Experiment -3

Aim :→ To find the real roots of the equations using Regular falsi method.

(i) Calculate the real root of the equation x^3-3x+5 by Regular falsi method, Correct upto 3 decimal places

(ii) Find out root of the equation x^3-5x-7 b/w 2 and 3 using Regular falsi method Correct upto 3 decimal places.

(iii) find the root of the equation $x^3+x^2+x+7=0$ using Regular falsi method correct upto 3 decimal places

Apparatus Required

(i) PC (Windows-11) (ii) MATLAB-2021a

Theory :→

Regular falsi method is a method to obtain solution of a Algebric and Transcendental equation and it is also known as method of false position. By solution we mean finding the roots of a equation and solving a equation $f(x)=0$ for x

Let $f(x)=0$ be the equation and α be the root of it, α being real, first we need to find a sufficiently small interval $[a_0, b_0]$ so that $f(a_0) \cdot f(b_0) < 0$, which implies the root α lies between a_0 and b_0.

Next we need to draw two perpendiculars at $x = a_0$ and $x = b_0$ on the axis - Let them cuts the function $Y=f(x)$ at $A(a_0, f(a_0))$ and $B(b_0, f(b_0))$, respectively. Then the equation of the straight line joining points A & B will be given by:

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$$\frac{y - f(a_0)}{x - a_0} = \frac{f(b_0) - f(a_0)}{b_0 - a_0} \quad \text{---(i)}$$

As $f(a_0)$ and $f(b_0)$ has opposite signs, the straight line AB must cut the x-axis. this point of intersection of AB and the x-axis. This will be considered as the next approximation to the root α.

To obtain this approximation say x_1, put $y=0$ in (i) we get

$$\frac{0 - f(a_0)}{x_1 - a_0} = \frac{f(b_0) - f(a_0)}{b_0 - a_0}$$

$$x_1 = a_0 - \frac{f(a_0) - (b_0 - a_0)}{f(b_0) - f(a_0)}$$

$$x_1 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)} \quad \text{---(ii)}$$

Now, if $f(a_0) \cdot f(x_1) < 0$, then $a_1 = a_0$ and $b_1 = x_1$. Otherwise $a_1 = x_1$ and $b_1 = b_0$. Thus, a new interval $[a_1, b_1]$ containing α is obtained and $[a_1, b_1] \subset [a_0, b_0]$ now, drawing a perpendicular at $x = x_1$ and continuing the same, a sequence $\{x_1, x_2, x_3 \dots\}$ is obtained. Thus the iterative formula for the regular falsi method is given by:

$$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}, \quad n = 0, 1, 2, \dots \quad \text{(iii)}$$

In regular falsi method, we use straight lines to determine the iterative values and all these values are within the initial estimated interval $[a_0, b_0]$ therefore, this method is an interpolation and hence sometime called linear interpolation.

Convergence of Regular Falsi

Let x_{n+1}, be the $(n+1)^{th}$ approximation to the root α of the equation $f(x)=0$. then, clearly either $x_n = a_n$ or $x_n = b_n$ resp.

$x_n = a_n$. then we can denote $b_n = x_{n-1}$. Let ε_{n+1} be the error in x_{n+1} so that $x_{n+1} = \alpha + \varepsilon_{n+1}$ then from the iterative formula, we get :-

$$\varepsilon_{n+1} = \frac{1}{2}\frac{f''(\alpha)}{f'(\alpha)}\varepsilon_n \varepsilon_{n-1} = c\varepsilon_n \varepsilon_{n-1} \quad -(iv)$$

$$c = \frac{1}{2}\frac{f''(\alpha)}{f'(\alpha)} \rightarrow (v)$$

Now to find order of convergence, let $\varepsilon_{n+1} = A\varepsilon_n^k$ using this we get

$$k = 1 + \frac{1}{k}$$

$$k^2 - k - 1 = 0$$

$$k = \frac{1}{2}(1 \pm \sqrt{5}) \approx 1.618 \text{ (taking the +ve sign)}$$

$$\varepsilon_{n+1} = A\varepsilon_n$$

Hence, the regular falsi has order of convergence 1.618

Important Guidelines for Computations:

(i) choose the interval $[a, b]$ so that $f(a).f(b) < 0$

(ii) Compute $x = \frac{af(b) - bf(a)}{f(b) - f(a)}$

(iii) If $f(a)$ and $f(x)$ have the same sign shift the point a to x otherwise, shift b to x.

Given problems :-

1) find the real root of the equation $x^3 - 3x - 5$ using regular falsi correct up to 3 decimal places.

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Solⁿ Let $f(x) = x^3-3x-5$, we see that $f(2) = -3<0$ and $f(3) = 13>0$ so there exits at least one positive root between 2 and 3. Hence, we set $[a_0, b_0] = [1\ 3]$. The computations for finding the root of the equation $x^3-3x-5=0$ is shown in table below:

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$	$f(x_{n+1})$
0	2	3	-3	13	2.1875	-1.095
1	2.1875	3	-1.095	13	2.2506	-0.3518
2	2.2506	3	-0.348	13	2.2704	-0.1084
3	2.2704	3	-0.1084	13	2.2764	-0.0329
4	2.2764	3	-0.0329	13	2.2782	-0.01
5	2.2782	3	-0.01	13	2.2788	-0.003
6	2.2788	3	-0.003	13	2.2789	-0.0009
7	2.2789	3	-0.0009	13	2.27899	-0.0003

Hence $\alpha = 2.2789$ which is the root of the equation $x^3-3x-5=0$

11) Find out root of the equation $x^3-5x-7=0$ b/w 2 and 3 using Regular Falsi method correct upto 3 decimal places.

Solⁿ Let $f(x) = x^3-5x-7$, we can see that $f(2) = -9<0$ and $f(3) = 5>0$ so there exists atleast one positive root between 2 and 3. Hence we set $[a_0, b_0] = [2, 3]$ the computations for finding the root of the equation $x^3-5x-7=0$ is shown in the table

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$	$f(x_{n+1})$
0	2	3	-9	5	2.6429	-1.7547
1	2.6429	3	-1.7547	5	2.7356	-0.2055
2	2.7356	3	-0.2055	5	2.7461	-0.0225
3	2.7461	3	-0.0225	5	2.7472	-0.0024
4	2.7472	3	-0.0024	5	2.7473	-0.0003

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Hence $\alpha = -2.7473$ which is the root of the equation $x^3 - 5x - 7 = 0$

(iii) Find the root of the equation $x^3 + x^2 + x + 7 = 0$ using Regula-falsi method correct upto three decimal places.

Soln Let $f(x) = x^3 + x^2 + x + 7$, we can see that $f(-3) = -14 < 0$ and $f(-2) = 1 > 0$ so there exists at least one root between -3 and -2. Hence, we set $[a_0, b_0] = [-3, -2]$ the computations for finding the root of the equation $x^3 + x^2 + x + 7 = 0$ is shown in the table below:

n	a_n	b_n	$f(a_n)$	$f(x_n)$	$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$	$f(x_{n+1})$
0	-3	-2	-14	1	-2.0667	0.3375
1	-3	-2.076	-14	0.3775	-2.0912	0.1371
2	-3	-2.0912	-14	0.1371	-2.1	0.0491
3	-3	-2.1	-14	0.0491	-2.1031	0.0175
4	-3	-2.1031	-14	0.0175	-2.1043	0.0062
5	-3	-2.1043	-14	0.0062	-2.1047	0.0022
6	-3	-2.1047	-14	0.0022	-2.148	0.0008
7	-3	-2.1048	-14	0.0008	-2.1049	0.0003

Hence $\alpha = -2.1049$ which is the root of the eqn $x^3 + x^2 + x + 7 = 0$

Results :-

(i) root of equation $x^2 - 3x - 5 = 0$ is $\alpha = 2.279$

(ii) root of equation $x^3 - 5x - 7 = 0$ is $\alpha = -2.747$ Ans

(iii) root of the equation $x^3 + x^2 + x + 7 = 0$ $\alpha = -2.105$ Ans

Code :-

clc;

clear all;

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F = input('enter the function :');
k = input('enter the limit in which we need to find the roots:');
for i = -k :1:k
  a = i ;
  if f(a) * f(b) < 0
    break
  end
end
b = a+1;
i = 2
tr = 0.00005;
 if f(a) > 0
  c = b ;
   b = a
   a = b
 end
 n(1) = a ;
while (1)
  n(i) = (a * f(b) - b * f(a)) /(f(b) - f(a));
  if f(n(i))) < 0
     a = n(i);
  else
    b = n(i);
  end
  if abs (n(i-1) - n(i)) < tr
     break
  end
  i = i+1
 end
 disp(n(i));
```

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Input
(i) enter the function : @ (n) [n^3-3*n-5]
enter the limit in which we need to find the root : 4

Output :-

$\alpha = 2.2790$

Input :-
(ii) enter the function: @(n)[n^3-5*x-7]
enter the limit in which we need to find the root of the equation: 4

Output :

$\alpha = 2.747$

Input :-
(iii) enter the function: @[n][n^3+n^2+n+7]
enter the limit in which we need to find the root of the equation: 4

Output :-

$\alpha = -2.105$

Observation :- In the above experiment we calculated the root of three different equations $n^3-3n-5=0$, $n^3-5n-7=0$, $n^2+n+7=0$ using Regular falsi by hand calculation and MATLAB programming. And the result obtained from hand calculation and output of the program completely matches

Conclusion :- In the above experiment we calculated the roots of equations using Regula falsi method by hand calculations and MATLAB Programming.

In this experiment, the results obtained by hand calculation for all three problems is successfully verified using MATLAB (2021a) as both the result of hand calculation for algebric equations and output of our program has matched successfully.

From above experiment we concluded that Regular falsi has several disadvantages.

(i) Regular falsi has order of convergence 1.68 and hence it needs more iterations for solution.

(ii) And in the method of Regular falsi every time we need to check the sign of function value f(x) for every iteration which is very tiresome work.

Reference: Ak Jalan and Utpal Sarkar "Numerical methods", University press 3rd edition, 2015

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