



# Tail risk in Bitcoin under the Basel framework

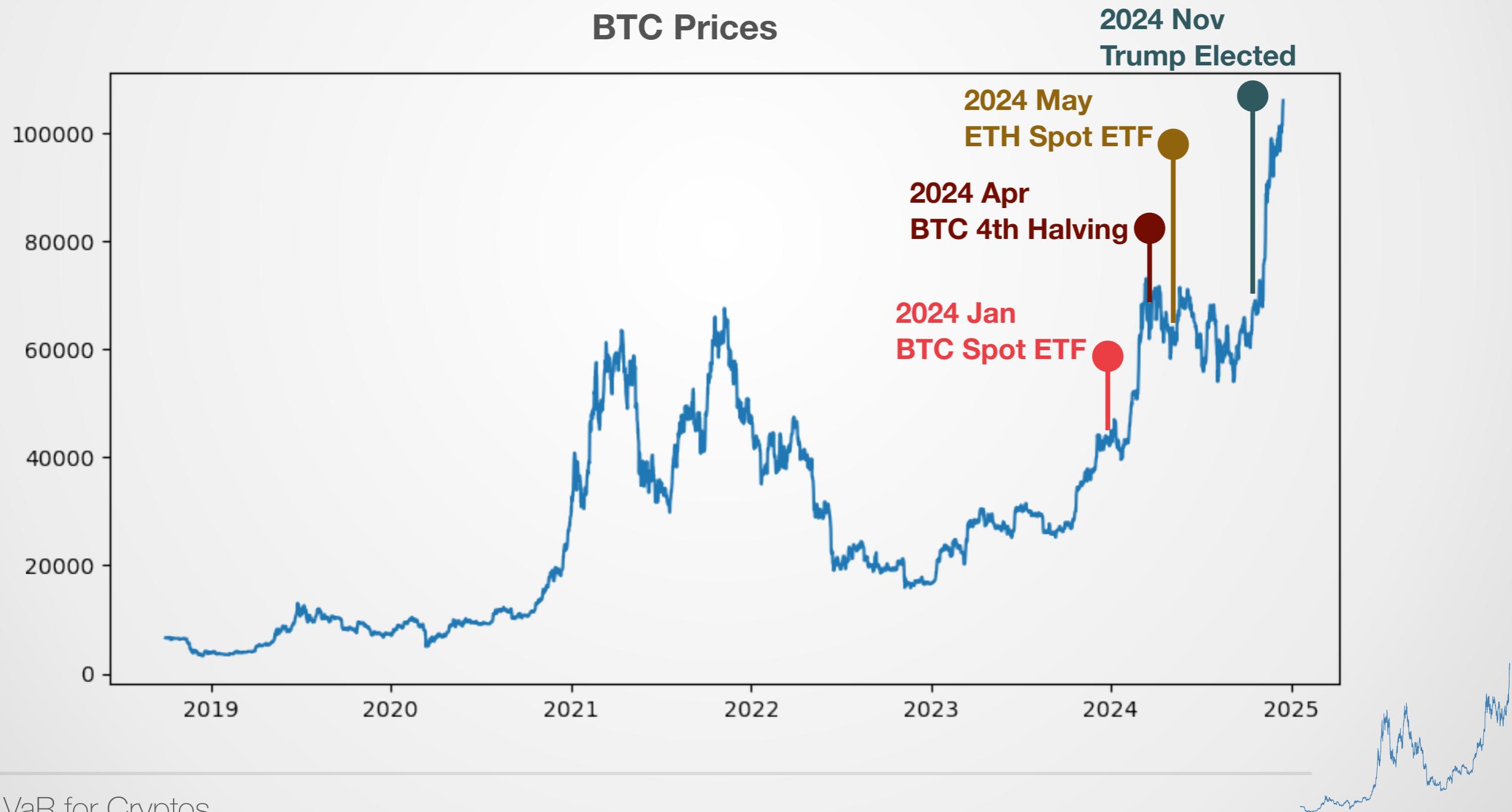
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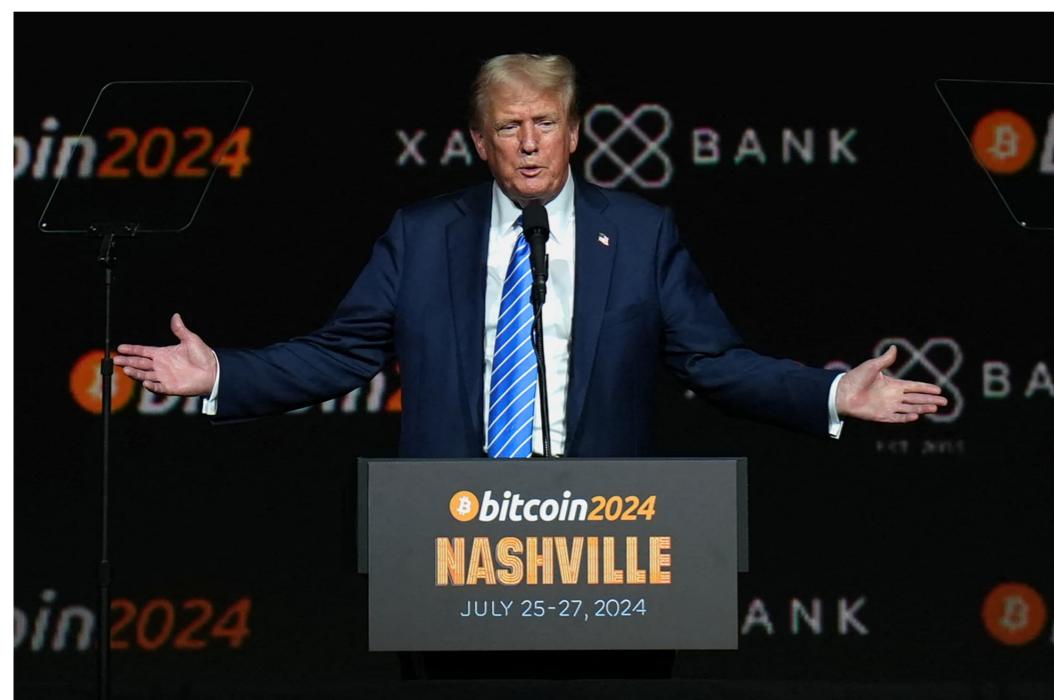
## SEC & US Presidential Election

- SEC: Bitcoin ETFs on January 10, 2024
- President TRUMP embraces the cryptocurrencies (CC)

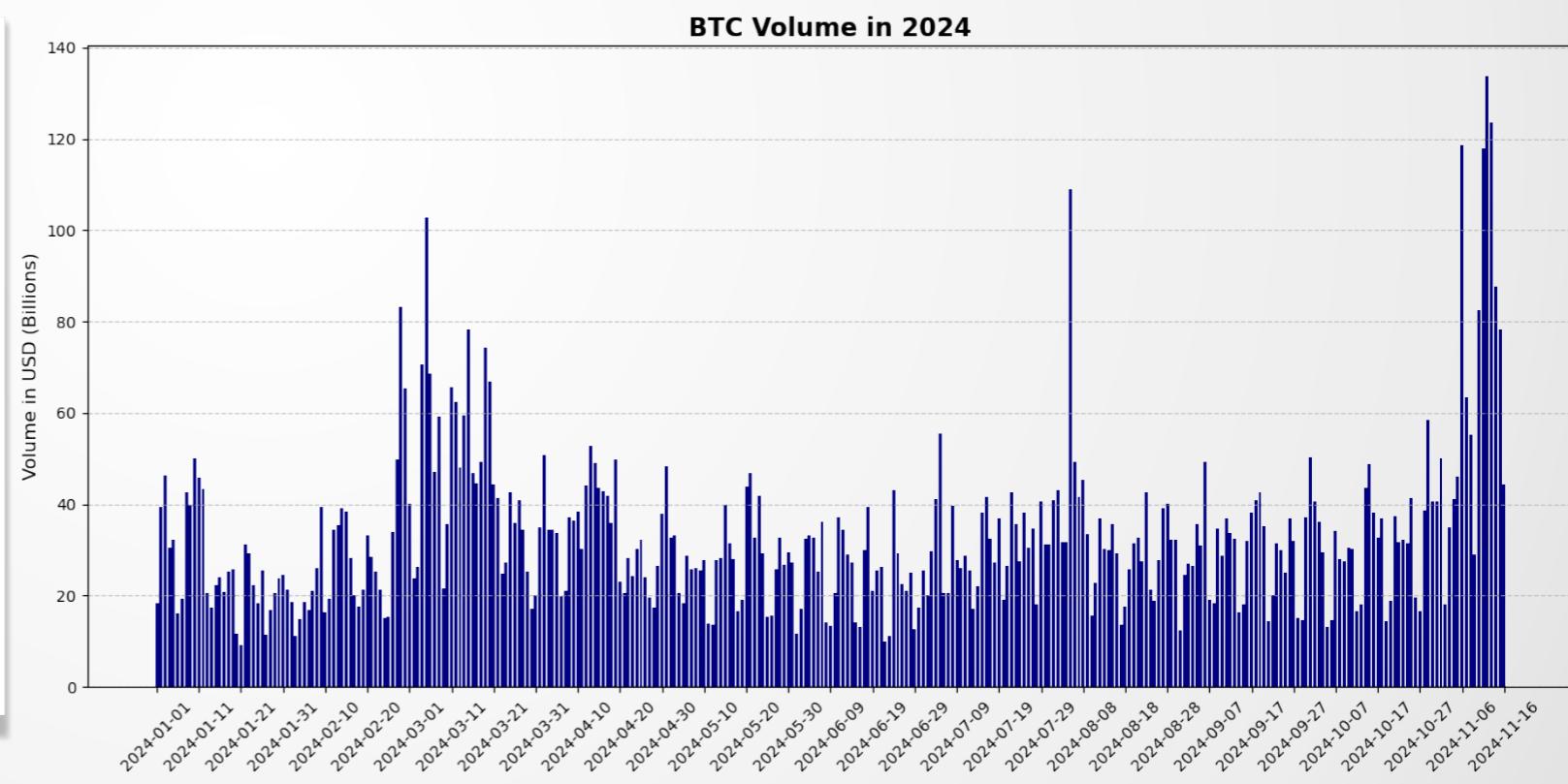


## Basel III and Capital Adequacy

- Basel III Compliance requires banks to hold adequate capital for market risk
- Challenge for Crypto-Assets: High volatility & jump risks make traditional models (e.g., BS) unreliable



Source: <https://www.cnbc.com>

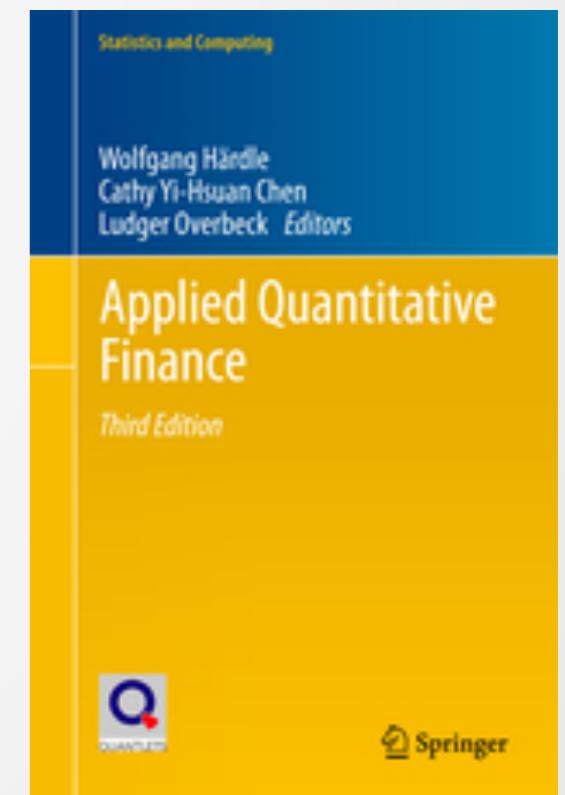
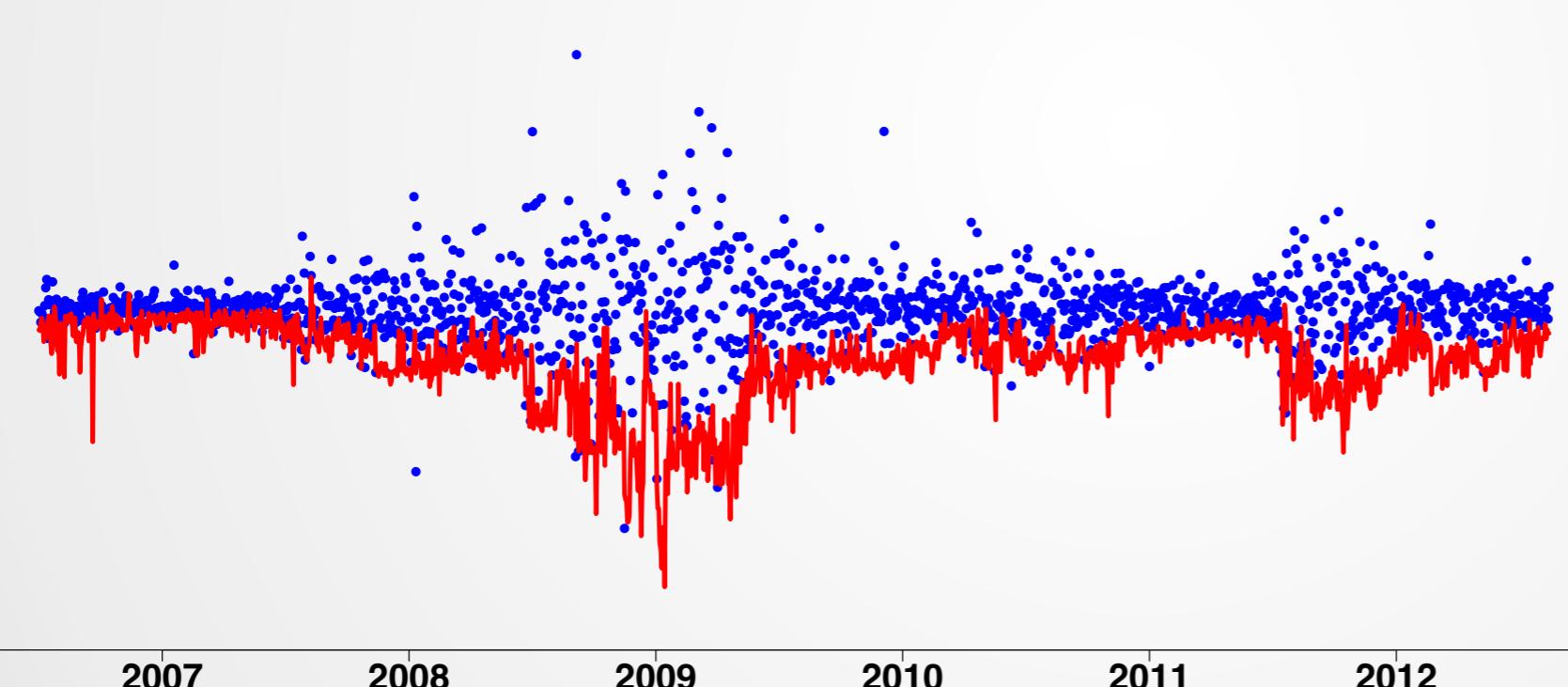


# Value at Risk (VaR): Maximum potential loss

- Confidence level  $\alpha$  (5% or 1%)

- $X_t$ , return at time  $t$ ,

$$\mathbb{P}(X_t \leq \text{VaR}_{t,\alpha}) \stackrel{\text{def}}{=} \alpha, \quad \alpha \in (0,1),$$



VaR of  $\alpha = 5\%$

## Expected Shortfall (ES): Average loss beyond the VaR

- ES at level  $\alpha$  is defined as the conditional expectation of loss given VaR exceedance:

$$ES_{t,\alpha} = \mathbb{E}[X_t | X_t \leq VaR_{t,\alpha}]$$

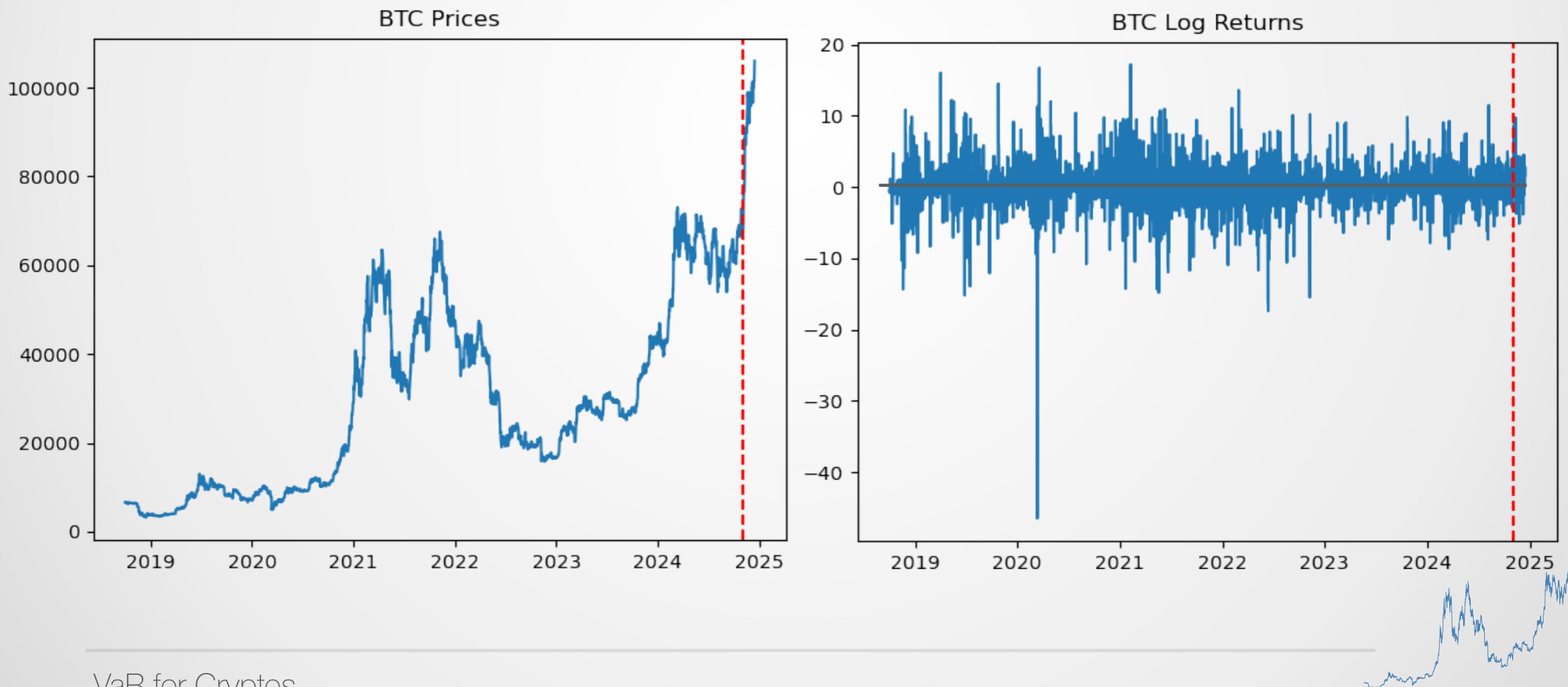
- ES incorporates information on the magnitude of extreme losses

Risk Measure	Focus	Limitation
VaR	Maximum potential loss at confidence level $\alpha$ .	Does not account for the severity of losses beyond the VaR threshold.
ES	Average loss beyond the VaR threshold.	More sensitive to tail assumptions, but more conservative.



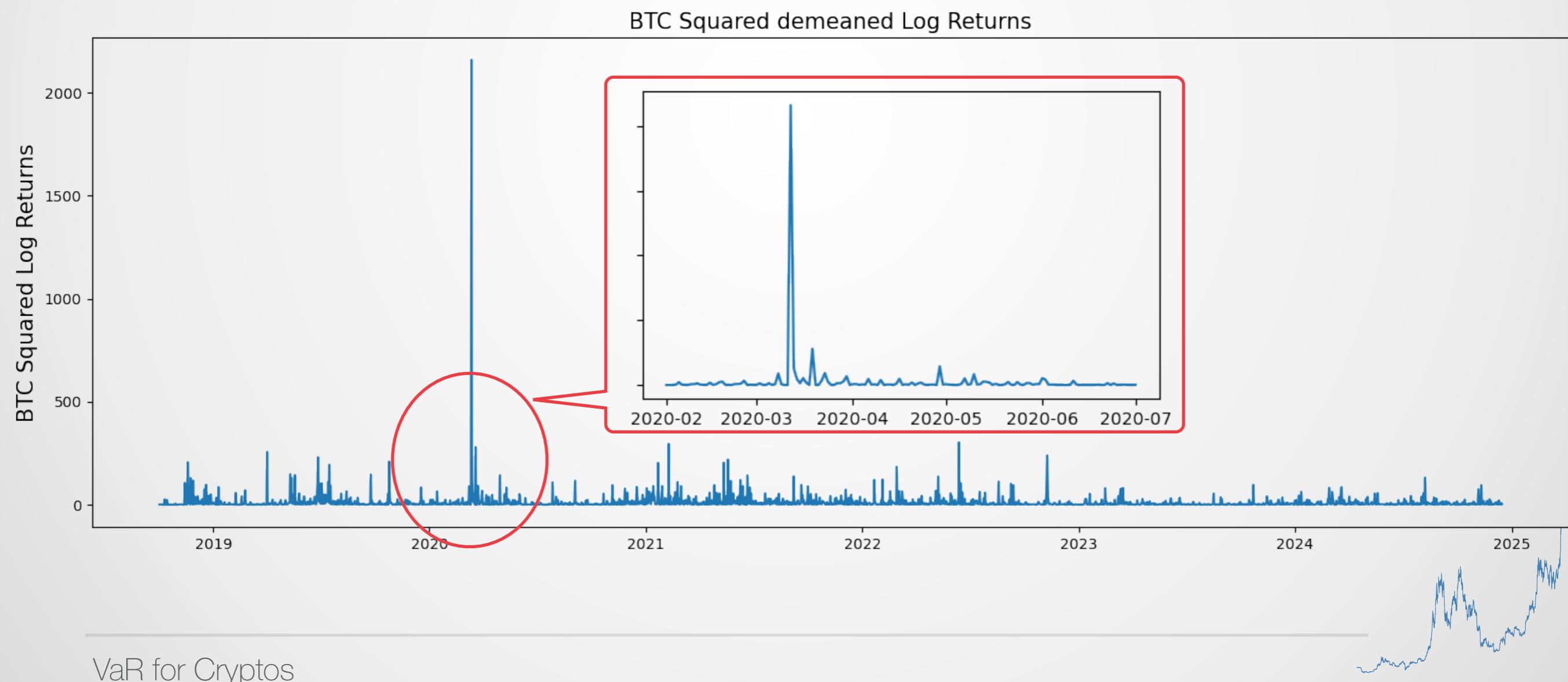
## Features of BTC returns

- BTC from Yahoo Finance: October 2018 to December 2024
- Volatility clustering
- Jumps in returns



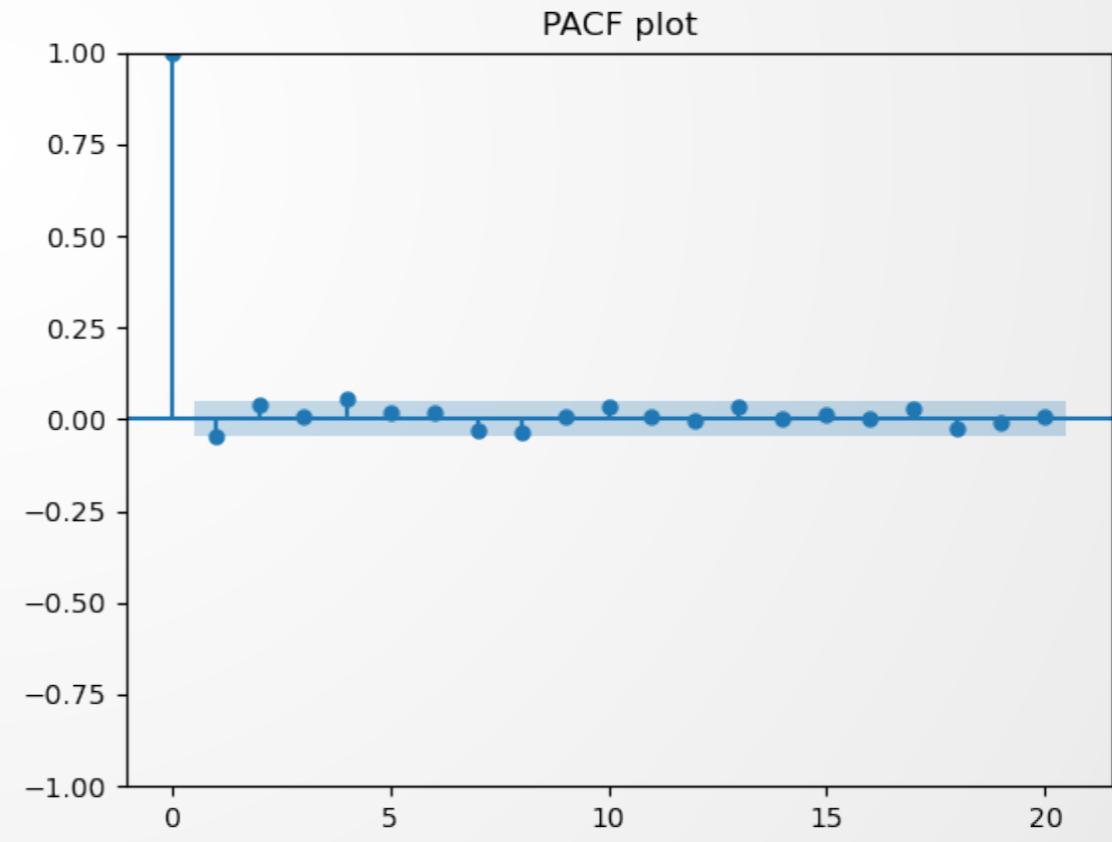
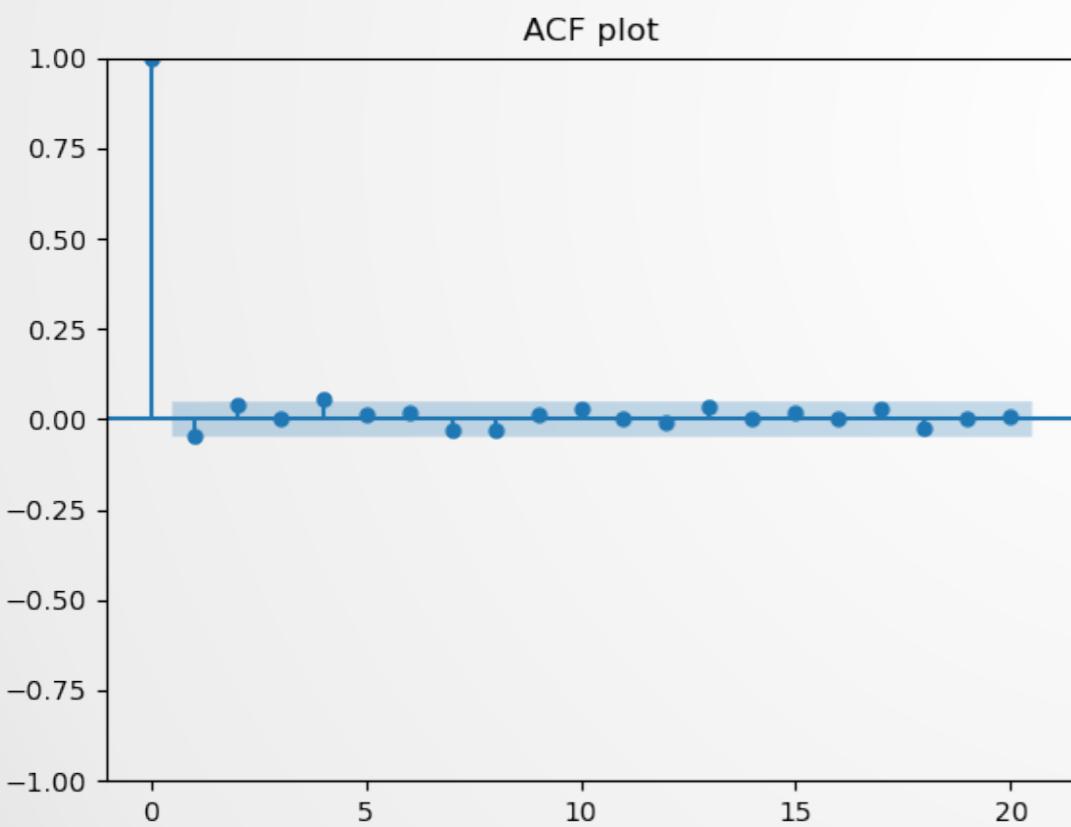
## Jumps in variance

- Squared demeaned returns: Mean-reverting in variance



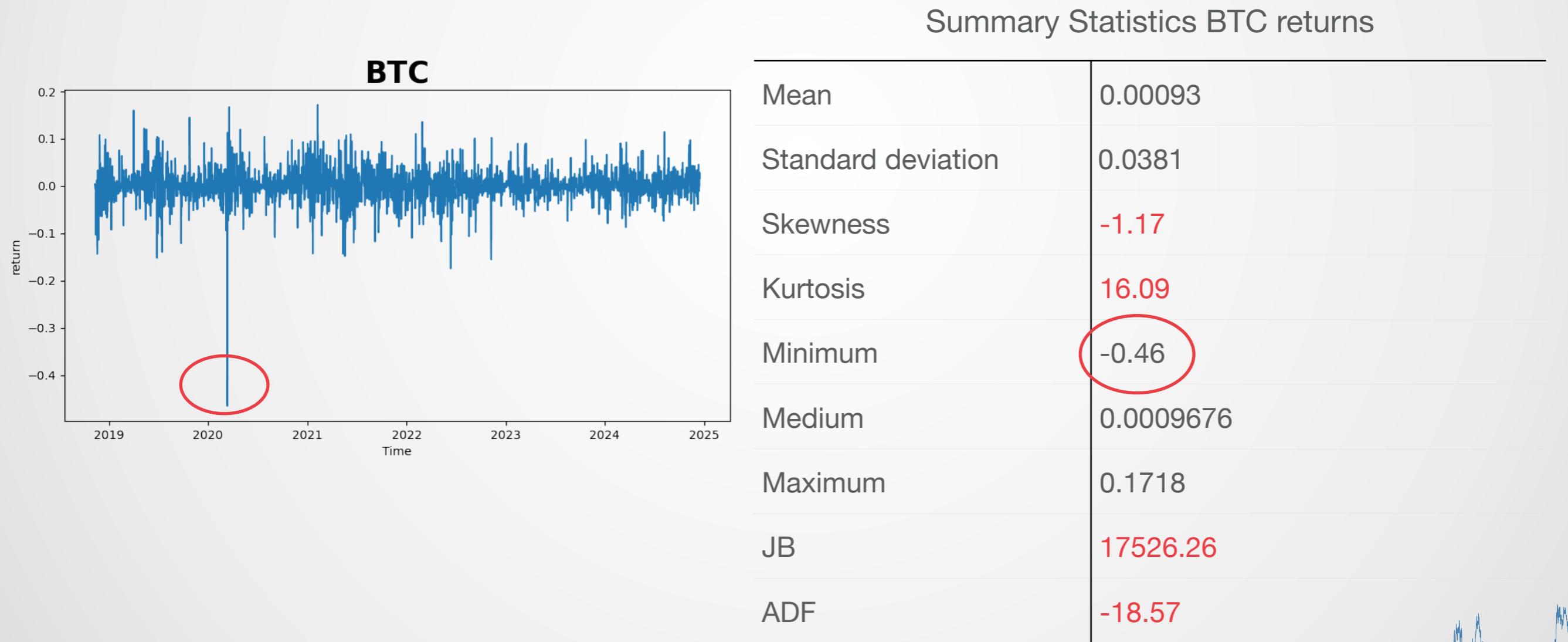
## ACF and PACF for BTC returns

- Both are zero lags, suggesting that the series does not exhibit any autoregressive (AR) or moving average (MA) effects.



# Tail Risk in Crypto Markets

- High kurtosis (16.09), left skewed (-1.17) → extreme jumps and losses
- JB test: Not normal distribution
- ADF test statistic of -18.57: stationary time series!



# Outline

1. Motivation ✓
2. Data ✓
3. Model
4. Empirical Results
5. Conclusion



## Stochastic Volatility with Correlated Jumps (SVCJ)

(1)  $d \log S_t = \mu dt + \sqrt{V_t} dW_t^s + Z_t^y dN_t$  : return process

(2)  $dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^v + Z_t^v dN_t$  : variance process

(3)  $Cov(dW_t^s, dW_t^v) = \rho dt$

(4)  $P(dN_t = 1) = \lambda dt$  : jump

(5)  $Z_t^y | Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2); Z_t^v \sim exp(\mu_v)$



# Stochastic Volatility with Correlated Jumps (SVCJ)

expected return

$$(1) \ d \log S_t = \boxed{\mu dt} + \sqrt{V_t} dW_t^s + Z_t^y dN_t : \text{return process}$$

CIR model

$$(2) \ dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^v + Z_t^v dN_t : \text{variance process}$$

$$(3) \ Cov(dW_t^s, dW_t^v) = \rho dt$$

$$(4) \ P(dN_t = 1) = \lambda dt : \text{jump}$$

$$(5) \ Z_t^y | Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2); \ Z_t^v \sim exp(\mu_v)$$



# Stochastic Volatility with Correlated Jumps (SVCJ)

expected log return

$$(1) \ d \log S_t = \mu dt + \sqrt{V_t} dW_t^s + Z_t^y dN_t : \text{log return process}$$

Mean reversion rate and level standard Brownian motions

$$(2) \ dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^v + Z_t^v dN_t : \text{variance process}$$

$$(3) \ Cov(dW_t^s, dW_t^v) = \rho dt$$

$$(4) \ P(dN_t = 1) = \lambda dt : \text{jump}$$

$$(5) \ Z_t^y | Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2); \ Z_t^v \sim \exp(\mu_v)$$



# Stochastic Volatility with Correlated Jumps (SVCJ)

- (1)  $d \log S_t = \mu dt + \sqrt{V_t} dW_t^s + Z_t^y dN_t$  : return process  
     random jump sizes, jump process with jump frequency  $\lambda$
- (2)  $dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_t^v + Z_t^v dN_t$  : variance process
- (3)  $Cov(dW_t^s, dW_t^v) = \rho dt$
- (4)  $P(dN_t = 1) = \lambda dt$  : jump
- (5)  $Z_t^y | Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2); Z_t^v \sim exp(\mu_v)$



## Stochastic Volatility with Correlated Jumps (SVCJ), Duffie (2000)

$$d \log S_t = \mu dt + \sqrt{V}_t dW_t^s + Z_t^y dN_t$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V}_t dW_t^v + Z_t^v dN_t$$

Stochastic Volatility with Jumps (SVJ), Bates (1996) Set  $Z_t^v = 0$ .

$$d \log S_t = \mu dt + \sqrt{V}_t dW_t^s + Z_t^y dN_t$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V}_t dW_t^v$$

Stochastic Volatility (SV), Heston (1993) Set  $\lambda = 0$ .

$$d \log S_t = \mu dt + \sqrt{V}_t dW_t^s$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V}_t dW_t^v$$

Black and Scholes (BS) Set  $\kappa = \theta = \sigma_V = 0$  and define  $Z_t^v = Z_t^y = 0$ .

$$d \log S_t = \mu dt + \sqrt{V}_t dW_t^s$$

$$dV_t = 0$$



# MCMC algorithm

## 1. For parameters : Draw ten parameters from prior distribution.

Draw  $\Theta_1^{(j)}$  from  $p\left(\Theta_1^{(j)} | Y, \Theta_2^{(j-1)}, \Theta_3^{(j-1)}, \dots, \Theta_K^{(j-1)}, V^{(j-1)}, J^{(j-1)}, Z^{V(j-1)}, Z^{Y(j-1)}\right)$

⋮

Draw  $\Theta_K^{(j)}$  from  $p\left(\Theta_K^{(j)} | Y, \Theta_1^{(j)}, \Theta_2^{(j)}, \dots, \Theta_{K-1}^{(j)}, V^{(j-1)}, J^{(j-1)}, Z^{V(j-1)}, Z^{Y(j-1)}\right)$

## 2. For jump times $t = 1, 2, \dots, T$ .

Draw  $J_t^{(j)}$  from  $p\left(J_t^{(j)} = 1 | Y, \Theta^{(j)}, V^{(j-1)}, Z^{V(j-1)}, Z^{Y(j-1)}\right)$

## 3. For jump sizes $t = 1, 2, \dots, T$ .

Draw  $Z_t^{V(j)}$  from  $p\left(Z_t^{V(j)} | Y, \Theta^{(j)}, V^{(j-1)}, J_t^{(j)}, Z_t^{Y(j-1)}\right)$

Draw  $Z_t^{Y(j)}$  from  $p\left(Z_t^{Y(j)} | Y, \Theta^{(j)}, V^{(j-1)}, J_t^{(j)}, Z_t^{V(j)}\right)$

## 4. For spot volatilities

Draw  $V_1^{(j)}$  from  $p\left(V_1^{(j)} | Y, \Theta^{(j)}, V_0^{(j)}, V_2^{(j-1)}, J^{(j)}, Z^{V(j)}, Z^{Y(j)}\right)$

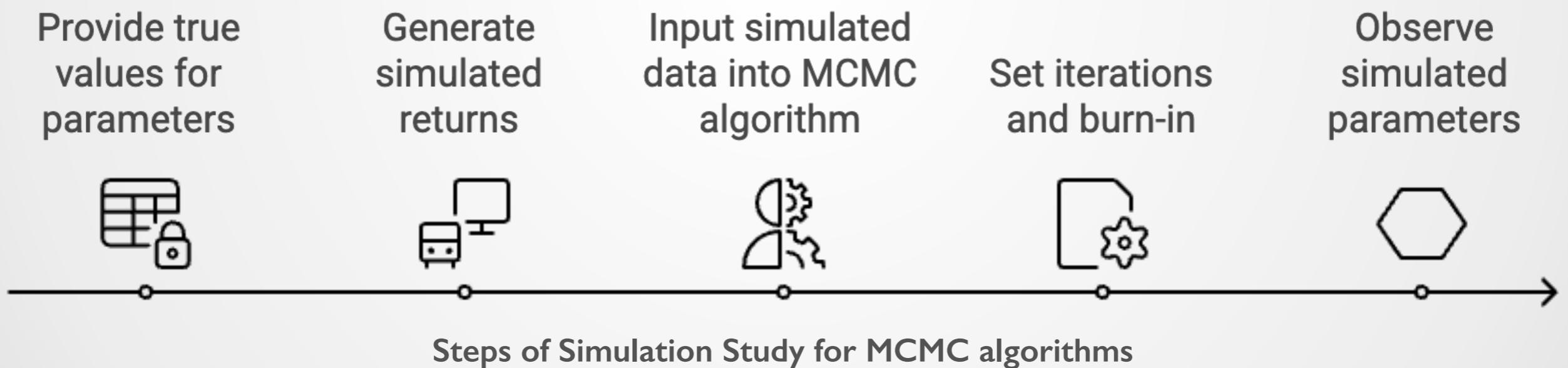
⋮

Draw  $V_T^{(j)}$  from  $p\left(V_T^{(j)} | Y, \Theta^{(j)}, V_{((t-1))}^{(j)}, V_{(T+1)}^{(j-1)}, J^{(j)}, Z^{V(j)}, Z^{Y(j)}\right)$



# Methodology

- Bayesian inference: posterior distribution of parameters
- Difficulty: posterior distribution typically does not have a closed form
- Employ Markov Chain Monte Carlo (MCMC)



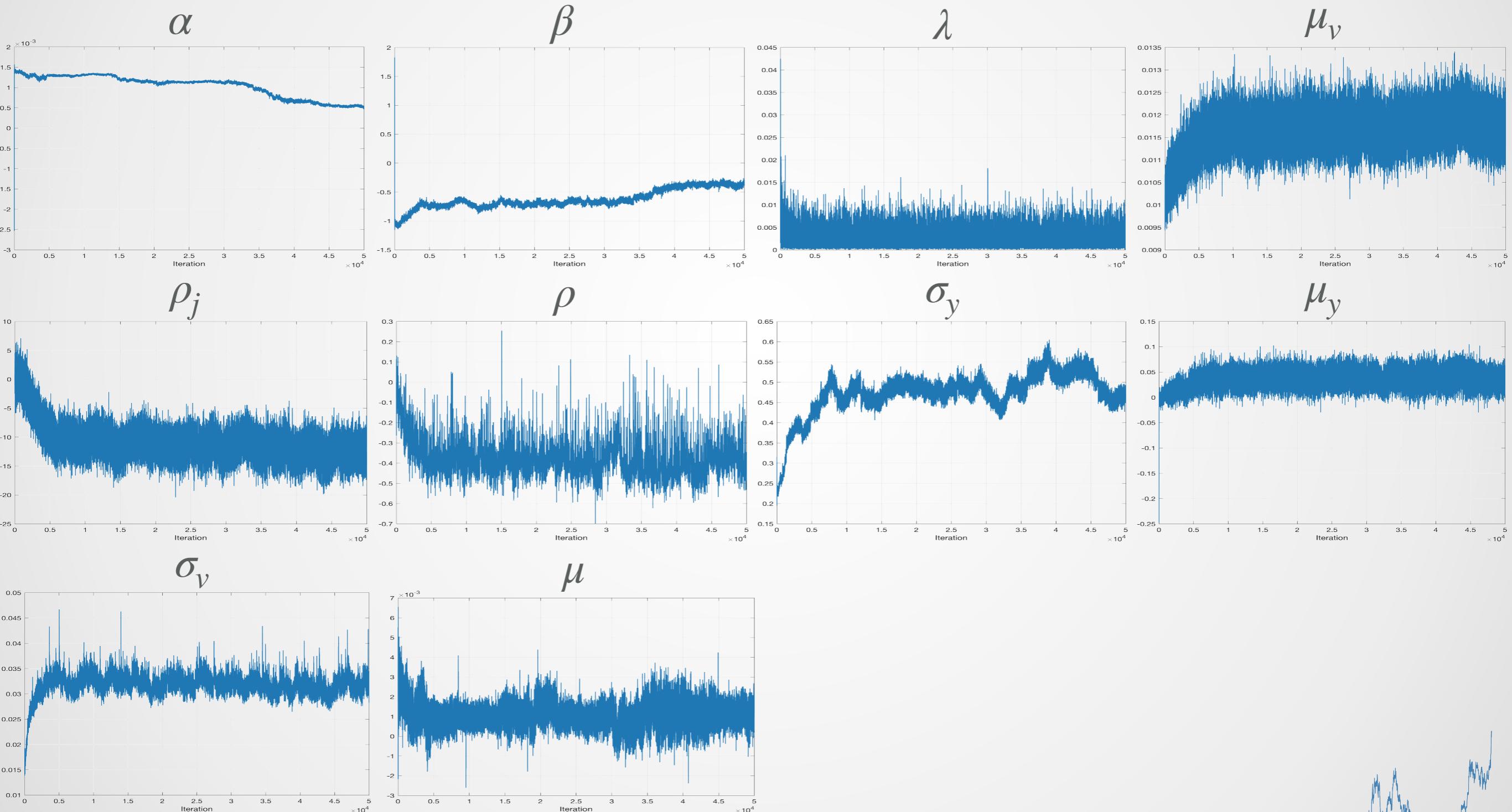
# Simulation studies for SVCJ

Parameters	True value				APE (%)	APE (%)	APE (%)
		T = 10	T = 100	T = 1,000	T = 10	T = 100	T = 1,000
$\mu$	0.0014	0.0069	0.0045	0.0017	392.9%	221.4%	21.4%
$\mu_y$	0.0009	0.0090	0.0062	0.0019	900.0%	588.9%	111.1%
$\sigma_y$	0.3166	-0.0283	0.00068	0.0023	108.9%	99.8%	99.3%
$\lambda$	0.0161	3.0906	0.3883	0.0399	19096.3%	2311.8%	147.8%
$\alpha$	0.0033	0.0479	0.0150	0.0019	1351.5%	354.5%	42.4%
$\beta$	0.000594	-8.0169E-05	5.2066E-07	2.7488E-05	113.5%	99.9%	95.4%
$\rho$	-0.7425	0.0162	0.0108	-0.0187	-102.2%	-101.5%	-97.5%
$\sigma_v$	-0.6245	-0.0390	0.00042	0.0039	-93.8%	-100.1%	-100.6%
$\rho_j$	0.0024	0.0184	0.0020	1.9950E-04	666.7%	16.7%	91.7%
$\mu_v$	-1.0821	-0.0015	0.0152	-0.0149	-99.9%	-101.4%	-98.6%
	0.0206	0.7141	0.0961	0.0100	3366.5%	366.5%	51.5%



# Trace plots after burn-in

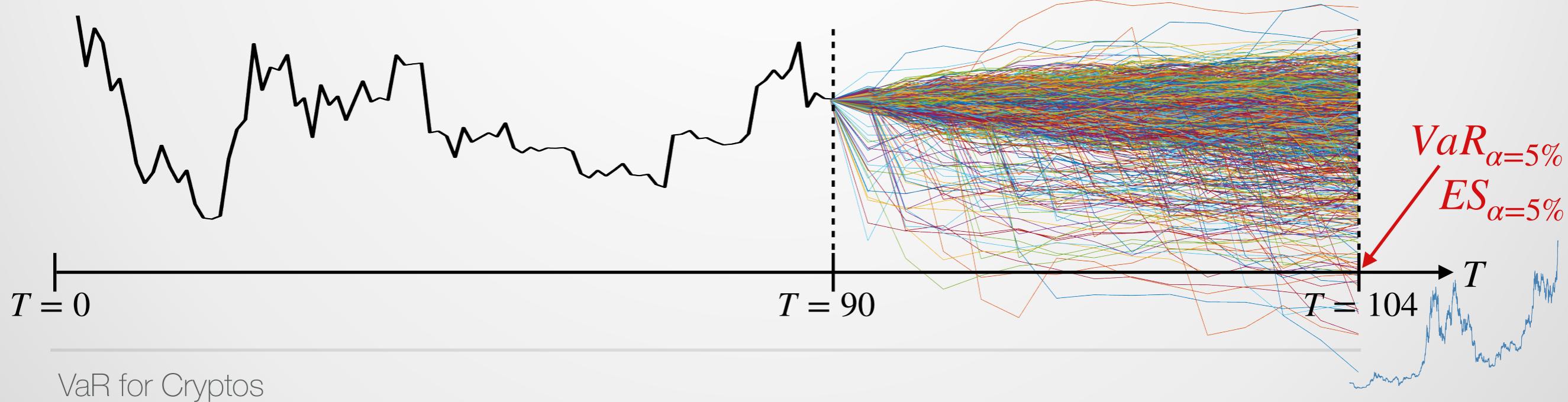
- Set number of iterations 50,000 and  $T = 1,000$ .



## Research plan

- Model: BS, SV, SVJ, SVCJ
- Confidence level  $\alpha : 5\% , 1\%$ ,
- Estimation windows  $H: 30, 90$
- Holding period (forecast horizon)  $h: 1, 14, 28$
- Rolling-window approach

Example rolling window visualization for  $H = 90, h = 14$



## Backtesting

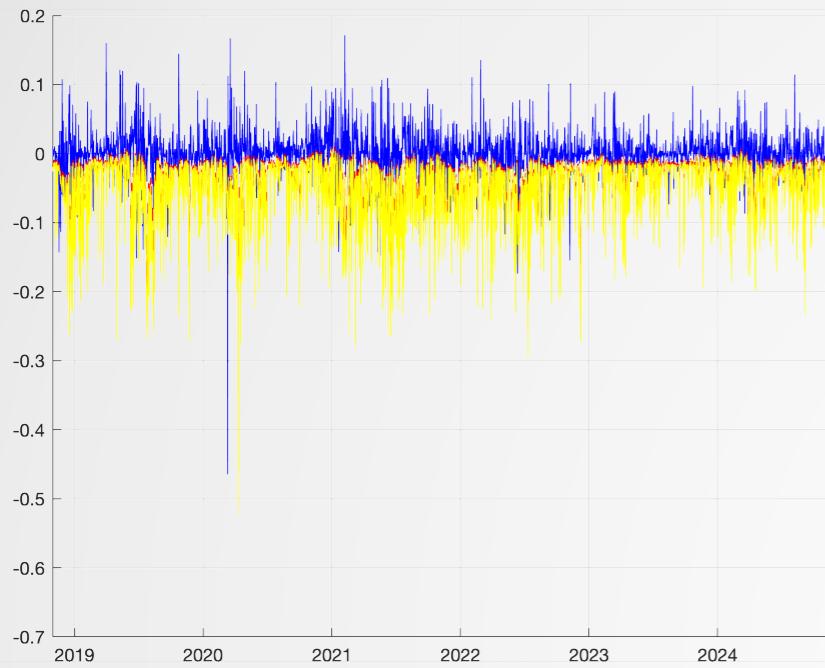
- An essential tool in assessing the accuracy and robustness of VaR models in risk management.
- Value-at-Risk (VaR)
  - ▶ Traffic Light Approach (Basel Committee on Banking Supervision, 1996)
  - ▶ Kupiec's POF test (Kupiec et al., 1995)
  - ▶ Christoffersen's Independence Test (Christoffersen, 2008): Check if exceedances occur independently across two consecutive days
- Expected Shortfall (ES) (Acerbi and Szekey, 2014)
  - ▶  $Z_1$  Test: test the average size of tail losses
  - ▶  $Z_2$  Test: test the variability of tail losses
  - ▶  $Z_3$  Test: test the model's ordering and distribution of tail losses
  - ▶ Embrechts evaluation: Measures average deviation between observed tail losses and predicted ES



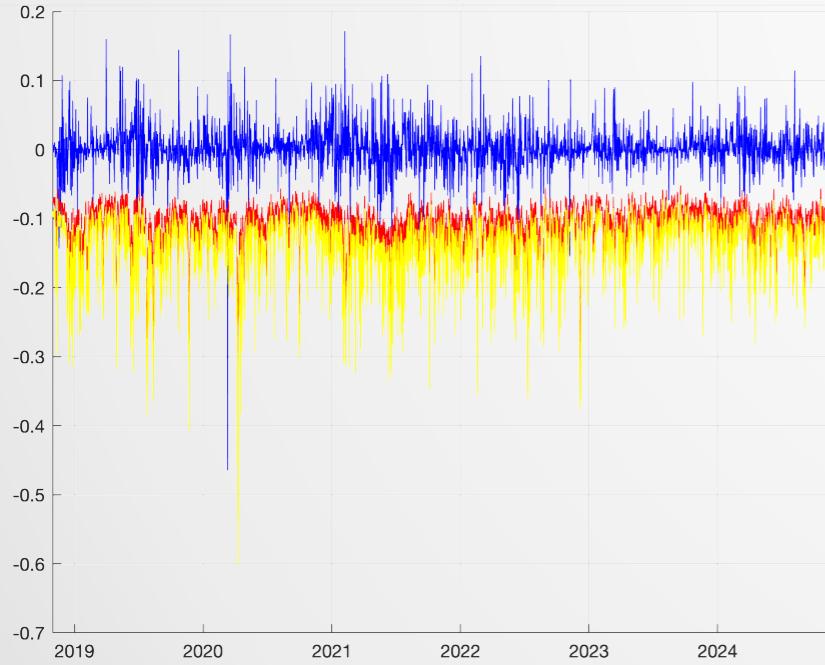
## Empirical Results

$$H = 30, h = 1, \alpha = 5\%$$

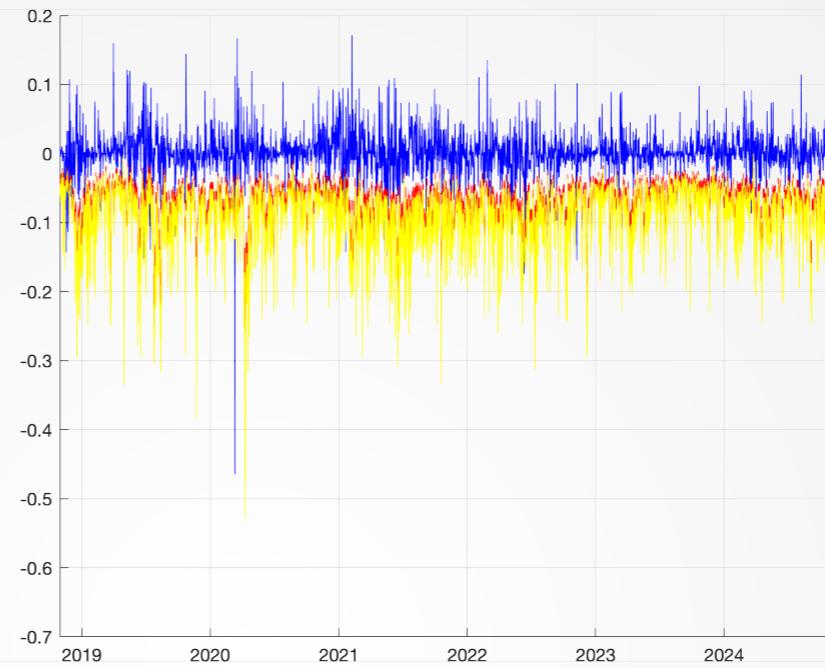
BS



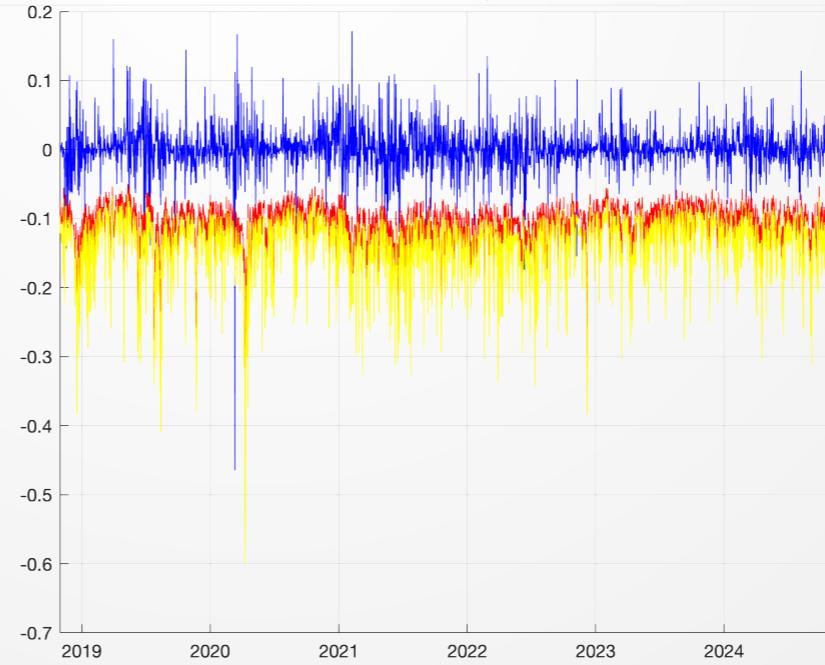
SVJ



SV



SVCJ



**h=1 Exceedance rate (%)**

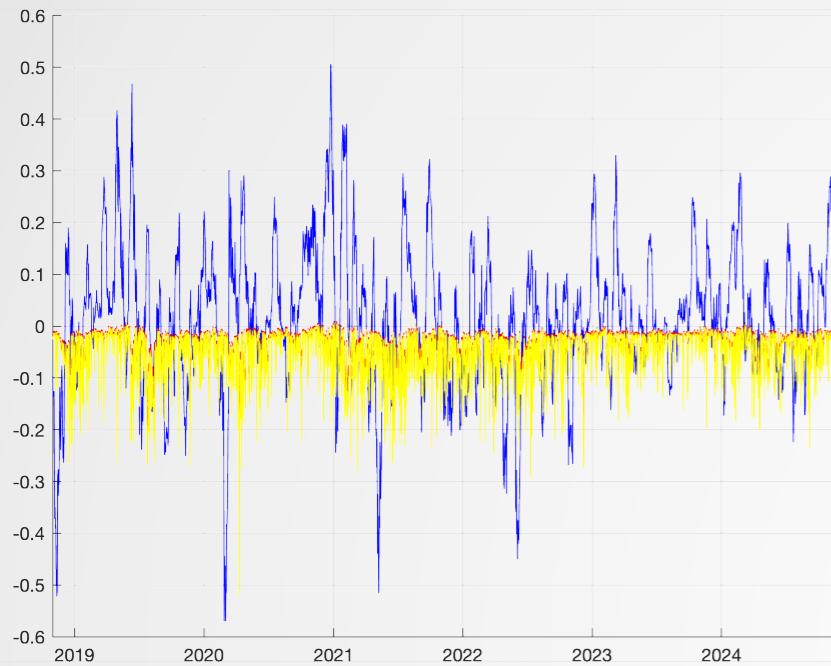
BS	15.9517%
SV	3.8427%
SVJ	0.9383%
SVCJ	0.9383%



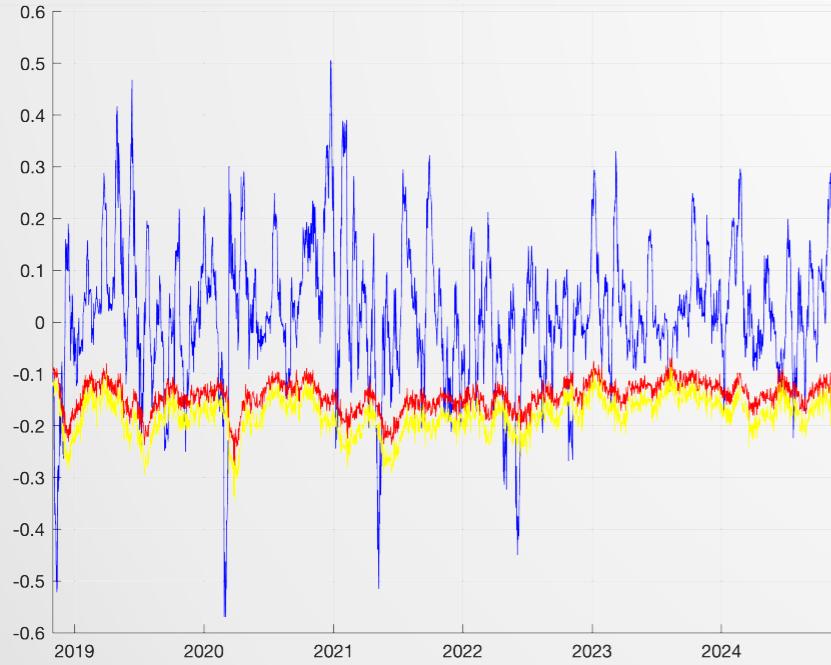
## Empirical Results

$$H = 30, h = 14, \alpha = 5\%$$

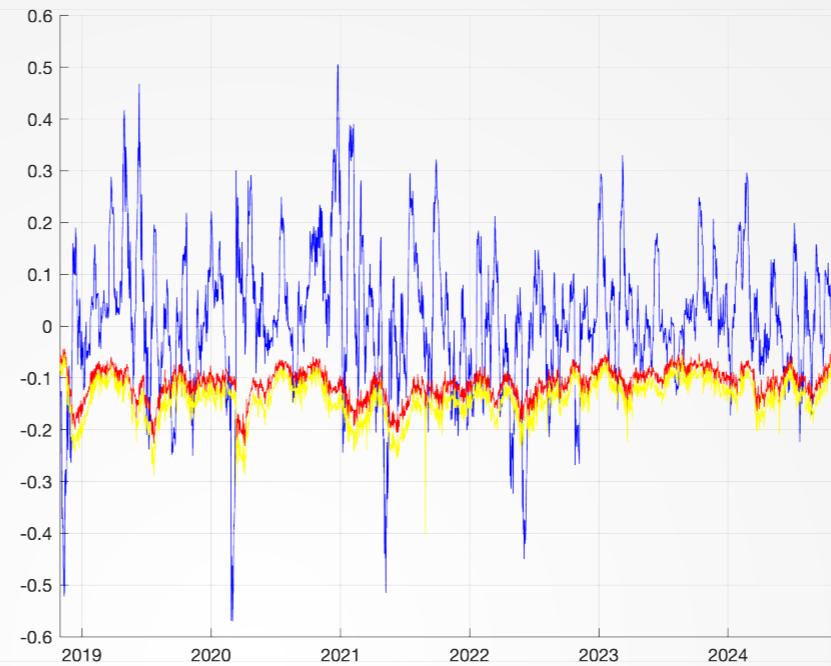
BS



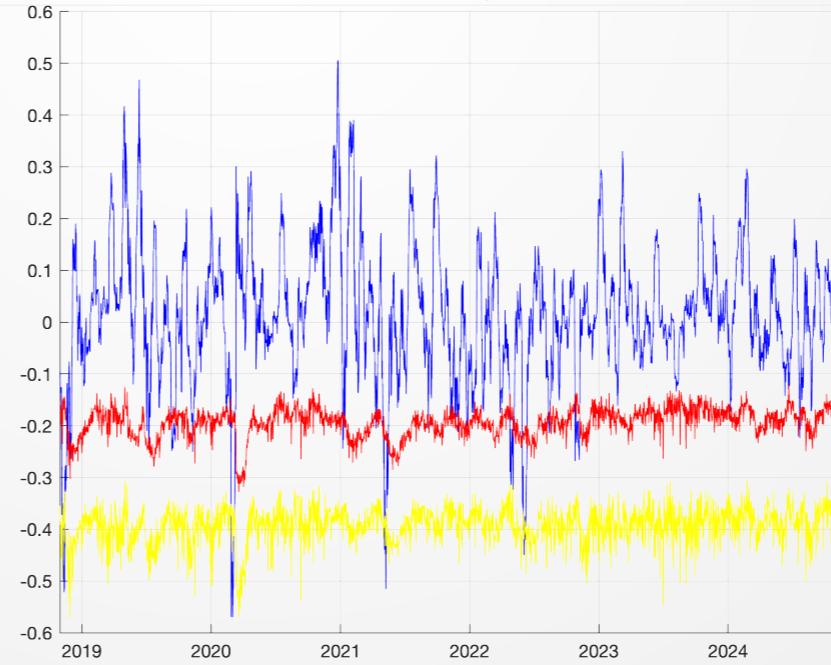
SVJ



SV



SVCJ



**h=14 Exceedance rate (%)**

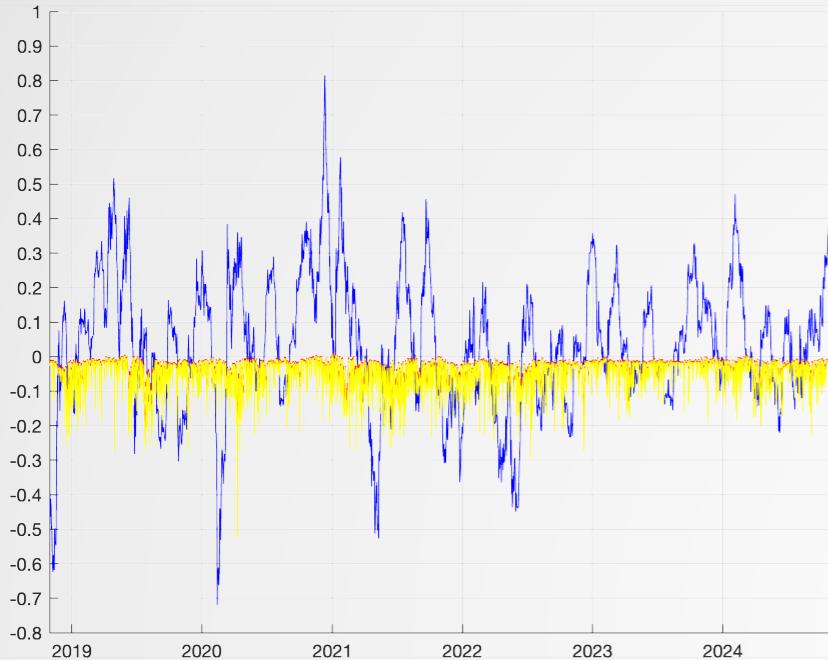
BS	29.4382%
SV	13.0787%
SVJ	9.5730%
SVCJ	5.1236%



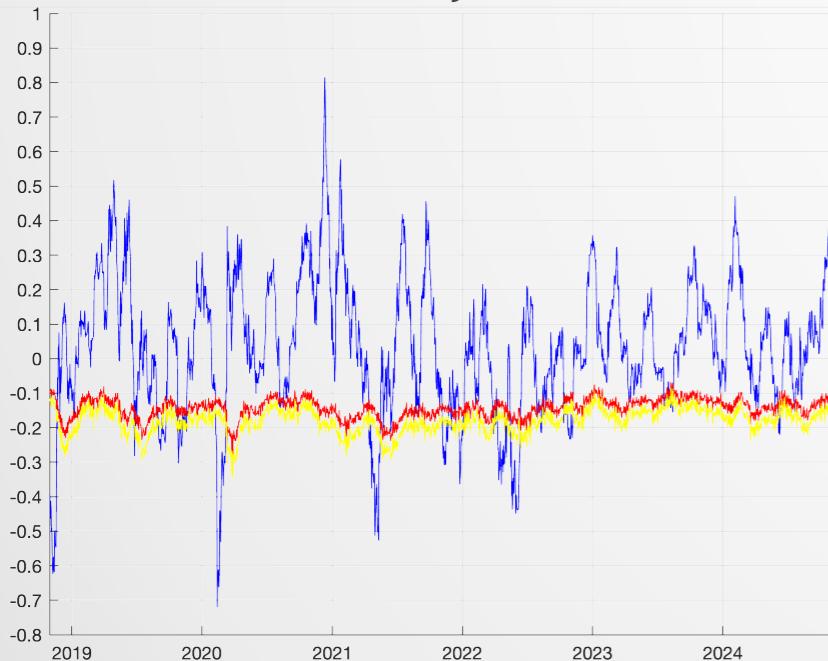
## Empirical Results

$$H = 30, h = 28, \alpha = 5\%$$

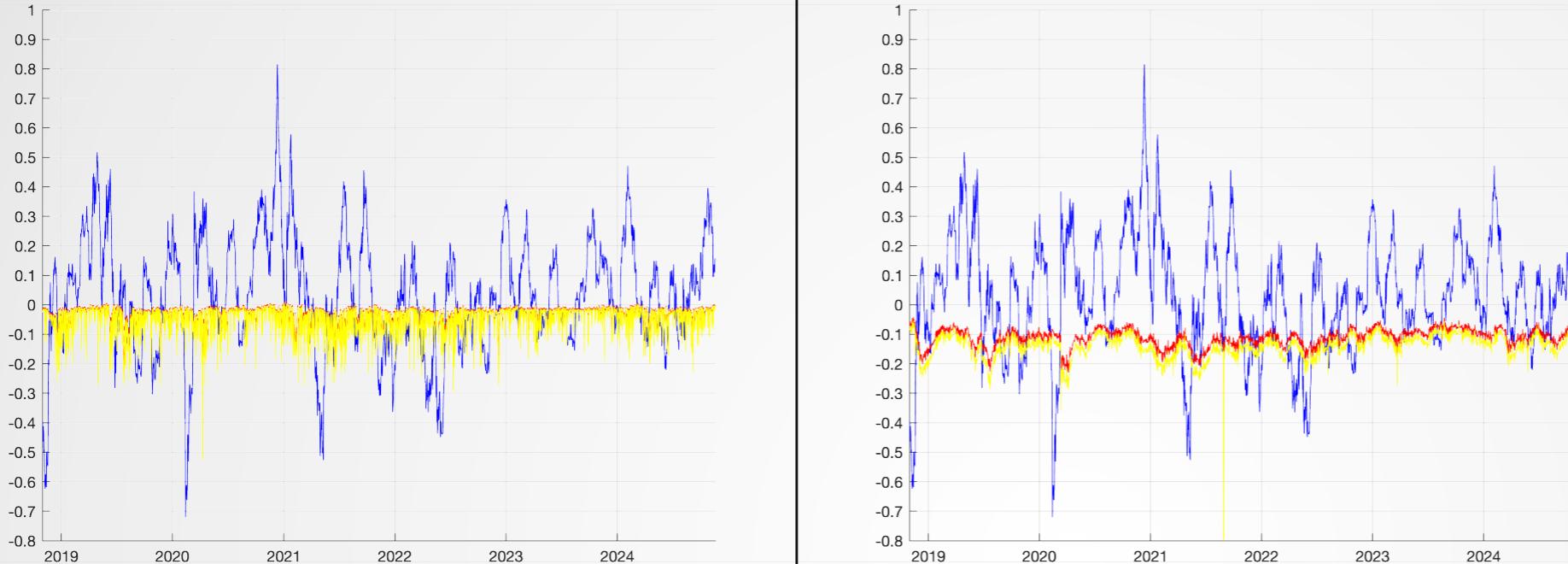
BS



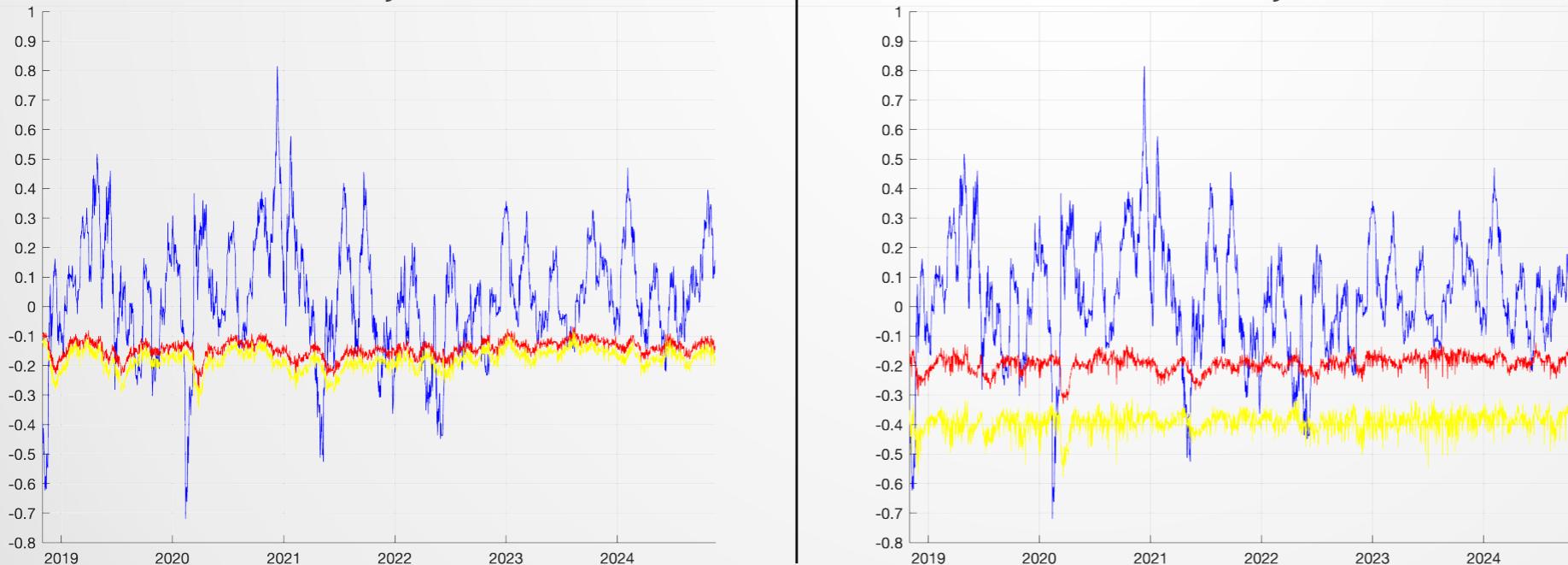
SVJ



SV



SVCJ



**h=28 Exceedance rate (%)**

BS	32.5192%
SV	18.5436%
SVJ	15.3777%
SVCJ	10.0407%



## Traffic Light Approach

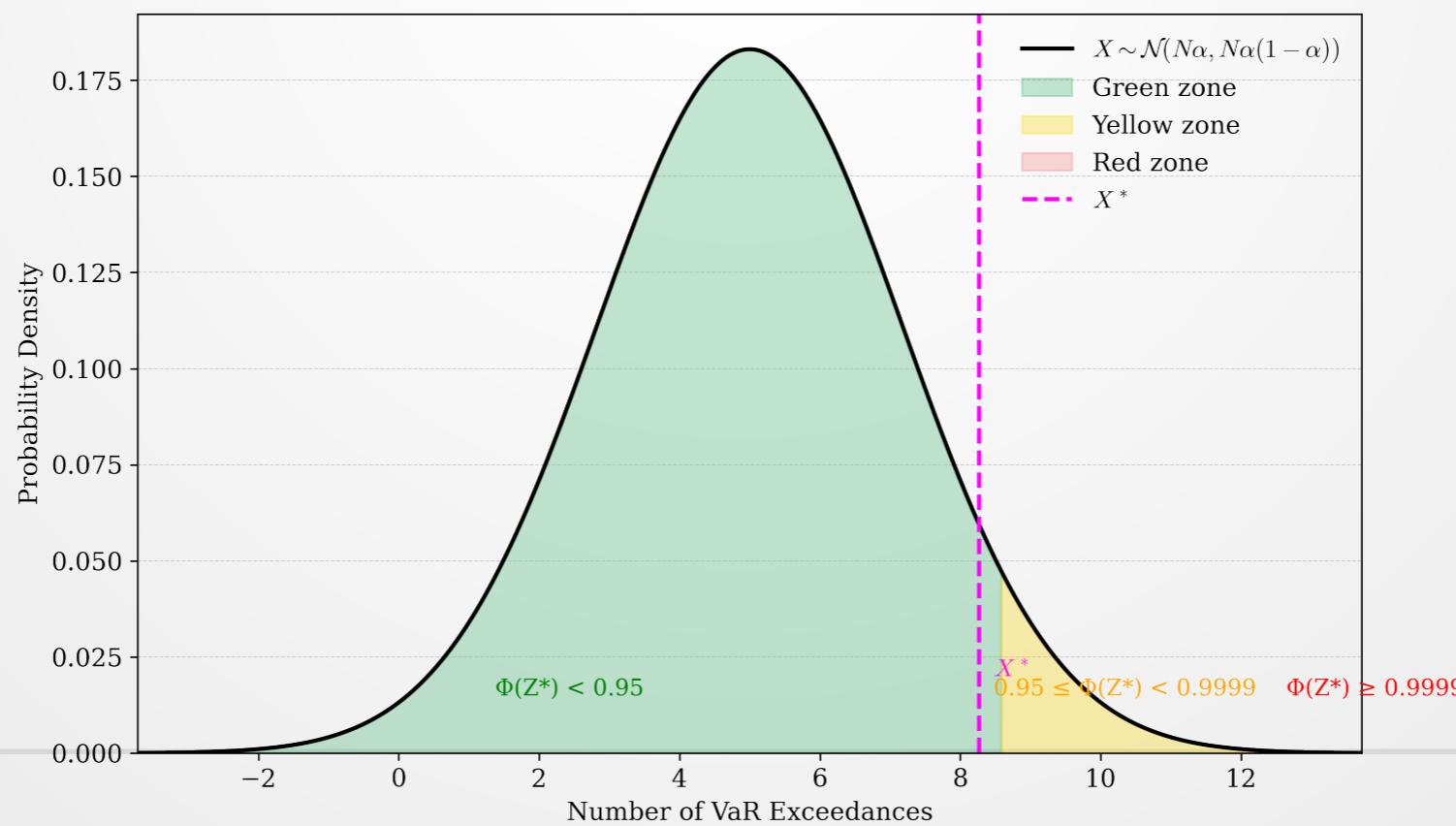
- Assume  $H_0$  is correct, let  $\mu = N\alpha$ ,  $\sigma = \sqrt{N\alpha(1 - \alpha)}$ .

- Suppose the realized value of  $X$  is  $X^*$ , CLT gives

$$P(X \leq X^*) = P\left(\frac{X - \mu}{\sigma} \leq \frac{X^* - \mu}{\sigma}\right) \approx P(Z \leq Z^*) = \Phi(Z^*)$$

- Classify VaR performance into three distinct zones:

- Green if  $\Phi(z) < 0.95$ , acceptable performance.
- Yellow if  $0.95 < \Phi(z) < 0.9999$ , potential issues.
- Red if  $\Phi(z) \geq 0.9999$ , unacceptable.



## Kupiec's POF-test (proportion of failure)

- The null hypothesis:

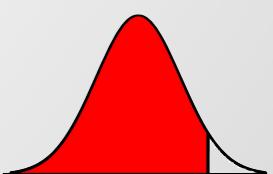
$$H_0 : p = \alpha,$$

where  $x$  is number of exceptions,  $n$  is total sample size,  
 $p$  is the true exception probability. We estimate  $\hat{p} = x/n$ .

- The binomial likelihood ratio test is conducted as:

$$LR_{POF} = -2\ln\left(\frac{(1-\alpha)^{n-x}\alpha^x}{(1-x/n)^{n-x}(x/n)^x}\right)$$

- If  $LR_{POF} > \chi^2_{1,(1-\alpha)}$  → reject  $H_0 : p = \alpha$   
 → the model is inadequately calibrated.
- Disadvantage: statistically weak for small sample sizes and examining testing only the failure rate and not the succession of occurrence.



## Christoffersen's Independence Test (Markov test)

- Exceedance indicator:

$$\text{Indicator } (I_t) = \begin{cases} 0 & \text{if VaR is not breached,} \\ 1 & \text{otherwise} \end{cases}$$

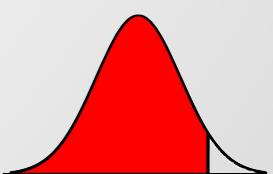
- The test statistic of independence test is:

$$LR_M = -2\ln \left( \frac{(1 - \pi)^{n_{00} + n_{10}} \pi^{n_{01} + n_{11}}}{(1 - \pi_0)^{n_{00}} \pi_0^{n_{01}} (1 - \pi_1)^{n_{10}} \pi_1^{n_{11}}} \right),$$

where  $\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}$ ,  $\pi_1 = \frac{n_{11}}{n_{10} + n_{11}}$ ,  $\pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}$

follows a  $\chi^2$  distribution with  $df = 1$ .

- $H_0 : \pi_0 = \pi_1$ , exceptions are independent across days.
- Disadvantage: It has limited power against clustering, as it only tests for the independence of exceptions on two consecutive days.



## Z<sub>1</sub> Test: testing ES after VaR

- Z<sub>1</sub> examines whether ES is accurate in the long run.
- From  $ES_{\alpha,t} = \mathbb{E}[X_t | X_t > VaR_{\alpha,t}]$ , can easily derive

$$\mathbb{E} \left[ \frac{X_t}{ES_{\alpha,t}} - 1 \mid X_t - VaR_{\alpha,t} > 0 \right] = 0$$

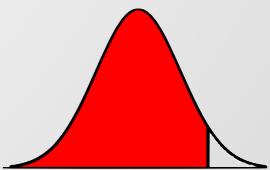
- Test statistic:

$$Z_1(\vec{X}) = \frac{\sum_{t=1}^T X_t I_t}{N_T ES_{\alpha,t}} - 1$$

- $H_0 : F_t^{[1-\alpha]} = P_t^{[1-\alpha]}, \forall t$

$H_1 : ES_{\alpha,t}^F \geq ES_{\alpha,t}, \forall t$  and  $>$  for some  $t$ .

$VaR_{\alpha,t}^F = VaR_{\alpha,t}, \forall t$



## Z<sub>2</sub> Test: testing ES directly

- Z<sub>2</sub> focusing on tail losses, identifies clustering effects in extreme losses.

- Test statistic:

$$Z_2(\vec{X}) = \sum_{t=1}^T \frac{X_t I_t}{T(1 - \alpha) ES_{\alpha,t}} - 1$$

- $H_0 : F_t^{[1-\alpha]} = P_t^{[1-\alpha]}, \forall t$

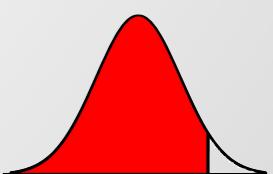
$H_1 : ES_{\alpha,t}^F \geq ES_{\alpha,t}, \forall t$  and  $>$  for some  $t$ .

$VaR_{\alpha,t}^F \geq VaR_{\alpha,t}, \forall t$

- If  $\mathbb{E}(Z_2) = 0$ , ES estimation model is correctly specified.

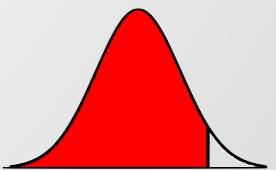
If  $\mathbb{E}(Z_2) > 0$ , the actual loss is greater than the expected ES, underestimates the actual underlying risk.

If  $\mathbb{E}(Z_2) < 0$ , overestimate risk.



## Z<sub>3</sub> Test: estimating ES from realized ranks

- If  $P_t$  is correct, we can backtest by checking if the observed ranks  $U_t = P_t(X_t)$  are i.i.d.  $U(0,1)$ .  
i.e. The model should not be able to "predict" tomorrow's losses.
- The rank  $U_t$  is the cumulated probability associated to the observed loss  $X_t$ .
- If the distribution of  $U_t$  deviates from a uniform distribution, it indicates that the risk model has systematic errors.
  - ▶ If  $U_t$  is skewed towards 0 (too many extreme losses): The risk model underestimates market risk
  - ▶ If  $U_t$  is skewed towards 1 (predicted losses are consistently larger than actual losses): The risk model is overly conservative.



$\alpha = 5\% , H = 30$ 

<b>h=1</b>	$\hat{p}$	VaR			ES				Embrechts evaluation
		Traffic Light	POF test	Independence test		Z1	Z2	Z3	
BS	15.95%	Red	367.5 (0.00)	0.0 (0.88)	1.56	-0.57	0.99	0.0353	
SV	3.84%	Green	6.8 (0.01)	3.5 (0.06)	0.35	-0.95	-0.59	0.0174	
SVJ	0.94%	Green	115.4 (0.00)	6.0 (0.01)	0.20	-0.99	-0.79	0.0324	
SVCJ	0.94%	Green	115.4 (0.00)	6.0 (0.01)	0.13	-0.99	-0.85	0.0297	
<b>h=14</b>									
BS	29.44%	Red	1388.6 (0.00)	853.2 (0.00)	6.95	1.46	8.42	0.1718	
SV	13.08%	Red	215.9 (0.00)	869.4 (0.00)	0.62	-0.78	-0.16	0.1180	
SVJ	9.57%	Red	78.2 (0.00)	703.9 (0.00)	0.38	-0.86	-0.48	0.0929	
SVCJ	5.12%	Green	0.1 (0.79)	508.0 (0.00)	-0.24	-0.96	-1.20	0.0904	
<b>h=28</b>									
BS	32.52%	Red	1671.9 (0.00)	1176.0 (0.00)	5.89	1.36	7.25	0.2422	
SV	18.54%	Red	520.7 (0.00)	1341.2 (0.00)	0.91	-0.63	0.28	0.1871	
SVJ	15.38%	Red	331.1 (0.00)	1245.6 (0.00)	0.56	-0.75	-0.18	0.1606	
SVCJ	10.04%	Red	92.7 (0.00)	1008.0 (0.00)	-0.17	-0.91	-1.08	0.0438	



## Empirical Results

$\alpha = 1\% , H = 30$

<b>h=1</b>	$\hat{p}$	VaR			ES				Embrechts evaluation
		Traffic Light	POF test	Independenc e test		Z1	Z2	Z3	
BS	11.04%	Red	760.5 (0.00)	0.1 (0.78)	1.39	-0.73	0.66	0.0609	
SV	2.14%	Red	22.3 (0.00)	0.8 (0.38)	0.34	-0.97	-0.63	0.0391	
SVJ	0.45%	Green	8.7 (0.00)	0.1 (0.76)	0.12	-0.99	-0.88	0.0169	
SVCJ	0.40%	Green	10.4 (0.00)	0.1 (0.79)	0.08	-1.00	-0.92	0.0176	
<b>h=14</b>									
BS	26.16%	Red	2836.0 (0.00)	709.4 (0.00)	3.52	0.20	3.72	0.2527	
SV	8.63%	Red	501.5 (0.00)	659.5 (0.00)	0.52	-0.87	-0.35	0.1889	
SVJ	4.85%	Red	173.1 (0.00)	486.9 (0.00)	0.39	-0.93	-0.54	0.1653	
SVCJ	0.27%	Green	16.9 (0.00)	29.0 (0.00)	-0.32	-1.00	-1.31	0.2955	
<b>h=28</b>									
BS	29.04%	Red	3280.4 (0.00)	978.9 (0.00)	4.48	0.61	5.08	0.3370	
SV	13.89%	Red	1084.3 (0.00)	1206.6	0.72	-0.76	-0.03	0.2862	
SVJ	9.91%	Red	628.8 (0.00)	958.0 (0.00)	0.48	-0.85	-0.37	0.2500	
SVCJ	0.81%	Green	0.8 (0.36)	103.3 (0.00)	-0.26	-0.99	-1.26	0.2242	



## Empirical Results

$\alpha = 5\% , H = 90$

h=1	$\hat{p}$	VaR			ES			Embrechts evaluation
		Traffic Light	POF test	Independence test	Z1	Z2	Z3	
BS	12.49%	Red	185.0 (0.00)	7.1 (0.01)	6.83	0.03	6.86	0.0324
SV	6.34%	Yellow	7.6 (0.01)	3.2 (0.07)	0.50	-0.90	-0.39	0.0212
SVJ	1.56%	Green	73.3 (0.00)	2.5 (0.11)	0.26	-0.98	-0.72	0.0186
SVCJ	1.65%	Green	68.6 (0.00)	0.2 (0.63)	0.28	-0.98	-0.70	0.0165
<b>h=14</b>								
BS	27.81%	Red	1207.7	788.0 (0.00)	4.82	0.70	5.53	0.1485
SV	15.20%	Red	314.5 (0.00)	955.7 (0.00)	0.55	-0.75	-0.21	0.1103
SVJ	13.16%	Red	213.9 (0.00)	871.0 (0.00)	0.46	-0.80	-0.34	0.1012
SVCJ	11.13%	Red	129.0 (0.00)	822.3 (0.00)	-0.03	-0.89	-0.92	0.0357
<b>h=28</b>								
BS	30.40%	Red	1429.5	1154.7	9.39	2.33	11.72	0.2090
SV	21.90%	Red	733.2 (0.00)	1352.7	0.77	-0.59	0.18	0.1694
SVJ	19.76%	Red	585.2 (0.00)	1260.6	0.64	-0.66	-0.02	0.1586
SVCJ	16.92%	Red	408.3 (0.00)	1218.9	0.10	-0.80	-0.70	0.0894



## Empirical Results

$\alpha = 1\% , H = 90$

<b>h=1</b>	$\hat{p}$	VaR			ES				Embrechts evaluation
		Traffic Light	POF test	Independen ce test		Z1	Z2	Z3	
BS	9.37%	Red	564.2 (0.00)	2.1 (0.15)	1.38	-0.78	0.60	0.0600	
SV	3.26%	Red	70.5 (0.00)	0.0 (0.83)	0.53	-0.95	-0.42	0.0479	
SVJ	0.69%	Green	2.4 (0.12)	0.2 (0.65)	0.31	-0.99	-0.68	0.0267	
SVCJ	0.51%	Green	6.6 (0.01)	0.1 (0.74)	0.64	-0.99	-0.35	0.0398	
<b>h=14</b>									
BS	23.28%	Red	2325.8	554.4 (0.00)	3.23	-0.01	3.22	0.2276	
SV	10.30%	Red	657.0 (0.00)	733.8 (0.00)	0.42	-0.85	-0.43	0.1786	
SVJ	7.85%	Red	414.5 (0.00)	613.8 (0.00)	0.39	-0.89	-0.50	0.1699	
SVCJ	2.22%	Red	24.1 (0.00)	234.2 (0.00)	-0.13	-0.98	-1.11	0.0389	
<b>h=28</b>									
BS	27.06%	Red	2880.3	941.9 (0.00)	3.80	0.31	4.11	0.2811	
SV	16.18%	Red	1337.3	1181.8	0.59	-0.74	-0.14	0.2341	
SVJ	13.34%	Red	990.8 (0.00)	1082.9	0.52	-0.80	-0.28	0.2230	
SVCJ	5.35%	Red	202.8 (0.00)	617.5 (0.00)	-0.13	-0.95	-1.08	0.0889	



## Conclusion

- Estimation window with  $H = 30$  produces more accurate VaR estimation, compared with  $H = 90$
- As the holding periods increases (e.g.,  $h = 14$  or  $28$ ), the SVCJ model outperforms others in predicting VaR.
- As the holding periods increases (e.g.,  $h = 14$  or  $28$ ), the SVJ and SVCJ models outperforms others in predicting ES.





# Tail risk in Bitcoin under the Basel framework

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