Expt. No. 1 Date

Aim :→ To find the solution of system of differential equation using euler's method and Euler's modified method :-

(i) Calculate $y(2.2)$, given $\frac{dy}{dx} = -xy^2$ and $y(2) = 1$, using euler's method

(ii) Calculate $y(0.1)$, given $\frac{dy}{dx} = \frac{y-x}{y+x}$ and $y(0) = 1$, using Euler's method.

(iii) Calculate $y(0.1)$ given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ using euler's method.

(iv) Calculate $y(0.1)$ using euler's modified method given, $y = x + y$ and $y(0) = 1$

(v) $\frac{dy}{dx} = \frac{1+y}{x}$, $y(1) = 2$, Calculate $y(1.2)$ using

Apparatus Required

(i) PC (Windows - 10, 11)

(ii) MATLAB

Theory :→ Euler's modified method is also called Range-Kutta method of first order. Let us consider a differential

$$\frac{dy}{dx} = f(x, y) \longrightarrow (i)$$

with the initial condition, $y_0 = y(x_0)$ —— (ii)

$y = \phi(x)$ be the solution of equation (i)

$$y = \phi(x) \quad \text{— (iii)}$$

$$y_{n+1} = \phi(x_{n+1}) = \phi(x_n + h)$$

$$\phi(x_n) + h\,\phi'(x_n) + \frac{h^2}{2}\phi''(x_n) + \dots$$

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$\approx \phi(x_n) + h\,\phi'(x_n)$, assuming h is a very small quantity.

$\Rightarrow y_n + h y'(x_n)$

$y_n + h f(x_n, y_n)$ — (iv)

Error in Euler's method :- The error (truncation error) commited in Euler's method is order of h^2, i.e. $O(h^2)$

Geometrical Interpretation of Euler's method :-

From equation (i) & (ii) we can show the solution y as curve PQ (shown in fig-1) the ordinate P, i.e. y_n is know so far y_{n+1} we can write.

$$\frac{y_{n+1} - y_n}{h} = \tan(\alpha) = \left(\frac{dy}{dx}\right)_{x_n, y_n}$$

$$= f(x_n, y_n)$$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

which is the recursion formula for Euler's method

Euler's modified formula :- To minimise the error in Euler's first formula, we modified the formula as given below:

$$y_{n+1} = \phi(x_n) + h\,\phi'(x_n) + \frac{h^2}{2!}\phi''(x_n) + \cdots$$

Differentiating (vi) with respect to x — (vi)

we have :-

$$\left(\frac{dy}{dx}\right)_{n+1} = \phi'(x_n) + h\,\phi''(x_n) + \frac{h^2}{2}\phi'''(x_n) + \frac{h^3}{3}\phi^{iv}(x_n)$$

$f(x_{n+1}, y_{n+1}) = \phi'(x_n) + h\phi''(x_n) + \ldots$

multiplying both sides by $h/2$, we get

$$\frac{h}{2} f(x_{n+1}, y_{n+1}) = \frac{h}{2}\phi'(x_n) + \frac{h^2}{2}\phi''(x_n) + \frac{h^3}{2.2!} + \phi'''(x_n)$$

$$+ \frac{h^4}{2.3!}\phi^{IV}(x_n) + \ldots \quad \to (vii)$$

Substracting (v) and (vii), we have

$$y_{n+1} - \frac{h}{2} f(x_{n+1}, y_{n+1}) = \phi(x_n) + \frac{h}{2}\phi'(x_n) + o(h^3)$$

$$y_{n+1} \approx y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right] \to (viii)$$

The equation (viii) is the equation of Euler's modified formula.

However to calculate y_{n+1}, we need the value of $f(x_{n+1}, y_{n+1})$ which can't be calculated as y_{n+1} is unknown. so, calculate y_{n+1} using Euler's first formula $y_{n+1} \approx y_n + hf(x_n, y_n)$

And denote this first expression as

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1}^{(r)} = y_n + \frac{h}{2}\left[f(x_{n+1}, y_{n+1}^{(r-1)}) + f(x_n, y_n)\right] \text{ where}$$

$r = 2, 3, \ldots$ for each $n = 0, 1, 2 \ldots$

this is also known as Runge Kutta method of second order

Given Equations

(i) Calculate $y(2.2)$ using Euler's method

given $\frac{dy}{dx} = x \cdot y^2$, $x_0 = 2$, $y_0 = 2$, $h = 0.05$

Solution

$f(x,y) = -xy^2$, $x_0 = 2$, $y_0 = 1$, $h = 0.05$

x	y	$f(x,y)$	$y_{n+1} = y_n + hf(x_n, y_n)$
2	1	-2	0.9
2.05	0.9	-1.6605	0.81698
2.10	0.81698	-1.40164	0.74689
2.15	0.74689	-1.199375	0.68692
2.20	0.68692		

$y(2.2) = 0.6869$

$y(2.2)$ i.e at $x = 2.2$ $y = 0.6869$

(iii) Calculate $y(0.1)$ given $\frac{dy}{dx} = \frac{y-x}{y+x}$ and $y(0) = 1$

Sol $f(x,y) = \frac{y-x}{y+x}$ $x_0 = 0$, $y_0 = 0$, $h = 0.02$

n	x	y	$f(x,y)$	$y_{n+1} = y_n + hf(x_n, y_n)$
0	0	1	1	1.02
1	0.02	1.02	0.9053	1.03923
2	0.04	1.03923	0.92587	1.05775
3	0.06	1.05775	0.89264	1.0756
4	0.08	1.0756	0.861543	1.09283
5	0.10	1.09283		

$y(0.1) = 1.0928$ //

(iii) Calculate $y(0.1)$, Given $y' = x^2 - y$ & $y(0) = 1$

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Sol^n $f(n,y) = n^2 - y$, $n_0 = 0$, $y_0 = 1$
$h = 0.02$

n	x	y	f(n,y)	$y_{n+1} = y_n + hf(n_n, y_n)$
0	0	1	-1	0.98
1	0.02	0.98	-0.9796	0.9604
2	0.04	0.9604	-0.9588	0.94123
3	0.06	0.94123	-0.9376	0.92248
4	0.08	0.92248	-0.91608	0.904158
5	0.10	0.904158		

$y\,0.1 = 0.904158$

iv) $\frac{dy}{dn} = 1 + \frac{y}{n}$, $y(1) = 2$, $y(1.2) = ?$

Sol^n $f(n) = 1 + \frac{y}{n}$, $n_0 = 1$, $y(1) = 2$, $y(1.2) = ?$

So, $n_1 = n_0 + h$, $h = 0.2$

$y(1) = y_0 + hf(n_0, y_0) = 2 + 0.2(3) = 2.6$

r	$y_1 = y_0 + (h/2)(f(n_0, y_0) + f(n, y, r-1))$
2	2.616667
3	2.618056
4	2.618171
5	2.618180
6	2.618181

$y(1.2) = 2.6182$

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$\frac{dy}{dx} = x + y$, $y(0) = 1$ find (0.1)

soln $f(x, y) = x + y$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

r	$y_1 = y_0 + h/2 \, [F(x_0, y_0) + f(x_1, y_{1, r-1})]$
2	1.1
3	1.1105
4	1.110525
5	1.11052625
6	1.11052631
7	1.11052631

Code:

(i) Euler's method

```
Clc
Clearall
f = input ('Enter the function:');
x1 = input ('Enter the initial value of x:');
y1 = input ('Enter initial value of y:');
h = input
for x = x1 : h : (a-h)
        y = y+h *f(x,y);
end
disp (y)
```

Input :→

(i) Enter function = @(x,y) [-x*y^2]

Enter initial value : 2

Enter initial value of y : 1

Enter step length = 0.05
Enter the finding point = 2.2

(ii) Enter the function @(x,y) [(y-x)/(y+x)]
Enter the initial value of x : 0
Enter the initial value of y = 1
Enter the step length = 0.02
Enter the finding point = 0.1

(iii) Enter the function : @(x,y) [x^2-y]
Enter the initial value of x = 0
Enter the initial value of y = 1
Enter the step length = 0.02
Enter the finding point = 0.01

Output of Euler's method:-
(i) y = 0.6869
(ii) y = 1.0928
(iii) y = 0.9042

ii) Euler's Modified method

```
clc;
Clearall;
f = input ('Enter the function:');
x1 = input('Enter initial value of x:');
y1 = input ('Enter initial value of y:');
h = input ('Enter step length:');
a = input
A = f(x1; y1);
y(1) = y1 + h * A;
while (1)
y(n+1) = y1 + (h/2) * (A + (f (x1 + h), (y(n)));
If abs (y(n+1) - y(n)) < 0.00001
break;
end; n++;
end
disp(y);
```

Input :→

(v) Enter the function : @ (x,y) (x+y)
Enter the initial value of x = 0
Enter the initial value of y = 1
Enter the step length = 0.1
Enter the finding point = 0.1

Output :→

1.1000   1.1100   1.1105   1.1105   1.1105

(iv) Enter the function: @ (x,y) (1+ y/x)
Enter initial value of x: 1
Enter initial value of y: 2
Enter step length: 0.2
Enter the finding point: 1.2

Output
y
2.6000 2.6167 2.6182 2.6182 2.6182

Conclusion:- In the above experiment we calculated the solution of differential equation using Euler's method and Euler's modified method by hand calculation and MATLAB programming

We have sucessfully verified the hand calculation of all the five problems using MATLAB (2021a) in the pc (windows-11)

And from above experiment we concluded that there are several advantages and disadvantages of Euler's method and Euler's modified method:-

References ?

7/3/22