Aim :- To find the real roots of the equations using secant method

(i) Find the real root of the equation $f(x) = x^3 - 3x - 5 = 0$ by secant method, correct upto three decimal places.

(ii) find the positive root of the equation $f(x) = x^3 - 4x - 9$ correct to 4 decimal places using secant method.

(iii) find the root of the equation $f(x) = 3x - \cos x - 1$ using secant method, correct up to 4 decimal places

Apparatus Required:

PC (windows 11) MATLAB 2021a

Theory :- Regular falsi method is used to find the roots of Algebric and Transcendental equations.

This method is quite similar to the regular falsi method except for the condition $f(a) \cdot f(x) < 0$. In this case, the interval at each iteration may not contain the root. The ratinale is that if the root lies between $x = a_0$ and $x = b_0$, then either a_0 or b_0 is kept fixed throughout the iterative process this is shown below:

Let us consider a sufficiently small interval $[a_0, b_0]$ so that condition $f(a_0) \cdot f(b_0) < 0$, which implies that the root α lies between a_0 and b_0. Then, the first aproximation x_1 is given by

$$x_1 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)}$$

Now. We have new interval $[a_1, b_1]$ where $a_1 = a_0$.

and $b_1 = x_1$. Note that in this case we fix the point a_0.
Therefore, the second aproximation x_2 is given by,

$$x_2 = \frac{a_1 f(b_1) - b_1 f(a_1)}{f(b_1) - f(a_1)} \quad \text{— (ii)}$$

Similarly, the $(n+1)^{th}$ approximation is given by

$$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}, \quad n = 1, 2, \ldots \quad \text{(iii)}$$

Convergence of Secant method:-
The order of convergence of Secant method is same as that of the Regular falsi method is i.e. 1.618

Given Problems:-

(i) Find the real root of the equation $x^3 - 3x - 5 = 0$ by secant method correct to 3 decimal places.

Solⁿ Let $f(x) = x^3 - 3x - 5$ we see that $f(2) = -3 < 0$ and $f(3) = 13 > 0$. So, there exist at least one positive root between 2 and 3. Hence, we set $[a_0, b_0] = [2, 3]$ the computations for finding the approximate value of the root by secant method is shown in the given table:-

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$	$f(x_{n+1})$
0	2	3	-3	13	2.1875	-1.0950
1	2.1875	3	-1.0950	13	2.2506	-0.3518
2	2.2506	3	-0.3518	13	2.2704	-0.0329
3	2.2704	3	-0.0329	13	2.2764	-0.0100
4	2.2764	3	-0.0100	13	2.2782	-0.0030
5	2.2782	3	-0.0030	13	2.2788	-0.0009
6	2.2788	3	-0.0009	13	2.2789	-0.0003

7	2.2789	3	−0.0003	13	2.790	−0.0001
8	2.790	3	−0.0001	13	2.790	−0.0003

Hence, $\alpha = 2.790$ which is root of Correct upto 3 decimal

(ii) Find the root of the equation $x^3 - 4x - 9 = 0$ correct upto 4 decimal ~~up~~ places using secant method.

Soln Let $f(x) = x^3 - 4x - 9 = 0$, we see that $f(3) = 6 > 0$ and $f(2) = -9 < 0$ so there exist a positive root between 2 and 3. Hence, we set $[a_0\ b_0]$ as $[2, 3]$. The computations for finding the root of equation $x^3 - 4x - 9 = 0$ is shown in the table below:

n	a_n	b_n	$f(a_n)$	$f(b_n)$	$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$	$f(x_{n+1})$
0	2	3	−9	6	2.6	−1.8240
1	2.6	3	−1.8240	6	2.6937	−0.2372
2	2.6933	3	−0.2372	6	2.7047	−0.0289
3	2.7049	3	−0.0289	6	2.7063	−0.0035
4	2.7063	3	−0.0035	6	2.7065	−0.0004
5	2.7065	3	−0.0004	6	2.7065	−0.0001

Hence, $\alpha = 2.7065$ which is root of the equation $x^3 - 4x - 9 = 0$

(iii) find the root of the equation $3x - \cos x - 1$ using secant method correct to 4 significant digits

Soln. Let $f(x) = 3x - \cos x - 1$. we see that $f(0) = -2 < 0$ and $f(1) = 1.4597 > 0$. so, there exist a positive root for the above equation b/w 0 and 1. Hence, we set $[a_0, b_0] = [0, 1]$. The computation for the root of the equation $3x - \cos x - 1 = 0$ is shown in the table below.

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n	an	bn	f(an)	f(bn)	$x_{n+1} = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n) - f(a_n)}$	$f(x_{n+1})$
0	0	1	-2	1.4597	0.41	-0.67
1	0.41	1	-0.67	1.4597	0.59	-0.061
2	0.59	1	-0.061	1.4597	0.6064	-0.0025
3	0.6064	1	-0.0025	1.4597	0.60707	-0.000113
4	0.60707	1	-0.000113	1.4597	0.6071064	-0.000045

Hence $\alpha = 0.6071$ is the root of equation $3x - \cos x - 1$

Results:-

i) $\alpha = 2.279$, is the root of the equation $x^3 - 3x - 5 = 0$

ii) $\alpha = 2.7065$ is the root of the equation $x^3 - 4x - 9 = 0$

iii) $\alpha = 0.6071$ is the root of the equation $3x - \cos x - 1 = 0$.

MATLAB Code:-

```
clc;
Clearall
F = input('enter the function:');
K = input('enter the limit:');
for i = -k:1:k
    a = i;
    if f(a) * f(a+1) < 0
        break
    end
end
b = a+1;
i = 2
tr = 0.00005;
if f(a) > 0
c = b;
b = a;
```

Teacher's Signature

```
a=b;
end
x(1) =a;
while(1)
 x(i)= (a * f(b) - b* f(a))/(f(b) ) - f(a));
   a=x(i);
  if abs (x(i-1)- x(i)) <tr
     break
  end
 i=i+1;
end
disp(x(i));
```

☆Input :→
(i) enter the function : [x^3 - 3*x-5]
enter the limit : 4
output :
α = 2.2790.

Input :→
(ii) enter the function : @[x^3 - 4 *3 -9]
enter the limit : 4

```
if f(a) >0
  c=b;
  b =a;
  a =b;
end
x(i)= a;
while (1)
```

Observation :- In the above we calculated the root of three equation in which two of them were algebric which are $x^3-3x-5=0$, $x^3-4x-9=0$ and one transcendental $3x-\cos x-1=0$ using secant method by hand calculation and MATLAB by hand calculation and MATLAB programming. The result obtained from the hand calculation and MATLAB programming completely matches.

Conclusion :- In the above experiment we calculated the roots of equations using secant method by hand calculation and MATLAB programming in MATLAB 2019a. In this experiment the results obtained by hand calculation for all three problems is successfully verified using MATLAB (2021a) as both the result of hand calculations for the roots of the equation and MATLAB program has matched successfully.

From above experiment we concluded that we can easily find the roots of a transcidental and algebric equation using secant method by writing method. While when we do hand calculations for roots using secant method then it takes more number of iterations as its order of convergence 1.618.

Reference :-

A K Jalan and Utpal Sarkar "Numerical methods" University press 3rd edition, 2015.

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