenovo ideapad 3(16s) Windows 11,
MATLAB 2021a.

Theory:

The Gauss seidel method is a improvisation of the Jacobi method. The method is named after mathematicians Carl Friedrich Gauss (1777-1855) and Philipp L Seidel (1821-1896). This modification often results in higher degree of accuracy within fewer iterations.

In Gauss Jacobi method, the value of $x_i^{(k)}$ obtained in the k th iteration remain unchanged until the entire $(k+1)$ th iteration has been calculated with the Gauss seidel, we use the new values, $x_i^{(k+1)}$ as soon as they are known for example, once we have computed $x_1^{(k+1)}$, from the first equation, its value is then used in the second equation to obtain the new $x_2^{(k+1)}$, and so on.

the system given by

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n &= b_n \end{aligned}$$

$$x_1 = \frac{1}{a_{11}} (b_1 - a_{12}x_2 - a_{13}x_3 - \dots - a_{1n}x_n)$$

$$x_2 = \frac{1}{a_{22}} (b_2 - a_{21}x_1 - a_{23}x_3 - \dots - a_{2n}x_n)$$

$$x_n = \frac{1}{a_{nn}} (b_n - a_{n1}x_1 - a_{n2}x_2 - \dots - a_{n,n-1}x_{n-1})$$

ix Gauss Seidel method: for each $k \geq 1$ generate the components n_i^k of x^k from $n^{(k-1)}$ by

$$n_i^{(k)} = \frac{1}{a_{ii}} \left[b_i - \sum_{j=1}^{i-1} a_{ij} n_j^{(k)} - \sum_{j=i+1}^n a_{ij} n_j^{(k-1)} \right]$$

for $i=1, 2, \dots, n$

Given Equation: is To find the soln of given equation by Gauss Seidel method.

$$3x + 4y + 15z = 54.8$$

$$x + 12y + z = 39.66$$

$$10x + y - 2z = 7.74$$

Soln:

First we have to rearrange the equation as because of the coefficient matrix of the given system is not diagonally dominant

$$\text{So, } 10x + y - 2z = 7.74$$

$$x + 12y + z = 39.66$$

$$3x + 4y + 15z = 54.8$$

Now,

$$x = \frac{1}{10} (7.74 - y + 2z)$$

$$y = \frac{1}{12} (39.66 - x - z)$$

$$z = \frac{1}{15} (54.8 - 3x - 4y)$$

n	n	y	z
1	0.114	3.2405	2.6344
2	0.9768	3.0041	2.6569
3	1.005	2.998	2.6524
4	1.0045	3.003	2.8524

So, $n \approx 1.0045$

$y \approx 3.003$

$z \approx 2.6524$

i) To find the solⁿ of given equation by Gauss seidel method

$$9n - 2y + z = 50$$

$$n + 5y - 3z = 18$$

$$-2n + 2y + 7z = 19$$

Here the coefficient matrix of the given system is diagonally dominant so we can rearrange the equations as,

$$n = \frac{1}{9} (50 + 2y - z)$$

$$y = \frac{1}{5} (18 - n + 3z)$$

$$z = \frac{1}{7} (19 + 2n + 2y)$$

So, $n \approx 6.1540$

$y \approx 4.3134$

$z \approx 3.2402$

MATLAB CODE :

clear all

clc

A = input('Enter the coefficient matrix A (In diagonal dominant):');

B = input('Enter the constant matrix B:');

P = [A B];

[row, col] = size(P);

x = zeros(row, 1);

c = zeros(row, 1);

Err = ones(row, 1);

tol = input('Enter the tolerance of error:');

while Err > tol

for m=1:row

$$x(m, 1) = (P(m, col) - \sum (A(m, :) + x(:, 1)) + A(m, m) x(m, 1)) / P(m, m);$$

$$Err(m, 1) = \text{abs}(l(m, 1) - x(m, 1));$$

$$c(m, 1) = x(m, 1);$$

end

end

disp('The required solution is :');

disp x

(1) INPUT:

Enter the coefficient matrix A (in diagonal dominant):

[10 1 -2; 1 12 1; 3 9 15]

Enter the constant matrix B: [7.74; 31.66; 54.8]

Enter the tolerance of error: 0.0001

OUTPUT

The required solution is

1.0045

3.0003

2.0524

(ii) INPUT:

Enter the coefficient matrix A (in diagonal dominant);

$[9 \ -2 \ 1; 1 \ 5 \ -3; -2 \ 2 \ 7]$

Enter the constant matrix B : $[50; 18; 19]$

Enter the tolerance of error: 0.0001

OUTPUT

The required solution is:

6.1541

4.3133

3.2402

Conclusion: we have done this experiment in MATLAB software. By this experiment we find the solution of linear equation by using Gauss-Seidel method and verify the result with MATLAB output successfully.

Reference:

B.S. Grewal, "Numerical method in engineering and science" by motilal learning and information LTC, 3rd Edition 2014

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