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Aim:→ To find the solution of given equation and study about 4th order R.K method with MATLAB programming.

Apparatus Required:→
MATLAB / PC

Theory:→ RK method is most widely used to solve the 1st order ordinary differential equation. this method gives better approximation than euler's method. This method will give more accuracy without performing more calculations one of the most significant advantage of Runga-Kutta ~~...~~ formulae is that it require the function value ~~of~~ some specified points.
Now consider ordinary differential equation of first order

$$\frac{dy}{dx} = f(x,y) \quad \text{- - - equation 1}$$

initial condition: at $x = x_0$, $y = y_0$

* the formulae of R.K method of 4th order.

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4); \quad n = 0,1,2,3.$$

Where,

$$K_1 = hf(x_n, y_n)$$

$$K_2 = hf(x_n + h/2, y_n + K_1/2)$$

$$K_3 = hf(x_n + h/2, y_n + K_2/2)$$

$$K_4 = hf(x_n + h, y_n + K_3)$$

the truncation errors in R-K method of second, third & fourth order methods are $O(h^3)$ and $O(h^5)$. The truncation error

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the truncation error in R-K method of 4th order is higher so accuracy of it is higher than first order second order and third order R-K method

Given Question

1) $\frac{dy}{dx} = -xy$ Calculate $y(0.2)$ and $y(0.4)$ with $y(0)=1$

Sol^n $\frac{dy}{dx} = -xy$

Let, $f(x_n, y_n) = -x_n y_n$

for $n=0$, $x_0 = 0$ and $y_0 = 1$

$x_1 = 0.2$

$h = x_1 - x_0 = 0.2 - 0 = \boxed{0.2}$

∴ using R-K method of 4th order

$k_1 = hf(x_0, y_0) = 0.2 \times (-0 \times 1) = 0$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$0.2 \times f(0.1, 1) = 0.2 \times (-0.1 \times 1) = \boxed{-0.02}$

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

$0.2 \times f(0.1, 0.99) = 0.2 \times (-0.1 \times 0.99)$

-0.0198

$k_4 = hf(x_0 + h, y_0 + k_3)$

$0.2 \times (-0.2 \times 0.9802)$

$= -0.039208$

So, now,

$y_1 = y_0 + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$

$1 + 1/6(0 + 2 \times (-0.02) + 2 \times (-0.0198)$

$\boxed{0.98019866667}$ -0.039208

$\therefore y_1 = 0.9802$

or $y(0.2) = 0.9802$

$\because$ for, $n=1$, $y_1 = 0.9802$, $x_1 = 0.2$ and $h = 0.2$

$k_1 = hf(x_1, y_1) = 0.2 \times (-0.2 \times 0.9802)$

$\therefore k_1 = -0.0392$

$k_2 = hf(x_1 + h/2, y_1 + k/2)$

$0.2 \times (-0.3 \times (0.960596))$

$\boxed{-0.057636}$

$k_3 = hf(x_1 + h/2, y_1 + k_2/2)$

$0.2 \times (-0.3 \times 0.951382)$

$\boxed{-0.057083}$

$k_4 = hf(x + h, y_1 + k_3)$

$0.2 \times (-0.4 \times (0.951658))$

$= -0.07385$

So now,

$\therefore y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 0.9802 + \frac{1}{6}(-0.0392 - 2 \times 0.057636 - 2 \times 0.057083 - 0.07385)$

$= 0.923114$

$\simeq 0.9231$

$y(0.4) \simeq \boxed{0.9231}$

2) $\frac{dy}{dx} = \frac{x^2+y^2}{10}$ calculation $x(0.1)$ and $x(0.2)$ with initial condition $y(0) = 1$

Soln Let $f(x_n, y_n) = \frac{x^2+y^2}{10}$

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for, $x = 0$

$x_0 = 0$ and $y_0 = 1$; $x_1 = 0.1$

$f(x_0, y_0) = \frac{0+1}{10} = 0.1$

$h = x_1 - x_0 = 0.1$

$k_1 = hf(x_0, y_0)$

$= 0.1 \times \left(\frac{0^2 + 1^2}{10}\right) \Rightarrow 0.01$

$k_2 = hf(x_0 + h/2, y_0 + k/2)$

$= 0.1 \times \left(\frac{0.05^2 + 1.005^2}{10}\right)$

0.010125

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

$0.1 \times \left(\frac{0.05^2 + 1.005\,0625^2}{10}\right)$

$= \boxed{0.010126}$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$\frac{0.1 \times (0.1^2 + (1.010126)^2)}{10}$

$\boxed{0.0103035}$

So, now

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$1 + \frac{1}{6}(0.01 + 2 \times (0.010125) + 2(0.010126 + 0.0103035$

$y_1 \approx 1.01013$

$\therefore y(0.1) = 1.0101$

Now for $y(0.2)$, taking $x = 1$

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$x_1 = 0.2$, $y_1 = 1.0101$ and $h = 0.1$

So, $k_1 = 0.1 \times f(x_1, y_1)$

$= \frac{0.1 \times (0.1^2 + 1.01012)}{10}$

$\Rightarrow 0.01030362$

$k_2 = 0.1 \times f(x_1 + h/2,\ y_1 + k_1/2)$

$\frac{0.1 \times (0.15^2 + 1.015286^2)}{10}$

$\boxed{0.010533}$

$k_3 = 0.1 \times f(x_1 + h/2,\ y_1 + k_2/2)$

$\frac{0.1 \times (0.15^2 + 1.01540092)}{10}$

$\boxed{0.010532}$

$k_4 = h \times f(x_1 + h,\ y_1 + k_3)$

$\frac{0.1 \times (0.2^2 + 1.020669^2)}{10}$

$= 0.010808$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$1.0101 + \frac{1}{6}(0.0103037 + 2 \times 0.010533 + 2 \times 0.01053 + 0.01081)$

$\Rightarrow 1.020609$

$y(0.2) \approx \boxed{1.0206}$

*3) $\frac{dy}{dx} = x^2 + xy$, $y(1) = 1$ and $h = 0.1$ find $y(1.1)$

$\Rightarrow$ Let $f(x,y) = x^2 + xy$

$f(x_0, y_0) = 1+1 = 2$

$x_0 = 1$ and $h = 0.1$

$y_0 = 1$ $\quad x_1 = 1.1$

Now, using fourth order R-K method

$k_1 = hf(x_0, y_0)$

$0.1 \times (2) = 0.2$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$0.1 \times f(1.05, 1 + 0.1)$

0.2365

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

$0.1 \times f(1.05, 1 + \frac{0.2365}{2})$

$= 0.24246$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$0.1 \times f(1.1, 1 + \frac{0.24246}{2})$

$= 0.29105$

So, now

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$1 + \frac{1}{6}[0.2 + 2 \times 0.2365 + 2 \times 0.24246 + 0.29105]$

$\boxed{y_1 = 1.2415}$

So,

$\boxed{y(1.1) \approx 1.2415}$

Coding

1. 4th order RK method

```
clear all;
clc
f = input('Enter function:');
h = input('Enter step length:');
```

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```
x = input ('Enter initial value of n;');
xn = input ('Enter final value of n:');
y = input ('Enter initial value of y:');
a = input (' Enter evaluating point:');
n = (a-n)/n;
for i=1:n
  if (n <= xn)
  k1 = n * f (n, y);
  k2 = n * f(x + n/2, y + k1/2);
  k3 = h * f (x + h/2, y + k2/2 );
  k4 = n* f(x+n , y+ k3);
   del y = (1/6) * (k1 +2* k2 + 2 * k3+ k4)
    y = y + del y
    x = x +h
 else
  break
  fprintf(' the value of x at x = %.0.4f is
                              = %.0.4f', x, y);
 end
end
```

i) Input :

Enter the function : @ (x, y) a(-x * y)

Enter step length : 0.2

Enter the initial value of x : 0

Enter the final value of x : 0.4

Enter the initial value of x : 1

Enter there evaluating point : 0.2 and 0.4

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Output

For evaluating point : 0.2

Y = 0.9802

For evaluating point : 0.4

Y = 0.9231

ii) Input :

Enter the function : @(x,y) (x^2 + y^2/10)

Enter the step length : 0.1

Enter the initial value of x : 0

Enter the final value of x : 0.2

Enter the initial value of y : 1

Enter the evaluating point ; 0.1 and 0.2

Output :

For evaluating point (0.1)

Y = 1.0101

∵ For evaluating point = 0.2

Y = 1.0206

iii) Input :-

Enter the function : @(x,y) (y^2 + (x*y))

Enter the step length : 0.1

Enter the initial value of x : 1

Enter the final value of x : 1.1

Enter the initial value of y : 1

Enter the evaluating point : 1.1