Aim:→ To find the solution of given equation using Taylor's method

Apparatus Required: Laptop - Asus Zenbook, windows 11, MATLAB R2021

Theory :→

Consider the first order equation $\frac{dy}{dx} = f(x,y)$ – (1)

Differentiating (1), we have

$$\frac{d^2y}{dx^2} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dx}$$

i.e. $y'' = f_x + f_y \cdot y'$ – (2)

Differentiating this successively, we can get y'', y^{iv} etc putting $x = x_0$ and $y = 0$ the value of (y_0'), $(y_0'')_0$, $(y''')_0$ can be obtained, Hence the taylor's series.

$$y = y_0 + (x - x_0)(y')_0 + \frac{(x-x_0)^2}{2!}(y'')_0 + \frac{(x-x_0)^3}{3!}(y''')_0 + \dots$$

gives the values of y for every value of x for which 3 converges

On finding the value y_1 for $x = x_1$ from 3, y', y'' etc can be evaluated at $x = x_1$ by means of (1), (2) etc. Then y can be expanded about beyond the range of convergence of series (3).

Given Equation :→

1) Using Taylor's method solve $\frac{dy}{dx} = 1+2y$ with $y(0) = 2$ at $x = 0.2$

Sol^n $x_0 = 0$, $y_0 = 2$ $\Rightarrow x - x_0 = 0.2 - 0 \Rightarrow 0.2$

$\frac{dy}{dx} = 1 + xy$

$y' = 1 + xy \Rightarrow y'_0 = 1 + 0 \times 2 = 1$

$y'' = y + xy' \Rightarrow y''_0 = 2 + 0 \times 1 = 2$

$y''' = y' + y' + xy''$
$\quad 2y' + xy''$ $\qquad y'''_0 = 2 \times 1 + 0 \times 2 = 2$

$y'''' = 2y'' + y'' + xy'''$ $\qquad y''''_0 = 3 \times 2 + 0 \times 2 = 6$
$\quad = 3y'' + xy'''$

$$y(0+x)' = y = y + (x-x_0)(y'_0) + \frac{(x-x_0)^2}{2!}(y''_0) + \frac{(x-x_0)^3}{3!}(y'''_0) + \frac{(x-x_0)^4}{4!}(y''''_0)$$

$$y(0.2) = 2 + \frac{(0.2)}{1!} + \frac{(0.2)^2}{2!}(2) + \frac{(0.2)^3}{3!}(2) + \frac{(0.2)^4}{4!} \times (6)$$

$= 2.2430$

$y(0.2) = 2.2430$

(2) Compute $y(1.1)$ and $y(1.2)$. Correct to 5 decimal places using the taylor's series method when $y(x)$ satisfies equation $\frac{dy}{dx} = xy$ with xy with $y(1) = 2$

Soln $y(1) = 2$ $\qquad y(1.1) = ?$ $\quad y(1.2) = ?$

$x_0 = 1, \ y_0 = 2$ $\qquad h_1 = x_1 - x_0 = 1.1 - 1 = 0.1$

$h_2 = x_2 - x_0 = 1.2 - 1 = 0.2$

$\frac{dy}{dx} = xy$

$y' = xy$

$y'' = y + x.y' \Rightarrow y'_0 = 1 \times 2 = 2$

$y''' = y' + y' + xy''$ $\qquad y''_0 = 2 + 1 \times 2 = 4$

$\quad = 2y' + xy''$ $\qquad y'''_0 = 2 \times 2 + 1 \times 4 = 8$

$y'''' = 2y'' + y'' + x.y'''$

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$3y'' + xy'''$ $\Rightarrow$ $y_0''' = 3 \times 4 + 1 \times 8 = 20$

$$y_1 = y_0 + h_1 y_0' + \frac{h_1^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h_1^4}{4!} y_0''''$$

$$y(1.1) = 2 + 0.1 \times 2 + \frac{(0.1)^2}{2!} \times 4 + \frac{(0.1)^3}{3!} \times 8 + \frac{(0.1)^4}{4!} \times 20$$

$$\Rightarrow y(1.1) = 2.2242$$

for

$$y(1.2) = 2 + (0.2) \times 2 + \frac{(0.2)^2}{2!} \times 4 + \frac{(0.2)^3}{3!} \times 8 + \frac{(0.2)^4}{4!} \times 20$$

$$y(1.2) = 2.49200$$

Q3) Solve using taylor's series method

$\frac{dy}{dx} = 2x + 3y^2$ with $y(0) = 0$ at $x = 0.2$

Sol^n $y(0) = 0$ $\quad x = 0.2$

$x_0 = 0, \ y_0 = 0$ $\quad h = x - x_0 = 0.2 - 0 = 0.2$

$$\frac{dy}{dx} = 2x + 3y^2$$

$y' = 2x + 3y^2$ $\Rightarrow y_0' = 2 \times 0 + 3 \times 0 = 0$

$y'' = 2 + 6y \cdot y'$ $\Rightarrow y_0'' = 2 + 6 \times 0 \times 0 = 2$

$y''' = 6(y')^2 + 6 \cdot y y''$ $\Rightarrow y_0''' = 6 \times (0)^2 + 6 \cdot 0 \times 2 = 0$

$y'''' = 12 y' \cdot y'' + 6 y'' \cdot y' + 6y \cdot y'''$

$\Rightarrow y_0'''' = 12 \times 0 \times 2 + 6 \times 2 \times 0 + 6 \times 0 \times 0$

$y_0''''' = 12 \times (y'')^2 + 12 \cdot y' \cdot y''' + 6(y'')^2 + 6 \cdot y' \cdot y''' + 6y' \cdot y''''$

$\Rightarrow y_0''''' = 48 + 24 = 72$

$$y = y_0 + h \cdot y_0' + \frac{h^2}{2!}(y_0'') + \frac{h^3}{3!} \times y_0''' + \frac{h^4}{4!}(y_0'''') + \frac{h^5}{5!}(y_0''''')$$

$$0 + (0.2) \times 0 + \frac{(0.2)^2}{2!} \times 2 + \frac{(0.2)^3}{3!} \times 2 + \frac{(0.2)^4}{4!} \times 0 + \frac{(0.2)^5}{5!} \times 72$$

$\Rightarrow 0.04992$

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$y(0.2) = 0.040192$

Q4) Using taylor's series method $y(0.1) = ?$ and $y(0.2) = ?$ if $y(x)$ satisfies $\frac{dy}{dx} = x - y^2$ where $y(0) = 1$

Sol^n $y(0) = 1$

$x_0 = 0, \; y_0 = 1$

$h_1 = x_1 - x_0$

$0.1 - 0 = 0.1$

$h_2 = x_2 - x_0 = 0.2 - 0$

$\Rightarrow 0.2$

$\frac{dy}{dx} = x - y^2$

$y' = x - y^2 \qquad \Rightarrow y'_0 = 0 - (1)^2 = -1$

$y'' = 1 - 2y \cdot y' \qquad y_0'' = 1 + 2 \cdot 1 \cdot 1 = 3$

$y''' = -2(y')^2 - 2y \cdot y'' \qquad y_0''' = -2(-1)^2 - 2 \cdot 1 \cdot 3$

-8

$y'''' = -4y' \cdot y'' - 2y'' \cdot y' - 2y \cdot y'''$

$\Rightarrow y_0'''' = 4 \cdot 1 \cdot 3 + 2 \cdot 3 \cdot 1 + 2 \cdot 1 \cdot 8$

$= 34$

$y = y_0 + h_1 \cdot y_0' + \frac{(h_1)^2}{2!}(y_0'') + \frac{(h_1)^3}{3!}(y_0''') + \frac{h_1^4}{4!}(y_0'''')$

$y(0.1) = 1 + (0.1)(-1) + \frac{(0.1)^2 \times 3}{2!} + \frac{(0.1)^3(-8)}{3!} + \frac{(0.1)^4(34)}{4!}$

$= 0.4138$

$\boxed{y(0.1) = 0.9138}$

$y(0.2) = 1 + (0.2)(-1) + \frac{(0.2)}{2!}(3) + \frac{(0.2)^3}{3!}(-8) + \frac{(0.2)^4 \, 34}{4!}$

$= 0.8516$

$\boxed{y(0.2) = 0.8516}$

MATLAB CODE:

```
clc
clear all
syms n y
f = input('Enter the function: ');
n0 = input('Enter the initial value of n: ');
y0 = input('Enter the initial value of y: ');
n = input('Enter the number of term: ');
for j=1:n
    fprintf('%d derivate:', j);
    fd(j) = input('Enter The value :');
end
n= input('Enter the final value of n:);
h=n-n0;
y=0
for i= 1:n
        y=y+((n^i)*fd(i))/factorial(i);
end
y=y0+((h^i)*fd(i))/factorial(i);
y=y0+y:
disp(y)
```

Input

Enter the function :@(n,y)(1+n*y)
Enter the initial value of n :0
Enter the initial value of y :2
Enter the number of term: 4
1 derivative ! Enter the value :1
2 derivative ! Enter the value :2

3rd derivative : Enter the value : 2
4 derivative : Enter the value : 6
Enter the find value of n : 0.2

Output:
2.2431

(2) for x = 1.1
INPUT
Enter the function @(n,y)(n+y)
Enter the initial value of n : 1
Enter the initial value of y : 2
Enter the number of term : 4
1 derivative : Enter the value : 2
2 derivative : Enter the value : 4
3 derivative : Enter the value : 8
4 derivative : Enter the value : 20
Enter the find value of n = 1.1

OUTPUT
2.2214

for n = 1.2
INPUT
Enter the find value of n = 1.2
OUTPUT
2~~.~~ = 2.4920

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for $n=0.2$

INPUT:

Enter the find value of $n=0.2$

OUTPUT

0.8516

CONCLUSION:

we have done this experiment in MATLAB Software. By this experiment we find the value of a function at a given value of n by using taylor's method and verify the result with MATLAB output successfully.

Reference:

B.S Grewal "Numerical methods in Engineering and Series" by mercury learning and information LLC, 3rd Edition 2014

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